DRB2.4 - Report on Eigen-mode/singular-mode characteristics and capacity of the propagation channel

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1 Document Control

1.1 Abstract

This report provides a description of the work performed in a joint activity involving the partners Eurecom and Aalborg University. The aim of the work was to investigate the eigen-mode/singular-mode characteristics and capacity of the propagation channel. These investigations were conducted by theoretical investigations combined with experimental study. The experimental study aimed at extracting the above information from measurement data. In the theoretical study, the information were gathered by means of theoretical analysis using the stochastic models derived within the framework of the joint work planned in WPRB1. The theoretical and experimental results are then compared. The expected outcome of this methodology was twofold: (1) a better understanding of the propagation mechanisms that boost or annihilate the number of channel degrees of freedom and the capacity of the propagation channel and (2) a validation of the above stochastic model regarding its ability to reproduce eigen-mode/singular-mode characteristics and capacity close to those observed in a real world scenario.

1.2 Keywords

Ultra-wideband, channel, entropy, capacity.

2 Introduction

This report provides a description of the work performed in a joint activity involving the partners Eurecom and Aalborg University. The aim of the work was to investigate the eigen-mode/singular-mode characteristics and capacity of the propagation channel. These investigations were conducted by theoretical investigations combined with experimental study. The experimental study aimed at extracting the above information from measurement data. In the theoretical study, the information were gathered by means of theoretical analysis using the stochastic models derived within the framework of the joint work planned in WPRB1. The theoretical and experimental results are then compared. The expected outcome of this methodology was twofold: (1) a better understanding of the propagation mechanisms that boost or annihilate the number of channel degrees of freedom and the capacity of the propagation channel and (2) a validation of the above stochastic model regarding its ability to reproduce eigen-mode/singular-mode characteristics and capacity close to those observed in a real world scenario.

In particular, in this work, a unified framework for Ultra-Wideband channel modeling based on the maximum entropy approach is provided. For a given set of constraints, a consistent model which takes into account the information at hand is obtained. Two cases are considered: channel power knowledge and knowledge of the covariance matrix. The channel power delay spectrum is also derived and the impact of the channel bandwidth is assessed through information theoretic considerations. The information variation slope with respect to the bandwidth is also studied, based on wideband measurements performed at Institut Eurecom

3 Technical Content

The main results of the work illustrated in this deliverable have been presented recently in the paper [1].

3.1 Maximum Entropy Modeling (MEM)

The problem of modelling wideband channels is crucial for the efficient design of wireless systems. Indeed, the wireless channel suffers from constructive/destructive interference signaling and therefore yields a randomized channel for which one has to attribute a joint probability distribution for the channel frequency response. In this contribution, we would like to provide some theoretical grounds to model the wideband channel based on a given state of knowledge. In other words, knowing only certain things related to channel (power, measurements), how to translate that information into a model for the channel? This question can be answered in light of the Bayesian probability theory [8] and the principle of maximum entropy. The principle of maximum entropy is at present the clearest theoretical justification in conducting scientific inference: we do not need a model, entropy maximization creates a model for us out of the information available. Choosing the distribution with greatest entropy avoids the arbitrary introduction or assumption of information that is not available. Note that this approach has been successfully used in spectrum analysis [9] and signal interpolation problems [2], [3].

3.1.1 MEM for channel power knowledge

Suppose in the following that the modeler has no knowledge about the frequency response of the wideband channel and would like, with this limited knowledge, to determine the number of clusters in the environment. The modeler knows however that the channel carries some power P and is stationary during the channel modelling phase.

Let $\{h_i\}_{i\in\mathbb{Z}}$ be the sequence of samples at frequencies $i\delta_f$ (δ_f is the frequency resolution) of the channel frequency response. In this case, the spectral autocorrelation function is defined as:

$$R(k) = E[h_i h_{i+k}^*] \tag{1}$$

and the power delay spectrum is defined as

$$P(\tau) = \sum_{k=-\infty}^{\infty} R(k) e^{-j2\pi\tau k}, \tag{2}$$

where $\tau = \frac{\hat{\tau}}{T_s}$ is the normalized delay and $\hat{\tau}$ is the delay in seconds.

In the following, we suppose that $\int_{-\frac{\tau_{max}}{2}}^{\frac{\tau_{max}}{2}} P(\tau)d\tau$ is the power of the channel equal to P and τ_{max} is the maximum delay. The modeler would like to take into account the power constraint without additional information that is not available. Maximizing the entropy of the process guarantees such a setting as one finds the sequence of autocorrelations that make the impulse response as white as possible. In other words, such an extrapolation places the least amount of structure in the channel.

For a Gaussian random process, with power delay spectrum $P(\tau)$, the entropy H is given by

$$H = \log(\pi e) + \int_{-\frac{1}{2}}^{\frac{1}{2}} \log(P(\tau) + \epsilon)d\tau, \tag{3}$$

where ϵ is an arbitrary small constant ($\epsilon \geq 0$) used to regularize the non-regular Gaussian process (h_i).

The modeler would like to maximize H under the constraint that the received power in a given delay interval is known. This statement can simply be expressed if one tries to maximize the following expression using Lagrange multipliers with respect to $R(k)$:

$$C = H - \mu_0(\int_{-\frac{\tau_{max}}{2}}^{\frac{\tau_{max}}{2}} P(\tau)d\tau - P) \tag{4}$$

$$\begin{aligned}
\frac{\partial C}{\partial R(k)} &= \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{P(\tau)+\epsilon}\frac{\partial P(\tau)}{\partial R(k)}d\tau - \mu_0 \int_{-\frac{\tau_{max}}{2}}^{\frac{\tau_{max}}{2}} e^{-j2\pi\tau k}d\tau, \\
&= \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{P(\tau)+\epsilon} e^{-j2\pi\tau k}d\tau + \frac{\mu_0}{j2\pi k}\left[e^{-j2\pi k\tau}\right]_{-\frac{\tau_{max}}{2}}^{\frac{\tau_{max}}{2}}, \\
&= \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{P(\tau)+\epsilon} e^{-j2\pi\tau k}d\tau - \mu_0\tau_{max}\mathrm{sinc}(k\pi\tau_{max}), \\
&= 0.
\end{aligned}$$

Let $Q(\tau) = \frac{1}{P(\tau)+\epsilon}$ and $q_k = \int_{-1/2}^{1/2} Q(\tau)e^{+j2\pi k\tau}d\tau$. We have

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} Q(\tau)e^{j2\pi\tau k}d\tau = \mu_0\tau_{max}\mathrm{sinc}(k\pi\tau_{max}) \tag{5}$$

$$\Leftrightarrow q_k = \mu_0\tau_{max}\mathrm{sinc}(k\pi\tau_{max}). \tag{6}$$

Thus, $Q(\tau) = \sum_{k=-\infty}^{\infty} \mu_0\tau_{max}\mathrm{sinc}(k\pi\tau_{max})e^{-j2\pi k\tau}$, which is constant for all $\frac{-\tau_{max}}{2} \leq \tau \leq \frac{\tau_{max}}{2}$. As a consequence,

$$P(\tau) = \begin{cases} \frac{P}{\tau_{max}}; & \frac{-\tau_{max}}{2} \leq \tau \leq \frac{\tau_{max}}{2} \\ 0 \quad ; & \text{elsewhere}. \end{cases}$$

and $R(k) = P\text{sinc}(k\pi\tau_{max})$ for all k. $P(\tau)$ is constant and vanishes on $[-\frac{1}{2}, +\frac{1}{2}]/[-\frac{\tau_{max}}{2}, +\frac{\tau_{max}}{2}]$. In other words if there is no knowledge except the maximum delay, the model gives an infinite number of clusters and the power is equally split across the different clusters. The methodology can be easily extended if the modeler has knowledge of the bandwidth (which determines the number of correlation coefficients $R(k)$) used.

3.1.2 MEM for covariance channel knowledge

In the following, we suppose that the modeler has knowledge (through measurements) of a finite number of frequency autocorrelation coefficients $R(k)$. The number of coefficients is determined by the measured bandwidth as well as the measurement resolution. Based on this knowledge, the modeler would like to derive a wideband model taking into account account the previous constraints and not more, i.e, the modeler would like to extrapolate the missing autocorrelation coefficients for deriving the power delay spectrum function. Using the same methodology as Section 3.1.1, the following theorem due to Burg [10] can be obtained:

Theorem 1 *The maximum entropy rate stochastic process* $\{h_i\}_{i\in\mathbb{Z}}$ *that satisfies the constraints*

$$E[h_i h_{i+k}] = R(k), \;\; k = 0, 1, ..., N, \;\; for\ all\ i, \tag{7}$$

is the N-th order Gauss-Markov process of the form

$$h_i = -\sum_{k=1}^{N} a_k h_{i-k} + Z_i, \tag{8}$$

where the Z_i are i.i.d. $\sim N(0, \sigma^2)$ and $a_1, a_2, ..., a_N, \sigma^2$ are chosen to satisfy Equation (7).

Remark: The theorem does not assume the h_i to be wide-sense stationnary.

A process satisfying (8) is also called autoregressive of order N ($AR(N)$). The coefficients $a_1, a_2, ..., a_N, \sigma^2$ are obtained by solving the Yule-Walker equations

$$R(0) = -\sum_{k=1}^{N} a_k R(-k) + \sigma^2, \tag{9}$$

$$R(l) = -\sum_{k=1}^{N} a_k R(l-k), \;\; l = 1, 2, ..., N. \tag{10}$$

Fast algorithms such as the Levinson and Durbin algorithm have been devised which exploit the special structure of these equations to efficiently calculate the coefficients $a_1, a_2, ..., a_N$ from the covariance samples $R(0), ..., R(N)$. The power delay spectrum of the Nth order Gauss-Markov process (8) is given by

$$P(\tau) = \frac{\sigma^2}{\left|1 + \sum_{k=1}^{N} a_k e^{-i2\pi k\tau}\right|^2}. \tag{11}$$

In general, from a finite set of N frequency measurements $[h_1^l, ...h_N^l]$ over a bandwidth of $N\delta_f$ (l is the lth channel realization), there are many ways to estimate the spectral autocorrelation coefficients. In the following, the sample autocorrelation function is defined as:

$$\hat{R}^N(k) = \frac{1}{L}\frac{1}{N-k}\sum_{l=1}^{L}\sum_{i=1}^{N-k} h_i^l (h_{i+k}^l)^*, \quad k \geq 0.$$

For a given N, we estimate and determine the autocorrelation function $\hat{R}^N(k)$, the autocorrelation coefficients $\hat{a}_k^{(N)}$ and the power delay spectrum $\hat{P}^N(\tau)$. As a consequence, the estimated entropy is given by:

$$\hat{H}^N = \log(\pi e) + \int_{-\frac{1}{2}}^{\frac{1}{2}} \log \frac{\sigma^2}{\left|1 + \sum_{k=1}^{N} a_k^{(N)} e^{-i2\pi k\tau}\right|^2} d\tau. \tag{12}$$

The roots of the power delay spectrum (11) determine the number of significant clusters. Practically, although the roots may exist, some may be not significant and therefore unnecessary to model. In order to assess the number of significant modelling coefficients, the information $\hat{H}^N$ contained in the process with respect to N need to be analysed.

3.2 Measurement Description

3.2.1 Measurement setup

The measurements were carried out in the Mobile Communications Laboratory of Institut Eurecom, which is a typical lab environment, rich in reflective and diffractive objects. The measurement device used is a wideband Vector Network Analyzer (VNA) which allows complex transfer function parameter measurements in the frequency domain, extending from 10 MHz to 20 GHz. This instrument has low inherent noise, less than -110 dBm for a measurement bandwidth of 10 Hz, and high measurement speed, less than 0.5 ms/point. The maximum number of equally-spaced frequency samples (amplitudes and phase) per measurement is equal to 2001. The measurement data were acquired and controlled remotely using RSIB protocol over Ethernet permitting off-line signal processing and instrument control in MATLAB.

In order to perform truly wideband measurements with sufficient resolution, several bands was concatenated by using consecutive measurements. In this contribution, measurements were performed from 3 GHz to 9 GHz by concatenating three groups of 2001 frequency samples per 2 GHz sub-bands (3-5 GHz,5-7 GHz,7-9 GHz). This yields a 1 MHz spacing between the frequency samples. Systematic and frequent calibration (remotely controlled) was employed to compensate the undesirable frequency-dependent attenuation factors that might affect the collected data. The wideband antennas employed in the measurements are omnidirectional in the vertical plane and have an approximate bandwidth of 7.5 GHz (varying from 3.1 GHz to 10 GHz). They were not perfectly matched across the entire band, with a Voltage Standing Wave Radio (VSWR) varying from 2 to 5.

3.2.2 Measurement environment

The data analyzed in Section 3.3 were collected in a Line of Sight (LoS) setting with measurements performed at spatially different locations. The experiment area is set by fixing the transmitting antenna on a mast at 1m above the ground and close to the VNA. The receiving antenna moves then to different locations on a horizontal linear grid of side 50 cm in steps of 5 cm. The transmitter antenna's height was varied by 5 cm up to 20 cm after completion of the measurements at various receiver positions. The transmitter and the receiver were separated by a distance of six meters.

3.3 Results

In the following, the scaling of channel uncertainty with respect to the bandwidth is analyzed. In Fig. 1, the variation of $\hat{H}^N$ is plotted for the LoS case as well as the Gaussian case (zero mean i.i.d frequency samples are generated in this case) versus $N\delta_f$ (the frequency band is $[3000, 3000 + N\delta_f]$ MHz). As one can observe, the channel uncertainty decreases with bandwidth. However, in the Gaussian case, additional information provided by the frequency samples does not lower the uncertainty as the samples are completely independent. Remarkably, the results show that in UWB settings and for a given channel representation complexity (here, the slope of the entropy), there is an optimum number of parameters to be chosen. In other words, AR modelling based on a limited number of parameters is adequate. In this respect, we have plotted in Fig. 2 and Fig. 3 the power delay spectrum based on 500 MHz and 6 GHz measurements. The results show that with increasing bandwidth, one is certainly able to capture the small variations but a great amount of the information is already included in the 500 MHz band. Note that the delay of 20 nanoseconds corresponds to the 6 meter distance between the transmitter and the receiver. Moreover, in both cases, the AR model fits the measured power delay spectrum response.

4 Conclusions

The work presented in this deliverable has been clearly defined since its beginning (see the Planning of Project WPRB.2 written at T0+12). It has resulted in a joint

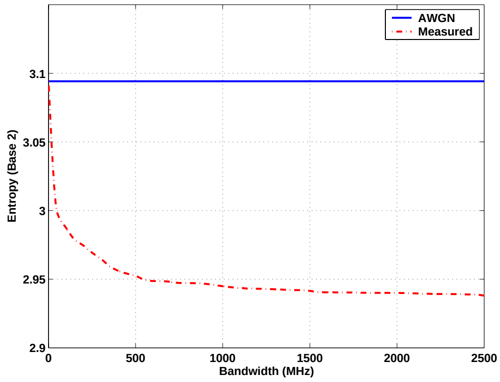

Figure 1: Entropy variation with respect to the bandwidth.

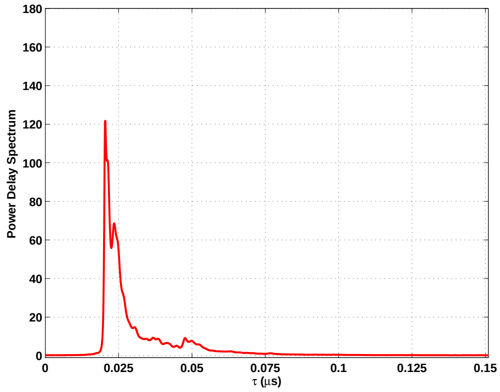

Figure 2: Power delay spectrum with 500 MHz bandwidth.

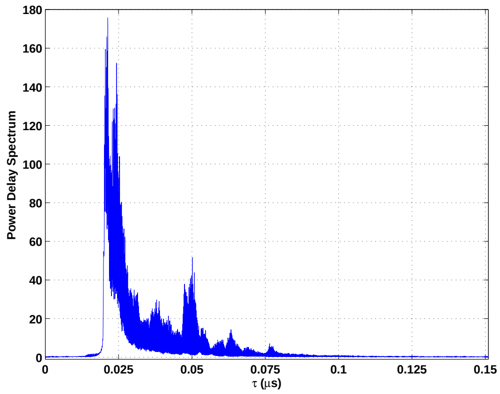

Figure 3: Power delay spectrum with 6 GHz bandwidth.

publication from the partners Eurecom and Aalborg University [8], whose contents have been illustrated in this document.

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