MOBILE LOCALIZATION FOR NLOS PROPAGATION

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ABSTRACT

Non-Line-of-Sight and multipath propagation conditions pose significant problems for most mobile terminal positioning approaches. In contrast, power delay profile fingerprinting (PDP-F) thrives on multipath propagation. This multipath extension of T(D)oA is based on matching an estimated power delay profile from one or several base stations (BSs) (or other transmitters (broadcast, ...)) with a memorized power delay profile map for a given cell. It is obvious that the overall location accuracy depends strongly of the quality of the PDP estimation. We propose exploiting the prior knowledge on the received signal structure to enhance the PDP estimation and increase the localization accuracy. Depending on the propagation environment, we propose a deterministic and Bayesian framework for PDP estimation. And, we investigate the application to fingerprinting based mobile localization.

keywords: positioning, localization, power delay profile, fingerprinting, non-LOS.

I INTRODUCTION

Mobile positioning systems have received significant attention in both research and industry over the past few years [1, 2]. Indeed, the localization of the mobile phone has become one of the most important features of communication systems due its various potential applications (effective intra and inter-system handoff, localization of emergency caller...). The basic function of localization system is to collect information about position-dependent parameters of a Mobile Station (MS) signal and to process that information to get a location estimate .

Conventional localization techniques aiming at higher accuracy than simple cell identification are organized in a two steps procedure [3]. The first step involves the measurement of certain physical parameters of the received signal (e.g. time, or time-difference, of arrival (ToA, TDoA), angle of arrival (AoA), signal strength...). The signal is assumed to be received under Line of Sight (LoS) conditions, in which case the parameters of multiple MS-BS links are required to have position identifiability. The second step combines multiple measurements from the link to a convenient number of Base Stations (BSs) to estimate the mobile position. Weiss et al. underline the sub-optimality of the two-step approach [4, 5]. In fact, the signal parameters are estimated separately and independently for each MS-BS link, ignoring the constraint that all measurements must correspond to the same source. Weiss et al. introduce the "Direct Position Determination (DPD) approach": the estimated channel impulse responses for each MS-BS link are processed jointly and the MS position is computed as the best match to all data simultaneously. Monte Carlo simulations demonstrate that the DPD method provides better localization accuracy (especially in the presence of multipath propagation/fading [6]), and allows to work in an extended (lower) SNR range.

The main cause of inaccuracies observed in conventional localizatio systems is the realistic propagation conditions imposed by the wireless channel: multipath propagation and often Non Line-of-Sight (NLoS) conditions. In fact, the conventional methods rely on the line-of-Sight path between a base station and the Mobile station. However, in an urban environment, a LoS condition (i.e. the LoS path being present) is rarely satisfied for three BSs at the same time. This fact degrades the localization performance (identifiability and accuracy) of conventional techniques and creates the need to develop more accurate techniques suited for these propagation. To alleviate this problem, Porretta et al. suggest tracking the MS position to obtain more reliable position estimates [7]. The combination of a ToA and AoA measurement allows localization identifiability from just one MS-BS link. Based on the ToA and AoA measurements, the MS location is estimated by following two alternative procedures. When the MS is in the LoS condition, the location is determined through the parameters relevant to the first path received at the BS (AoA and ToA). On the other hand, under the NLOS condition, the MS position is determined by minimizing a given cost function (taking into account the ToA, the AoA, and the coordinates of the obstacles found along the AoA for the first N paths). An alternative approach is proposed by Nájar et al. [8]. In LoS condition, the estimation of the ToA of the LoS and a NLoS path allows the determination of an offset (bias) between the two ToAs. This bias is then subtracted to the ToA of the NLoS path during NLoS conditions to provide an estimate of the LoS ToA. In general, the position estimate accuracy and its identifiability can always be improved by adding a Kalman filtering stage to track the location trajectory, on the basis of brute position estimate. The use of the Kalman filter allows the tracking, not only of the position and the velocity of the mobile, but also of the ToA bias caused by multipaths, and NLoS conditions.

While previous techniques try to reduce the multipath and the NLOS effects, it cannot be eliminated, and the errors it produces are difficult to predict. So that, some location methods, e.g. Enhanced Signal Strength and Location Fingerprinting have been designed to obtain optimal performance in urban environment. Those techniques not only overcome the problems related to the propagation environment, but also take advantage from the temporal diversity of the wireless channel. The idea is to use a previously collected or predicted signal database (location dependent parameters) form the coverage area. The phone measures the same parameters, and sends it to the location server in the network. The position is then determined by a correlation algorithm, which compares the measured signal parameters with the information stored in the database. The Enhanced Signal Strength (ESS) method is based on this principle, and has allowed the deployment of personal locator systems in PHS service areas in Japan. The position of the mobile is determined using the signal strength of preferably three to five base stations. From this input plus information from the base station database, the system can calculate the position of the MS [1]. The database is built by simulating the signal propagation characteristics of every wireless transmitting antenna in the area of interest. Heikki et al. propose building the signal strength database through measurements instead of computation [9]. Instead of exploiting signal strength, the Location Fingerprinting (LF) (introduced by U.S. Wireless Corp. of San Ramon, Calif.) relies on signal structure characteristics [1, 16, 17, 18]. By combining multipath pattern with other characteristics, the LF creates a signature unique to a given location. The position of the mobile is determined by matching the transmitter's signal characteristics to an entry of the database. For LF, multipoint signal reception is not required: the system can use data for only a signal point to determine location. Ahonen and Eskelinen suggest using the measured Power Delay Profiles (PDPs) in the database [10]. Thus, the location estimation is possible by using only one BS due to the additional information provided by the PDP, i.e., amplitudes and delays of the multipath components.

Notations: upper- and lower-case boldface letters denote matrices and vectors, respectively. $(.)^T$ and $(.)^H$ represent the transpose and the transpose-conjugate operators. $E\{.\}$ is the statistical expectation, and tr $\{.\}$ is the trace operator.

II PDP FINGERPRINTING FOR MOBILE LOCATION

Location Fingerprinting is a general location method that can be applied to any cellular or WLAN network. The key idea is to store signal structure information, from the whole coverage area of the location system, in a database. The database should contain collected or predicted position dependent signal information (a position signature), called fingerprints, with a resolution comparable to the accuracy that can be achieved with the method. The MS measures the same parameters, and sends it to the location server in the network. The position is then determined by a correlation algorithm, which compares the measured signal parameters with the information stored in the database. The correlation algorithm measures a kind of the likelihood that the MS signal comes from a given position. In such a way, the fingerprinting based approaches can be interpreted as an extension of the DPD location scheme [4, 5] to multi-paths propagation environment.
A relevant issue with the location fingerprinting is the choice of the signal fingerprints. Any location-dependent signal information that can be measured by the MS or the BSs is useful for the location fingerprinting technique. The signal fingerprints could include signal strength, signal time delay, or even channel impulse response. Ahonen and Eskelinen suggest using the measured PDPs as a signal fingerprints for UMTS systems[]. The power delay profile shows the power and the arrival times of the different ray-paths between the selected transmitter and the selected receiver. The received impulse response between a MS and the BS:

$$h(t,\tau) = \sum_{l=1}^{L} A_l(t)\, e^{j\varphi_l(t)}\, p(\tau - \tau_l(t)) \qquad (1)$$

where L denotes the number of paths, $p(t)$ is the convolution of the transmit and receive filters, $\tau_l(t)$, $A_l(t)$ and $\varphi_l(t)$ are respectively the delay, the fading amplitude and the phase of the l^{th} path. The path delay and fading amplitude vary slowly with the position (almost constant if the mobile moves around a given position); whereas the fading phase varies rapidly. If the MS moves slowly, one can assume delays and fading amplitudes are constant over T channel observations, but not the fading phases. Thus, the estimated CIR can be written as

$$\widehat{h}(t,\tau) = \sum_{l=1}^{L} A_l\, e^{j\varphi_l(t)}\, p(\tau - \tau_l) + v(t,\tau) \qquad t = 1:T \qquad (2)$$

where $v(t,\tau)$ is a Gaussian additive white noise. Classically, the PDP is estimated by averaging the square of the CIR taps, i.e.,

$$\widehat{PDP}(\tau) = \frac{1}{T}\sum_{t=1}^{T}\left|\widehat{h}(t,\tau)\right|^2 \qquad (3)$$

However, if the mobile moves rapidly and/or some paths are not resolvable (due to the limited bandwidth of the pulse-shape $p(t)$, paths contributions can overlap), the averaging gives a poor PDP estimation, and then a poor location accuracy. In the following, we propose exploiting the prior information of the channel structure to enhance the PDP estimation.

III DETERMINISTIC PDP ESTIMATION FOR PDP-F

In this section, a structural priors for the wireless channel is considered such as the multipath propagation model (in (2)), and the prior knowledge of the pulse-shape. The parameters τ_l, A_l, and $\varphi_l(t)$ are considered unknown deterministic parameters. The exploitation of this structural information leads to the following two-step precedure:

- First, estimate the model parameters by optimizing the Maximum Likelihood criterion

$$\widehat{\tau}_l, \widehat{A}_l, \widehat{\varphi}_l(t) = \arg\max_{\tau_l, A_l, \varphi_l(t)} \sum_{t=1}^{T}\sum_{\tau=1}^{N}\left\|\widehat{h}(t,\tau) - \sum_{l=1}^{L} A_l\, e^{j\varphi_l(t)}\, p(\tau-\tau_l)\right\|^2 \qquad (4)$$

- Then, construct the PDP estimate as

$$\widehat{PDP}(\tau) = \sum_{l=1}^{L}\widehat{A}_l^2\, p^2(\tau - \widehat{\tau}_l) \qquad (5)$$

Minimizing (4) leads to a difficult non-linear optimization problem. Although the least-squares problem (4) is separable in the complex path amplitude $A_l e^{j\varphi_l(t)}$, a difficulty arises from imposing that A_l does not depend on t. To have a tractable solution, we propose a two step optimization scheme. First, we estimate the paths delays τ_l and the complex fading coefficients $b_l(t) = A_l e^{j\varphi_l(t)}$. Then, the constant fading amplitudes and the varying phases are extracted from the varying complex coefficients $b_l(t)$ using an LS based technique.
For the clarity of the algorithm description, we shall consider matrix notation. Equation (2) becomes

$$\widehat{\mathbf{h}}(t) = \underbrace{[\mathbf{p}_{\tau_1}\cdots\mathbf{p}_{\tau_L}]}_{\mathbf{P}_{\boldsymbol{\tau}}}\underbrace{\begin{bmatrix} A_1\, e^{j\varphi_1(t)} \\ \vdots \\ A_L\, e^{j\varphi_L(t)}\end{bmatrix}}_{\mathbf{b}(t)} + \mathbf{v}(t) \qquad t = 1:T$$

where $\widehat{\mathbf{h}}(t) = \left[\widehat{h}(t,t_0)\cdots\widehat{h}(t,t_0+(N-1)t_s)\right]^T$, and similarly for $\mathbf{v}(t)$, $\boldsymbol{\tau} = [\tau_1\cdots\tau_L]^T$, and
$\mathbf{p}_\tau = [p(t_0-\tau)\cdots p(t_0+(N-1)t_s-\tau)]^T$. N is the channel impulse length, and t_0 is the sampling period. Note that $\mathbf{p}_\tau^T = \mathbf{p}_\tau^H$
The paths delays $\boldsymbol{\tau}$ and fading coefficients $\mathbf{b}(t)$ should minimize:

$$\widehat{\boldsymbol{\tau}}, \widehat{\mathbf{b}}(t) = \arg\min_{\boldsymbol{\tau},\mathbf{b}(t)}\sum_{t=1}^{T}\left\|\widehat{\mathbf{h}}(t) - \mathbf{P}_{\boldsymbol{\tau}}\mathbf{b}(t)\right\|^2 \qquad (6)$$

The problem is quadratic in $\mathbf{b}(t)$, leading to the estimates $\widehat{\mathbf{b}}(t) = \left(\mathbf{P}_\tau^T\mathbf{P}_\tau\right)^{-1}\mathbf{P}_\tau^T\widehat{\mathbf{h}}(t)$ for a given $\boldsymbol{\tau}$. The resulting problem for $\boldsymbol{\tau}$ is non-linear:

$$\widehat{\boldsymbol{\tau}} = \arg\min_{\boldsymbol{\tau}}\sum_{t=1}^{T}\widehat{h}^H(t)\mathcal{P}_{P_\tau}^{\perp}\widehat{h}^H(t) \qquad (7)$$

where $\mathcal{P}_{P_\tau} = \mathbf{P}_\tau\left(\mathbf{P}_\tau^T\mathbf{P}_\tau\right)^{-1}\mathbf{P}_\tau^T$, $\mathcal{P}_{P_\tau}^{\perp} = \mathbf{I} - \mathcal{P}_{P_\tau}$ represent the projection on the column space of $\mathbf{P}_\tau$, and its orthogonal subspace.
We propose estimating these parameters by exploiting the sparse nature of the CIR through the use a Matching Pursuit (MP) algorithm. The MP has been used in a variety of applications [11], and particular to derive accurate channel estimates [12]. Using the standard form of the MP algorithm, we first find the delay τ_1, such as $\mathbf{p}_{\tau_1}$ is that best aligned with the different channel realizations $\mathbf{h}^{(0)}(t) = \widehat{\mathbf{h}}(t)$. Then, for each channel realization, the projection of $\mathbf{h}^{(0)}(t)$ along $\mathbf{p}_{\tau_1}$ is removed from $\mathbf{h}^{(0)}(t)$ and the residual $\mathbf{h}^{(1)}(t)$ is found. Now, the delay

τ_2 which best aligns $\mathbf{p}_{\tau_2}$ and $\mathbf{h}^{(1)}(t)$ is computed and a new residual $\mathbf{h}^{(2)}(t)$ is formed. The algorithm proceeds by sequentially choosing the column that best matches the residual until some termination criterion is met.
In the previous step, we have ignored the constraint that $\forall l \;\; b_l(t), \;\; t = 1:T$, have the same magnitude. In fact, the estimated complex fading coefficient (corresponding to the l^{th} path) can be written as:

$$\widehat{b}_l(t) = A_l e^{j\varphi_l(t)} + \widetilde{b}_l(t) \quad t = 1:T \tag{8}$$

where $\widetilde{b}_l(t)$ are the fading estimation errors. If the paths are resolvable (path contributions do not overlap much), the estimation errors can be assumed to be white Gaussian. In this case, a Maximum Likelihood (ML) formulation leads to the following LS problem:

$$\begin{aligned} \widehat{A}_l, \widehat{\varphi}_l(t) &= \arg \min_{A_l, \varphi_l(t)} \sum_{t=1}^{T} \left| \widehat{b}_l(t) - A_l e^{j\varphi_l(t)} \right|^2 \\ &= \arg \min_{A_l, \varphi_l(t)} \sum_{t=1}^{T} \left| \widehat{b}_l(t) e^{-j\varphi_l(t)} - A_l \right|^2 . \end{aligned} \tag{9}$$

Thus, the fading amplitudes and phases are estimated as:

$$\begin{cases} \widehat{A}_l = \dfrac{1}{T} \displaystyle\sum_{t=1}^{T} \left| \widehat{b}_l(t) \right| \\ \widehat{\varphi}_l(t) = \text{angle} \left(\widehat{b}_l(t) \right) \;\; / \;\; e^{j\widehat{\varphi}_l(t)} = \widehat{b}_l(t) / \left| \widehat{b}_l(t) \right| \end{cases} \tag{10}$$

Finally, the refined PDP estimate is computed as in (5).

A Deterministic PDP Fingerprinting for Mobile Localization

Obviously, the overall location accuracy depends strongly of the fingerprint estimation quality. Particularly, the accuracy of the PDP estimation is affected by two major sources of impairment [14]:

- Additive noise: because the input signal is recorded in presence of noise, the estimated CIR (then the PDP) is always corrupted by a random fluctuation.
- Outlying noise: if the SNR at a given delay falls below a given threshold, the corresponding PDP component will contain almost no useful information on the source localization: these values must be interpreted as outlying components.

Using the simulation environment described in [15], we investigate the effect of the PDP estimation on the location accuracy of the PDP-fingerprinting. Figure 1 compares the RMSE of the PDP-fingerprinting (function of the input SNR) where the PDP is estimated using the non-parametric scheme (as in (3)) or the parametric deterministic model (as in (5)). We remark that the parametric PDP estimation outperforms the non-parametric scheme. In fact, exploiting the prior knowledge of the pulse-shape increases the robustness of the estimation scheme to additive noise outlying components (by ignoring paths with low energy). This leads to more accurate PDP estimation, and thus better location performance.

IV Bayesian PDP Estimation for PDP-F

If the propagation paths are resolvable (in delay), the deterministic approach in (4) is appropriate. However, if this is not the case, the channel taps are the superpositions of different paths arriving at almost the same delay, i.e.,

$$A_l(t) e^{j\varphi_l(t)} = \sum_{k=1}^{K_l} A_{l,k} e^{j\varphi_{l,k}(t)}. \tag{11}$$

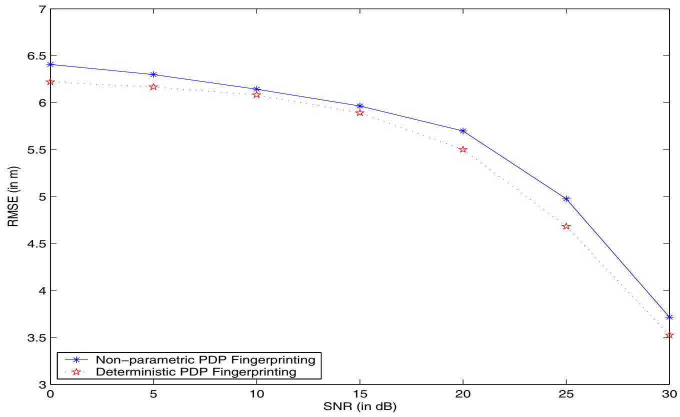

Figure 1: Positioning accuracy for PDP-F vs. SNR (using non-parametric and deterministic PDP estimation schemes)

Therefore, modeling the fading amplitudes as deterministic quantities is no longer appropriate and a Bayesian modeling is warranted. In this section, we assume that the complex fading vector $\mathbf{b}(t)$, and the additive noise $\mathbf{v}(t)$ are independent i.i.d. zero-mean Gaussian vector processes, i.e.,

$$\begin{matrix} \mathbf{b}(t) \sim \mathcal{N}\left(\mathbf{0}, \mathbf{C_b}\right) \\ \mathbf{v}(t) \sim \mathcal{N}\left(\mathbf{0}, \sigma_v^2 \mathbf{I}_N\right) \end{matrix} \;\; , \;\; t = 1 \cdots T \tag{12}$$

where $\mathcal{N}\left(\mathbf{0}, \mathbf{C}\right)$ denotes the zero-mean complex normal distribution with covariance matrix $\mathbf{C}$, $\mathbf{C}_b = \text{diag}\left(\sigma_{b,1}^2 \cdots \sigma_{b,L}^2\right)$ is a diagonal matrix characterizing the covariance of the random complex fading amplitudes.
The statistical model (12) implies that the $\widehat{\mathbf{h}}(t)$ are modelled as i.i.d. complex Gaussian vectors with $\widehat{\mathbf{h}}(t) \sim \mathcal{N}\left(\mathbf{0}, \mathbf{C_h}\right)$, $\mathbf{C_h} = \mathbf{P}_\tau \mathbf{C}_b \mathbf{P}_\tau^T + \sigma_v^2 \mathbf{I}_N$. Thus, whereas in the deterministic case the channel is parameterized by path delays and amplitudes, the Bayesian model parameterizes the channel with path delays and powers. The considered approach is Bayesian for $\mathbf{h}(t)$, but Maximum Likelihood for the parameters $\boldsymbol{\tau}$ and $\mathbf{C}_b$. To distinguish from the deterministic ML approach in the previous section, the ML approach considered here will be called Rayleigh ML.
Taking into account the statistical model, the likelihood of the channel parameters is given by:

$$L\left(\boldsymbol{\tau}, \mathbf{C}_b\right) \propto -T \ln\left(\det \mathbf{C}_h\right) - \sum_{t=1}^{T} \widehat{\mathbf{h}}^H(t) \mathbf{C}_h^{-1} \widehat{\mathbf{h}}(t) \tag{13}$$

Maximizing (13) (with respect to $\boldsymbol{\tau}$, $\mathbf{C}_b$) is again a difficult non-linear problem. In this section, we will not elaborate on the global maximization of the Rayleigh likelihood.
Using the Bayesian structure, the PDP is parameterized by the time delay and the fading variance of the different paths. During the creation and the maintenance of the database, these parameters are estimated and stored at quantized positions of the coverage area. The likelihood that the received signal $\widehat{\mathbf{h}}(t)$ $t = 1 : T$ comes from a MS located around the position corresponding to the p^{th} database entry is:

$$L\left(\widehat{\mathbf{h}}_1 \cdots \widehat{\mathbf{h}}_T | \tau^{(\mathbf{p})}, \mathbf{C}_b^{(p)}\right) \propto -\ln\left(\det \mathbf{C}_h^{(p)}\right) - \text{tr}\left\{\mathbf{C}_h^{-1} \widehat{\mathbf{C}}_h\right\} \tag{14}$$

where $\mathbf{C}_h^{(p)}$ is the channel covariance matrix computed using $\tau^{(\mathbf{p})}$ and $\mathbf{C}_b^{(p)}$ (the time delay and amplitude covariance stored at the p^{th} database entry). $\widehat{\mathbf{C}}_h = \dfrac{1}{T} \displaystyle\sum_{t=1}^{T} \widehat{\mathbf{h}}(t) \widehat{\mathbf{h}}^H(t)$ is the observed sampling covariance matrix. The MS position is the selected by maximizing this likelihood, i.e.,

$$\hat{p} = \arg\max_p L\left(\widehat{\mathbf{h}}_1, \cdots, \widehat{\mathbf{h}}_T | \tau^{(\mathbf{p})}, \mathbf{C}_b^{(p)}\right) \quad (15)$$

This leads to a one step location approach tacking into account the constraints imposed by the MS location. Prior information on the MS location (using for example a tracking scheme) can be exploited to reduce the optimization subspace and avoid potential positioning ambiguities.

Differentiating the likelihood leads to

$$\begin{aligned} \partial L(\tau, \mathbf{C}_b) &= \partial\left(-\ln(\det \mathbf{C}_h) + \text{tr}\left\{\mathbf{C}_h^{-1}\widehat{\mathbf{C}}_h\right\}\right) \\ &= -\frac{1}{2}\,\partial\left\|\mathbf{C}_h^{-\frac{1}{2}}\left(\widehat{\mathbf{C}}_h - \mathbf{C}_h\right)\mathbf{C}_h^{-\frac{H}{2}}\right\|_F^2 \end{aligned} \quad (16)$$

where $\|\mathbf{C}\|_F$ and $\mathbf{C}^{\frac{1}{2}}$ denote respectively the Frobenius norm and the square root of the matrix $\mathbf{C}$. Thus, the maximum likelihood leads to the Optimally weighted Covariance Matching (OCM) method [13]. The parameters are chosen to match the whole covariance matrix, and not only the PDP (the diagonal elements). Remark also that if the channel impulse response is sufficiently sparse (pulse-shape supports do not overlap), the covariance matrix $\mathbf{C}_h$ is diagonal, and the deterministic and Bayesian estimation techniques coincide.

On the other hand, prior information on the signal structure is available, and can be exploited to enhance the estimation of the observed covariance matrix. Different levels of structural information can be considered: subspace decomposition, and high resolutions methods can be used to emphasis the prior structure of the observed covariance matrix. Exploiting the prior structure improves the localization accuracy and resolution, and defines intermediate approaches between the classic geometric and mapping techniques.

A Local identifiability

To investigate the local identifiability of Bayesian PDP-Fingerprinting, we assume that paths are well separated, which implies

$$\mathbf{p}_{\tau_i}^T \mathbf{p}_{\tau_j} \approx \sigma_p^2 \delta_{i,j} \quad (17)$$

where σ_p^2 is the pulse-shape energy, and $\delta_{i,j}$ is the Kronecker delta function. Under this assumption, $\frac{1}{\sigma_p}\mathbf{P}_\tau$ becomes an orthogonal matrix and the determinant of $\mathbf{C}_h$ does not depend on the path delays:

$$\det(\mathbf{C}_h) = \prod_{l=1}^{L}\left(\sigma_p^2\sigma_{b,l}^2 + \sigma_v^2\right) \quad (18)$$

On the other hand, using the matrix inversion lemma (ShermanMorrisonWoodbury formula), one can also show that

$$\mathbf{C}_h^{-1} = \sigma_v^{-2}\mathbf{I}_N - \sigma_v^{-2}\sum_{l=1}^{L}\frac{\sigma_{b,l}^2}{\sigma_{b,l}^2\sigma_p^2 + \sigma_v^2}\mathbf{p}_{\tau_l}\mathbf{p}_{\tau_l}^H \quad (19)$$

Using (18) and (19), the Fisher Information Matrix (FIM) can be derived for the parameters $\tau_l,\ \sigma_{b,l}^2,\ l = 1 : l$. On the other hand, differentials in these channel parameters can be coupled to differentials in the position (dx, dy). By assuming a certain scenario for obstacle positions and path attenuation exponent, this leads to a FIM for the estimation of (dx, dy). One can show that this FIM is non-singular (with a probability one over a random scenario distribution) for $L \geq 2$. Thus using one BS, the mobile localization is locally identifiable if we consider at least two paths, which is consistent with the identifiability results derived in the framework of classic geometric localization (in the LoS conditions, at least 2 BSs are needed for local identifiability).

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