

# Comparison of two analog feedback schemes for transmit side MIMO channel estimation

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**Abstract**—System performance can be quite improved by timely full channel state information at the transmitter (CSIT), especially for multidimensional channels as MIMO. For obtaining full CSIT, it is necessary to feed back the transmit channel knowledge from the receiver to the transmitter. There are two schemes to feed back the transmit channel knowledge: One is that channel estimation is done at the receiver with feeding back the channel estimate result subsequently; the other is that the received training signals are fed back to the transmitter with channel estimation subsequently. In this paper, we assume that the channel is quasi-static Rayleigh-fading, the channel knowledge is fed back from the receiver to the transmitter in a time-discrete uncoded linear analog way without any quantization on the information, and the least-square estimator is used. We give explicit expressions of estimate error of two feedback schemes. The two schemes are compared with respect to mean-square error. It comes out that the channel-estimate-based feedback scheme is less imperfect than the received-signal-based feedback scheme with respect to mean-square error. Nevertheless, the difference is trivial at high SNR.

## I. INTRODUCTION

It is well-known that the channel knowledge at the transmitter can make a big difference to MIMO systems. Systems can benefit much from the timely full CSIT in channel capacity and link performance. For obtaining the full CSIT, there are two schemes to feed back the transmit channel knowledge: One is that the channel estimation is done at the receiver with feeding back the channel estimate subsequently; the other is that the received training signals are fed back to the transmitter with channel estimation subsequently. Both two schemes can apply to either FDD (Frequency Duplex Division) mode or TDD (Time Duplex Division) mode systems. For TDD systems, although there is the feature of channel reciprocity, it still needs the feedback of transmit channel knowledge in fact, which is different to the supposal in [1], due to the electronic discrepancy between the transmit circuitry and the receive circuitry. However, as a merit of TDD system, the operations of channel knowledge feedback can be done at a quite low rate than in FDD systems [4].

The transmit channel knowledge to be fed back is a kind of time-discrete amplitude-continuous source. There are three types of time-discrete transmission to feed it back: Digital transmission with quantization loss [3], HDA (Hybrid Digital-Analog) transmission with part quantization loss [2] and discrete-time analog transmission without any quantization loss [1]. Among them, the analog transmission is the fastest

way with the least feedback delay and also feasible with less complexity, which has been studied in [5], [6], etc..

Although there is no quantization loss in analogue transmission, noises exist in both transmit process and feedback process. Also, the noise in estimating feedback channel matrix should be considered. Thus, the full CSIT we acquired by feedback schemes are imperfect. The mean-square error of the CSIT estimate can be a measure to see which feedback scheme is less imperfect.

In this paper, we suppose the transmit channel knowledge is fed back in analogue, i.e., uncoded and linearly. The least-square estimator is employed for estimation. We derive out the explicit expressions of estimate errors of two analog feedback schemes and compare them with respect to mean-square error in the scenario of quasi-static independent Rayleigh-fading flat MIMO channel with additive white noises.

## II. FRAMEWORKS

### A. Procedures of the channel-estimate-based feedback scheme (scheme I)

Fig.1(a) shows the block diagram of the channel-estimate-based feedback scheme. In this scheme, the training matrix  $\mathbf{S}$  is transmitted at the transmitter A under the power constraint  $P_a$ . The receiver B does least-square estimate, namely "pseudo" ML estimate, for  $\mathbf{H}$ . The scaling factor, which will be transmitted from B to A in a reliable non-linear way, is figured out based on the channel estimate  $\hat{\mathbf{H}}$ 's energy and transmit power constraint  $P_b$  in the feedback process.  $\hat{\mathbf{H}}'$  is acquired by multiplying the scaling factor to the raw channel estimate  $\hat{\mathbf{H}}$ . Then, the scaled channel estimate  $\hat{\mathbf{H}}'$  is spread and spatial-multiplexed at B linearly and fed back to the transmitter A. The received feedback signals is demultiplexed and despread to  $\hat{\hat{\mathbf{H}}}'$  and descaled to  $\hat{\hat{\mathbf{H}}}$  at A.  $\hat{\hat{\mathbf{H}}}$  is taken as the imperfect estimate of full CSIT.

### B. Procedures of the received-signal-based feedback scheme (scheme II)

Fig.1(b) shows the block diagram of the received-signal-based feedback scheme. In this scheme, the training matrix  $\mathbf{S}$  is transmitted by the transmitter A under the power constraint  $P_a$ . The scaling factor is figured out based on the received signal matrix  $\mathbf{R}$ 's energy and the transmit power constraint  $P_b$  at B in the feedback process.  $\mathbf{R}$  is scaled to  $\mathbf{R}'$  and fed back

to the transmitter A after being spread and spatial-multiplexed at the receiver B. The received feedback signals is recovered to  $\widehat{\mathbf{R}}$  at the transmitter A, and then the least-square estimation is done on  $\widehat{\mathbf{R}}$  for obtaining  $\widehat{\mathbf{H}}$ .  $\widehat{\mathbf{H}}$  is taken as the imperfect estimate of full CSIT.

### III. ESTIMATE ERRORS

The error of estimating CSIT is caused by the noise  $\mathbf{Z}_b$  in transmit process, the noise  $\mathbf{Z}_a$  in feedback process and the noise  $\mathbf{Z}_g$  while learning feedback channel.  $\mathbf{Z}_g$  causes the estimate error of  $\mathbf{G}$ , -  $\widetilde{\mathbf{G}}$ .

The MIMO system investigated in this work consists of  $N_a$  and  $N_b$  antennas at A and B, respectively. In transmit process, the training matrix  $\mathbf{S}$  is emitted by  $N_a$  antennas at A and received by  $N_b$  antennas at B. In feedback process, the feedback symbols are emitted by  $N_b$  antennas at B and received by  $N_a$  antennas at A. We suppose  $N_a \geq N_b$ .

The training matrix  $\mathbf{S}$  of size  $Q \times N_a$  is over  $Q$  symbol periods and space-orthogonal. Namely,

$$\mathbf{S}^H \mathbf{S} = \frac{QP_a}{N_a} \mathbf{I}. \quad (1)$$

Denote the transmit channel matrix  $\mathbf{H}$  of size  $N_a \times N_b$  as

$$\begin{aligned} \mathbf{H} &= \begin{pmatrix} \mathbf{H}_{(\cdot,1)} & \mathbf{H}_{(\cdot,2)} & \cdots & \mathbf{H}_{(\cdot,N_b)} \end{pmatrix} \\ &= \begin{pmatrix} \mathbf{H}_{(1,\cdot)} \\ \mathbf{H}_{(2,\cdot)} \\ \vdots \\ \mathbf{H}_{(N_a,\cdot)} \end{pmatrix} \end{aligned} \quad (2)$$

where  $\mathbf{H}_{(i,\cdot)}$  is the  $i$ -th row vector of size  $1 \times N_b$ , and  $\mathbf{H}_{(\cdot,j)}$  is the  $j$ -th column vector of size  $N_a \times 1$ .

The transmit channel state information can also be denoted by column vectors below.

$$\mathbf{h} = \begin{pmatrix} \mathbf{H}_{(\cdot,1)} \\ \vdots \\ \mathbf{H}_{(\cdot,N_b)} \end{pmatrix}, \quad \mathbf{h}^\circ = \begin{pmatrix} \mathbf{H}_{(1,\cdot)}^T \\ \vdots \\ \mathbf{H}_{(N_a,\cdot)}^T \end{pmatrix}. \quad (3)$$

Analogously, the received signals at B can be denoted in three forms, the matrix  $\mathbf{R}$  of size  $Q \times N_b$ , the length- $QN_b$  column vector  $\mathbf{r}$ , and the same-length column vector  $\mathbf{r}^\circ$ .

For deriving out MSE expressions, we introduce permutation matrices herein,

$$\mathbf{P}_1 = \begin{pmatrix} \mathbf{u}_1 & & O \\ & \ddots & \\ O & & \mathbf{u}_1 \\ & \vdots & \\ \mathbf{u}_{N_a} & & O \\ & \ddots & \\ O & & \mathbf{u}_{N_a} \end{pmatrix}, \mathbf{P}_2 = \begin{pmatrix} \mathbf{v}_1 & & O \\ & \ddots & \\ O & & \mathbf{v}_1 \\ & \vdots & \\ \mathbf{v}_Q & & O \\ & \ddots & \\ O & & \mathbf{v}_Q \end{pmatrix}. \quad (4)$$

$\mathbf{P}_1$  is a permutation matrix of size  $N_a N_b \times N_a N_b$  where  $\mathbf{u}_i$  is a length- $N_a$  row vector whose  $i$ -th entry is 1 and the others

are 0s.  $\mathbf{P}_2$  is a permutation matrix of size  $QN_b \times QN_b$  where  $\mathbf{v}_i$  is a length- $Q$  row vector with similar definition to  $\mathbf{u}_i$ 's.

Then,

$$\mathbf{h}^\circ = \mathbf{P}_1 \mathbf{h}, \quad \mathbf{r}^\circ = \mathbf{P}_2 \mathbf{r}. \quad (5)$$

The training-signal-transmission model can be written as

$$\mathbf{r} = \begin{pmatrix} \mathbf{S} & & O \\ & \ddots & \\ O & & \mathbf{S} \end{pmatrix} \mathbf{h} + \mathbf{z}_b. \quad (6)$$

$\begin{pmatrix} \mathbf{S} & & O \\ & \ddots & \\ O & & \mathbf{S} \end{pmatrix}$  is denoted by the block diagonal matrix  $\mathbf{S}'$  in following.

The scaling factors of both two schemes are supposed to be transmitted from B to A in some non-linear way precisely and reliably. The errors of received scaling factors at A are neglected.

Note that for the terseness of expressions, the size of the feedback channel matrix  $\mathbf{G}$  is  $N_a \times N_b$  as  $\mathbf{H}$ , but not  $N_b \times N_a$  conventionally.

#### A. The estimate error of the channel-estimate-based feedback scheme

Each antenna at B feeds back the estimate of CSIT related to one receive antenna at B to A.

The total of complex-valued parameters to be fed back in scheme I is  $N_a N_b$ . Assume the feedback delay is  $TQ$  symbol periods. Then, the number of replicas of each parameter is  $TQ/N_a$ , denoted by  $M_1$ ,  $M_1 \in \mathcal{N}$ .

After the least-square estimator at B, the channel estimate

$$\widehat{\mathbf{h}} = \mathbf{h} + \mathbf{S}'^+ \mathbf{z}_b \quad (7)$$

where  $(\mathbf{X})^+$  denotes pseudo-inverse of the matrix  $\mathbf{X}$  whose rank is the number of column vectors.

Thus,

$$\widehat{\mathbf{h}}^\circ = \mathbf{h}^\circ + \mathbf{P}_1 \mathbf{S}'^+ \mathbf{z}_b, \quad (8)$$

$$\widetilde{\mathbf{h}}^\circ = \mathbf{P}_1 \mathbf{S}'^+ \mathbf{z}_b. \quad (9)$$

The scaling factor

$$\begin{aligned} \alpha_1 &= \sqrt{\frac{TQP_b}{M_1 E_{\widehat{\mathbf{H}}}}} \\ &= \sqrt{\frac{N_a P_b}{E_{\widehat{\mathbf{H}}}}}. \end{aligned} \quad (10)$$

After scaling,

$$\widehat{\mathbf{H}}' = \alpha_1 \widehat{\mathbf{H}}. \quad (11)$$

The received feedback at A can be written as

$$\mathbf{Y}_{(i,\cdot)}^T = \begin{pmatrix} \mathbf{G} \\ \vdots \\ \mathbf{G} \end{pmatrix} \widehat{\mathbf{H}}_{(i,\cdot)}'^T + \mathbf{Z}_a_{(i,\cdot)}^T \quad (12)$$

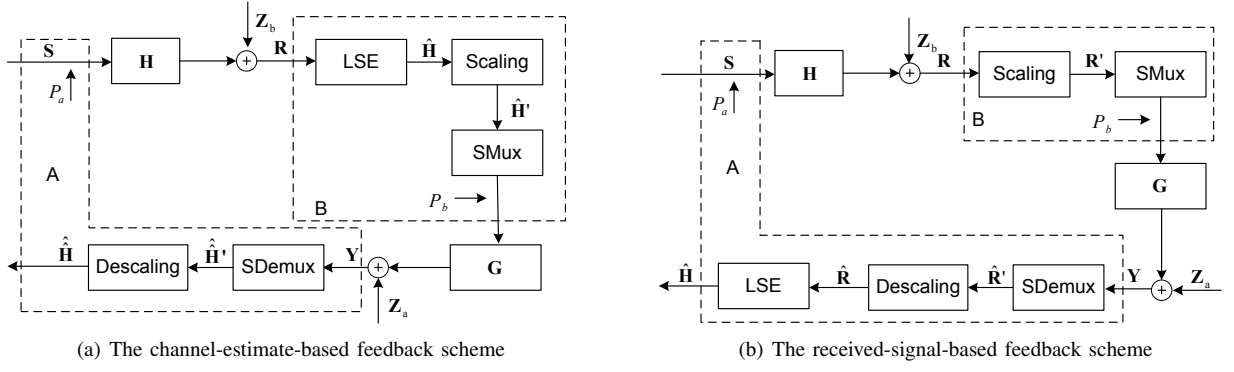


Fig. 1. Block diagrams of two feedback schemes

where  $\begin{pmatrix} \mathbf{G} \\ \vdots \\ \mathbf{G} \end{pmatrix}$  is composed of  $M_1 \mathbf{G}$  and denoted by  $\mathbf{G}'_1$  in the following.

We can thus write

$$\mathbf{y}^\circ = \begin{pmatrix} \mathbf{G}'_1 & & O \\ & \ddots & \\ O & & \mathbf{G}'_1 \end{pmatrix} \hat{\mathbf{h}}'^\circ + \mathbf{z}_a^\circ \quad (13)$$

where  $\begin{pmatrix} \mathbf{G}'_1 & & O \\ & \ddots & \\ O & & \mathbf{G}'_1 \end{pmatrix}$  is the block diagonal matrix composed of  $N_a \mathbf{G}'_1$  and denoted by  $\mathbf{G}''_1$  in following.

After the zero-forcing spatial demultiplexing and descaling, the imperfect CSIT obtained at A is

$$\hat{\mathbf{h}}^\circ = \mathbf{h}^\circ + \tilde{\mathbf{h}}^\circ - \hat{\mathbf{G}}_1'' + \tilde{\mathbf{G}}_1'' \mathbf{h}^\circ - \hat{\mathbf{G}}_1'' + \tilde{\mathbf{G}}_1'' \tilde{\mathbf{h}}^\circ + \frac{1}{\alpha_1} \hat{\mathbf{G}}_1'' + \mathbf{z}_a^\circ. \quad (14)$$

Thus, the error of the CSIT estimate in scheme I is

$$\begin{aligned} \tilde{\mathbf{h}}_1^\circ &= -\hat{\mathbf{G}}_1'' + \tilde{\mathbf{G}}_1'' \mathbf{h}^\circ + (\mathbf{I} - \hat{\mathbf{G}}_1'' + \tilde{\mathbf{G}}_1'') \tilde{\mathbf{h}}^\circ + \frac{1}{\alpha_1} \hat{\mathbf{G}}_1'' + \mathbf{z}_a^\circ \\ &= -\hat{\mathbf{G}}_1'' + \tilde{\mathbf{G}}_1'' \mathbf{h}^\circ + (\mathbf{I} - \hat{\mathbf{G}}_1'' + \tilde{\mathbf{G}}_1'') \mathbf{P}_1 \mathbf{S}' + \mathbf{z}_b + \frac{1}{\alpha_1} \hat{\mathbf{G}}_1'' + \mathbf{z}_a^\circ \end{aligned} \quad (15)$$

where

$$\hat{\mathbf{G}}_1'' + \tilde{\mathbf{G}}_1'' = \begin{pmatrix} \hat{\mathbf{G}} + \tilde{\mathbf{G}} & & O \\ & \ddots & \\ O & & \hat{\mathbf{G}} + \tilde{\mathbf{G}} \end{pmatrix} \quad (16)$$

with  $N_a$  diagonal blocks of  $\hat{\mathbf{G}} + \tilde{\mathbf{G}}$ .

Namely,

$$\begin{aligned} \tilde{\mathbf{h}}_1 &= -\mathbf{P}_1^T \hat{\mathbf{G}}_1'' + \tilde{\mathbf{G}}_1'' \mathbf{P}_1 \mathbf{h} + \mathbf{P}_1^T \hat{\mathbf{G}}_1'' + \tilde{\mathbf{G}}_1'' \mathbf{P}_1 \mathbf{S}' + \mathbf{z}_b \\ &\quad + \frac{1}{\alpha_1} \mathbf{P}_1^T \hat{\mathbf{G}}_1'' + \mathbf{z}_a^\circ. \end{aligned} \quad (17)$$

*B. The estimate error of the received-signal-based feedback scheme*

Each antenna at B feeds back the received training-signals of one receive antenna at B to A.

The total of complex-valued parameter to be fed back in scheme II is  $N_b Q$ . The feedback delay is assumed as long as in the other scheme for comparison,  $TQ$  symbol periods. Then, the number of replicas of each parameter is  $T$ .

The scaling factor,

$$\alpha_2 = \sqrt{\frac{QP_b}{E_R}}. \quad (18)$$

After scaling,

$$\mathbf{R}' = \alpha_2 \mathbf{R}. \quad (19)$$

The received feedback at A can be written as

$$\mathbf{Y}_{(i,\cdot)}^T = \mathbf{G}'_2 \mathbf{R}'_{(i,\cdot)}^T + \mathbf{Z}_{a(i,\cdot)}^T \quad (20)$$

where  $\mathbf{G}'_2$  is composed of  $T \mathbf{G}$  like in scheme I.

After descaling,

$$\begin{aligned} \hat{\mathbf{r}}^\circ &= \frac{1}{\alpha_2} \hat{\mathbf{G}}_2'' + \mathbf{y}^\circ \\ &= \mathbf{r}^\circ - \hat{\mathbf{G}}_2'' + \tilde{\mathbf{G}}_2'' \mathbf{r}^\circ + \frac{1}{\alpha_2} \hat{\mathbf{G}}_2'' + \mathbf{z}_a^\circ \end{aligned} \quad (21)$$

where  $\mathbf{G}''_2$  is the block diagonal matrix composed of  $Q \mathbf{G}'_2$ . Namely,

$$\hat{\mathbf{r}} = \mathbf{r} - \mathbf{P}_2^T \hat{\mathbf{G}}_2'' + \tilde{\mathbf{G}}_2'' \mathbf{P}_2 \mathbf{r} + \mathbf{P}_2^T \frac{1}{\alpha_2} \hat{\mathbf{G}}_2'' + \mathbf{z}_a^\circ \quad (22)$$

After the least-square estimator at A,

$$\begin{aligned} \hat{\mathbf{h}} &= \mathbf{S}' + \hat{\mathbf{r}} \\ &= \mathbf{h} + \mathbf{S}' + \mathbf{z}_b - \mathbf{S}' + \mathbf{P}_2^T \hat{\mathbf{G}}_2'' + \tilde{\mathbf{G}}_2'' \mathbf{P}_2 (\mathbf{S}' \mathbf{h} + \mathbf{z}_b) \\ &\quad + \frac{1}{\alpha_2} \mathbf{S}' + \mathbf{P}_2^T \hat{\mathbf{G}}_2'' + \mathbf{z}_a^\circ \end{aligned} \quad (23)$$

Thus, the error of the CSIT estimate in scheme II is

$$\begin{aligned} \tilde{\mathbf{h}}_2 &= -\mathbf{S}' + \mathbf{P}_2^T \hat{\mathbf{G}}_2'' + \tilde{\mathbf{G}}_2'' \mathbf{P}_2 \mathbf{S}' \mathbf{h} \\ &\quad + \mathbf{S}' + \mathbf{P}_2^T \hat{\mathbf{G}}_2'' + \tilde{\mathbf{G}}_2'' \mathbf{P}_2 \mathbf{z}_b \\ &\quad + \frac{1}{\alpha_2} \mathbf{S}' + \mathbf{P}_2^T \hat{\mathbf{G}}_2'' + \mathbf{z}_a^\circ. \end{aligned} \quad (24)$$

#### IV. COMPARISON AND ANALYSIS

The estimate-error expressions of two feedback schemes derived in Section III can be written as sums of three column-vector terms,

$$\begin{aligned}\tilde{\mathbf{h}}_1 &= \mathbf{t}_{1,h} + \mathbf{t}_{1,z_b} + \mathbf{t}_{1,z_a}, \\ \tilde{\mathbf{h}}_2 &= \mathbf{t}_{2,h} + \mathbf{t}_{2,z_b} + \mathbf{t}_{2,z_a},\end{aligned}\quad (25)$$

where

$$\begin{aligned}\mathbf{t}_{1,h} &= -\mathbf{P}_1^T \widehat{\mathbf{G}}_1'' + \widetilde{\mathbf{G}}_1'' \mathbf{P}_1 \mathbf{h}, \\ \mathbf{t}_{1,z_b} &= \mathbf{P}_1^T \widehat{\mathbf{G}}_1'' + \mathbf{G}_1'' \mathbf{P}_1 \mathbf{S}' + \mathbf{z}_b, \\ \mathbf{t}_{1,z_a} &= \frac{1}{\alpha_1} \mathbf{P}_1^T \widehat{\mathbf{G}}_1'' + \mathbf{z}_a^\circ, \\ \mathbf{t}_{2,h} &= -\mathbf{S}' + \mathbf{P}_2^T \widehat{\mathbf{G}}_2'' + \widetilde{\mathbf{G}}_2'' \mathbf{P}_2 \mathbf{S}' \mathbf{h}, \\ \mathbf{t}_{2,z_b} &= \mathbf{S}' + \mathbf{P}_2^T \widehat{\mathbf{G}}_2'' + \mathbf{G}_2'' \mathbf{P}_2 \mathbf{z}_b, \\ \mathbf{t}_{2,z_a} &= \frac{1}{\alpha_2} \mathbf{S}' + \mathbf{P}_2^T \widehat{\mathbf{G}}_2'' + \mathbf{z}_a^\circ.\end{aligned}\quad (26)$$

It can be seen that three terms of each expression are uncorrelated to each other with respect to average of noises and real channel states.

Thus, the mean-square errors of CSIT estimates of two schemes,

$$\begin{aligned}\epsilon_1^2 &= \mathcal{E}(\|\mathbf{t}_{1,h}\|_F^2) + \mathcal{E}(\|\mathbf{t}_{1,z_b}\|_F^2) + \mathcal{E}(\|\mathbf{t}_{1,z_a}\|_F^2), \\ \epsilon_2^2 &= \mathcal{E}(\|\mathbf{t}_{2,h}\|_F^2) + \mathcal{E}(\|\mathbf{t}_{2,z_b}\|_F^2) + \mathcal{E}(\|\mathbf{t}_{2,z_a}\|_F^2),\end{aligned}\quad (27)$$

where  $\mathcal{E}(\cdot)$  refers to the expectation with respect to the noises and real channel states.

Assume both noises in transmit process and feedback process are additive white noises, whose distributions are i.i.d. spacially and temporally, with variance  $\sigma_{z_a}^2$  and  $\sigma_{z_b}^2$  respectively. Assuming independent Rayleigh fading, the entries of channel matrices  $\mathbf{H}$  and  $\mathbf{G}$  are i.i.d zero-mean Gaussian random variables with variance  $\sigma_h^2$  and  $\sigma_g^2$  respectively.

In terms of properties of trace function,

$$\begin{aligned}\mathcal{E}(\|\mathbf{t}_{1,h}\|_F^2) &= \mathcal{E}(\|\mathbf{t}_{2,h}\|_F^2) \\ &= N_a \sigma_h^2 \mathcal{E}_{g,z_g}(\|\widehat{\mathbf{G}}^+ + \widetilde{\mathbf{G}}\|_F^2)\end{aligned}\quad (28)$$

$$\begin{aligned}\mathcal{E}(\|\mathbf{t}_{1,z_b}\|_F^2) &= \mathcal{E}(\|\mathbf{t}_{2,z_b}\|_F^2) \\ &= \frac{N_a^2 \sigma_{z_b}^2}{Q P_a} \mathcal{E}_{g,z_g}(\|\widehat{\mathbf{G}}^+ + \mathbf{G}\|_F^2)\end{aligned}\quad (29)$$

$$\begin{aligned}\mathcal{E}(\|\mathbf{t}_{1,z_a}\|_F^2) &= \frac{N_a^2 N_b (Q P_a \sigma_h^2 + N_a \sigma_{z_b}^2) \sigma_{z_a}^2}{T Q^2 P_a P_b} \\ &\quad \cdot \mathcal{E}_{g,z_g}(\text{tr}\{(\widehat{\mathbf{G}}^H \widehat{\mathbf{G}})^{-1}\})\end{aligned}\quad (30)$$

$$\begin{aligned}\mathcal{E}(\|\mathbf{t}_{2,z_a}\|_F^2) &= \frac{N_a^2 N_b (P_a \sigma_h^2 + \sigma_{z_b}^2) \sigma_{z_a}^2}{T Q P_a P_b} \\ &\quad \cdot \mathcal{E}_{g,z_g}(\text{tr}\{(\widehat{\mathbf{G}}^H \widehat{\mathbf{G}})^{-1}\})\end{aligned}\quad (31)$$

Comparing (30) and (31), it is apparent to see that when  $Q = N_a$ ,  $\mathcal{E}(\|\mathbf{t}_{1,z_a}\|_F^2) = \mathcal{E}(\|\mathbf{t}_{2,z_a}\|_F^2)$ ; when  $Q > N_a$ ,  $\mathcal{E}(\|\mathbf{t}_{1,z_a}\|_F^2) < \mathcal{E}(\|\mathbf{t}_{2,z_a}\|_F^2)$ .

Therefore, the mean-square error of CSIT estimate in scheme I is equal to that in scheme II when  $Q = N_a$  and

less than that in scheme II when  $Q > N_a$ . The reason of the difference is that the effectiveness ratio of the effective information energy to the disturbing source energy in scheme I is  $\frac{Q P_a \sigma_h^2}{N_a \sigma_{z_b}^2}$  but in scheme II is  $\frac{P_a \sigma_h^2}{\sigma_{z_b}^2}$ . When  $Q = N_a$ , the two ratios are equal. As  $Q$  increases, the effectiveness ratio in scheme I is increasing monotonically with  $Q$  and the ratio in scheme II keeps the same, so the scheme I performs better.

Considering  $\mathbf{G}^H \mathbf{G}$  is a central Wishart matrix on the assumption of i.i.d. Rayleigh fading, in terms of Lemma 2.10 in [7],

$$\mathcal{E}_{g,z_g}(\text{tr}\{(\mathbf{G}^H \mathbf{G})^{-1}\}) = \frac{N_b}{(N_a - N_b) \sigma_h^2} \quad (32)$$

Assuming the feedback channel matrix  $\mathbf{G}$  is estimated at A by an analogous least-square estimator as  $\mathbf{H}$ , all entries of  $\widehat{\mathbf{G}}$  are zero-mean Gaussian-distributed random variables with variance  $\sigma_g^2 + \frac{N_a N_b}{Q_g P_b} \sigma_{z_g}^2$  independently and identically. Thus, the difference between mean-square errors of CSIT estimates of two feedback schemes

$$\begin{aligned}\Delta_\epsilon &= \epsilon_2^2 - \epsilon_1^2 \\ &= \frac{N_a^2 N_b^2}{(N_a - N_b) T Q \rho_a \rho_b \sigma_g^2} \left(1 - \frac{N_a}{Q}\right) \\ &= \frac{N_a^2 N_b^2}{(N_a - N_b) T Q \rho_a \rho_b (\sigma_g^2 + \frac{N_a N_b}{Q_g P_b} \sigma_{z_g}^2)} \left(1 - \frac{N_a}{Q}\right)\end{aligned}\quad (33)$$

where  $\rho_a = P_a / \sigma_{z_b}^2$  and  $\rho_b = P_b / \sigma_{z_a}^2$ .

#### V. NUMERICAL RESULTS AND DISCUSSIONS

Assume there is no noise in estimating the feedback channel matrix  $\mathbf{G}$ , i.e.,  $\sigma_{z_g}^2 = 0$ ,  $\widehat{\mathbf{G}} = \mathbf{G}$ ,  $\widetilde{\mathbf{G}} = \mathbf{0}$ . In such case, the mean-square estimate errors of two schemes are

$$\begin{aligned}\epsilon_1^2 &= \frac{N_a^2 N_b}{Q \rho_a} + \frac{N_a^2 N_b^2 \sigma_h^2}{(N_a - N_b) T Q \sigma_g^2 \rho_b} + \frac{N_a^3 N_b^2}{(N_a - N_b) T Q^2 \rho_a \rho_b \sigma_g^2} \\ \epsilon_2^2 &= \frac{N_a^2 N_b}{Q \rho_a} + \frac{N_a^2 N_b^2 \sigma_h^2}{(N_a - N_b) T Q \sigma_g^2 \rho_b} + \frac{N_a^2 N_b^2}{(N_a - N_b) T Q \rho_a \rho_b \sigma_g^2}\end{aligned}\quad (34)$$

Assume there are four transmit antenna and two receive antennas, the transmit SNR  $\rho_a$  is 15dB, the receive SNR  $\rho_b$  is 10dB,  $T = 1$ , and  $\sigma_h^2 = \sigma_g^2 = 1$ .

From the plot in Fig.2, we can see that the difference between  $\epsilon_1^2$  and  $\epsilon_2^2$  is trivial at high SNR. If we assume A is a base station and B is a mobile station, when the difference of  $\epsilon_1^2$  and  $\epsilon_2^2$  is negligible, the received-signal-based feedback scheme is preferable for reducing the complexity overhead at mobile station.

#### VI. CONCLUSION

In this paper, we have presented two uncoded analog channel-knowledge feedback schemes, the channel-estimate-based analog feedback scheme and the received-signal-based analog feedback scheme, based on least-square estimators to obtain the full CSIT. We have derived out the explicit expressions of CSIT-estimate errors and mean-square errors

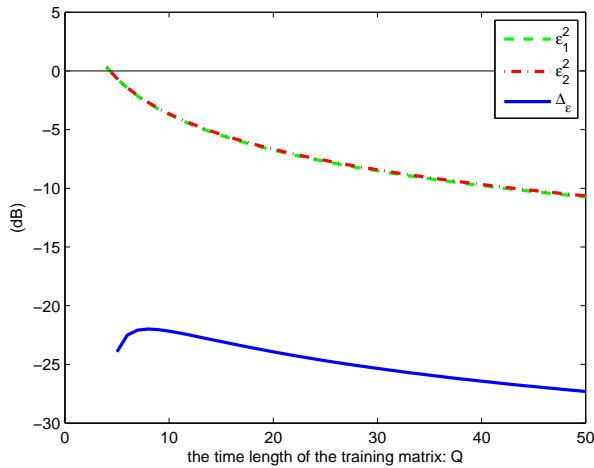


Fig. 2. averaged square estimate errors of two feedback schemes and the difference curve

of two schemes. Comparing the two expressions of mean-square errors, we have shown that the channel-estimate-based feedback scheme is less imperfect than the received-signal-based feedback scheme with respect to mean-square error when the training period is longer than  $N_a$  symbol periods. The reason of the difference is that the effectiveness ratio of effective information to disturbing source in the feedback information in the channel-estimate-based feedback scheme is higher than that in the received-signal-based feedback scheme when the training period is longer than  $N_a$  symbol periods. Nevertheless, the difference between them is trivial at high SNR as shown by numerical results.

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