BOUNDS ON THE THROUGHPUT CAPACITY OF WIRELESS AD HOC NETWORKS WITH NON-UNIFORM TRAFFIC

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***Abstract*—We establish lower bounds on the capacity of wireless ad hoc networks with two types of non-uniform traffic patterns. We first focus on the impact of traffic patterns where local communication predominates and show the improvement in terms of per user-capacity over ad hoc networks with unbounded average communication distances. We then study the capacity of hybrid wireless networks, where long-distance relaying is performed by a fixed overlay network of base stations. We investigate the scaling of capacity versus the number of nodes and the density of base stations in the area of the network. The throughput capacity results under these two scenarios hold with probability one as the number of nodes goes to infinity.**

I. INTRODUCTION

The study of wireless ad hoc networks has recently received significant attention. A purely ad hoc network is a collection of wireless nodes forming a network without the use of any existing network infrastructure or centralized coordination. In [1], Gupta and Kumar determined the scaling of capacity of these networks under simplified propagation and traffic assumptions. They showed that given n nodes randomly located in the unit disk and an uniform traffic pattern (i.e. that nodes are equally likely to communicate with any other node in the network), the aggregate capacity is of $\Theta(\sqrt{n})$ allowing optimal scheduling and relaying of packets. The nodes are however assumed to be fixed throughout the duration of the communication sessions. Because of their assumptions regarding interference and measure of connectivity, their result is not information theoretic. Xie and Kumar relaxed these assumptions in [2], and proposed another upper bound on the total rate of communication in the network. Based on the assumption of a minimum distance between nodes and a power loss exponent $\alpha > 6$, it is shown that the transport capacity is asymptotically bounded by the sum of the transmit power of the nodes in the network, mainly for domains of size $\Theta(n)$, transport capacity scales as $O(n)$, leading to a rate of communication which is again sublinear. More recently, it has been shown that even with information-theoretically optimal coding strategies, the per-user capacity still diminishes to zero [3]. The upper bounds have been derived for $\alpha > d \vee 2(d-2)$, where d is the dimension of the network, for the uniform traffic pattern and for extended networks (i.e., the number of users per unit area is constant, and increasing number of users implies increase in geographical area, which is a scenario studied in [2] but with different assumptions as seen above, namely for $\alpha > 6$ or $\alpha > 2(d+1)$ since the result in [2] was shown for planar networks). In [4], the model in [1] was modified to take into account mobility and using only one-hop relaying, an $\Theta(n)$ throughput was obtained for a mobile ad hoc network. Even with limited mobility, i.e., when nodes move on large circles, it was shown in [5] that the throughput of ad hoc wireless networks can be enhanced. In [6] [7], the capacity of a three-dimensional wireless ad hoc network is studied. These results provide expressions for the ad hoc network capacity and determine the scalability of such networks as the number of nodes increases to infinity.

As can be concluded from the studies referenced previously, the performance limitation of an ad hoc network comes first from the long-range peer-to-peer communication (that causes excessive interference) and second the increase in relayed traffic in the case of multi-hop transmissions. let $\overline{L}$ be the mean distance traversed by a packet and r be the common transmission range (which is proportional to transmit power) and each node has a randomly chosen destination to which it wishes to send $\lambda(n)$ bits/s. Then each packet has to take $\frac{\overline{L}}{r}$ hops to reach destination. This creates $\frac{\overline{L}\lambda(n)}{r}$ bits/s of traffic per use for other nodes, and if each link is capable of W bits/s, we should have $\lambda(n) \leq \frac{Wr}{\overline{L}}$. The right-hand side is proportional to range, so it appears that increasing range should increase throughput. But increased range causes more interference and loss of packets (spatial concurrency, simulataneous transmissions), and too small a range increases relay traffic.

In general, the transmitter-receiver pairs are not arbitrarily close to each other and an important physical insight from [1] is the need for multiple hops to reach the destination. Because the large majority of traffic carried by the nodes is relayed traffic, in [4] each packet is constrained to make at most two hops and transmission is limited to nearest neighbors. But since source and destination are nearest neighbors only for a very small frac-

The work reported here was partially supported by the National Competence Center in Research on Mobile Information and Communication Systems (NCCR-MICS), a center supported by the Swiss National Science Foundation under grant number 5005-67322. This research is also partially supported by Eurecom and its industrial partners: Hasler Stiftung, Swisscom, France Telecom, La Fondation Cegetel, Bouygues Telecom, Thales, ST Microelectronics, Hitachi Europe and Texas Instruments.

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tion of time, the transmission is spread to a large number of intermediate mobile relay nodes, and whenever they get close to the final destination, they hand the packets off to the final destination. Suppose now that the transmitter-receiver pairs of nodes are close to each other, then reliable communication will cause little interference to the other nodes and the scenario is essentially that of a set of non-interfering point-to-point communication systems, or transmitter-receiver pairs communicating through a small number of hops. The studies cited above assume an uniform traffic pattern, where each pair of nodes is equally likely to communicate, so that packet path length grows with the physical dimensions of the network leading to a growth in the relay load (since the number of hops to reach destination increases). This assumption may not be true in large networks, where users communicate mostly with physically nearby nodes: users in the same department in an university, the same group in a company, and even in the case of telephony, users communicate mostly with neighbors in the same city (or even district) rather than users in other countries.

In [8], traffic patterns that allow the per node capacity to scale well with the size of the network are discussed. The local traffic pattern is scalable where the expected path length clearly remains constant as the network size (equivalently the number of nodes in the case of a large network) grows. [9] is illustrating the impact of an exponentially decaying traffic pattern and the relay load on the throughput in the context of a decentralized system with retransmission protocols. In [10], the capacity under a different traffic pattern is studied. There is only one active source-destination pair, while all other nodes serve as relay, helping the transmission between the source and the destination nodes. The capacity is shown to scale as $O(\log n)$. In this paper, we continue the investigation along the lines of [1] but show the impact of traffic pattern on the throughput capacity. We are able, by using a simple deterministic scheme, to derive a lower bound on the per-node capacity of an ad hoc wireless network where local communication predominates.

The pessimistic results of [1] dampened the early enthusiasm for ad-hoc networks which would eliminate the need for infrastructure like base-station. In this work we also investigate a hybrid wireless network, a tradeoff between a purely ad hoc network and a cellular one. In the latter, data is always forwarded through the base-station, whereas in our model of a hybrid network, a cellular mode (data forwarded from source to destination through the base-station) and a pure ad hoc mode (data forwarded from source to destination using multi-hop relaying communications) coexist. The primary interest is to reduce the transmit power of mobile terminals through multi-hop relaying. In [11], the introduction of a sparse network of basestations was shown to help in improving the network connectivity. In [12], the scaling behavior of the throughput capacity of a hybrid network is studied under two particular routing strategies. It was shown that an effective improvement of a hybrid mode over a pure ad hoc mode is provided only if the number of base stations scales faster than the square-root of the number of nodes in the network. Here, we assess the tradeoff between the number of base stations in the network and the increase in the throughput capacity of a hybrid ad hoc network due to the additional infrastructure. We assume that base stations are connected to each other by a wired network, and are regularly placed within the ad hoc network. Terminal nodes are reaching the base stations through multi-hop communication. On the other hand, the link from the base stations to the terminals (down-link) can be achieved by single-hop communication, since we assume there is no power constraint for basestations.

The outline of the paper is as follows: In section II we specify the ad hoc network and problem model. In section III we describe the constructive communication scheme. Section IV deals with the throughput capacity expressions for both locally predominant communication and hybrid networks. Finally, in Section V we draw some conclusions.

II. Network and Problem Model

We consider a random network where n nodes are distributed uniformly on a two-dimensional area, a square of area $\Theta(n)$ (similar network model is considered in [13] with unit area). This is a large network where the number of nodes is increasing with the area of the network leading to a fixed density of nodes per area (similar scenario in [2] [3]), whereas in the Gupta and Kumar model [1], the density of nodes is increasing with the number of nodes and the area is fixed. The large network model is more realistic since one would not expect nodes to get arbitrarily close by letting the number of nodes become very large. We assume that all nodes can act as both transmitters and receivers, and each node wants to communicate with another node chosen randomly and independently among the rest. Therefore there exist n communicating pairs of nodes. Moreover, the nodes are static, relative to the scale time of communication.

In the system we are considering, each node can transmit over a common wireless channel of bandwith W. As this model incorporates the distance between nodes, the propagation model is described by the signal attenuation due to the distance r between the transmitter and the receiver, proportional to $r^{-\alpha}$, where α is the power loss exponent (positive number typically $\alpha > 2$ which is the usual model outside a small neighborhood of the transmitter). Each node transmits with a common power P. Let $\{X_t : t \in \mathcal{T}\}$ be the set of transmitting nodes at a given time, and suppose that node X_i transmits to a node X_j, then the signal to interference and noise ratio (SINR) at node X_j is given by:

$$\gamma_{ij} = \frac{P|X_i - X_j|^{-\alpha}}{N_0 + \sum_{\substack{k \neq i \\ k \in \mathcal{T}}} P|X_k - X_j|^{-\alpha}} \quad (1)$$

where N_0 is the thermal noise at receiver node j. We take the transmission rate as the Shannon's formula $C_{ij} = W \log_2(1 + \gamma_{ij})$ (where single-user decoding is assumed, i.e., each decoder treats the signals from other users as noise, and the single-user decoder for each node has perfect knowledge of the channel gain and the total interference power, i.e., noise and interfering user traffic).

III. CONSTRUCTIVE COMMUNICATION SCHEME

A. The cell partition

We partition the area of the network in a set of regular cells, and each cell is a square of side length $c(n)$. We impose that nodes transmit only to nodes in the same cell or in (the eight) neighboring cells adjacent to its cell. This is a local communication and all other cells far away could simultaneously transmit with reduced interference. Motivated by cellular architecture, we introduce a parameter K which corresponds to a reuse factor in a cellular system as in [13]. Indeed, all the cells that are a vertical and horizontal distance of exactly some multiple of K, can transmit simultaneously as depicted in (fig.1). Then, we

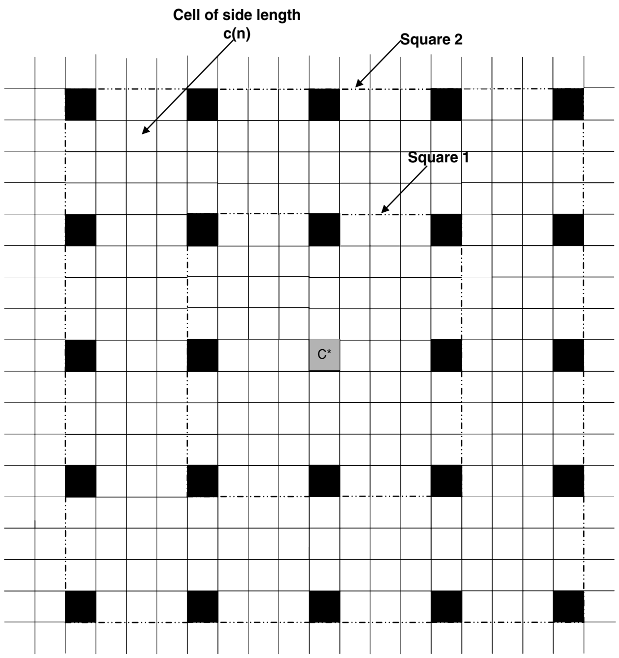

Fig. 1. An example of network area partition with $K = 4$. All nodes in the shaded cells can transmit simultaneously to the eight neighboring cells.

choose a finite length time-division scheduling scheme of K^2 slots ($K > 2$), in which each cell is assigned one slot to transmit. This scheduling between cells ensures that transmissions from a cell do not interfere with transmissions in simultaneously transmitting cells. Nodes in the same group of nodes transmit with reduced interference, and the distance from an interferer to a receiver is at least $c(n)$. (fig.2) shows an example of the cell partition with $K = 5$, where the distance transmitter-receiver is always less than the distance interferer-receiver. When a cell becomes active, packets that are relayed or originated from this cell are scheduled one after the other (one packet by Source-Destination pair).

B. The routing strategy

The packet routing is as follows: a packet is relayed from the cell containing the source to the cell containing the destination in a sequence of hops. In each hop, the packet is transferred

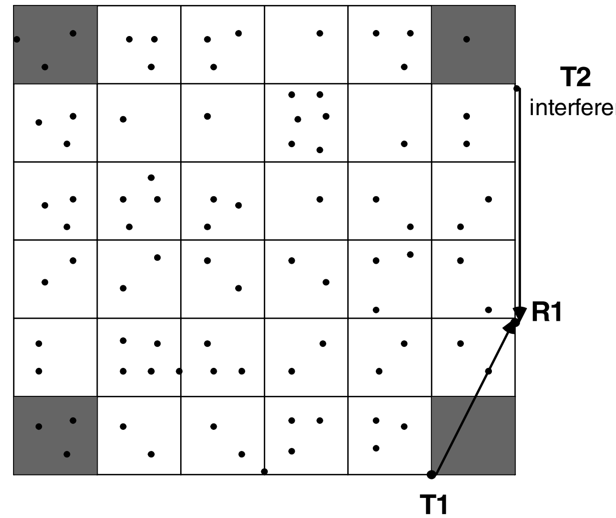

Fig. 2. An example of network area partition with $K = 5$ showing the maximal distance of a single hop (T1-R1) which is always less than the distance interferer-receiver T2-R1.

from one cell to another, in the order in which cells intersect the straight line connecting the source to the destination. To make relaying of traffic between cells feasible, it is required that every cell contains at least one node with high probability.

Lemma 1: For $c(n) = \sqrt{3 \log n}$, no cell is empty with high probability as n is large.

Proof: For a network of area $\Theta(n)$ where n is the number of nodes in the network, with cells of side length $c(n)$, the probability that a cell is empty is equal to $\left(1 - \frac{c(n)^2}{n}\right)^n$. By using the union bound, we have:

$$\begin{aligned} \Pr[\textit{at least one cell is empty}] &\leq Q(n)\left(1 - \frac{c(n)^2}{n}\right)^n \\ &= \frac{n}{c(n)^2}\left(1 - \frac{c(n)^2}{n}\right)^n \\ &\overset{(a)}{\leq} \frac{n}{c(n)^2}\exp(-c(n)^2) \qquad (2) \end{aligned}$$

where $Q(n)$ is the number of cells in the network area and it is equal to $\frac{n}{c(n)^2}$, and inequality (a) is by using $1 - x \leq \exp(-x)$. We obtain for $c(n) = \sqrt{3 \log n}$:

$$\begin{aligned} \Pr[\textit{at least one cell is empty}] &\leq \frac{1}{3n^2 \log n} \\ &\rightarrow 0 \qquad (3) \end{aligned}$$

and the result is proven for sufficiently large n.

Lemma 2: If the power loss exponent $\alpha > 2$, the SINR at node j and then the rate transmission of pair (i, j) is asymptotically lower bounded by:

$$C_{ij}(n) \geq k_1 \qquad (4)$$

for all $j = 1, \cdots, n$, the index i is for all nodes in the neighboring cells of the cell containing node j as explained above.

Proof: As specified by the routing strategy and the time-division scheme, intuitively such a bound on the SINR exists. We need first to derive a lower bound on the useful signal. Under the routing strategy, each node can transmit only to nodes in the same cell or nodes in neighboring cells. Under this assumption, the maximum distance between a transmitter and a receiver is $\sqrt{5}c(n)$. The useful signal is then bounded by:

$$P|X_i - X_j|^{-\alpha} \geq P(15\log n)^{-\frac{\alpha}{2}} \quad (5)$$

Let us bound the interference. Consider a particular cell c^*. If one node from this cell is transmitting, all others simultaneous transmissions may occur in cells belonging to the same set of cells that are a vertical and horizontal distance of exactly some multiple of K. Actually, the interfering cells are placed along the perimeter of concentric squares, whose center is c^*, and each square contains $(2lK+1)^2, l = 1, 2..., S(n)$ cells and $2lK, l = 1, 2..., S(n)$ interfering cells as depicted in (fig.1), where $S(n)$ is the number of such concentric squares. For example, the first concentric square contains 8 interfering cells, whereas the second concentric square contains 16 interfering cells, for the particular case where $K = 4$. Each node in the intended cell c^* transmits information packets to nodes in the eight neighboring cells. Then, the distance between these nodes (the possible receivers in the eight adjacent cells) and the interfering ones is at least $l(K-2)c(n), l = 1, 2..., S(n)$. As we are considering a lower bound, we take the worst-case and we neglect the edge effects. Then, the number of concentric squares (irrespective of the position of the intended cell, since the worst case is when the intended cell is at one corner of the area) is at most $S(n) \leq \left\lceil \frac{\sqrt{\frac{n}{\log n}}}{K} \right\rceil$. We proceed in upper bounding the interference at the receiver:

$$\begin{aligned} I &= \sum_{\substack{k \neq i \\ k \in \mathcal{T}}} P|X_k - X_j|^{-\alpha} \\ &\leq \sum_{i=1}^{S(n)} \frac{2PKi}{[i(K-2)c(n)]^{\alpha}} \\ &= \frac{2PK}{[(K-2)c(n)]^{\alpha}} \sum_{i=1}^{S(n)} i^{1-\alpha} \\ &\leq \frac{2PK}{[(K-2)c(n)]^{\alpha}} \left[1 + \int_1^{S(n)} x^{1-\alpha} \mathrm{d}x\right] \\ &\overset{(\alpha \geq 2)}{=} \frac{2PK}{[(K-2)c(n)]^{\alpha}} \left(\frac{\alpha-1}{\alpha-2}\right) \\ &\quad + S(n)^{2-\alpha} \frac{2PK}{[(K-2)c(n)]^{\alpha}} \frac{1}{2-\alpha} \\ &\overset{(\alpha > 2)}{\leq} \frac{cPK(\log n)^{-\frac{\alpha}{2}}}{(K-2)^{\alpha}} \end{aligned} \quad (6)$$

where c is a positive number. The thermal noise N_0 is negligible as $n \to \infty$, and by combining (5) with (6), the $SINR(n)$ is lower bounded by $SINR_{min}(n)$ which is a constant and as $C_{ij}(n) = W\log_2(1 + SINR_{min}(n))$, we obtain the result (4).

IV. Throughput Capacity Expressions

A. Local traffic pattern

The information packets that are relayed through a particular cell create load for the nodes in the cell, and it is important to compute the maximum number of routes passing through any cell. This helps us estimate how much traffic, apart from its own, each cell has to relay, and the reduction in the node-throughput induced by the relay traffic. We recall that a route is the collection of cells a source will use to forward packets to a destination following the straight line connecting the source to the destination (hence a route is a S-D line).

Lemma 3: The number of routes passing through any cell is $O(\log n)$ for $\frac{\overline{L}}{c(n)} \leq 1$, whereas it is on the order $O(\overline{L}\sqrt{\log n})$ for $\frac{\overline{L}}{c(n)} > 1$. Result that happens with high probability as n is large.

Proof: We try first to show it intuitively. Let X(s-d) be the S-D distance, $\overline{L} = E[X(\text{s-d})]$, the average path length and n being very large. Each S-D line will traverse a mean number of $O\left(\frac{\overline{L}}{c(n)}\right)$ cells. Moreover, we consider n pairs S-D. Then, $O\left(n\frac{\overline{L}}{c(n)}\right)$ is the mean number of times all cells are traversed by S-D lines, and we can conclude that $O\left(\frac{n\overline{L}}{c(n)\text{number of cells}}\right)$ is the mean number of routes passing through a cell. For an uniform traffic pattern, the path length is $O(\sqrt{n})$ in a large network of area $\Theta(n)$, and as the number of cells is $O\left(\frac{n}{3\log n}\right)$, the mean number of lines passing through a cell is $O\left(\frac{n\overline{L}}{c(n)\text{number of cells}}\right) = O(\sqrt{n\log n})$ as in the Gupta-Kumar model. In [8], traffic patterns that allow the throughput capacity to scale with the network size are discussed. For local traffic patterns, the expected path length (S-D distance) remains constant as the network size grows. Actually, for a local traffic pattern (power decaying law), the path length is $O(1)$. One can notice that for a path length order smaller than a cell side length (i.e $\frac{\overline{L}}{c(n)} = O\left(1/\sqrt{3\log n}\right) \leq 1$), the source-destination line is completely included in the cell. Even if the line is inside the cell, traffic should be scheduled for this pair S-D, and we should count that at least one line is intersecting the cell. We take it into account by replacing $\frac{\overline{L}}{c(n)}$ by $\left\lceil\frac{\overline{L}}{c(n)}\right\rceil = 1$. The mean number of routes passing through a cell is $O(\log n)$ for $\frac{\overline{L}}{c(n)} \leq 1$, whereas it is on the order $O(\overline{L}\sqrt{\log n})$ for $\frac{\overline{L}}{c(n)} > 1$.

We proceed as in [9] to compute the number of routes passing through a cell. Let Z_i be a Bernoulli random variable indicating if the cell is used by the S-D pair i to relay packets to the destination. Then, the number of routes passing through any cell is:

$$L_n = \sum_{i=1}^{n} 1_{\{Z_i=1\}} \quad (7)$$

Moreover, we note that:

$$\begin{aligned}\Pr(Z_i = 1) &= O\left(\frac{\text{\# of cells traversed by a route}}{\text{\# total of cells}}\right)\\ &= \begin{cases} O\left(\frac{\overline{L}c(n)}{n}\right) & \text{if } \frac{\overline{L}}{c(n)} > 1 \\ O\left(\frac{c(n)^2}{n}\right) & \text{if } \frac{\overline{L}}{c(n)} \leq 1 \end{cases}\\ &= p \qquad (8)\end{aligned}$$

We need now to bound the actual number of routes going through any cell. Neglecting the edge effects, and using the fact that L_n is a Binomial random variable with parameters (p, n) (recall that we consider n S-D pairs), we use a Chernoff bound to obtain, for $\delta > 0$, $t > 0$, $\frac{\overline{L}}{c(n)} \leq 1$

$$\begin{aligned}\Pr(L_n > \delta logn) &\leq \frac{E[\exp(tL_n)]}{\exp(t\delta \log n)} \qquad (9)\\ &= \frac{(1+(e^t-1)p)^n}{\exp(t\delta \log n)}\\ &\stackrel{(a)}{\leq} \exp\left(np(e^t-1) - t\delta \log n\right)\\ &\stackrel{(8)}{=} \exp\left(c(n)^2(e^t-1) - t\delta \log n\right)\end{aligned}$$

where (a) is by using $(1+x) \leq \exp(x)$. Taking $t = 1$, $\delta = 3e$ in (9), we obtain:

$$\Pr(L_n > 3e \log n) \leq \frac{1}{n^3} \qquad (10)$$

Similarly for $\delta = \sqrt{3}e$, $t = 1$, $\frac{\overline{L}}{c(n)} > 1$,

$$\Pr\left(L_n > \sqrt{3}e\overline{L}\sqrt{logn}\right) \leq \frac{1}{n^3} \qquad (11)$$

We need now to prove that the above (10), (11) bounds hold for all cells with high probability as n gets large. Let us call E_i the event that the number of lines passing through cell i does not exceed bounds (10), (11). Then,

$$\begin{aligned}\Pr\left(\bigcap_{i=1}^{|\mathcal{C}_n|} E_i\right) &= 1 - \Pr\left(\bigcup_{i=1}^{|\mathcal{C}_n|} E_i^c\right)\\ &\stackrel{(a)}{\geq} 1 - |\mathcal{C}_n| \Pr(E_i^c) \qquad (12)\\ &\stackrel{(b)}{\geq} 1 - n\varepsilon(n) \qquad (13)\\ &\to 1 \qquad (14)\end{aligned}$$

where $\mathcal{C}_n$ is the set of all cells, (a) is from the union of events bound, (b) is from the fact that there are at most n cells in the network and $\varepsilon(n)$ are the bounds (10), (11). Similarly and by the Borel-Cantelli Lemma since $\sum_{n=1}^{\infty} \frac{1}{n^3} < \infty$, we conclude that the number of routes passing through any cell does not exceed almost surely the bounds in (10), (11). We are now ready to state the following result (c', c'', c_1, c_2 are positive constant).

Theorem 1: For a large ad hoc network of n nodes, the scheme described above achieves a per-node throughput capacity (with high probability as n gets large):

$$\begin{aligned}\lambda(n) &= \begin{cases} c' \frac{1}{\overline{L}c(n)} & \text{if } \frac{\overline{L}}{c(n)} > 1 \\ c'' \frac{1}{c(n)^2} & \text{if } \frac{\overline{L}}{c(n)} \leq 1 \end{cases} \qquad (15)\\ &= \begin{cases} c_1 \frac{1}{\overline{L}\sqrt{\log n}} & \text{if } \frac{\overline{L}}{c(n)} > 1 \\ c_2 \frac{1}{\log n} & \text{if } \frac{\overline{L}}{c(n)} \leq 1 \end{cases}\end{aligned}$$

Proof: We recall that the throughput capacity is computed over all possible time-space scheduling of transmissions and paths. A per node throughput is called *feasible* if there exist satisfying time-space scheduling and routing paths. We denote by $\lambda(n)$ the maximum feasible throughput with high probability as n gets large.

By Lemma 2, we guarantee a constant rate to all communications. Lemma 3 bounds the number of routes each cell needs to serve. By Lemma 1, each cell will contain at least one node to forward the packets of these routes. Due to the time division, each cell will be active every one of K^2 slots. Then each path is guaranteed, with high probability as n gets large, a rate of $\frac{k_1}{K^2 L_n}$. Combining these results yields a proof of Theorem 1.

Moreover one can notice that for $\frac{\overline{L}}{c(n)} > 1$, the maximum power is on the order of the average power, mainly $O(c(n)^\alpha) = O\left((\log n)^{\frac{\alpha}{2}}\right)$; whereas for $\frac{\overline{L}}{c(n)} \leq 1$, the maximum power is on the order $O\left((\log n)^{\frac{\alpha}{2}}\right)$ and the average power is $O\left(\overline{L}^\alpha\right)$ showing the benefit of having a very local traffic pattern.

In [1], it was shown that any upper bound on the transport capacity for arbitrary networks under the protocol model is also an upper bound on the transport capacity for random networks under the physical model. For a domain of area $\Theta(n)$ applying the results of [1], the transport capacity is bounded as follows (we scale the upper bound by $\sqrt{area}$):

$$\lambda n\overline{L} \leq c_3 Wn \ bit\text{-}meters/sec \qquad (16)$$

where c_3 is a positive constant, $\overline{L}$ is the average source destination distance. This leads to an upper bound on the per-node throughput on the order $O\left(\frac{1}{\overline{L}}\right)$. Similarly in [1], the lower and upper bound do not coincide under the physical model.

B. Hybrid wireless networks

A hybrid wireless network is formed by placing a sparse network of base stations (or access points, gateways) in an ad hoc network. These base stations are assumed to be connected by a high bandwith wired network, and only to relay the packets since they do not generate any data traffic themselves. In addition to n nodes randomly located within a square of area $\Theta(n)$, $f(n)$ base stations are regularly placed within the network area. These base stations divide the network area in $f(n)$ squares that we call clusters. We have then, a collection of $f(n)$ clusters, each of which has a base station placed in the middle of it as shown in (fig.3). As stated before, a base station is never

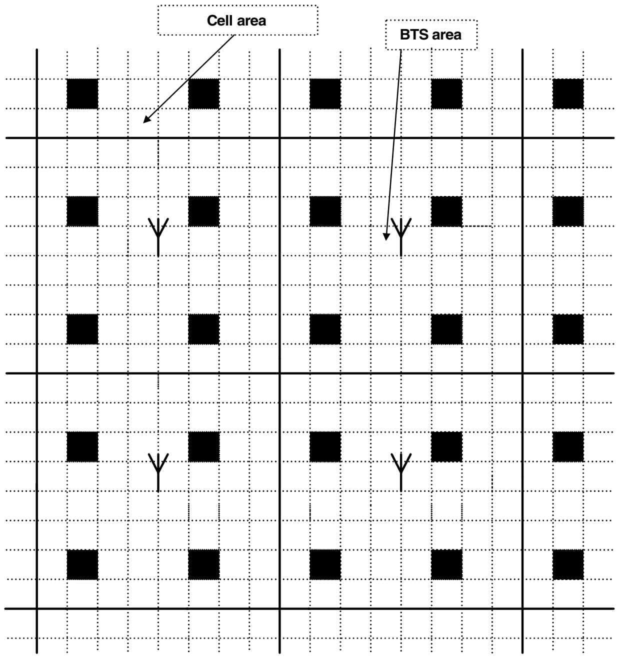

Fig. 3. A hybrid wireless network with base stations regularly placed in the middle of a BTS area (cluster).

the initiator of a data transmission, but a relay that acts as a gateway between various clusters. Moreover, the infrastructure network is assumed to be an infinite capacity backbone and to have relatively abundant bandwith and resources. The base stations are not power constrained and have the ability to reach any node within the cluster, whereas the nodes are power limited. A packet that reaches a base station tunnels through another base station closest to the destination. Because of our subdivision of the network area in $f(n)$ mutually exclusive clusters, each wireless node is close to only one base station. Within the same cluster, data transmissions are carried out without the use of the base station. Data are forwarded from the source to the destination in a multi-hop fashion. Transmissions to nodes in other clusters are carried out by routing the data via the infrastructure (base stations). Data are first transmitted from the source to the closest base station (the base station of the cluster) in a multi-hop fashion (it means in an ad hoc manner since nodes are power limited); the base station then transmits the data through the wired infrastructure to the base station closest to the destination, which finally transmits the data to the destination directly (since the base station is not power constrained). The transmissions within any mode (ad hoc mode or infrastructure mode) do not interfere. The ad hoc mode and the infrastructure mode go through different sub-channels. Similarly, the infrastructure sub-channel can be divided into up-link and down-link parts. It means that the RF is built such that an ad hoc transmitter is attached to each base station and that a BTS receiver is attached to each node.

In order to derive the throughput capacity of a hybrid wireless network, we use the deterministic scheme described above and the technical Lemmas derived in Section III. We keep the same cell partition as described in Section III. The network of ad hoc nodes, excluding the base stations is required to be connected since it is desirable to have an ad hoc network which can function without any infrastructure. The cell size was determined by the condition that no cell is empty as n gets large, i.e., we have a standalone ad hoc network that can provide connection between any pair of ad hoc nodes without the support of any infrastructure. We do not change the transmission policy (each node in a cell can transmit to a node in the same cell or in the neighboring cells), therefore we do not require that each node is connected with high probability to a base station. The latter will be reached in a multi-hop fashion (ad hoc mode). On the top of this partition, we add clusters, where each cluster contains a base station and a number of cells depending on the number $f(n)$ of base stations in the network. We assume that $f(n) < \frac{n}{4c(n)^2}$, otherwise we have a purely cellular system where each node can reach a base station directly since each node will have a base station within its range and the ad hoc mode (relaying done by nodes) is not needed (the distance source base station is less than $c(n)$ the range of a node, this is the case for example when a base station is placed within each cell of size $c(n)^2$ of our partition). Since we are assuming a frequency division of intra-cell, up-link and down-link data transmissions, there is no interference between the three types of traffic. However, within a sub-channel, interference exists between the same type of traffic. Interference between adjacent clusters may be reduced by employing frequency reuse as in the case of a cellular network. Whereas for the ad hoc transmissions, we showed in Section III a spatial transmission schedule that ensures simultaneous transmissions with reduced interference. Actually, the cells are spatially divided into K^2 (a constant number) different groups. Each group is allocated a slot in a round robin fashion, and each cell will be able to transmit once every fixed amount of time with reduced interference. Then, Lemma 2 is still valid for our analysis of hybrid wireless networks.

From Theorem 1, the throughput capacity of a wireless ad hoc network is completely determined by the number of routes each cell needs to serve. Each cell relays the intra-cluster traffic (if the source and the destination are inside the same cluster, data transmissions are done in an ad hoc (multi-hop) fashion) and the traffic to reach the base station (for a source willing to communicate with a destination not belonging to the same cluster, data transmissions are sent in a multi-hop fashion to the closest base station, which is routing data through the infrastructure until the destination). We are assuming now an uniform traffic pattern as in [1]. Sources and the corresponding destinations are randomly and independently placed in the network area. The probability that a node and its corresponding destination are located in the same cluster area is $\frac{1}{f(n)^2}$ (this happens with high probability as n gets large). We conclude that the number of S-D pairs belonging to the same cluster area (thus communicating in a pure ad hoc mode) is $\frac{n}{f(n)^2}$, whereas the number of S-D pairs communicating through the infrastructure is $\frac{n}{f(n)}\left(1 - \frac{1}{f(n)}\right)$. Neglecting the edge effects and bottlenecks around base stations, and using the results of Lemma 3, the number of routes passing through a cell is $O\left(\frac{(\#\text{cells traversed by a route})\ (\#\text{S-D pairs})}{\#\text{total of cells}}\right)$. Assuming that the S-D or the S-BTS mean path is on the order $O\left(\sqrt{\text{cluster area}}\right) = O\left(\sqrt{\frac{n}{f(n)}}\right)$, the number of cells traversed by a route is $O\left(\sqrt{\frac{n}{f(n)}}\frac{1}{c(n)}\right)$ and the total number of cells in a cluster area is $O\left(\frac{n}{f(n)c(n)^2}\right)$. We obtain that the number of routes passing through a cell due to the intra-cluster traffic is $O\left(\frac{c(n)\sqrt{n}}{f(n)^{\frac{3}{2}}}\right)$, whereas the number of routes due to the traffic S-BTS is $O\left(\frac{c(n)\sqrt{n}(f(n)-1)}{f(n)^{\frac{3}{2}}}\right)$. We are now ready to state the following result which proof stems from the technical Lemmas derived above.

Theorem 2: For a hybrid wireless network of n nodes and $f(n)$ base stations regularly placed within the network area, and under the deterministic scheme and the routing strategy described above, the per-node throughput capacity is:

$$\lambda(n) = c_4\frac{f(n)^{\frac{1}{2}}}{c(n)\sqrt{n}} = c_5\sqrt{\frac{f(n)}{n\log n}} \tag{17}$$

with high probability as n gets large (c_4, c_5 are positive constant).

Suppose now that $f(n) = O\left(\frac{n}{c(n)^2}\right) = O\left(\frac{n}{3\log n}\right)$, this is mainly the case where each cell contains a base station. Long-distance relaying is performed by the infrastructure (no need for ad hoc mode). Since the number of nodes per area is constant (fixed density), we have $O(3\log n)$ nodes per cell. We can schedule each node in the network without any conflict by a scheduling of length $3K^2\log n$, and the per node throughput is on the order $O\left(\frac{1}{\log n}\right)$. (17) gives the same result. Remember that in the case of local traffic pattern where the mean path length is less than $c(n)$, a similar per node throughput was obtained. This is mainly due to the fact that the scenario is essentially of a set of point-to-point communication systems, where data transmissions unlikely use relaying to reach destination. The only way to increase the throughput is then to reduce the cell size (in the case of local traffic pattern).

Moreover for a cellular system with $f(n)$ base stations ($f(n) < \frac{n}{4c(n)^2}$) where each node communicates directly with a base station (no ad hoc mode), the average power of each node is on the order $O\left(\left(\frac{n}{f(n)}\right)^{\frac{\alpha}{2}}\right)$ (where $\sqrt{\frac{n}{f(n)}}$ is the mean source base station distance in the setting described above), this is higher than the average power for a hybrid wireless network with the same number of base stations and under the same setting where the average power of each node is $O\left((\log n)^{\frac{\alpha}{2}}\right)$, showing the benefit of a hybrid wireless network over a purely cellular system.

V. Conclusions

Following [1], we construct an elementary scheme that achieves the throughput capacity of a large ad hoc wireless network with high probability as the number of nodes increases. The proofs being made simple and more intuitive (we do not resort to the Vapnik-Chervonenkis Theorem for example), we were able to study the asymptotic behavior of ad hoc wireless networks under local traffic pattern and hybrid wireless networks.

For local traffic pattern we show the effect of the mean S-D distance on the throughput. Moreover there is a limit in the throughput improvement as the mean path length becomes smaller than the cell side length. Mainly for a local traffic pattern for which $\frac{\overline{L}}{c(n)} \leq 1$, we obtain a per-node throughput larger than $\frac{1}{\log n}$, whereas it grows faster than $\frac{1}{\overline{L}\sqrt{\log n}}$ for $\frac{\overline{L}}{c(n)} > 1$. It seems that the way to increase the throughput capacity is by relaxing the connectivity condition, mainly by decreasing $c(n)$, the cell side length.

In this paper, we address also the benefits of using a hybrid wireless network in terms of per-node capacity. The base stations are regularly placed within the network area, and the analysis is based on the subdivision of the network into $f(n)$ clusters, where $f(n)$ is the number of base stations in the network. Moreover, the infrastructure network is assumed to have relatively abundant bandwith and resources. Inside each cluster, the communications are done in a pure ad hoc mode, whereas if the source and the destination do not belong to the same cluster, packets first reach the base station in a multi-hop fashion and tunnels through the infrastructure to the closest base station to the destination. We obtain a per node throughput larger than $\sqrt{\frac{f(n)}{n\log n}}$(e.g. $f(n) = (\log n)^2$, we obtain $\sqrt{\frac{\log n}{n}}$). The gain in performance is mainly due to the reduction in the mean number of hops from source to destination.

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