Power allocation Policies over multi-band/multi-user Cognitive Radio System

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***Abstract*— This paper[1] addresses the performance limits of a multi-band/multi-user frequency fading channels. We firstly study the optimal diversity-multiplexing trade-off of such systems when channel state information is available at the receiver only (CSIR). However, when channel state information is made available at the receiver and at the transmitter (CSIT), the transmitter knows the channel gains and thus will not send bits unless they can be decoded correctly. Accordingly, there is no more notion of capacity versus outage in this case where the transmitter sends bits that cannot be decoded. We introduce the notion of outage capacity and derive optimal outage-rate trade-off for some commonly used power allocations and conclude on their ability to optimize rate and/or outage with respect to classical uniform power allocation.**

I. Introduction

Cognitive radio is an emerging approach to implement efficient reuse of the licensed spectrum by detecting unoccupied spectrum bands and adapting the transmission to those bands while avoiding the interference to primary users. This novel approach to spectrum access introduces unique functions at the physical layer: reliable detection of primary users and adaptive transmission over a wide bandwidth. From this brief definition, it is obvious that the cognitive module in the transmitter must work in a harmonious manner with the cognitive modules in the receiver. In order to maintain this harmony between the cognitive radio's transmitter and receiver at all times, we need a feedback channel connecting the receiver to the transmitter. Through the *feedback channel*, the receiver is enabled to convey information on the performance of the forward link to the transmitter. The cognitive radio is, therefore, by necessity, an example of a *feedback communication system* [1], [2]. Many works are now oriented to design smart terminals able to detect the available bands and to allocate in a smart way the available power based on the channel state information (CSI).
Furthermore, it turns out necessary to keep in mind the two primary objectives of cognitive radio:

- highly reliable communications whenever and wherever needed;
- efficient utilization of the radio spectrum.

Accordingly, it is of major interest for cognitive radio systems to guarantee a more intensive and efficient spectrum use and, at the same time, a high quality of service (QoS). This is exactly the question we tackled here. In fact, we propose to study the limit performances of a slow (block) frequency selective fading wide band (or multi-band) system in terms of optimal outage-rate trade-off when CSI is made available. This work is motivated by the idea that in an integrated network, non real-time applications will benefit from maximizing the capacity, and at the same time, real-time applications (such as voice and video) will benefit from a Quality Of Service (QoS) guarantee by minimizing the outage probability.
Moreover, for different channel information assumptions, there are different definitions of channel capacity, depending on whether capacity characterizes the maximum rate averaged over all fading states or the maximum constant rate that can be maintained in all fading states (with or without some probability of outage). A comprehensive survey of these concepts can be found in [3]. The notion of *information outage probability* defined as the probability that the instantaneous mutual information of the channel is below the transmitted code rate was introduced in [4]. Accordingly, the outage probability is:

$$P_{out}(R) = P\{I(\mathbf{x};\mathbf{y}) \leq R\} \tag{1}$$

Where $I(\mathbf{x};\mathbf{y})$ is the mutual information of the channel between the transmitted vector $\mathbf{x}$ and the received vector $\mathbf{y}$ and R is the target data rate in $(bits/s/Hz)$. Reliable communication can therefore be achieved when the mutual information of the channel is strong enough to support the target rate R. Additional definitions related to outage probability are those of:

- *Zero outage capacity* : also called *delay-limited capacity*. It represents the maximum data-rate R for which the minimum outage probability is zero.
- *Outage capacity* : is the maximum target rate that can be achieved over the channel with an outage probability less than ε.

In this work, we firstly focus our analysis on the optimal diversity-multiplexing trade-off of a multi-band/multi-user frequency fading channel where CSI is made available at the receiver only (CSIR). In the second part, by considering CSIT, the transmitter can adapt its transmission strategy relative to this knowledge by transmitting at the target rate specified by the application with an error-free transmission. Thus, there

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is no more notion of *diversity* as defined in [5]. In fact, the transmitter will not send bits unless they can be decoded correctly at the receiver. We will adopt this framework to characterize the performance of some commonly used PA policies in terms of outage capacity and at the same time, show the potential gain of such coding schemes with respect to classical uniform power allocation.

The remainder of the paper is organized as follows. Section II describes the system model. In Section III, we analyze the performance limits of such a system when considering perfect CSIT. Simulation results are provided in Section IV and Section V concludes the paper.

II. MULTI-BAND/MULTI-USER SYSTEM MODEL

Consider a downlink communication with a single transmitter (the base station) sending independent information to multiple receivers (the users). The baseband frequency model for downlink channel with K users is:

$$y_k(f) = h_k(f)x_k(f) + n_k(f); \qquad k = 1,...,K \tag{2}$$

where $h_k(f) = h_{k,l}$ is the fading process of user k at the lth block fading where $k = 1,...,K$ and $l = 1,...,L$. We statistically model the channel h to be i.i.d distributed over the KL rayleigh fading coefficients and $\mathbb{E}\left\{\left|h_{k,l}\right|^2\right\} = 1$. The additive gaussian noise n at the receiver is i.i.d circularly symmetric and $n_k \sim \mathcal{CN}(0,N_0)$). We assume that the channel stays constant over a large number of transmissions (a transmission burst) and then changes to a new value based on the fading distribution, (i.e. the slow fading scenario) and is known by the receiver (CSIR). Such a system model is especially suitable for multi-carrier/multi-band cognitive radio systems where diversity can be obtained by coding across the symbols in different sub-carriers available for each user, like in Orthogonal Frequency Division Multiplexing Access (OFDMA).

III. RATE VERSUS OUTAGE PROBABILITY PERFORMANCE

In this section, we will analyze the performance limits of the underlying system. If the transmitter has no CSI, then all it can do is to transmit with equal power irrespective of the channel gain. The relevant metric in this case is the optimal diversity-multiplexing trade-off. However, if it has access to CSI, the transmitter can adapt its transmission strategy relative to this knowledge by transmitting at the target rate specified by the application with an error-free transmission. Thus, there is no notion of diversity as defined in [5]. In fact, the transmitter will not send bits unless they can be decoded correctly at the receiver. The relevant metric in this case is the outage capacity which could be viewed as a trade-off between rate and outage.

A. CSI at the receiver only

In this section, we study the underlying problem from a diversity order point of view and try to find the optimal rate-outage trade-off. We consider a slow fading channel and derive the optimal diversity-multiplexing trade-off for this channel where the CSI is made available at the receiver only. Traditionally, this tradeoff has been studied for a multiple antenna channel (space diversity) by exploiting the independence of faded signal paths, with the spatial multiplexing gain attainable by parallel transmission. Thus, in [5], Zheng and Tse define a multiplexing gain r and a diversity gain d if the data rate, R, and the probability of outage, P_{out}^k, as functions of SNR, satisfy:

$$d = \lim_{\mathrm{SNR}\to\infty} -\frac{\log_2(P_{out}^k(\mathrm{SNR}))}{\log_2(\mathrm{SNR})} \tag{3}$$

$$r = \lim_{\mathrm{SNR}\to\infty} \frac{\log_2(R(\mathrm{SNR}))}{\log_2(\mathrm{SNR})} \tag{4}$$

Where data-rate R is defined as below:

$$R = r\log_2 \mathrm{SNR}; \qquad r \in [0,1] \tag{5}$$

According to these definitions, one can interpret the diversity-multiplexing trade-off as a trade-off between reliability and data rate of a given system. We adopt this framework to characterize how to deal with the ISI and, at the same time, how to best exploit the inherent frequency diversity provided by the frequency selective channel. A common way to study the underlying trade-off is to compute the reliability function from the theory of error exponents.

1) Single Carrier: Let us, firstly, consider a multi-user/single carrier system and derive the optimal diversity-multiplexing trade-off corresponding. The probability of outage of such a system at a fixed target rate R is:

$$\begin{aligned} P_{out}^k &= P\left\{log_2\left(1+\mathrm{SNR}\,|h_k|^2\right) \le R\right\}; \qquad k = 1,...,K \\ &= P\left\{|h_k|^2 \le \tfrac{\mathrm{SNR}^r-1}{\mathrm{SNR}}\right\}; \qquad k = 1,...,K \end{aligned}$$

Notice that $|h_k|^2$ is exponentially distributed with probability density function : $p_{|h_k|^2} = e^{-t}$, yielding at high SNR-regime:

$$\begin{aligned} P_{out}^k(r,\mathrm{SNR}) &\simeq \left[1 - exp\left(-\mathrm{SNR}^{r-1}\right)\right] \\ &\simeq \mathrm{SNR}^{r-1} \end{aligned}$$

The optimal diversity-multiplexing trade-off in this case is:

$$d(r) = (1-r); \qquad r \in [0,1]$$

Hence, this scheme provides a diversity gain of order 1 for each user k. This suggests that a multi-band/multi-carrier fading system would have better performances in terms of optimal diversity-multiplexing trade-off.

2) Multiple Carrier: The outage probability for each user k where $k = 1,...,K$ over the L sub-carrier is:

$$P_{out}^k(r,\mathrm{SNR}) = P\left\{\sum_{l=1}^{L}\log_2\left(1+\mathrm{SNR}\left|h_{k,l}\right|^2\right) \le LR\right\}; \tag{6}$$

Considering that outage occurs when user k can not support the target rate R over the L fading gains available, equation

(6) can be tightly upper bounded by:

$$P_{out}^{k}(r,\mathrm{SNR}) \simeq P\left\{log_2\left(1+\mathrm{SNR}\left|h_{k,l}\right|^2\right) \le R\right\}^L \qquad (7)$$
$$\simeq P\left\{\left|h_{k,l}\right|^2 \le \frac{2^R-1}{\mathrm{SNR}}\right\}^L$$

The quality of this approximation as well as the parameters which make it possible to make can be found in [6].

$$P_{out}^{k}(r,\mathrm{SNR}) \simeq \left[1-exp\left(-\mathrm{SNR}^{r-1}\right)\right]^L$$

$$\simeq \mathrm{SNR}^{L(r-1)}$$

and the optimal diversity-multiplexing trade-off is:

$$d(r) = L(1-r); \qquad r \in [0,1]$$

Hence, frequency fading channels achieve an L-fold diversity gain over the single carrier performance at every multiplexing gain r. This result appears as a counter intuitive result since one would expect that the ISI will degrade the received signal. This suggests that ISI channel would have good performances in term of optimal diversity-multiplexing trade-off especially. Figure (1) depicts optimal diversity-multiplexing trade-off of some commonly used coding scheme over each user k. A suboptimal coding scheme is *repetition code*, which consists of repeating the same codeword over the the L carriers. In spite of its obvious sub-optimality, repetition code might be an option because of its simplicityWe notice that the multiple carrier transmission scheme presents an upper bound to achievable performance. In fact, the maximum achievable multiplexing gain of repetition code is equal to $\frac{1}{L}$ due to the use of only one degree of freedom among the L available. Notice that all the results listed above assume optimal coding over the L fading gains. Thus, each user k can will benefit from maximizing the capacity, and at the same time, will benefit from a QoS guarantee by minimizing the outage probability. Accordingly, these optimal diversity-multiplexing trade-off permit to ensure an optimal radio resource management for a given input device.

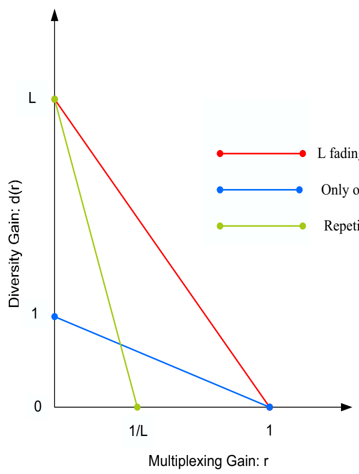

Fig. 1. Optimal Diversity-Multiplexing Trade-off.

B. CSI at the receiver and the transmitter

Throughout this section, we assume the same system model as in (2) with the assumption that CSI is made available at the receiver and at the transmitter. In this case, the transmit power could be controlled as a function of the channel gain to maximize the outage capacity. The problem here is to design power-allocation policies that minimizes outage probability on a given fading channel. This problem (among others) was addressed in [7], where it was shown that the best power-allocation scheme is to use no transmit power if the channel is below a threshold, and to use power allocation above this threshold. We will adopt this framework to find the transmission scheme that minimizes the outage probability for a given data-rate R under an average power constraint. The probability of outage, over the L fading gains available, is defined as below:

$$P_{out}^{k} = P\left\{\frac{1}{L}\sum_{l=1}^{L}\log_2(1+\mathrm{SNR}P_{k,l}\mid h_{k,l}\mid^2) \le R\right\} \qquad (8)$$

where $P_{k,l}$ denotes [2] the power allocated to the lth block fading of user k subject to the average power constraint on each user:

$$\frac{1}{L}\sum_{l=1}^{L}P_{k,l} = \overline{P}; \qquad k = 1,\ldots,K \qquad (9)$$

Without loss of generality, we take $\overline{P} = 1$ and analyze performances of some commonly used power-allocation policies in terms of outage capacity.

1) Truncated Channel Inversion policy (TCI): Let us firstly consider a sub-optimal power adaptation strategy where the transmitter uses the CSIT to maintain the received SNR constant irrespective of the channel gain. Thus, with exact channel inversion, there is zero-outage probability. However, this strategy would not be efficient especially when the channel is very bad. Consequently, we will allow to inverse the channel only below a certain cutoff value γ_0. The truncated channel inversion (TCI) power allocation in this context is:

$$P_l^{TCI} = \frac{1}{\gamma_l}, \qquad \gamma_l \ge \gamma_0$$

Where γ_l is defined as:

$$\gamma_l = \frac{|h_l|^2}{N_0}; \qquad l \in [1,L] \qquad (10)$$

By solving the power constraint on γ_0 in (9), and from the asymptotic expansion of $E_i(x)$ in [8], we obtain[3]:

$$\mathrm{SNR} = E_i\left(\frac{\gamma_0}{\mathrm{SNR}}\right)$$
$$\simeq -\log\left(\frac{\gamma_0}{\mathrm{SNR}}\right); \qquad \texttt{at high SNR regime}$$

[2]Throughout the rest of the paper, we will find it convenient to denote by $P_{k,l}$ the power allocation policy indexed by $l = 1,\ldots,L$ and $k = 1,\ldots,K$ rather than $P_{k,l}(h_{k,l})$.

[3]$E_i(x)$ is the exponential integral function defined as: $E_i(x) = \int_x^{+\infty}\frac{e^{-t}}{t}dt$.

Then, $\gamma_0 \simeq \text{SNR}.exp\,(-\text{SNR})$. The outage probability can be written as:

$$P^k_{out}(R,\text{SNR}) = P\left\{\frac{1}{L}\sum_{l=1}^{L}\log_2(1+\text{SNR}.P^{TCI}_{k,l}.\mid h_{k,l}\mid^2)\leq R\right\}$$

$$= \sum_{l=0}^{L} P\{l \leq L.R\}.\Gamma^l.(1-\Gamma)^{L-l}$$

$$= \sum_{l=0}^{L} \Pi(l,LR).\Gamma^l.(1-\Gamma)^{L-l}$$

Where:

$$\Pi(i,j) = \begin{cases} 1, & i \leq j \\ 0, & \text{otherwise} \end{cases}$$

And

$$\Gamma = P\left\{\mid h_{k,l}\mid^2 \geq \frac{\gamma_0}{\text{SNR}}\right\}$$

$$= exp(-\frac{\gamma_0}{\text{SNR}})$$

2) Water-filling policy (WF): This strategy stands in contrast to the case of TCI where more power is allocated when the channel is bad than when the channel is good. The optimal power allocation which maximizes the transmission rate here is solution of the optimization problem:

$$\max_{P_{k,l},\ldots,P_{k,L}} \left\{\frac{1}{L}\sum_{l=1}^{L}\log_2(1+\text{SNR}P_{k,l}\mid h_{k,l}\mid^2)\right\},$$

subject to:

$$\begin{cases} \frac{1}{L}\sum_{l=1}^{L} P_{k,l} = 1, \\ \\ P_{k,l} \geq 0 \end{cases} \tag{11}$$

The optimal solution computed by applying the Lagrangian leads to the well known water-filling (WF) power allocation, namely:

$$P^{WF}_{k,l} = \left(\frac{1}{\gamma_0} - \frac{1}{\gamma_l}\right)^+. \tag{12}$$

Where γ_l is defined as in (10) and γ_0 is the the Lagrange's multiplier satisfying:

$$\frac{1}{\gamma_0}\int_{\frac{\gamma_0}{\text{SNR}}}^{+\infty} e^{-t}dt - \frac{1}{\text{SNR}}\int_{\frac{\gamma_0}{\text{SNR}}}^{+\infty}\frac{e^{-t}}{t}dt = 1 \tag{13}$$

By substituting the exponential integral and solving the integral in (13), we obtain:

$$\frac{1}{\gamma_0}exp\left(-\frac{\gamma_0}{\text{SNR}})\right) - \frac{1}{\text{SNR}}E_i\left(\frac{\gamma_0}{\text{SNR}}\right) = 1 \tag{14}$$

The outage probability is given by [4]:

$$P^k_{out}(R,\text{SNR}) = P\left\{\frac{1}{L}\sum_{l=1}^{L}\log_2(1+\text{SNR}P^{WF}_{k,l}\mid h_{k,l}\mid^2)\leq R\right\}$$

$$= \sum_{n=0}^{L} P\left\{\frac{1}{L}\sum_{l=n+1}^{L}\log_2(\frac{\text{SNR}}{\gamma_0}\mid h_{k,l}\mid^2)\leq R\right\}.$$

$$\Gamma^{L-n}.(1-\Gamma)^n \tag{15}$$

[4] $(x)^+ = \max(0,x)$.

A cautionary note is in order here. An exact form for expression above is substantially more complex. Thus, a closed form of (15) is available in [9] but can be numerically sensitive to compute. In this work, we investigate the assumption that outage occurs when each of the fading channel can not support the target rate R. Thus, equation (15) can be tightly upper bounded by:

$$P^k_{out}(R,\text{SNR}) \simeq \sum_{n=0}^{L} P\left\{\log_2(\frac{\text{SNR}}{\gamma_0}\mid h_{k,l}\mid^2)\leq \frac{LR}{L-n}\right\}^{L-n}.$$

$$\Gamma^{L-n}.(1-\Gamma)^n$$

$$\simeq \sum_{n=0}^{L} P\left\{\mid h_i\mid^2 \leq 2^{\frac{LR}{L-n}}.\frac{\gamma_0}{\text{SNR}}\right\}^{L-n}.\Gamma^{L-n}.(1-\Gamma)^n$$

$$\simeq \sum_{n=0}^{L} exp\left(2^{\frac{LR}{L-n}}\frac{\gamma_0}{\text{SNR}}\right)^{L-n}.\Gamma^{L-n}.(1-\Gamma)^n$$

Thus, for a given data-rate R, SNR and L, one can determine the suspected outage probability on the communication. On the other hand, by defining an outage probability (as a QoS coefficient), we can conclude on the outage capacity allowed by this system knowing the SNR and the number of sub-channels L.

3) Hayes' Policy (H.P): Instead of analyzing the policy that maximizes the rate (water-filling), let us now focus on the strategy that minimizes the BER. The optimum power allocation that minimizes the BER of an uncoded system on fading channel was studied in [10] and defined as following:

$$P^{HP}_{k,l} = \begin{cases} \frac{1}{\gamma_l}ln\left(\frac{\gamma_l}{\gamma_0}\right), & \gamma_i \geq \gamma_0 \\ 0, & \text{otherwise} \end{cases} \tag{16}$$

Where γ_l is defined as in (10) and γ_0 is chosen to satisfy the average power constraint in (9). Substituting (16) in (9) we find that γ_0 must satisfy:

$$\frac{1}{\text{SNR}}\int_{\frac{\gamma_0}{\text{SNR}}}^{+\infty}\frac{e^{-t}}{t}\ln\left(\frac{\text{SNR.t}}{\gamma_0}\right)dt = 1$$

Note that we will show in section IV by numerical results that γ_0 lies near zero as SNR increases. On the other hand, by considering the same assumptions than for WF policy, the outage probability can be written as:

$$P^k_{out}(R,\text{SNR}) = P\left\{\frac{1}{L}\sum_{l=1}^{L}\log_2(1+\text{SNR}P^{HP}_{k,l}\mid h_{k,l}\mid^2)\leq R\right\}$$

$$\simeq \sum_{n=0}^{L}\frac{\gamma_0}{\text{SNR}}.exp\left(2^{\frac{LR}{L-n}-1}\right)^{L-n}.\Gamma^{L-n}.(1-\Gamma)^n$$

Thus, HP outage probability decreases with a factor equal to $\frac{\gamma_0}{\text{SNR}}.exp\left(2^{\frac{LR}{L-k}-1}\right)^{L-k}$ while for WF policy outage probability decreases as $exp\left(2^{\frac{LR}{L-k}}\frac{\gamma_0}{\text{SNR}}\right)^{L-k}$ for $k = 0,\ldots,L$. It is clear that WF would have better performance than HP in term of outage-rate trade-off. Notice here that TCI presents an indicator function as a factor depending on the data-rate R as mentioned before.

IV. Simulations and Results

Figure 2 depicts outage probabilities of respective P.A policies presented in this paper over each user. The cut-off values γ_0 where numerically obtained trough a dichotomous algorithm. One observes that TCI policy presents the worst behavior as SNR increases. This is due to the power control policy chosen here. In fact, by maintaining the received SNR constant irrespective of the fading gain, channel inversion policy does not exploit the available diversity. While Hayes'policy bad behavior can be explained by the optimization problem which focuses on minimizing the BER. Furthermore, it should be rather intuitive that the problem of minimizing the outage probability in (11) is equivalent to that of maximizing the mutual information $I(\mathbf{x};\mathbf{y})$ subject to the same average power constraint. The optimal power allocation for this problem was shown to be a mixture of TCI (for very low outage values) and water-filling allocation (for higher outage values). However, the optimal strategy is carried out by TCI and water-filling for lower outage values. On the other hand, it is straightforward that an optimal power allocation strategy affords a significant performance gain over the constant-power strategy at low SNR-region. The intuition is that when there is little transmit power, it is much more effective to expend it on the strongest fading gain of the system rather than spread the power evenly across all modes. However, the transmit and receive strategies associated with TCI may be easier to implement (lower complexity) than the water-filling schemes. Next, in figure 3, we consider the high SNR-regime. We see that as a result already noted in the literature, as SNR $\rightarrow \infty$, it is well known that the water-filling and the constant power strategies yield almost the same performance.

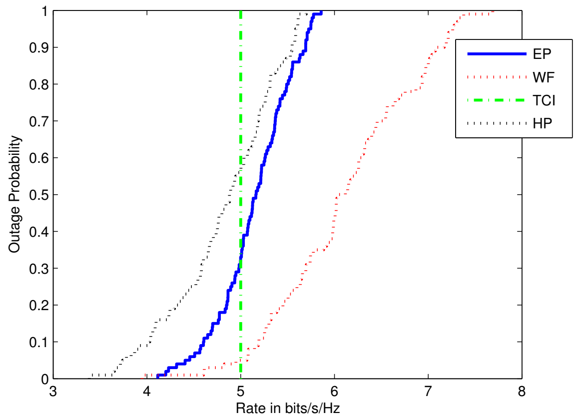

Fig. 2. Outage Probabilities of different Power allocation policies for $L = 5$ at SNR = 5 dB.

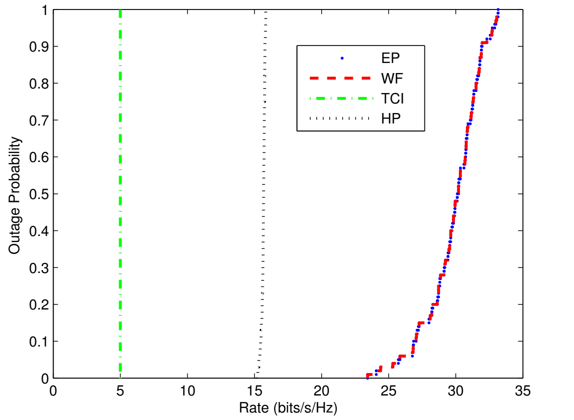

Fig. 3. Outage Probabilities of different Power allocation policies for $L = 5$ at SNR = 20 dB.

V. Conclusion

An important issue in cognitive radio systems is the design of techniques that exploit the inherent variability of the channel across time, frequency, and space. Diversity and multiplexing schemes appear as a useful solution to exploit the wireless variations of the channel. In this paper, we focus our attention on the optimal constant-rate coding schemes that minimizes the information outage probability over a multi-band/multi-user system when CSIT is available. Interestingly, with appropriate power allocation, one can increase the performance of classical trade-off. Thus, we showed that W.F achieves the optimal information outage probability, but, at high SNR-regime, reach the same performance provided by the multi-band channel without CSIT over each user.

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