Resource allocation in multicell wireless networks: Some capacity scaling laws

David Gesbert (1), Marios Kountouris (1-2)
(1) Eurecom Institute, Sophia-Antipolis gesbert@eurecom.fr
(2) France Telecom Research and Development, Issy Les Moulineaux

***Abstract*— We address the optimization of the sum rate performance in multicell interference-limited wireless networks where access points are allowed to cooperate in terms of joint resource allocation. The resource allocation policies considered here combine power control and user scheduling. Although very promising from a conceptual point of view, the optimization of the sum rate (network capacity) hinges, in principle, on tough issues such as computational complexity and the requirement for heavy receiver-to-transmitter channel information feedback across all network cells. However, we show that, in fact, distributed algorithms are actually obtainable in the asymptotic regime where the numbers of users per cell is allowed to grow to infinity. Additionally, using extreme value theory, we provide scaling laws for upper and lower bounds for the network capacity (as the number of users grows large), corresponding to two forms of distributed resource allocation schemes. We show these bounds are in fact identical asymptotically. This remarkable result suggests that distributed resource allocation is practically possible, with vanishing loss of network capacity if enough users exist.**

I. Introduction

Traditional resource allocation in multicell wireless networks follows a *divide and conquer* strategy. First, network-wide frequency planning is used to allow the fragmentation of the network area into smaller zones more or less isolated from each other from a radio point of view. Then, the link efficiency *in a given cell* is optimized via a careful design of the PHY/MAC layers, exploiting advanced processing such as efficient FEC coding, multiple antennas transceivers, fast link adaptation protocols, interference canceling, and multiuser diversity-oriented scheduling [1]. These are single cell based techniques however and multicell coordination remains limited. Multicell Power control and dynamic channel assignement methods do exist and help alleviate the problem of interference. However the majority of used techniques are geared toward achieving a given signal to noise plus interference (SINR), common to all users, rather than maximizing the network's throughput [2].

In this work we consider a wireless network where the cells are allowed to cooperate so as to maximize the total capacity (sum of rates achieved by simultaneously transmitting users). The considered cooperation is limited to resource allocation in the form of power control and user scheduling. In principle a more advanced form of information theoretic cooperation using distributed coding could further increase the capacity [3] however this aspect is not considered here.

A simple intuitive idea behind multicell resource allocation is to exploit the large amount of spatial and multiuser diversity offered by the extra multicell dimension in order to optimize the network performance at all times. Clearly the potential gains comes with great challenges. One is the complexity associated with the joint optimization of a large number of parameters (slot assignement, power levels, ..). Another one is the need for the joint processing of multicell channel state information which necessitates huge cell-to-cell signaling overhead. This makes global network coordination hard to realize, especially in fast mobile settings. Despite the challenges, some recenty published work suggests possible techniques for low complexity and distributed resource allocation. Examples of such approaches include game theoretic algorithms with pricing [4], resource allocation based on quantized power levels, and iterative/greedy capacity maximization techniques. An overview of such techniques is available in [5]. Nonetheless it remains that such approaches are suboptimal.

In this paper we address the problem of capacity-optimal resource allocation in the form of multicell power control and scheduling. We investigate upper and lower bounds on the maximum network capacity provided by resource allocation in interference-free and full-powered interference scenarios, respectively. Interestingly the solution to the multicell scheduling and power control is fully distributed in both scenarios. We study these bounds in the asymptotic regime where the number of users per cell is allowed to grow large while the number of cells remain fixed. We introduce scaling laws of capacity for this asymptotic regime, based on extreme value theory and recently published work in the different context of opportunistic single cell beamforming. We show the scaling law for the upper and lower bounds on capacity are in fact identical. Our results suggest that very simple distributed resource allocation algorithms could be used, and that the capacity loss with respect to the optimal resource allocation solution is negligible if the number of users is large enough.

II. Network model and assumptions

We consider a wireless network featuring a number of transmitters and receivers. Among these, there are N transmit-receive active pairs, which are simultaneously selected for transmission by the scheduling protocol at any considered instant of time, others remaining silent. In this network the i-th transmitter, denoted T_i in Fig.1, sends a message which is intended to the i-th receiver R_i only. However R_i is being interfering from all $T_j, \forall j \neq i$ due to full reuse of spectral resource. This setup can be seen as an instance of the interference channel, the analysis of which is a famously difficult problem in information theory [6]. The situation depicted above can be that of a cellular network with reuse factor one (say e.g. the downlink with T_i being access points (AP) or base stations). It can also depict a single-hop peer-to-peer communication network. Below, we focus on the cellular example. This paper addresses joint resource allocation between cells in terms of joint power control and user scheduling.

A. Signal Model

We consider N time-synchronized cells, and U_n users randomly distributed over each cell. Within each cell, we consider an orthogonal multiple access scheme so that a *single* user is supported on any given spectral resource slot (resource slots are time or frequency slots in TDMA/FDMA, or code in orthogonal CDMA). Single antenna devices are considered. Generalizations to MIMO-aided spatial division multiple access will be considered elsewhere. On any given spectral resource slot, shared by all N cells, let $u_n \in \{1, .., U_n\}$ be the index of the user that is granted access to the slot in cell n.

We denote the complex downlink channel gain between the i-th AP and user u_n of cell n by $\alpha_{u_n,i}$. We hereby focus on the downlink but all ideas here carry over to the uplink as well unless otherwise stated. The local channel state information (CSI) is assumed perfect at the receiver side. This information is also fedback to the control unit responsible for resource allocation, either in a centralized or distributed manner (see later). The received signal Y_{u_n} at user u_n is given by

$$Y_{u_n} = \alpha_{u_n,n} X_{u_n} + \sum_{i \neq n}^{N} \alpha_{u_n,i} X_{u_i} + Z_{u_n},$$

where X_{u_n} is the message-carrying signal from the serving AP, subject to a peak power constraint P_{max}. $\sum_{i\neq n}^{N} \alpha_{u_n,i} X_{u_i}$ is the sum of interfering signals from other cells and Z_{u_n} is the additive noise or extra interference. Z_{u_n} is modeled for convenience as white Gaussian with power $\mathbb{E}|Z_{u_n}|^2 = \sigma^2$.

III. The Multicell Resource Allocation Problem

As stated above, intra-cell multiple access is orthogonal, while intercell multiple access is simply superposed, due to full reuse of spectrum. The resource allocation problem considered here consists in *power allocation* and *user scheduling* subproblems. Importantly we focus on *capacity maximizing* resource allocation policies, rather than *fairness-oriented* ones. In this setting the optimization of resource in the various resource slots decouples and we can consider the power allocation and user scheduling maximizing the capacity in any one slot, independently of other slots. Fairness issues are touched upon in [5]. A few useful definitions follow.

Definition 1: A ***scheduling vector*** $\boldsymbol{U}$ contains the set of users simultaneously scheduled across all N cells in the same slot:

$$\boldsymbol{U} = [u_1\, u_2\, \cdots\, u_n\, \cdots\, u_N]$$

where $[\boldsymbol{U}]_n = u_n$. Noting that $1 \leq u_n \leq U_n$, the feasible set of scheduling vectors is given by $\Upsilon = \{\boldsymbol{U} \mid 1 \leq u_n \leq U_n \ \forall\ n = 1, \ldots, N\}$.

Definition 2: A ***transmit power vector*** $\boldsymbol{P}$ contains the transmit power values used by each AP to communicate with its respective user:

$$\boldsymbol{P} = [P_{u_1}\, P_{u_2}\, \cdots\, P_{u_n}\, \cdots\, P_{u_N}]$$

where $[\boldsymbol{P}]_n = P_{u_n} = \mathbb{E}|X_{u_n}|^2$. Due to the peak power constraint $0 \leq P_{u_n} \leq P_{\max}$, the feasible set of transmit power vectors is given by $\Omega = \{\boldsymbol{P} \mid 0 \leq P_{u_n} \leq P_{\max} \ \forall\ n = 1, \ldots, N\}$.

A. Capacity optimal resource allocation

The merit (or utility) associated with a particular choice of a scheduling vector and power allocation vector is measured via the set of signal to noise and intereference ratios (SINR) observed by all scheduled users simultaneously. $\Gamma([\boldsymbol{U}]_n, \boldsymbol{P})$ refers to the SINR experienced by the receiver u_n in cell n as a result of power allocation in all cells, and is given by:

$$\Gamma([\boldsymbol{U}]_n, \boldsymbol{P}) = \frac{G_{u_n,n} P_{u_n}}{\sigma^2 + \sum_{i\neq n}^{N} G_{u_n,i} P_{u_i}}, \tag{1}$$

where $G_{u_n,i} = |\alpha_{u_n,i}|^2$ is the channel power gain from cell i to receiver u_n.

In data-centric applications, a reasonable choice of utility is a monotonically piece-wise increasing function of the SINR, reflecting the various coding rates implemented in the system. With an idealized link adaptation protocol, the utility eventually converges to a smooth function reflecting the user's instantaneous rate in Bits/Sec/Hz. For the overall network utility we consider [6]:

$$\mathcal{C}(\boldsymbol{U}, \boldsymbol{P}) \triangleq \frac{1}{N} \sum_{n=1}^{N} \log\Big(1 + \Gamma([\boldsymbol{U}]_n, \boldsymbol{P})\Big). \tag{2}$$

The capacity optimal resource allocation problem can now be formalized simply as:

$$(\boldsymbol{U}^*, \boldsymbol{P}^*) = \arg\max_{\substack{\boldsymbol{U}\in\Upsilon \\ \boldsymbol{P}\in\Omega}} \mathcal{C}(\boldsymbol{U}, \boldsymbol{P}), \tag{3}$$

The optimization above can be seen as generalizing known approaches in two ways. First the capacity maximizing scheduling problem has been considered (e.g. [1]), but in general not jointly over multiple cells. Second, the problem above extends the classical multicell power control problem (which usually rather aims at achieving SINR balancing) to include joint optimization with the scheduler.

The problem in (3) presents the system designer with many degrees of freedom to boost system capacity but also with several serious challenges. First the problem above is non convex and standard optimization techniques do not apply directly. On the other hand an exhaustive search of the $(\boldsymbol{U}, \boldsymbol{P})$ pairs over the feasible set is prohibitive. Finally, even if computational issues were to be resolved, the optimal solution still requires a central controller updated with instantaneous inter-cell channel gains which would create acute signaling overhead issues in practice. The central question of this paper thus arises: Can we extract all/some of the gain related to multicell resource allocation (compared with single cell treatment) within reasonable complexity and signaling constraints? Inspection of the recent literature reveals that this is a hot research issue with many possible tracks of investigation including use of modified capacity metrics, game theoretic approches, reuse partitioning, power shaping and power quantizing (see e.g. [5] and references therein). Below, we investigate theoretical answers on this question by means of so-called *scaling laws* of the capacity, obtained via extreme value theory. This study reveals both surprising and promising answers.

Interestingly, other work exists on analyzing the scaling law of capacity in interference-limited networks, including recently submitted [7]. In such work, a similar metric is used related to the sum capacity of simultaneously active links. Importantly though, in their work the scaling is in terms of growing number of links (or cells for a cellular network), rather than growing number of users per cell in a multiple access scheme as is studied here. Thus multi-user diversity scheduling is not exploited and fundamentally different scaling laws are obtained in the two cases.

IV. Network capacity: Bounds and asymptotic results

Let us consider a system with a large number of users in each cell. For the sake of exposition we shall assume $U_n = U$ for all n, where U is asymptotically large, while N remains fixed. We expect a growth of the sum capacity $\mathcal{C}(\boldsymbol{U}^*, \boldsymbol{P}^*)$ with U thanks to the *multicell* multiuser diversity gain[1]. Thus we are interested in how the *expected* sum capacity *scales* with U. To this end we shall use several interpretable bounding arguments. We consider two channel gain models. The first considers a symmetric distribution of gains to all users from their serving AP. In the other one, an additional random distance-dependent path loss is accounted for.

A. Bounds on network capacity

The simple bounds below hold in the asymptotic and non asymptotic regimes.

Upper bound: An upper bound (ub) on the capacity for given resource allocation vectors (non necessarily optimal ones) is obtained by simply ignoring intercell interference effects:

$$\mathcal{C}(\boldsymbol{U}, \boldsymbol{P}) \leq \frac{1}{N} \sum_{n=1}^{N} \log\left(1 + \frac{G_{u_n,n} P_{u_n}}{\sigma^2}\right). \tag{4}$$

In the absence of interference, the optimal capacity is clearly reached by transmitting at a level equal to the power constraint, i.e. $\mathbf{P}_{max} = [P_{max}, .., P_{max}]$ and selecting the user with the largest channel gain in each cell (maximum rate scheduler), thus giving the following upper bound on capacity:

$$\mathcal{C}(\boldsymbol{U}^*, \boldsymbol{P}^*) \leq \mathcal{C}^{ub} = \frac{1}{N} \sum_{n=1}^{N} \log\left(1 + \Gamma_n^{ub}\right). \tag{5}$$

where the upper bound on SINR is given by:

$$\Gamma_n^{ub} = \max_{u_n=1..U} \{G_{u_n,n}\} P_{max}/\sigma^2 \tag{6}$$

Lower bound: A lower bound on the optimal capacity (in the presence of interference) $\mathcal{C}(\boldsymbol{U}^*, \boldsymbol{P}^*)$ can be derived by restricting the domain of optimization. Namely, by restricting the power allocation vector to full power P_{max} in all transmitters, we have

$$\mathcal{C}(\boldsymbol{U}^*, \boldsymbol{P}^*) \geq \mathcal{C}^{lb} = \mathcal{C}(\boldsymbol{U}^*_{FP}, \mathbf{P}_{max}) \tag{7}$$

where $\boldsymbol{U}^*_{FP}$ denotes the optimal scheduling vector assuming full power everywhere, defined by

$$\boldsymbol{U}^*_{FP} = \arg\max_{\boldsymbol{U} \in \Upsilon} \mathcal{C}(\boldsymbol{U}, \boldsymbol{P}_{max}), \tag{8}$$

Note that the n-th cell's user in $\boldsymbol{U}^*_{FP}$ is found easily via:

$$[\boldsymbol{U}^*_{FP}]_n = \arg\max_{\boldsymbol{U} \in \Upsilon} \Gamma_n^{lb} \tag{9}$$

where Γ_n^{lb} is a lower bound on the best SINR given by:

$$\Gamma_n^{lb} = \max_{u_n=1..U} \frac{\{G_{u_n,n}\} P_{max}}{\sigma^2 + \sum_{i \neq n}^{N} G_{u_n,i} P_{max}} \tag{10}$$

B. Channel models

For clarity of exposition we make certain assumptions on the system model. However some of these assumptions are purely technical and could be relaxed without altering the fundamental results. We assume a cellular network where APs are regularly located with cell radius R. In this sense, the cells are assumed to be circular with each base being at the center of it, although this assumption is not critical to this study (i.e. similar conclusions can be obtained for hexagonal cell etc.).

The basic channel model consists in the product between a variable representing the path loss and a variable representing the fast fading coefficient: Let $G_{u_n,i} = \gamma_{u_n,i} |h_{u_n,i}|^2$, $u_n = 1..U, i = 1..N$ be the set of power gains where $\gamma_{u_n,i}$ is a path loss coefficient and $h_{u_n,i}$ is the normalized complex fading coefficient. A generic path loss model is given by [8]:

$$\gamma_{u_n,i} = \beta d_{u_n,i}^{-\epsilon} \tag{11}$$

where β is scaling factor, ϵ is the path loss exponent (usually with $\epsilon > 2$), and $d_{u_n,i}$ is the distance between user u_n and AP i.

Note that we assume as preamble a user-to-AP assignement strategy resulting in all users being served by the AP with the smallest path loss (but of course not necessarily that giving the least fast fading). We consider in turn two channel models. In

[1] the multicell multiuser diversity gain can be seen as a generalization of the conventional multiuser diversity [1] to multicell scenarios with joint scheduling

the first one, denoted as, *symmetric network*, all users served by a given AP are assumed to be located at the same distance from that AP (equal average SNR). In the second one, denoted simply as *non-symmetric network*, the users are subject to a location dependent path loss, which will affect their chances of being selected by the scheduler (unequal average SNR).

C. Capacity scaling with large U in symmetric network

We analyze the scaling of capacity $\mathcal{C}(\boldsymbol{U}^*, \boldsymbol{P}^*)$ via the scaling of the bounds $\mathcal{C}^{lb}$ and $\mathcal{C}^{ub}$, with increasing U. We just focus on the performance in cell n, as other cells are expected to behave similarly under equal number of users U. We provide here sketches of proof. Detailed proofs are provided in the companion full length version of this paper [9].

Interestingly, for the symmetric network, we can reuse extreme value theory results [10] developped specifically in the context of single cell opportunistic beamforming [11], [12] and transposed here to the case of networks with multicell interference.

First, the following results provides insight into the interference-free scaling of SINR and capacity respectively.

1) Scaling laws for interference-free case:

Lemma 1: Let $G_{u_n,n} = \gamma_{u_n,n}|h_{u_n,i}|^2$, $u_n = 1..U, n = 1..N$, where $\gamma_{u_n,n} = \gamma_n$. Assume $|h_{u_n,n}|^2$ is Chi-quare distributed with 2 degrees of freedom ($\chi^2(2)$) (i.e. $h_{u_n,n}$ is a unit-variance complex normal random variable). Assume the $|h_{u_n,n}|^2$ are i.i.d across users. Then for fixed N and U asymptotically large, the upper bound on the SINR in cell n scales like

$$\Gamma_n^{ub} \approx \frac{P_{max}\gamma_n}{\sigma^2}\log U \tag{12}$$

Sketch of proof: This result is a reuse of a well known result for single cell opportunistic scheduling which states that the maximum of U $\chi^2(2)$ random variables behaves like $\log U$ for large U. See for instance [11].

Theorem 1: Let $G_{u_n,n} = \gamma_{u_n,n}|h_{u_n,n}|^2$, $u_n = 1..U, n = 1..N$, where $\gamma_{u_n,n} = \gamma$. This means that all cells are assumed to enjoy an identical link budget. Assume $|h_{u_n,n}|^2$ is Chi-quare distributed with 2 degrees of freedom ($\chi^2(2)$). Assume the $|h_{u_n,n}|^2$ are i.i.d across users. Then for fixed N and U asymptotically large, the average of the upper bound on the network capacity scales like

$$E(\mathcal{C}^{ub}) \approx \log\log U \tag{13}$$

where the expectation is taken over the complex fading gains.

Sketch of proof: This result is a reuse of results for single cell opportunistic scheduling found in [11], [12] among others. See e.g. [12], Theorem 1.

2) Scaling laws for full-powered interference case: We now turn to the behavior of interference limited networks by exploring the lower bounds given for SINR and capacity.

Lemma 2: Let $G_{u_n,i} = \gamma_{u_n,i}|h_{u_n,i}|^2$, $u_n = 1..U, n = 1..N$, where $\gamma_{u_n,n} = \gamma_n$, $\gamma_{u_n,i} = \beta d_{u_n,i}^{-\epsilon}$ for $i \neq n$. Assume $|h_{u_n,i}|^2$ is Chi-quare distributed with 2 degrees of freedom ($\chi^2(2)$). Assume the $|h_{u_n,i}|^2$ are i.i.d across users, cells. Then for fixed N and U asymptotically large, the lower bound on the SINR in cell n scales like

$$\Gamma_n^{lb} \approx \frac{P_{max}\gamma_n}{\sigma^2}\log U \tag{14}$$

Sketch of proof: To obtain this result, one uses the fact that users in cell n are served by their closest AP. An upper bound on the interference power then given by $\sum_{i\neq n}^{N}\beta R^{-\epsilon}|h_{u_n,i}|^2 P_{max}$. This gives a further lower bound on Γ_n^{lb} given by

$$\Gamma_n^{lb} \geq \Gamma_n^{lb2} = \max_{u_n=1..U} \frac{\gamma_{u_n,n}|h_{u_n,n}|^2 P_{max}}{\sigma^2 + \sum_{i\neq n}^{N}\beta R^{-\epsilon}|h_{u_n,i}|^2 P_{max}} \tag{15}$$

$$\Gamma_n^{lb2} = \max_{u_n=1..U}\gamma_{u_n,n}P_{max}\omega_{u_n,n} \tag{16}$$

where $\omega_{u_n,n}$ denotes the normalized SINR:

$$\omega_{u_n,n} = \frac{|h_{u_n,n}|^2}{\sigma^2/P_{max} + \beta R^{-\epsilon}\sum_{i\neq n}^{N}|h_{u_n,i}|^2} \tag{17}$$

Given $\gamma_{u_n,n} = \gamma_n$ is constant, the scaling law of Γ_n^{lb2} can be obtained by exploiting the similarity between $\omega_{u_n,n}$ and the SINR expression in the single cell opportunistic beamforming problem reported in ([12], Lemma 4). This gives $\Gamma_n^{lb2} \approx P_{max}\gamma_n \log U/\sigma^2$. Finally Γ_n^{lb} is bounded above and below by two expressions (respectively the interference-free Γ_n^{ub} and Γ_n^{lb2}) with same scaling law and must satisfy itself the same scaling law.

Theorem 2: Let $G_{u_n,i} = \gamma_{u_n,i}|h_{u_n,i}|^2$, $u_n = 1..U, n = 1..N$, where $\gamma_{u_n,n} = \gamma$, $\gamma_{u_n,i} = \beta d_{u_n,i}^{-\epsilon}$ for $i \neq n$. Assume $|h_{u_n,i}|^2$ is Chi-quare distributed with 2 degrees of freedom ($\chi^2(2)$). Assume the $|h_{u_n,i}|^2$ are i.i.d across users, cells. Then for fixed N and U asymptotically large, the average of the lower bound on the network capacity scales like

$$E(\mathcal{C}^{lb}) \approx \log\log U \tag{18}$$

Sketch of proof: Assuming the result in Lemma 2 holds, this result can be proved in similar way as ([12], Theorem 1).

From the bounding results and from theorems 1 and 2 above, the following conclusion is trivially obtained:

Theorem 3: Let $G_{u_n,i} = \gamma_{u_n,i}|h_{u_n,i}|^2$, $u_n = 1..U, n = 1..N$, where $\gamma_{u_n,n} = \gamma_n$, $\gamma_{u_n,i} = \beta d_{u_n,i}^{-\epsilon}$ for $i \neq n$. Assume $|h_{u_n,i}|^2$ is Chi-quare distributed with 2 degrees of freedom ($\chi^2(2)$). Assume the $|h_{u_n,i}|^2$ are i.i.d across users, cells. Then for fixed N and U asymptotically large, the average of the network capacity with optimum power control and scheduling scales like

$$E(\mathcal{C}(\boldsymbol{U}^*, \boldsymbol{P}^*)) \approx \log\log U \tag{19}$$

Theorems 1 and 2 suggest that, in a symmetric multi-cell network, the capacity obtained with optimal multicell scheduling in both an interference-free environment and an environment with full interference power have identical scaling laws in $\log\log U$. This result bears analogy to the results by [12] which indicate that in a single cell broadcast channel with random beamforming and opportunistic scheduling, the degradation caused by inter-beam interference tends becomes negligible when the number of users becomes large. Here the multicell interference becomes negligible because the optimum scheduler tends to select users who have both large instantaneous SNR and *small* interference power.

Furthermore, since a system where the full power is allocated at all transmitters is asymptotically optimal (in terms of scaling law), a simple procedure based on (9) is also asymptotically optimal. Interestingly, this procedure is completely distributed as only local CSI is exploited by each user and fedback to its serving AP only. The SINRs are computed during a preamble phase where all APs are asked to transmit pilot or data symbols at full power. These results come as a complement to previously reported findings [13] which propose a near optimal power allocation scheme, for fixed number of users, where a fraction of the transmitters are selected to be turned off while the rest operate at full power. It was observed experimentally there that the fraction of off cells would go to zero when the number of users grows large. The analysis of scaling of capacity provides a theoretical justification to this intuitevly appealing behavior.

We now turn to a non symmetric network where users can experience different average SNR values depending on their position and conduct a similar analysis. However we will see that different scaling rates are obtained compared with the symmetric network case.

D. Capacity scaling with large U in non-symmetric network

We assume the path loss is determined by the user's distance to the emitting AP, both serving and interfering. We consider a uniform distribution of the population in each cell. Thus $d_{u_n,n}$ is a random variable with non-uniform distribution $f_D(d) = 2d/R^2$, in $[0, R]$. Further, this random variable can be considered i.i.d across users and cells, if users in each cell are dropped randomly in the network. Assuming $R = 1$ for normalization, the distribution of $\gamma_{u_n,n} = \beta d_{u_n,i}^{-\epsilon}$ is given by

$$f_\gamma(g) = \frac{2}{\epsilon}(\frac{g}{\beta})^{-\frac{2}{\epsilon}}\frac{1}{g} \quad \text{with} \quad g \in [\beta, \infty) \tag{20}$$

and zero elsewhere. In order to get upper and lower bounds on performance, we are interested in the behavior of the extreme value of the products of random variables $\gamma_{u_n,n}|h_{u_n,n}|^2$ and $\gamma_{u_n,n}\omega_{u_n,n}$ respectively, where $\omega_{u_n,n}$ is defined as per (17).

The distribution of $\gamma_{u_n,n}$ shown in (20) is remarkable in that it differs stronly from fast fading distributions, due to its *heavy tail* behavior. Heavy tail is also observed in other *large scale* fading models such as log normal shadowing for instance. The distribution of $\gamma_{u_n,n}$ can be classified as *regularly varying* [10] (see below for the definition). An interesting aspect of regularly varying distributed random variable (R.V.) is that they are stable with respect to multiplication with independent R.V. with finite moments (such as the distribution of $|h_{u_n,n}|^2$ or that of $\omega_{u_n,n}$), as pointed out by the following theorem shown by Breiman [14]:

Theorem 4: Let X and Y two independent R.V. such that X is regularly varying with exponent $-a$, i.e. the cdf of X, $F_X(x)$, is such that $\frac{1-F_X(x)}{1-F_X(tx)} \to t^a$ when $t \to \infty$. Assuming Y has finite moment $E(Y^a)$, then the tail behavior of the product $Z = XY$ is governed by:

$$1 - F_Z(z) \to E(Y^a)(1 - F_X(z)) \quad \text{when } z \to \infty \tag{21}$$

The idea behind this theorem is that when multiplying a heavy tail regularly varying R.V. with another one with finite moment, one obtains a heavy tail R.V. whose tail behavior is similar to the first one, up to a scaling. We now apply this result to $X = \gamma_{u_n,n}$ and Y given by $Y = |h_{u_n,n}|^2$ for the interference free case and $Y = \omega_{u_n,n}$ for the full-powered interference case, respectively. In both cases, the tail behavior of $Z = XY$ can then be characterized by the following lemma:

Lemma 3: Let $X = \gamma_{u_n,n}$ be a R.V. with distribution given by (20). Let Y be an independent R.V. such that $E(Y^{\frac{2}{\epsilon}}) < \infty$. Then the tail of $Z = XY$ is governed by:

$$1 - F_Z(z) \to E(Y^{\frac{2}{\epsilon}})\left(\frac{\beta}{z}\right)^{\frac{2}{\epsilon}} \quad \text{when } z \to \infty \tag{22}$$

Proof: A direct application of Theorem 4 with X having distribution shown in (20).

Under the lemma above, Z is seen to be regularly varying with exponent $-\frac{2}{\epsilon}$. Following [10], Z can be classified to be of *Frechet* type, for which extreme value theory results exist.

1) Scaling law for interference-free case: We can now derive scaling laws for the interference-free case in a non-symmetric network:

Theorem 5: Let $h_{u_n,n}$, $u_n = 1..U$ be i.i.d Gaussian distributed unit-variance random variables. Assuming that $\gamma_{u_n,n}$ is i.i.d., distributed as per (20), for $n = 1..N$. Then for fixed N and U asymptotically large, the interference-free SNR scales like:

$$\Gamma_n^{ub} \approx \kappa U^{\frac{\epsilon}{2}} \tag{23}$$

where $\kappa > 0$ is a scaling factor which depends on β, ϵ, P_{max} and N.

Sketch of proof: From Lemma 3 we have that, given $X = \gamma_{u_n,n}$ and $Y = |h_{u_n,n}|^2$, $Z = XY$ has a regularly varying distribution with exponent $-\frac{2}{\epsilon}$. Then we invoke Gnedenko's theorem [15] given in appendix here. We find that $a_U = \beta E(Y^{\frac{2}{\epsilon}})^{\frac{\epsilon}{2}} U^{\frac{\epsilon}{2}}$ where a_U is defined in the appendix. From this, the scaling rate of (23) is derived.

From the scaling of SNR above, we can infer that the upper bound on capacity will behave like:

$$E(\mathcal{C}^{ub}) \approx \frac{\epsilon}{2}\log U \quad \text{for large } U \tag{24}$$

2) Scaling law for full-powered interference case: We can derive the scaling laws for the lower bounds of SINR and capacity by following a strategy similar to Sec.IV-D.1 but simply replacing the R.V. $|h_{u_n,n}|^2$ by the R.V. $\omega_{u_n,n}$ which also has bounded moments. We obtain the following result:

Theorem 6: Let $h_{u_n,i}$, $u_n = 1..U, i = 1..N$ be i.i.d Gaussian distributed unit-variance random variables. Assuming that $\gamma_{u_n,n}$ is i.i.d, distributed as per (20), for $n = 1..N$. Then for fixed N and U asymptotically large, the lower bound on SINR scales like:

$$\Gamma_n^{lb} \approx \kappa' U^{\frac{\epsilon}{2}} \tag{25}$$

where $\kappa' > 0$ is a scaling factor which depends on β, ϵ, P_{max} and N.

Sketch of proof: We use the same proof as for Theorem 5, with $X = \gamma_{u_n,n}$ but this time $Y = \omega_{u_n,n}$.

Finally, from Theorem 6, we infer that the upper bound on capacity for a non-symmetric network behaves also like:

$$E(\mathcal{C}^{lb}) \approx \frac{\epsilon}{2} \log U \tag{26}$$

As in the case of the symmetric network, the results above (24) and (26) suggest that multicell interference, no matter how strong, does not affect the scaling of the network capacity, if enough users exist *and* capacity optimal scheduling is applied. Furthermore, for a network with path loss-based average SNR, the maximum capacity behaves like

$$\mathcal{C}(\boldsymbol{U}^*, \boldsymbol{P}^*) \approx \frac{\epsilon}{2} \log U \tag{27}$$

Additionally, in this case too, a suboptimal but fully distributed resource allocation based on constant (full) power transmission at all transmitters and scheduling policy based on (9) will actually result in the best possible scaling law of capacity for the network. Finally we obtain a much greater rate growth than in the case of the symmetric network. This is due to the amplified multiuser diversity gain due to the presence of unequal path loss.

V. Numerical evaluation

We validate the asymptotic behavior of the multicell sum rate when U grows large with Monte Carlo simulations. We use a small network with $N = 3$ cells with both the symmetric distribtion of average SNR and the path-loss based average SNR with unit cell radius and $\beta = 1/16$, $\epsilon = 4$, $P_{max} = 1$, $\sigma^2 = 0.02$. iid flat Rayleigh fading is considered. In both cases we compute the upper and lower bound on capacity (see Fig.2 and 3 and observe the identical rate growth, further suggesting that the capacity obtained with exhaustive user and power level selection also has the same growth rate (in $\log U$ for the symmetric network and in $\log \log U$ for the non symmetric network.

VI. Conclusions

We present an extreme value theoretic analyis of network capacity for maximum sum rate multicell power allocation and user scheduling. We derive scaling laws of capacity when the number of users per cell grows large, both in cases where the users have same average SNR and path loss dependent SNR. We show that in both cases, 1-the effect of intercell interference tends to be negligible asymptotically, and 2-should intercell interference be considered, an asymptotically optimal allocation procedure is given based on full power allocation at all transmitters, which is furthermore completely distributed. We show that the growth rate of capacity is much faster in the case of a system with unequal distance-based average SNR.

Appendix

The following theorem is due to Gnedenko [15] (1943) and states the following property for regularly verying distributions:

Theorem 7: Let Z_i an i.i.d random process. Then Z_i has a regularly varying distribution with exponent a if and only if

$$\lim \Pr\{\max_{i=1..U} Z_i \leq a_U x\} = e^{-x^{-a}} \quad \text{when } U \to \infty \tag{28}$$

where a_U is a sequence such that $1 - F_Z(a_U) = \frac{1}{U}$.

References

[1] R. Knopp and P. Humblet, "Information capacity and power control in single-cell multiuser communications," in *Proc. IEEE Int. Conf. on Comm.*, Seattle, June 1995, pp. 331–335.

[2] G. J. Foschini and Z. Miljanic, "A simple distributed autonomous power control algorithm and its convergence," *IEEE Trans. Veh. Technol.*, vol. 42, 1993.

[3] G. J. Foschini, H. Huang, K. Karakayali, R. Valenzuela, and S. Venkatesan, "The value of coherent base station coordination," in *Proc. Conference on Information Sciences and Systems (CISS)*, 2005.

[4] E. Altman, T. Boulogne, R. El-Azouzi, T. Jimenez, and L. Wynter, "A survey on networking games in telecommunication," *Comput. Oper. Res.*, vol. 33, pp. 286–311, Feb. 2006.

[5] D. Gesbert, S. Kiani, G. Oien, and A. Gjendemsjo, "Adaptation, coordination, and distributed resource allocation in interference-limited wireless networks," *Proceedings of the IEEE*, to appear 2007.

[6] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. New York: Wiley, 1991.

[7] M. Ebrahimi, M. Maddah-Ali, and A. Khandani, "Throughput scaling in decentralized single-hop wireless networks with fading channels," *IEEE Trans. on Information Theory*, submitted Oct. 2006.

[8] W. C. Y. Lee, *Mobile Communications Engineering*. New York: McGraw Hill, 1982.

[9] D. Gesbert, S. Kiani, and M. Kountouris, "Joint power control and user scheduling in multicell wireless networks: capacity scaling laws," *in preparation*, 2006.

[10] H. A. David and H. N. Nagaraja, *Order Statistics, third ed.* New York: John Wiley and Sons, 2003.

[11] P. Viswanath, D. N. Tse, and R. Laroia, "Opportunistic beamforming using dumb antennas," *IEEE Transactions on Information Theory*, vol. 48, no. 6, pp. 1277–1294, June 2002.

[12] M. Sharif and B. Hassibi, "On the capacity of MIMO broadcast channel with partial side information," *IEEE Trans. Inform. Theory*, vol. 51, no. 2, pp. 506–522, Feb. 2005.

[13] S. G. Kiani, G. E. Øien, and D. Gesbert, "Maximizing multi-cell capacity using distributed power allocation and scheduling," to appear in Proc. IEEE Wireless Communications and Networking Conference (WCNC'07), Hong Kong, China, March 2007.

[14] L. Breiman, "On some limit theorems similar to the arc-sin law," in *Theory of Probability and its applications, 10.*, 1965, pp. 323–331.

[15] B. Gnedenko, "Sur la distribution limite du terme maximum d'une serie aleatoire," *Annales Mathematiques*, vol. 44, pp. 423–453, 1943.

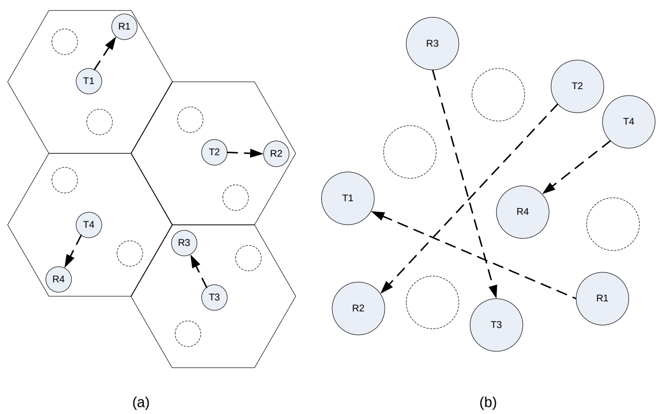

Fig. 1. Snapshot of network model, with $N = 4$ interfering pairs of transmitters T_i and receivers R_i. The cellular model (a) and the single-hop peer-to-peer or adhoc model (b) give rise to equivalent mathematical models. Dashed circles refer to silent users while solid circles refer to access points or users selected by the scheduler.

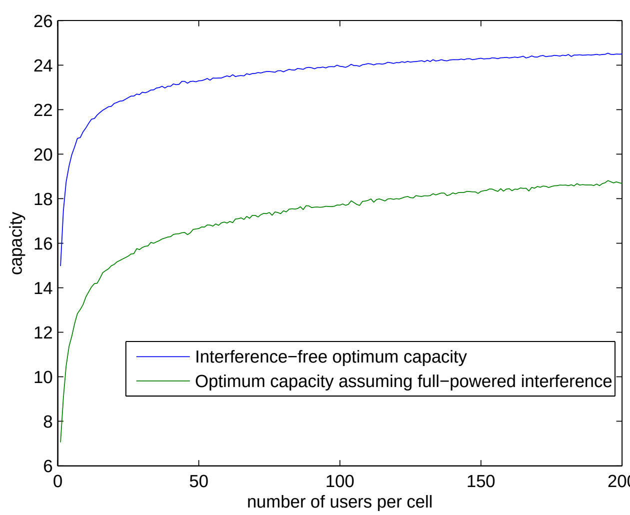

Fig. 2. Scaling of upper and lower bounds of capacity versus U for a symmetric network ($N = 3$)

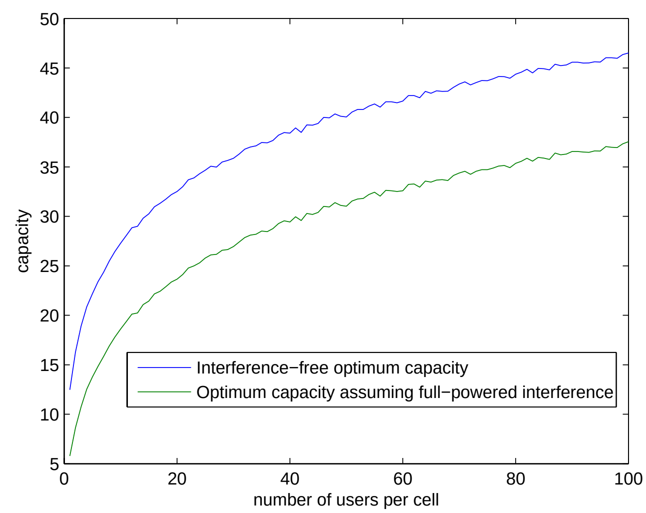

Fig. 3. Scaling of upper and lower bounds of capacity versus U for a non-symmetric network (unequal average SNR) ($N = 3$).