On the Design of Scalar Feedback Techniques for MIMO Broadcast Scheduling

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***Abstract*— MIMO broadcast channels with partial channel knowledge at the transmitter are considered, obtained via limited-rate feedback. Given a maximum orthogonality factor ϵ between transmit beamforming vectors ($\epsilon = 0$ if orthogonal beamforming) and variable number of active beams, the design of scalar feedback metrics for user scheduling is studied. These metrics provide an estimate of the received signal-to-noise plus interference ratio (SINR) and can take the form of either an upper or lower bound on the SINR. A closed-form expression for the sum-rate is provided, showing the performance of each metric in different scenarios. Analytical and simulation results are shown in order to identify the regions where SDMA provides higher rates than TDMA.**

I. Introduction

Multiple-input multiple-output (MIMO) systems can significantly increase the spectral efficiency by exploiting the spatial degrees of freedom created by multiple antennas. The capacity gain of multiuser MIMO systems is highly dependent on the available CSIT. While having full CSI at the receiver can be assumed, this assumption is not reasonable at the transmitter side. Several limited feedback approaches have been considered in point-to-point systems [1], [2], [3], where each user sends to the transmitter the index of a quantized version of its channel vector from a codebook. An extension for MIMO broadcast channels is made in [4], in which each mobile feeds back a finite number of bits regarding its channel realization at the beginning of each block based on a codebook. An SDMA extension of opportunistic beamforming [5] using partial CSIT in the form of individual signal-to-interference-plus-noise ratio (SINR) is proposed in [6], achieving optimum capacity scaling for large number of users.

In this paper, we consider the finite rate feedback model of [4] for the case when $K \geq M$. In this scenario, information on the channel direction is not sufficient in order to perform user scheduling, and thus additional scalar metrics are fed back to the base station, providing instantaneous channel quality information (CQI).

In [7],[8] a simple scalar feedback metric is proposed for user scheduling. A scheme with similar metric is also reported in [9]. In this paper, we generalize these results to scalar feedback metrics incorporating information on the number of active beams and predertermined orthogonality properties of the beamforming vectors. These metrics provide an estimate of the received signal-to-noise plus interference ratio (SINR) and can take the form of either an upper or lower bound on the SINR. The advantages and disadvantages of such techniques are presented, evaluating their performances in terms of sum rate.

II. System Model

We consider a multiple antenna broadcast channel consisting of M antennas at the transmitter and $K \geq M$ single-antenna receivers. The received signal y_k of the k-th user is mathematically described as

$$y_k = \mathbf{h}_k^H \mathbf{x} + n_k, \quad k = 1, \dots, K \tag{1}$$

where $\mathbf{x} \in \mathbb{C}^{M\times 1}$ is the transmitted signal, $\mathbf{h}_k \in \mathbb{C}^{M\times 1}$ is an i.i.d. Rayleigh flat fading channel vector, and n_k is additive white Gaussian noise at receiver k. We assume that each of the receivers has perfect and instantaneous knowledge of its own channel $\mathbf{h}_k$, and that n_k is independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian with zero mean and unit variance. The transmitted signal is subject to an average transmit power constraint P, i.e., $\mathbb{E}\{\|\mathbf{x}\|^2\} = P$. Let $\mathbf{H} \in \mathbb{C}^{K\times M}$ refer to the concatenation of all channels, $\mathbf{H} = [\mathbf{h}_1 \ \mathbf{h}_2 \dots \mathbf{h}_K]^H$, where the k-th row is the channel of the k-th receiver. Let $\mathcal{S}$ denote the set of users selected for transmission at a given time slot, with cardinality $|\mathcal{S}| = M_o$, $1 \leq M_o \leq M$. Then $\mathbf{H}(\mathcal{S})$, $\mathbf{W}(\mathcal{S})$, $\mathbf{s}(\mathcal{S})$, $\mathbf{y}(\mathcal{S})$ are the concatenated channel vectors, beamforming vectors, uncorrelated data symbols and received signals respectively for the set of scheduled users $\mathcal{S}$. When concatenating the beamforming matrix $\mathbf{W}(\mathcal{S})$ prior to transmission, the signal model can be described as follows

$$\mathbf{y}(\mathcal{S}) = \mathbf{H}(\mathcal{S})\mathbf{W}(\mathcal{S})\mathbf{s}(\mathcal{S}) + \mathbf{n} \tag{2}$$

The beamforming matrix is given by

$$\mathbf{W}(\mathcal{S}) = \mathbf{V}(\mathcal{S})\Lambda(\mathcal{S})^{1/2} \tag{3}$$

where the columns of $\mathbf{V}(\mathcal{S})$ are the normalized beamforming vectors and $\Lambda(\mathcal{S})$ is a diagonal power allocation matrix. At the k-th mobile, the received signal is given by

$$y_k = \sqrt{\frac{P}{M_o}} \sum_{i \in \mathcal{S}} \mathbf{h}_k^H \mathbf{w}_i s_i + n_k, \qquad k = 1, \ldots, K \tag{4}$$

Hence, the SINR of user k is

$$SINR_k = \frac{|\mathbf{h}_k^H \mathbf{w}_k|^2}{\sum_{i \in \mathcal{S}, i \neq k} |\mathbf{h}_k^H \mathbf{w}_i|^2 + \frac{M_o}{P}} \tag{5}$$

We focus on the ergodic sum rate (SR) which, assuming Gaussian inputs, is equal to

$$SR = \mathbb{E}\left\{ \sum_{k \in \mathcal{S}} \log\left[1 + SINR_k\right] \right\} \tag{6}$$

III. Linear Beamforming with Limited Feedback

Joint linear beamforming and scheduling is performed in a system where limited feedback is present at the transmitter side. The feedback conveyed by each user to the base station consists of channel direction information (CDI) based on a predetermined codebook, and a scalar metric with channel quality information (CQI) used to perform user scheduling.

In such systems, design of appropriate scalar metrics in scenarios with realistic number of users and average SNR values remains a challenge. These metrics must contain information of the users' channel gains as well as channel quantization errors, as discussed in [10]. If the users have additional knowledge of the beamforming technique used at the transmitter side, an estimate on the multiuser interference at the receiver can be computed. This information can be encapsulated together with the channel gain, quantization error and average noise power into a scalar metric ξ, which consists of an estimate on the SINR. In our work, we consider such scalar feedback strategies, as discussed in detail in next section. User selection is carried out based on these metrics and the users' spatial properties, obtained from channel quantizations.

As simple transmission technique we consider transmit matched filtering (TxMF), which consists of using as normalized beamforming vectors the quantized channel directions of users scheduled for transmission. The normalized channel vector of user k to be quantized is $\overline{\mathbf{h}}_k = \mathbf{h}_k / \|\mathbf{h}_k\|$, which corresponds to the channel direction. A B-bit quantization codebook $\mathcal{V}_k$ is considered, containing $L = 2^B$ unit norm vectors in $\mathbb{C}^M$, which is assumed to be known to both the receiver and the transmitter. Similarly to [2], [3], we assume that each receiver quantizes its channel to the vector that maximizes the inner product

$$\mathbf{v}_k = \arg\max_{\mathbf{v} \in \mathcal{V}_k} |\overline{\mathbf{h}}_k^H \mathbf{v}|^2 = \arg\max_{\mathbf{v} \in \mathcal{V}_k} \cos^2(\angle(\overline{\mathbf{h}}_k, \mathbf{v})) \tag{7}$$

Each user sends the corresponding quantization index back to the transmitter through an error-free and zero-delay feedback channel using B bits. Note that this model is equivalent to the finite rate feedback model proposed by [2],[4].

The optimal vector quantizer is difficult to find and the solution to this problem is not yet known. As codebook design goes beyond the scope of the paper, we adopt the geometrical framework presented in [3]. The resulting quantization error is defined as $\sin^2\theta_k = \sin^2(\angle(\bar{\mathbf{h}}_k, \mathbf{v}_k)) = 1 - \left|\bar{\mathbf{h}}_k^H \mathbf{v}_k\right|^2$ [3], [11], where $\mathbf{v}_k$ is the quantized channel direction of user k. Using this framework, the cumulative distribution function (CDF) of the quantization error is given by [3], [11]

$$F_{\sin^2\theta_k}(x) = \begin{cases} \delta^{1-M} x^{M-1} & 0 \leq x \leq \delta \\ 1 & x > \delta \end{cases}$$

where $\delta = 2^{-B/(M-1)}$.

Let the orthogonality factor ϵ denote the maximum degree of non orthogonality between two unit-norm vectors. The columns of the normalized beamforming matrix $\mathbf{V}(\mathcal{S})$ are constrained to be ϵ-orthogonal and thus $\left|\mathbf{v}_i^H \mathbf{v}_j\right| \leq \epsilon \;\forall\, i, j \in \mathcal{S}$. An outline of the proposed scheduling algorithm is shown in Table I. In case M_o users with ϵ-orthogonality can not be found, the algorithm stops and distributes the power equally among the scheduled users, setting $M_o = |\mathcal{S}|$. Note that this greedy algorithm is equivalent to the one proposed in [12], [13], [14]. The first user is selected from the set $\mathcal{Q}^0 = \{1, \ldots, K\}$ as the one having the highest channel quality, i.e., $k_1 = \arg\max_{k \in \mathcal{Q}^0} \xi_k$. For $i = 1, \ldots, M_o - 1$, the $(i+1)$-th user is selected as $k_{i+1} = \arg\max_{k \in \mathcal{Q}^i} \xi_k$ among the user set $\mathcal{Q}^i = \left\{1 \leq k \leq K : |\mathbf{v}_k^H \mathbf{v}_{k_j}| \leq \epsilon, 1 \leq j \leq i\right\}$.

The number of active beams for transmission M_o and orthogonality factor ϵ are system parameters fixed by the Base Station (BS) that can be adapted in order to maximize the system sum rate.

TABLE I
Outline of Scheduling Algorithm

MS
Compute & Feedback ξ_k
quantization index $i \in \{1, \ldots, L\}$

BS

Initialize Set $\mathcal{S} = \emptyset$

Loop For $i : 1 \ldots M_o$ repeat

Set $\xi_{max}^i = 0$

Loop For $k : 1 \ldots K, k \notin \mathcal{S}$ repeat

If $\xi_k > \xi_{max}^i$ and $\left|\mathbf{v}_k^H \mathbf{v}_j\right| \leq \epsilon \,\forall j \in \mathcal{S}$

$\xi_k \to \xi_{max}^i$ and $k_i = k$

Select $k_i \to \mathcal{S}$

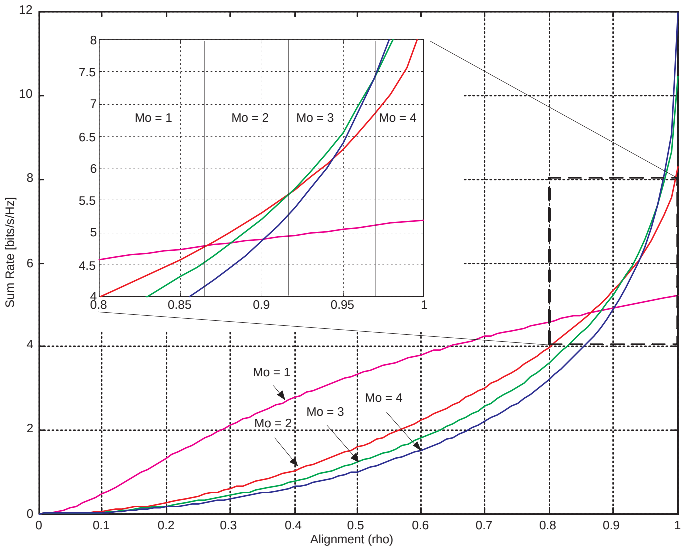

Fig. 1. Sum rate as a function of the alignment (rho) for $M = 4$ antennas, variable number of active beams M_o, orthogonality factor $\epsilon = 0.1$ and $SNR = 10$ dB.

IV. Scalar Feedback Metrics

In this section, we present the design guidelines of scalar feedback metrics based on an estimate of the expected SINR. Knowledge of the channel realization, quantization error, average transmit power, noise power, orthogonality factor ϵ and number of active beams M_o can be combined at the receiver in order to obtain a lower bound on the SINR. In [15], a metric containing these elements is proposed, which consists of an exact lower bound with the purpose of avoiding outage events in the communication. Here we consider instead a statistical lower bound which provides tighter bounds on the average SINR and thus higher performance for user scheduling.

Including the system parameters ϵ and M_o in the SINR feedback has advantages and disadvantages. As we show through simulations, taking into account ϵ in the SINR computation may mask the contribution of the channel gains in the SINR expression, hence reducing the benefits of multiuser diversity. However, this loss is negligible for practical number of active users. On the other hand, computing an SINR lower bound can be beneficial from the network point of view. The total amount of feedback overhead can be reduced by appropriately setting minimum desired SINR thresholds. Hence, in a practical system each user may send feedback to the base station only if a minimal QoS can be guaranteed. Scalar feedback without encapsulated ϵ and M_o information is also presented, in the form of an upper bound on the SINR.

Metric I: Lower Bound on SINR

For user k and index set $\mathcal{S}$, the multiuser interference can be expressed as $I_k(\mathcal{S}) = \sum_{i\in\mathcal{S}, i\neq k} \frac{P}{M_o}|\mathbf{h}_k^H \mathbf{v}_i|^2 = \frac{P}{M_o}\|\mathbf{h}_k\|^2 \overline{I}_k(\mathcal{S})$, where $\overline{I}_k(\mathcal{S})$ denotes the interference over the normalized channel $\overline{\mathbf{h}}_k$. Define $\overline{I}_{UB_k}$ as the upper bound on $\overline{I}_k$ and $\theta_k = \angle(\overline{\mathbf{h}}_k, \mathbf{v}_k)$. Based on the work developed in [10] for arbitrary orthogonality between beamforming vectors, we propose a metric which, when averaged over the statistics of $\|\mathbf{h}_k\|$ and $\cos\theta_k$, yields a lower bound on the average SINR. The proposed feedback metric for the k-th user is given by

$$\xi_k^I = \frac{\|\mathbf{h}_k\|^2 \cos^2\theta_k}{\|\mathbf{h}_k\|^2 \overline{I}_{UB_k} + \frac{M_o}{P}} \tag{8}$$

where

$$\overline{I}_{UB_k} = \alpha_k \cos^2\theta_k + \beta_k \sin^2\theta_k + 2\gamma_k \sin\theta_k \cos\theta_k \tag{9}$$

and

$$\begin{aligned} \alpha_k &= \tfrac{(M_o-1)^2}{M-1}\epsilon^2 \\ \beta_k &= \tfrac{(M_o-1)}{M-1}\left[1 + (M_o-2)\epsilon\right] \\ \gamma_k &= \tfrac{(M_o-1)^2}{M-1}\epsilon \end{aligned} \tag{10}$$

Metric II: Upper Bound on SINR

As a particular case of the above metric, we consider $\epsilon = 0$ and $M_o = M$ in the metric computation, which can be interpreted as an upper bound on the SINR when equal power allocation is performed at the transmitter. The resulting metric becomes

$$\xi_k^{II} = \frac{\|\mathbf{h}_k\|^2 \cos^2\theta_k}{\|\mathbf{h}_k\|^2 \sin^2\theta_k + \frac{M}{P}} \tag{11}$$

which was proposed in parallel in [7], [8], [9].

In Fig. 1, an approximated bound on the system sum rate is plotted as a function of the alignment $\rho = \cos\theta_k$, computed as $SR \approx M_o \log(1 + \xi_k^I)$. The system under consideration is assumed to have $M = 4$ antennas, $\epsilon = 0.1$ and average $SNR = 10$ dB. The sum rate is evaluated for different number of active beams to observe the impact of appropriately choosing M_o. The system with $M_o = 1$ exhibits better performance for low and intermediate values of ρ, i.e. TDMA provides higher rates than SDMA in most cases. Only for large values of ρ, $M_o > 1$ provides higher rates, which in practice occurs for large number of quantization bits B or large number of users K. Since the amount of bits B is generally low due to bandwidth limitations, SDMA will be chosen over TDMA when $M_o > 1$ users with small quantization errors can be found, with higher probability as the number of users in the cell increases. As the parameter ϵ increases, the crossing points of the curves in Fig. 1 shift to the right and thus the range for which TDMA performs better also increases. This is due to the fact that the bound in ξ_k^I becomes looser for increasing ϵ values. As shown in this example, for $\epsilon > 0$ there exist M possible modes of transmission, i.e. $M_o = 1, \ldots, M$. However, for the case of $\epsilon = 0$ as considered in ξ_k^{II}, it can be proven that the modes of transmission exhibiting higher rates are reduced to 2, namely $M_o = 1, M$.

V. Lower Bound on the Sum Rate

Denoting the lower bound on SINR of equation (8) as s, we derive an approximation on its CDF using mathematical tools from [16], which is given by

$$F_s(s) \approx 1 - \frac{e^{\frac{-M_o s}{P(1-\alpha s)}}}{\delta^{M-1}\left(1+m\right)^{M-1}} \tag{12}$$

where $m = \frac{2\gamma s\left[\gamma s+\sqrt{\gamma^2 s^2+(1-\alpha s)\beta s}\right]+(1-\alpha s)\beta s}{(1-\alpha s)^2}$

Note that the CDF above is a generalization for arbitrary ϵ and M_o of the CDF derived in [8]. Let the ordered variate $s_{i:K}$ denote the i-th largest among K i.i.d. random variables. From known results of extreme order statistics [17], we have that the CDF of $s_1 = \max_{1\leq i\leq K} s_{i:K}$ is $F_{s_1} = (F_s(s))^K$. According to the proposed user selection algorithm, the SINR of the first selected user is the maximum SINR over K i.i.d. random variables. However, at the i-th selection step (i-th beam) the search space gets reduced since the ϵ-orthogonality condition needs to be satisfied. Hence, the i-th user is selected over K_i i.i.d. random variables yielding a CDF for the maximum SINR given by $F_{s_i} = (F_s(s))^{K_i}$. Its mean value can be approximated as

$$\mathbb{E}(s_i) \approx \int_0^{1/\alpha} 1-(F_s(s))^{K_i} ds \tag{13}$$

An approximation of K_i can be calculated through the probability that a random vector in $\mathbb{C}^{M\times 1}$ is ϵ-orthogonal to a set with $i-1$ vectors in $\mathbb{C}^{M\times 1}$, which is equal to $I_{\epsilon^2(i,M-i)}$ [13], $I_x(a,b)$ being the regularized incomplete beta function. By using the law of large numbers [14], we can find the following approximation:

$$K_i \approx K I_{\epsilon^2}(i-1, M-i+1) \tag{14}$$

Using Jensen's inequality and solving the integral in equation (13) for the CDF of s described in (12), we obtain the following sum-rate approximation as a function of ϵ and M_o

$$R_{M_o} \approx \sum_{i=1}^{M_o} \log_2\left[1+\sum_{n=1}^{K_i}\frac{(-1)^{n-1}}{\delta^{n(M-1)}}\binom{K_i}{n}\frac{1}{\alpha}\left(1+\frac{Cn}{\alpha}e^{\frac{Cn}{\alpha}}E_i\left(-\frac{Cn}{\alpha}\right)\right)\right] \tag{15}$$

where $C = \frac{M_o}{P} + (M-1)\beta$, α and β are as described in equation (10) and the exponential integral function is used, defined as $E_i(x) = \int_{-x}^{\infty}\frac{e^{-t}}{t}dt$. Note that, as a particular case of the equation above, a simpler expression can be derived for $M_o = 1$, given by

$$R_1 \approx \log_2\left[1+\sum_{n=1}^{K}\frac{(-1)^{n-1}}{\delta^{n(M-1)}}\binom{K}{n}\frac{P}{n}\right] \tag{16}$$

In Fig. 2 the approximated sum rate is plotted as a function of the number of active beams M_o and orthogonality factor ϵ, for $K = 35$ users, $SNR = 10$ dB and a simple codebook with $B = 1$ bit. Note that, in this particular scenario, TDMA provides better rates than SDMA regardless of the value of ϵ. In this context, the number of users is too low to favor SDMA transmission, which is consistent with the results obtained in previous section. It can be easily verified from equations (15) and (16) that given an arbitrary ϵ, SDMA outperforms TDMA asymptotically with the number of users K

$$\lim_{K\to\infty}\frac{R_{M_o}}{R_1} = M_o \tag{17}$$

VI. Numerical results

In this section, we evaluate through simulations the sum rate performance of the systems based on metrics I and II, for $M = 3$ antennas and $B = 6$ bits. The system using metric I is assumed to appropriately set M_o and ϵ both for transmision and metric computation, maximizing the sum rate for each K and SNR pair. On the other hand, the scheme with metric II uses optimal ϵ values in each scenario. For comparison, the performances of random beamforming [6] and TxMF with perfect CSIT and exhaustive-search user selection are provided.

Fig. 3 shows a performance comparison in terms of sum rate versus number of users for $SNR = 10$ dB, in a cell with realistic number of active users. The scheme based on metric I (lower bound) provides slightly better performance than the one using metric II, exploiting the benefits of having variable number of active beams.

Fig. 4 shows similar performances for the schemes with metric I and II in the low-mid SNR region, in a setting with $K = 10$ users. Since in the simulated system the number of codebook bits B is not increased proportionally to the average SNR, as discussed in [4], the scheme using metric II ($M_o = M$) exhibits an inteference-limited behavior, flattening out at high SNR. As the SNR increases, the scheme using metric I converges to a TDMA solution, i.e. choosing $M_o = 1$, which provides linear growth with the SNR.

VII. Conclusions

We studied a multiple antenna broadcast channel in which partial CSIT is conveyed via a limited rate feedback channel. We presented scalar feedback metrics which, combined with efficient joint scheduling and linear beamforming, can achieve a large portion of the optimum capacity by exploiting multiuser diversity. As shown through simulations, the scalar metric based on an SINR lower bound provides better sum rates than the one based on an upper bound, provided that the number of active beams and ϵ-orthogonality is appropritely set in each scenario. In addition, feedback based on an SINR lower bound can easily enable the possibility of reducing the amount of feedback overhead by setting minimum desired SINR thresholds.

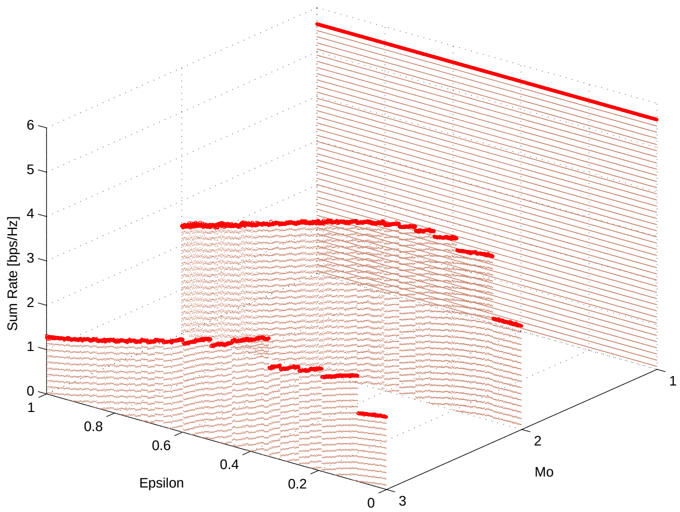

Fig. 2. Sum rate as a function of the orthogonality factor ϵ and number of active beams M_o for $K = 35$ users, $SNR = 10$ dB and $B = 1$ bit.

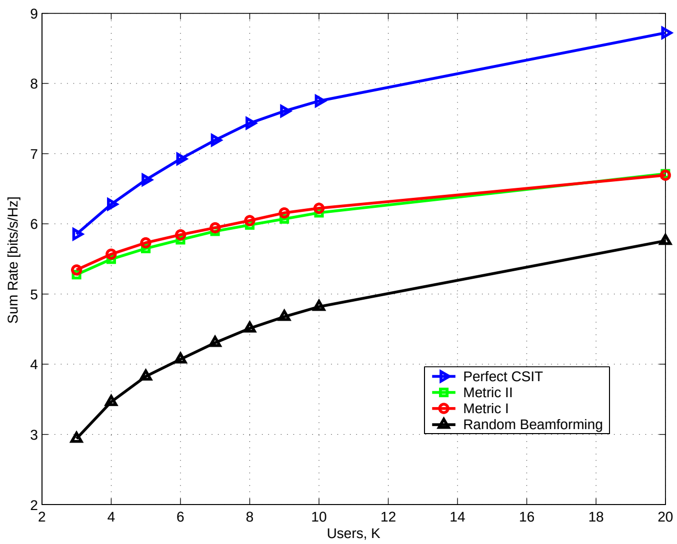

Fig. 3. Sum rate as a function of the number of users for $B = 6$ bits, $M = 3$ transmit antennas and $SNR = 10$ dB.

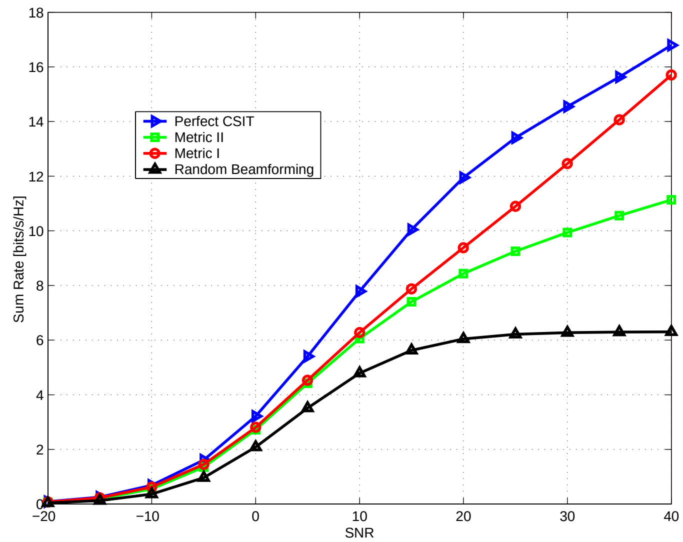

Fig. 4. Sum rate versus average SNR for $B = 6$ bits, $M = 3$ transmit antennas and $K = 10$ users.

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