ARQ Based Half-Duplex Cooperative Diversity Protocol

Tarik Tabet * S. Dusad † Raymond Knopp ‡

I. Introduction

Distributed antennas can be used to provide a mean to combat fading with a similar flavor as that of space diversity. This kind of reliability obtained by the creation of virtual antennas is referred to as cooperative diversity because the terminals share their resources to get the information across to the destination. Such schemes have attracted significant attention recently, and a variety of cooperation protocols have been studied and analyzed in various papers like [1] [2] [3] [4].

Recently, the authors of [5] extended the Zheng-Tse formulation [6] and characterized the three dimensional diversity-multiplexing-delay tradeoff in MIMO ARQ channels. They established that delay can be exploited as a potential source for diversity. Thus, retransmission protocol is an appealing scheme to combat fading and its performance has been recently studied in decentralized ad hoc networks [7]. Inspired by [5], we propose a new scheme for transmission in relay channel utilizing the ARQ to increase the diversity gain. We look at the tradeoff in the high snr (Signal-to-Noise Ratio) regime and point out the gain achieved by the ARQ. Moreover it is reasonable to assume the ARQ, as the receiver just needs to send back a one bit feedback to signify success or failure.

Following the setup in [2], the terminals are constrained to employ half-duplex transmission, i.e. they cannot transmit and receive simultaneously. The source and the relay are allowed to transmit in the same channel using cooperative protocols not relying on orthogonal subspaces. This is in contrast to [2], where the available bandwidth is divided into orthogonal channels allocated to the transmitting terminals. In the dynamic decode and forward scheme proposed in [4] the communication is across one block of fixed length l, where l is asymptotically large. In our setting the ARQ permits the use of communication over a variable number of blocks (henceforth referred to as number of rounds) of fixed length where the number of blocks used depend on the quality of the channel and are upper bounded by a fixed number L. If the destination is not able to decode at the end of these L blocks an outage is declared.

The outline of the paper is as follows. We introduce the channel model and the details of the algorithm in Section II. Section III contains a summary of the useful results and notations used in the rest of the paper. The actual achievable tradeoff for this protocol is analyzed and presented in Section IV for both long term and short term quasi-static channels. Finally we summarize and present a few concluding remarks and future directions in Section V.

II. System Model and Setting

In this work, we consider communication over a relay network with one relay node (R) assisting the transmission of a source(S) destination(D) pair as described in Fig. 1. Each link has i.i.d. circularly symmetric complex Gaussian zero mean channel gain h_{sd}, h_{sr}, h_{rd}, and these channels are independent. Moreover we assume that each decoder has perfect knowledge of the channel gain. Perfect channel state information at the receivers implies that the S-R channel is known to the relay node, while the individual S-D R-D channels are known to the destination node. The channel state information (CSI) is assumed to be absent at the node which is transmitting. Moreover, perfect synchronization is assumed between nodes, which requires some form of distributed pilot signals in practice.

We investigate two scenarios for the channel gains: 1) long-term static channel, where the fading is constant for all the channels over all retransmission (ARQ) rounds, and changes independently when the transmission of the current information message is stopped; 2) short-term static channel where the fading for all the channels is constant over each transmission round (or block) of the ARQ protocol and is an i.i.d process across successive rounds. The ARQ protocol considered in this work is a form of incremental redundancy as studied in [7] [8]. The transmission queue at the source is assumed to be infinite (not concerned by stability issues). The

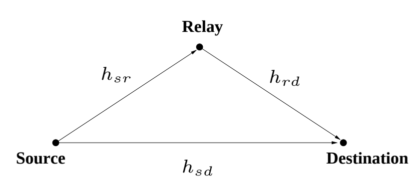

Fig. 1. System Model

* Mobile Communications laboratory, School of Computer and Communication Sciences, Ecole Polytechnique Fédérale de Lausanne, 1015 Lausanne, Switzerland. Email: tarik.tabet@epfl.ch. T. Tabet is supported by the National Competence Center in Research on Mobile Information and Communication Systems (NCCR-MICS), a center supported by the Swiss National Science Foundation under grant number 5005-67322.

†LTHC-LICOS, School of Computer and Communication Sciences, Ecole Polytechnique Fédérale de Lausanne, 1015 Lausanne, Switzerland. Email: sanket.dusad@epfl.ch.S. Dusad was supported in part by SNSF Grant #200021-105640/1

‡Mobile Communications Department, Eurécom Institute, Sophia-Antipolis, France Email: raymond.knopp@eurecom.fr R. Knopp was supported by Eurécom and its industrial partners Hasler Stiftung, Swisscom, France Telecom, La Fondation Cegetel, Bouygues Telecom, Thales, ST Microelectronics, Hitachi Europe and Texas Instruments

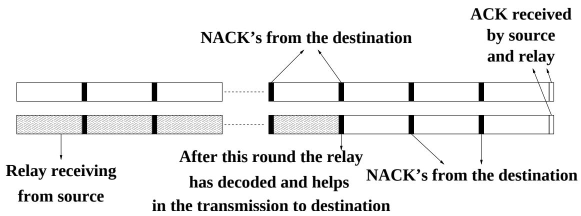

Fig. 2. Message as seen by the destination

information message of b bits is encoded using a space-time code with code book $\mathcal{C} \subset \mathbb{C}^{2\times LN}$, where N is the number of channel uses taken to transmit one round and L is the maximum number of rounds that can be used to transmit the b information bits. We let $\mathcal{C}_l$ for $l = 1, \cdots, L$ denote the punctured space-time code of length lN obtained from $\mathcal{C}$ by deleting the last $(L-l)N$ columns of the space time code.

The protocol utilizes the ARQ as follows. The receiver feeds back a one bit success/failure indication to both the relay and the source. If the relay decodes before the destination then knowing the codebook $\mathcal{C}$ it begins transmitting the second row of the codebook $\mathcal{C}$ to the destination. Thus effectively it becomes a MISO channel increasing the diversity. If the destination decodes before the relay, it just sends the feedback to the source and relay and the source moves on to transmitting the next message. The source moves on to the next information message in the transmission queue either if L rounds have been exhausted for the message or if the destination sends success feedback. If successful decoding occurs at the l-th transmission, the effective coding rate for the current codeword is R/l bit/dim where $R = b/N$. In incremental redundancy, the receiver has memory of the past signals since it accumulates mutual information.

As defined above, the information message is encoded by a space-time encoder, and mapped in a sequence of L blocks, $\{\mathbf{x}_l \in \mathcal{C}^{2\times N} : l = 1, \ldots, L\}$, and the transmission is as in a MIMO system, where the columns of $\mathbf{x}_l = \begin{bmatrix} \mathbf{x}_{sd} & \mathbf{x}_{rd} \end{bmatrix}^T$ are transmitted in parallel by the source and the relay. Each symbol of the transmitted codeword has unit power constraint. Let us call $\mathcal{T}_r$ a random variable denoting the block in which the relay was able to decode the source information message. Then, the signal model of our channel is given by:

$$\mathbf{y}_l^d = \sqrt{\frac{snr}{2}}\mathbf{h}_l\mathbf{x}_l + \mathbf{n}_l^d \tag{1}$$

where l stands for the retransmission round, $\{\mathbf{y}_l^d \in \mathcal{C}^{1\times N}\}$ is the received signal block by the destination, and $\{\mathbf{n}_l^d \in \mathcal{C}^{1\times N}\}$ is the channel noise assumed to be temporally and spatially white with i.i.d entries $\sim \mathcal{N}_{\mathbb{C}}(0,1)$. The channel of the l-th round is characterized by the matrix $\{\mathbf{h}_l \in \mathcal{C}^{1\times 2}\}$ as follows:

$$\mathbf{h}_l = \begin{cases} \begin{bmatrix} h_{sd} & 0 \end{bmatrix} & \text{if } l \in [1, \mathcal{T}_r] \\ \begin{bmatrix} h_{sd} & h_{rd} \end{bmatrix} & \text{if } l \in [\mathcal{T}_r + 1, L] \end{cases} \tag{2}$$

The received signal at the relay for $l = 1, \ldots, \mathcal{T}_r$ is given by:

$$\mathbf{y}_l^r = \sqrt{\frac{snr}{2}} h_{sr;l}\mathbf{x}_{sd;l} + \mathbf{n}_l^r \tag{3}$$

III. Useful results and Notations

The symbol $\doteq$ will be used to denote the exponential quality, i.e. $f(snr) \doteq snr^b$ to denote:

$$\lim_{snr\to\infty} \frac{\log f(snr)}{\log snr} = b$$

The trade-off between diversity and multiplexing was formally defined and studied in the context of point-to-point coherent communications in [6]. A family of codes $C(snr)$ of block length T, with one code for each snr level, is said to have a diversity gain of d and spatial multiplexing gain of r if

$$r = \lim_{snr\to\infty} \frac{R(snr)}{\log\, snr} \quad , \quad d = -\lim_{snr\to\infty} \frac{\log\, P_e(snr)}{\log\, snr}$$

where $R(snr)$ is the rate of the code $C(snr)$ and $P_e(snr)$ is the average error probability.

We define the effective rate in a different manner as follows. Let $\mathcal{T}_d$ be a random variable denoting the stopping time of the transmission of the current message at the destination. Let E be the event that the mutual information at a particular decoder crosses the transmission rate R, i.e, $E_l = \{\sum_{i=1}^{l} I_i > R\}$ for $l = 1, \ldots, L-1$. Then, we have:

$$\begin{aligned} \Pr(\mathcal{T}_d = l) &= \Pr(\overline{E_{d,1}}, \ldots, \overline{E_{d,l-1}}, E_{d,l}) \\ &= \Pr(\overline{E_{d,1}}, \ldots, \overline{E_{d,l-1}}) - \Pr(\overline{E_{d,1}}, \ldots, \overline{E_{d,l}}) \\ &= \Pr(\overline{E_{l-1}}) - \Pr(\overline{E_l}) \end{aligned} \tag{4}$$

where we used the fact that the random sequence I_l is non-decreasing with probability 1, and $\overline{E_l} \subseteq \overline{E_m}$ for $l \leq m$ leading to $\Pr(\overline{E_1}, \ldots, \overline{E_l}) = \Pr(\overline{E_l})$. We have also $\Pr(\overline{E_0}) = 1$, and $\Pr(\mathcal{T}_d = L) = \Pr(\overline{E_{d,L-1}})$.

Let e_l denote the event that the destination makes an error in decoding at the end of the l^{th} round. Then it can be shown that,

$$\begin{aligned} P_e &= \sum_{l=1}^{L} \Pr\left(e_l, \mathcal{T}_d = l\right) \\ &\leq \sum_{l=1}^{L} \Pr\left(e_l, E_{d,l}\right) + \Pr\left(e_L, \overline{E_{d,L}}\right) \\ &= \sum_{l=1}^{L} \Pr\left(e_l, E_{d,l}\right) + \Pr\left(e_L | \overline{E_{d,L}}\right) \Pr\left(\overline{E_{d,L}}\right) \\ &\leq \sum_{l=1}^{L} \Pr\left(e_l, E_{d,l}\right) + \Pr\left(\overline{E_{d,L}}\right) \end{aligned} \tag{5}$$

Over here the destination can try to decode at the end of each round less than L. If it is able to decode it sends feedback to the source otherwise it waits for more rounds to allow the mutual information to accumulate. The above expression can be seen as,

$$P_e \leq \sum_{l=1}^{L} \Pr\left(\text{error at}\, l, \text{no outage at}\, l\right) + \Pr\left(\text{outage at}\, L\right) \tag{6}$$

It can be shown that in this expression only the second term dominates as the first one can be made arbitrarily small for sufficiently large N. Hence,

$$P_e \leq \Pr\left(\text{outage at}\, L\right) \leq \Pr\left(\text{outage at}\, l < L\right) \tag{7}$$

The effective multiplexing rate is then defined as,

$$\begin{aligned} r_e &= \lim_{snr\to\infty} \frac{R(snr)}{\left(\sum_{l=0}^{L-1} \Pr(\overline{E_l})\right) \log\, snr} \\ &= \lim_{snr\to\infty} \frac{R(snr)}{\left(1+\sum_{l=1}^{L-1} \Pr(\overline{E_l})\right) \log\, snr} \\ &= \frac{r}{\left(1+\sum_{l=1}^{L-1} \Pr(\overline{E_l})\right)} \end{aligned} \quad (8)$$

The channels are Rayleigh fading, i.e. $\gamma_i = |h_i|^2, i \in sd, sr, rd$ is exponentially distributed with unit mean. Defining $\mu_i = -\frac{\log\, \gamma_i}{\log\, snr}$ we note that μ_i is distributed as,

$$f_{\mu_i}(\mu) = \log(snr) snr^{-\mu} \exp(-snr^{-\mu}) \quad (9)$$

which, in the high snr gives:

$$f_{\mu_i}(\mu) \doteq \begin{cases} snr^{-\mu} & \text{for } \mu \geq 0 \\ 0 & \text{for } \mu < 0 \end{cases} \quad (10)$$

At high snr, we have $(1+snr\gamma_i) \doteq snr^{(1-\mu_i)^+}$. Let us define $\mathcal{A}$ as the set describing the outage event. Then, for independent random variables $\underline{\mu} = [\mu_1, \ldots, \mu_n]$, the outage probability is given by:

$$P_{out} \doteq \int_{\mathcal{A}} f(\underline{\mu}) \mathrm{d}\underline{\mu} \doteq snr^{-d} \quad (11)$$

where:

$$d = \inf_{(\mu_1,\ldots,\mu_n)\in\mathcal{A}} \sum_{j=1}^{n} \mu_j \quad (12)$$

From this, it follows that

$$\Pr(\log(1+\gamma snr) < r\log(snr)) \doteq snr^{-(1-r)} \quad (13)$$

and the following results can be obtained

$$\begin{aligned} &\Pr\left(\sum_{i=1}^{l} \log(1+\gamma_i snr) < r\log(snr)\right) \\ &\doteq \Pr\left(\sum_{i=1}^{l} (1-\mu_i)^+ < r\right) \\ &\doteq \int_{\mathcal{A}} snr^{-\sum_{i=1}^{l}\mu_i} \mathrm{d}\underline{\mu} \\ &\doteq snr^{-d} \end{aligned} \quad (14)$$

where the set $\mathcal{A} = \{\underline{\mu} : \sum_{i=1}^{l}(1-\mu_i)^+ < r\}$ describes the outage event and d is given by:

$$d \doteq \inf_{\underline{\mu}\in\mathcal{A}} \sum_{i=1}^{l} \mu_i \doteq l\left(1-\frac{r}{l}\right) \quad (15)$$

IV. Tradeoff Curves

In this section we derive the tradeoff curves for the case of the long term quasi-static and short term quasi-static channels. We assume that all the channels are constant over all the ARQ blocks. Since we are in the high snr regime we ignore the factor 2 and use $snr \doteq \frac{snr}{2}$ for the remaining sections of the paper. In this case the instantaneous average mutual informations for the j^{th} blocks are given by:

$$I_1^j = I^j(\mathbf{x}_{sd,j}; \mathbf{y}_j^d | h_{sd,j}) = \log(1+snr\gamma_{sd,j}) \quad (16)$$

$$\begin{aligned} I_2^j &= I(\mathbf{x}_{sd,j}, \mathbf{x}_{rd,j}; \mathbf{y}_l^d | h_{sd,j}, h_{rd,j}) \\ &= \log(1+snr(\gamma_{sd,j}+\gamma_{rd,j})) \\ &\doteq \log(1+snr^{(1-\mu_{sd,j})}+snr^{(1-\mu_{rd,j})}) \\ &\doteq \log(1+snr^{(1-\min(\mu_{sd,j},\mu_{rd,j}))}) \end{aligned} \quad (17)$$

$$I_3^j = I(\mathbf{x}_{sr,j}; \mathbf{y}_r^d | h_{sr,j}) = \log(1+snr\gamma_{sr,j}) \quad (18)$$

A. Long term static channel

For a long term static channel the instantaneous average mutual informations do not vary from one round to another. Denote their common values as I_1, I_2 and I_3. At round l, the outage probability for this cooperative channel depends on the fact that the relay was able to decode the message from the source. From (4), we have:

$$\begin{aligned} \Pr(\mathcal{T}_r = k) &= \Pr((k-1)I_3 < r\log(snr)) \\ &\quad - \Pr((k)I_3 < r\log(snr)) \\ &\doteq snr^{-(1-r/(k-1))} - snr^{-(1-r/k)} \end{aligned} \quad (19)$$

And the outage probability for the ARQ relay long-term static channel is,

$$\Pr_{out}(l) = \sum_{k=1}^{L} \Pr_{out|\mathcal{T}_r=k}(l) \Pr(\mathcal{T}_r = k) \quad (20)$$

$$\begin{aligned} &\doteq \sum_{k=1}^{l-1} \Pr(kI_1 + (l-k)I_2 < r\log(snr)) \Pr(\mathcal{T}_r = k) \\ &\quad + \sum_{k=l}^{L} \Pr(lI_1 < r\log(snr)) \Pr(\mathcal{T}_r = k) \\ &\doteq snr^{-d_{out}^{lt}(r,l)} \end{aligned} \quad (21)$$

$$d_{out}^{lt}(r,l) = \begin{cases} (1-r) & \text{for } l = 1 \\ (1-\frac{r}{l}) + (1-\frac{r}{l-1}) & \text{for } l \neq 1,3 \\ 2-5r/6 & \text{for } l = 3, r < \frac{3}{4} \\ 5/2-3r/2 & \text{for } l = 3, r \geq \frac{3}{4} \end{cases} \quad (22)$$

We use the following result but omit the lengthy proof for space considerations,

$$\begin{aligned} &\Pr(kI_1 + (l-k)I_2 < r\log(snr)) \\ &\quad \doteq \begin{cases} snr^{-2(1-r/l)} & \text{for } k \leq \lfloor l/2 \rfloor \\ snr^{-(l-r)/k} & \text{for } \lfloor l/2 \rfloor < k \leq l-1 \end{cases} \end{aligned} \quad (23)$$

B. Short term static channel

Unlike in the case of long term static channel, the instantaneous mutual informations defined above vary from one block to the other. We have:

$$\begin{aligned} \Pr(\mathcal{T}_r = k) &= \Pr\left(\sum_{i=1}^{k-1} I_3^i < r\log(snr)\right) \\ &\quad - \Pr\left(\sum_{i=1}^{k} I_3^i < r\log(snr)\right) \\ &\doteq snr^{-(k-1)(1-r/(k-1))} - snr^{-k(1-r/k)} \end{aligned}$$

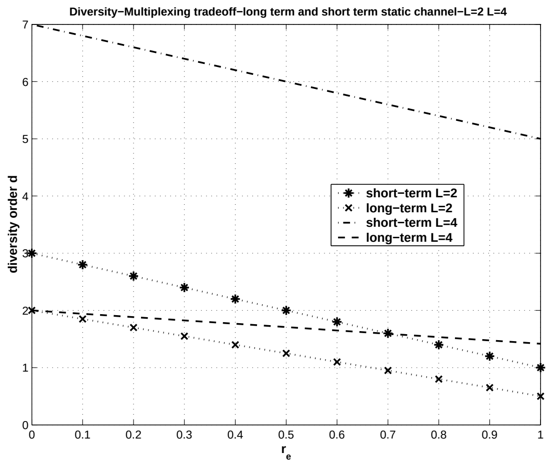

Fig. 3. The diversity-multiplexing tradeoff for different values of the maximum number of ARQ rounds for the short term and long term static channel.

And the outage probability for the ARQ relay short-term static channel is:

$$\Pr_{out}(l) = \sum_{k=1}^{L} \Pr_{\mathrm{out}|\mathcal{T}_r=k}(l)\Pr(\mathcal{T}_r = k) \tag{24}$$

$$\doteq \sum_{k=1}^{l-1} \Pr\left(\sum_{i=1}^{k} I_1^i + \sum_{i=k+1}^{l} I_2^i < r\log(snr)\right)\Pr(\mathcal{T}_r = k) + \sum_{k=l}^{L} \Pr\left(\sum_{i=1}^{l} I_1^i < r\log(snr)\right)\Pr(\mathcal{T}_r = k)$$

$$\doteq snr^{-d_{out}^{st}(r,l)} \tag{25}$$

$$d_{out}^{st}(r,l) = \begin{cases} (1-r) & \text{for } l = 1 \\ l\left(1-\frac{r}{l}\right) + (l-1)\left(1-\frac{r}{(l-1)}\right) & \text{for } l \neq 1 \end{cases} \tag{26}$$

We again use the following result and omit the proof for space considerations,

$$\sum_{k=1}^{l-1} \Pr\left(\sum_{i=1}^{k} I_1^i + \sum_{i=k+1}^{l} I_2^i < r\log(snr)\right) \doteq \begin{cases} snr^{-(2l-k)(1-r/l)} & \text{for } k \leq \lfloor l/2 \rfloor \\ snr^{-l(l-r)/k} & \text{for } \lfloor l/2 \rfloor < k \leq l-1 \end{cases} \tag{27}$$

Note that the way we have defined the effective rate earlier (8) and from the expressions above for both the short term and long term static channel, it follows that:

$$r_e = \frac{r}{\left(1 + \sum_{l=1}^{L-1} snr^{-d_{out}(r,l)}\right)}$$

$$\Longrightarrow r_e \doteq r \tag{28}$$

Hence the achievable diversity-multiplexing-delay tradeoff for the ARQ relay channel for the long term static and short term static relay channel is,

$$d_{out}^{lt}(r_e, L) = d_{out}^{lt}(r, L),\ d_{out}^{lt}(r_e, L) = d_{out}^{lt}(r, L) \tag{29}$$

V. CONCLUSION

We notice that by increasing the value of the retransmission rounds L, the diversity-multiplexing tradeoff curve for the long term static channel is getting flat. Since the channel fades independently to a new realization in each round for the short-term static channel, transmission in each new round gives additional diversity which explains the multiplicative L and $L-1$ factors in the diversity expression. Note that the factor is $(L-1)$ (both in the multiplication and the division) in the second term as the relay has to wait for at least one round before it can start transmitting to the destination. The reason that this multiplicative does not show up in the case of the long term quasi-static channel is the channel is constant over all ARQ rounds and there is no time diversity benefit. But still there is a gain in the diversity because of the relay to destination channel and because of the ARQ protocol (the division factor l in r/l and respectively $l-1$). Note that $d_{out}^{lt}(0,l) = 2$ for all $l \neq 1$. Thus, the long-term static channel is limiting the performance at low multiplexing-gains, which motivates the use of the power control as a future work.

We also propose to investigate the extension of these schemes to the case of multiple relays relaying the information for a single source destination pair. This protocol can then be applied to ad-hoc TDMA wireless networks where in each slot all the remaining nodes in the network act as relays for a particular source destination pair. Another future direction is the investigation of the the impact of multiple antennas (in particular two antennas at the receiver (base station) considering the practical implications) where the source and the relay collaborate to reach the base station.

REFERENCES

[1] A. Sendonaris, E. Erkip and B. Aazhang, “User Cooperation Diversity. Part I. System Description,” *IEEE Transactions On Communications,*, pp. 1927-1938, Vol. 51, Issue: 11, Nov 2003.

[2] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, “Cooperative Diversity in Wireless Networks: Efficient Protocols and Outage Behavior,” *IEEE Trans. Inform. Theory*, Vol. 50, no. 12, pp. 3062-3080, Dec. 2004.

[3] R. U. Nabar, H. Bölcskei, and F. W. Kneubhler, “Fading relay channels: Performance limits and space-time signal design,” *IEEE Journal on Selected Areas in Communications*, Vol. 22, Issue: 6, Aug 2004.

[4] K. Azarian, H. El Gamal, and P. Schniter, ”On the achievable diversity-multiplexing tradeoff in half-duplex cooperative channels” , *submitted to the IEEE Transactions on Information Theory*, July 2004.

[5] H. El Gamal, G. Caire, and M. O. Damen, ”The MIMO ARQ channel: Diversity-multiplexing-delay tradeoff”, *submitted to the IEEE Transactions on Information Theory*, Nov. 2004.

[6] L. Zheng and D. Tse, “Diversity and Multiplexing: A Fundamental Trade-off in Multiple Antenna Channels”, *IEEE Transacations on Information Theory*, vol.49, pp. 1073-1096, May 2003.

[7] T.Tabet and R.Knopp, ”Spatial Throughput of Multi-hop Wireless Networks Under Different Retransmission Protocols”, *in Proc. Allerton Conf. on Communication, Control, and Computing*, Monticello, IL, Oct. 2004

[8] Caire, G., Tuninetti, D., “The Throughput of Hybrid-ARQ Protocols for the Gaussian Collision Channel ” *IEEE Transactions on Information Theory*, Volume: 47 Issue: 5, Pages: 1971 -1988, Jul. 2001.