Low SNR Analysis for MIMO Broadcast Channels with different types of CSIT

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***Abstract*— We consider a multi-antenna broadcast channel with more users K than transmit antennas operating in the low SNR regime. By applying low-power analysis tools, we derive sum-rate and energy efficiency bounds for dirty paper coding (DPC) and time-division multiple access (TDMA) with full channel state information at the transmitter (CSIT), as well as for random multiuser beamforming with partial CSIT. The optimality of multiuser transmission is established and large K asymptotic analysis is provided. Finally, we analytically show the efficiency of random beamforming and determine the optimal number of users to transmit to in the low SNR regime.**

I. INTRODUCTION

Multiple-input multiple-output (MIMO) communication systems have recently attracted particular attention mainly due to their high spectral efficiency. Although dirty paper coding (DPC) is capacity-achieving in MIMO broadcast channels (BC), its high complexity makes it difficult to be implemented. Downlink linear precoding, although suboptimal, has been shown to achieve a large portion of DPC capacity, exhibiting reduced complexity. Furthermore, random multiuser beamforming (RBF) [1] is shown to yield the optimal capacity growth for large number of users, even with partial channel state information at the transmitter (CSIT).

In this paper, we utilize the low-power analysis tools developed by Verdú [2] to quantify the performance difference in terms of sum-rate and energy efficiency among optimal DPC, time-division multiple access (TDMA) and the simpler RBF in an i.i.d. Rayleigh fading environment. We consider a multiple antenna broadcast channel with M antennas at the transmitter and $K \geq M$ single-antenna receivers. The base station is subject to an average power constraint P and we assume that each of the receivers has perfect CSI.

At asymptotic low SNR, the sum rate $C(P)$ of a given scheme is well approximated by an affine function of $\frac{E_b}{N_0}$ in dB as

$$C\left(\frac{E_b}{N_0}\right) \approx S_0/3|_{dB}\left(\frac{E_b}{N_0}|_{dB} - \frac{E_b}{N_0}_{min}|_{dB}\right) + \epsilon \qquad (1)$$

where ϵ are lower-order terms and $3|_{dB} = 10\log_{10} 2$. The slope S_0 and the minimum energy per information bit required for reliable communication (normalized to the background noise) $\frac{E_b}{N_0}_{min}$, are defined as:

$$S_0 = -\frac{2[\dot{C}(0)]^2}{\ddot{C}(0)} \quad \text{and} \quad \frac{E_b}{N_0}_{min} = \frac{\log_e 2}{\dot{C}(0)} \qquad (2)$$

where $\dot{C}(0)$ and $\ddot{C}(0)$ are the first and second derivative, respectively, of the capacity $C(P)$ in nats/dimension at $P = 0$. The spectral efficiency $C(\frac{E_b}{N_0})$ is defined as $C(\frac{E_b}{N_0}) = C(P)$ and $P = C(\frac{E_b}{N_0})\frac{E_b}{N_0}$.

II. MAIN RESULTS

A. DPC and TDMA

We first present the performance of DPC and TDMA with full CSIT for low to moderate number of users.

Proposition 1: At low SNR, the minimum energy per information bit required for reliable communication for DPC and TDMA with full CSIT and $K \geq M$ are characterized by

$$\frac{E_b}{N_0}_{min}^{(dpc)} \geq \frac{\log_e 2}{K \cdot Y} \quad \text{and} \quad \frac{E_b}{N_0}_{min}^{(tdma)} \geq \frac{\log_e 2}{K \cdot Y} \qquad (3)$$

where $Y = \frac{B_M(\ldots,(i-1)!H_{M(K-1)+1,i},\ldots)}{(M-1)!(M(K-1)+1)}$, $B_M(\ldots, x_i, \ldots)$ are the M-th complete Bell polynomials, and $H_{n,i}$ is the generalized harmonic number of order n of i.

The above bound is accurate for small M and K. For M, K increasing, a tighter low bound, which equals to $\frac{\log_e 2}{M + \frac{K-1}{\sqrt{2K-1}}\sqrt{M}}$, can be found. The ratio of the slope S_0 of DPC to the TDMA slope is equal to M, implying that DPC provides a gain of $10\log_{10} M$ dB compared to TDMA, or equivalently, a factor of M in rate (nats/s/Hz) for the same power.

Proposition 2: At low SNR and $K >> M$, the minimum energies per information bit for DPC and TDMA with full CSIT are given by

$$\frac{E_b}{N_0}_{min}^{(dpc)} \geq \frac{\log_e 2}{\log(K \log^M K)} \text{ and } \frac{E_b}{N_0}_{min}^{(tdma)} \geq \frac{\log_e 2}{\log(K \log^M K)}$$

Evidently, with K increasing, a decreasing $\frac{E_b}{N_0}_{min}$ is required for reliable communication.

B. Random multiuser beamforming (RBF)

We study the sum rate and the energy efficiency of RBF operating in the power-limited regime.

Proposition 3: The low-SNR regime of random beamforming with partial CSIT ($K \geq M$) is characterized by

$$\frac{E_b}{N_0}_{min}^{(RBF)} = \frac{\log_e 2}{H_K} \quad \text{and} \quad S_0 = \frac{M \cdot H_K^2}{2B_4(\ldots,(i-1)!H_{K,i},\ldots)} \qquad (4)$$

where H_n is the n-th harmonic number.

A system employing RBF requires higher $\frac{E_b}{N_0}_{min}$ compared to DPC and TDMA. However, transmitting to more than one user is energy-efficient, as the spectral efficiency slope of RBF is higher than that of TDMA (but lower than the slope of DPC).

Let now $\mathcal{M}$ denote the number of active beams ($1 \leq \mathcal{M} \leq M$). At low SNR, the expected sum rate of RBF is given by

$$\mathcal{R} = -K\mathcal{M}\log_2 e \sum_{k=0}^{K-1} \binom{K-1}{k} (-1)^k \frac{e^{k'}}{k+1} \cdot \text{Ei}(-k') \approx P \cdot H_K$$

where $k' = \mathcal{M}(k+1)/P$ and $\text{Ei}(x)$ is the exponential integral. Thus, at low SNR and fixed K, the expected sum rate of RBF is maximized when $\mathcal{M} = M$, implying that serving M users simultaneously is the optimal sum-rate maximizing scheduling strategy.

REFERENCES

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