Multi-Polarized MIMO Communications: Channel Model, Mutual Information and Array Optimization

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***Abstract*— This paper considers the use of $n_r \times n_t$ dual-polarized arrays for improving the mutual information of MIMO communications. We first develop an elegant model of the multi-polarized MIMO channel, which allows to account for spatial correlation and depolarization effects. The major advantage of the model lies in its analytical expression, whose parameters have a clear physical meaning. In a second part, we investigate the ergodic mutual information of multi-polarized schemes, and derive a condition upon which multi-polarized transmissions offer a higher capacity than equivalent uni-polarized schemes using the same number of antennas under the same overall array dimensions.**

I. INTRODUCTION

Multiple-Input Multiple-Output (MIMO) broadband wireless communication systems are now becoming part of future standards. However, many investigations have shown that antenna spacings of at least half a wavelength at the customer premise equipment and ten wavelengths at the base station are typically required for achieving significant MIMO gains. As as consequence, using arrays of co-located orthogonally-polarized antennas appears as a space- and cost-effective alternative [1], [2]. It is intuitively expected that orthogonal polarizations offer a complete separation between individual channels, fully canceling both transmit and receive correlations. At the same time, depolarization mechanisms reduce the receive energy, decreasing the average signal-to-noise ratio (SNR).

Despite a number of recent studies focusing on spatial channel models, only a limited number of papers have addressed the polarization issue [1], [3]–[8], theoretically or experimentally, mostly because the (de-)coupling effect between orthogonal polarizations is a complex mechanism.

In this paper, we deal with multi-polarized $n_r \times n_t$ systems, i.e. both the transmit and receive arrays are made of $n_t/2$ and $n_r/2$ dual-polarized spatially separated sub-arrays (with orthogonal polarizations). We compare such schemes to equivalent uni-polarized systems, keeping the number of antennas (or RF chains) equal to n_t and n_r. In Section II, we review a few analytical models for dual-polarized channels, and propose a simple model combining the effects of space and polarization separations. In Section III, we compare uni-polarized to multi-polarized communications under a mutual information perspective, and derive simple criteria at high and arbitrary SNR.

II. MULTI-POLARIZED CHANNEL MODELING

The use of antennas with different polarizations may lead to power and correlation imbalance between the elements of the channel matrix. For 2×2 channels, the channel matrix corresponds to a system for which both the transmit and receive arrays are made of two antennas, co-localized or not, with orthogonal polarizations. The polarizations at both ends do not need to be identical. For example, the transmit polarization scheme may be vertical-horizontal (denoted as VH), while the received scheme is chosen as slanted $\big(\pm 45$ degrees).

To account for depolarization in analytical representations, we must consider depolarization caused by the non-ideal antennas as well as by the scattering medium. Regarding the first effect, it is evident that the cross-polar discrimination (XPD) of the antennas is easily included in a physical model by means of the cross-polar antenna pattern. Analytically, this can be approximated on the average by a scalar antenna depolarization factor. The latter is used to build an antenna depolarization matrix, which pre- or post-multiplies the channel matrix (similarly to a coupling matrix). In what follows, we assume that antenna cross-polar coupling is negligible, i.e. that antenna XPDs at both ends are infinite, and we focus on Rayleigh channels.

In that case, the channel matrix is denoted as $\mathbf{H}_\times$ and should account for three mechanisms:

- the spatial correlation arising from the finite spacing between the antennas (if dual-polarized antennas are co-located, this correlation is equal to one),
- the gain imbalance between the various co- and cross-polar components,
- the (de)correlation between all pairs of co- and cross-polar antennas arising only from the polarization difference (i.e. for co-located dual-polarized antennas).

A. Dual-Polarized Rayleigh Channels

In a first approach [9], [10], $\mathbf{H}_\times$ is decomposed by isolating the impact of depolarization on the channel gains, yielding

$$\mathbf{H}_\times = \big|\mathbf{X}\big| \odot \mathbf{H}^{'}, \tag{1}$$

where $\big|\cdot\big|$ is the element-wise absolute value and $\odot$ is the Hadamard product. Naturally, $\mathbf{X}$ depends on the polarization scheme. For a slanted-to-slanted scheme (± 45 degrees at both Tx and Rx), it is naturally given by

$$|\mathbf{X}_{\pm 45^\circ \rightarrow \pm 45^\circ}| = \begin{bmatrix} 1 & \sqrt{\chi} \\ \sqrt{\chi} & 1 \end{bmatrix}. \quad (2)$$

where χ is the global real-valued depolarization factor ($0 < \chi < 1$) for a slanted-to-slanted scheme. What is important to notice is that $\mathbf{H}^{'}$ still includes two correlation mechanisms (space and polarization). Hence, it is generally not equal to an equivalent uni-polarized transmission matrix $\mathbf{H}$ (i.e. with the same antenna spacings, all polarizations being then identical). As a result, $\mathbf{H}^{'}$ is some hybrid matrix, whose covariance does not explicitly depend on the spatial and polarization correlations.

In a more general modeling approach, a second model explicitly separates spacing-related and polarization-related effects. The separation is thus operated based on the physical mechanisms (space versus polarization) rather than on their impact (gain versus correlation). Subsequently, the dual-polarized Rayleigh channel matrix may be rewritten as

$$\mathbf{H}_{\times} = \mathbf{H} \odot \mathbf{X}. \quad (3)$$

In (3), $\mathbf{H}$ is modeled as a uni-polarized correlated Rayleigh channel, while $\mathbf{X}$ models both the correlation and power imbalance impacts of scattering-induced depolarization. It is important to stress that $\mathbf{X}$ only models the power imbalance and the phase-shifts between the four channels, but does not contain fading. Normalized fading (i.e. with unit average power) is entirely modeled by $\mathbf{H}$.

A practical model of $\mathbf{H}$ [11] relies on the transmit and receive correlation matrices, $\mathbf{\Theta}_t$ and $\mathbf{\Theta}_r$, and is given by

$$\mathbf{H} = \mathbf{\Theta}_r^{1/2} \mathbf{H}_w \mathbf{\Theta}_t^{1/2}, \quad (4)$$

where $\mathbf{H}_w$ is the classical i.i.d. complex Gaussian matrix.

A relatively general model of the channel matrix for VH-to-VH downlink transmission [12] is given by

$$\mathrm{vec}\left(\mathbf{X}^H_{\mathrm{VH}\rightarrow\mathrm{VH}}\right) = \begin{bmatrix} 1 & \sqrt{\mu\chi}\vartheta^* & \sqrt{\chi}\sigma^* & \sqrt{\mu}\delta_1^* \\ \sqrt{\mu\chi}\vartheta & \mu\chi & \sqrt{\mu}\chi\delta_2^* & \mu\sqrt{\chi}\sigma^* \\ \sqrt{\chi}\sigma & \sqrt{\mu}\chi\delta_2 & \chi & \sqrt{\mu\chi}\vartheta^* \\ \sqrt{\mu}\delta_1 & \mu\sqrt{\chi}\sigma & \sqrt{\mu\chi}\vartheta & \mu \end{bmatrix}^{1/2} \mathrm{vec}\left(\mathbf{X}_w^H\right), \quad (5)$$

where

- μ and χ represent respectively the co-polar imbalance and the scattering XPD, and are assumed to be constant,
- σ and ϑ are the receive and transmit correlation coefficients (i.e. the correlation coefficients between VV and HV, HH and HV, VV and VH or HH and VH),
- δ_1 and δ_2 are the cross-channel correlation coefficients caused by the use of orthogonal polarizations, i.e. δ_1 is the correlation between the VV and the HH components, and δ_2 is the correlation between the VH and the HV components (both are typically low, see [8]),
- $\mathbf{X}_w$ is a 2×2 matrix whose four elements are independent circularly symmetric complex exponentials of unit amplitude, $e^{j\phi_k}$, $k = 1, \ldots, 4$, the angles ϕ_k being uniformly distributed over $[0, 2\pi)$.

In the following, we assume for simplicity that the correlation coefficients σ and ϑ between any cross-polar component (VH or HV) and any co-polar component (VV or HH) are equal to zero, although they might actually be slightly higher [8]. Furthermore, if $\delta_1 = \delta_2 = 0$, we may also write that $\mathbf{H}_\times = \left|\mathbf{X}\right| \odot \mathbf{H}_w$.

For alternative polarization schemes, the Rayleigh channel matrix is simply obtained by applying adequate rotations, as outlined in [12].

B. Multi-Polarized Rayleigh Channels

Arbitrary $n_r \times n_t$ schemes (for even values of n_t and n_r) are modeled by considering that the transmit (resp. receive) array is made of $n_t/2$ (resp. $n_r/2$) dual-polarized sub-arrays (each sub-array is identical and is made of two co-located antennas with orthogonal polarizations). In that case, the transmission between any transmit sub-array to any receive sub-array can be derived from (3) and reads as

$$\mathbf{H}_\times = h\mathbf{X}, \quad (6)$$

where h is the scalar channel representing the transmission between the locations of the considered transmit and receive sub-arrays (remember that each sub-array is made of two co-located orthogonally-polarized antennas). Hence, the global channel matrix is represented as

$$\mathbf{H}_{\times, n_r \times n_t} = \mathbf{H}_{n_r/2 \times n_t/2} \otimes \mathbf{X}, \quad (7)$$

where the covariance of $\mathbf{H}_{n_r/2\times n_t/2}$ is the spatial covariance related to the spacing between the sub-arrays, and $\mathbf{X}$ is the 2×2 dual-polarized matrix modeled by (5). Again, $\mathbf{X}$ only models the power imbalance and the phase-shifts between the four dual-polarized channels.

To simplify the analysis of such channels, we assume from now on that $\mu = 1$. In that case, it is relatively straightforward to compute the eigenvalues of $\mathbf{X}\mathbf{X}^H$, given by

$$\eta_{1,2} = A \pm \sqrt{A^2 + B}, \quad (8)$$

where

$$A = 1 + \chi + \chi|\delta_2|\sqrt{1 - |\delta_2|^2}\cos\left(\phi_2 - \phi_3 + \arg\{\delta_2\}\right) + |\delta_1|\sqrt{1 - |\delta_1|^2}\cos\left(\phi_1 - \phi_4 + \arg\{\delta_1\}\right), \quad (9)$$

and

$$\begin{aligned} B \;=\; & 2\chi|\delta_2||\delta_1|\cos\left(2\phi_1 - 2\phi_2 + \arg\{\delta_1\} - \arg\{\delta_2\}\right) \\ + \; & \chi^2|\delta_1|\sqrt{1-|\delta_2|^2}\cos\left(2\phi_1 - \phi_2 - \phi_3 + \arg\{\delta_1\}\right) \\ + \; & 2\chi|\delta_2|\sqrt{1-|\delta_1|^2}\cos\left(\phi_1 - 2\phi_2 + \phi_4 - \arg\{\delta_2\}\right) \\ + \; & 2\chi^2\sqrt{1-|\delta_2|^2}\sqrt{1-|\delta_1|^2}\cos\left(\phi_1 - \phi_2 - \phi_3 + \phi_4\right) \\ - \; & 2|\delta_1|\sqrt{1-|\delta_1|^2}\;\cos\left(\phi_1 - \phi_4 + \arg\{\delta_1\}\right) - 1 - \chi^2 \\ - \; & 2\chi^2|\delta_2|\sqrt{1-|\delta_2|}\cos\left(\phi_2 - \phi_3 + \arg\{\delta_2\}\right). \end{aligned} \quad (10)$$

Note that if $\delta_1 = \delta_2 = 0$, the eigenvalues simplify into

$$\eta_{1,2} = 1 + \chi \pm \sqrt{2\chi(1+\cos\psi)}, \tag{11}$$

where $\psi = \phi_1 - \phi_2 - \phi_3 + \phi_4$ is a random angle uniformly distributed over $[0, 2\pi)$. These eigenvalues are needed in the later developments.

III. Single vs. Multiple Polarizations: Mutual Information Analysis

A. Problem Statement

We want to compare the mutual information (MI) of two MIMO systems

- using the same numbers of antennas on both sides ($n_t = n_r = n$),
- with uniform linear arrays having the same total length (denoted by L_t and L_r respectively for the transmit and receive arrays) in both cases.

The first system is made of uni-polarized arrays with n equi-spaced antennas whereas for the second system the Tx and Rx arrays are made of $n/2$ dual-polarized equi-spaced sub-arrays (each sub-array is identical and is made of two co-located antennas with orthogonal polarizations).

Because we use the total length as a constraint, we must define a spatial correlation model. In what follows, we simply assume that the antenna correlation is an exponential function of the spacings (d_t and d_r) [13], hence it is given by e^{-d_t/Δ_t} at the Tx side, and e^{-d_r/Δ_r} at the Rx side (Δ_t and Δ_r are characteristic distances proportional to the spatial coherence distance at each side). We further assume that the spatial correlation is separable (i.e. the Kronecker model may be used) with the Tx and Rx correlation matrices respectively expressed as,

$$\mathbf{\Theta}_t = \begin{bmatrix} 1 & e^{-d_t/\Delta_t} & \dots & e^{-(n-1)d_t/\Delta_t} \\ e^{-d_t/\Delta_t} & 1 & \dots & e^{-(n-2)d_t/\Delta_t} \\ \vdots & & \ddots & \\ e^{-(n-1)d_t/\Delta_t} & e^{-(n-2)d_t/\Delta_t} & \dots & 1 \end{bmatrix} \tag{12}$$

$$\mathbf{\Theta}_r = \begin{bmatrix} 1 & e^{-d_r/\Delta_r} & \dots & e^{-(n-1)d_r/\Delta_r} \\ e^{-d_r/\Delta_r} & 1 & \dots & e^{-(n-2)d_r/\Delta_r} \\ \vdots & & \ddots & \\ e^{-(n-1)d_r/\Delta_r} & e^{-(n-2)d_r/\Delta_r} & \dots & 1 \end{bmatrix} \tag{13}$$

Combining all assumptions, it can be shown that the determinant of, say, $\mathbf{\Theta}_t$ reads as

$$\begin{aligned} \det\mathbf{\Theta}_t &= \left(1 - e^{-2d_t/\Delta_t}\right)^{n-1}, & (14)\\ &= \left(1 - e^{-\frac{2L_t}{(n-1)\Delta_t}}\right)^{n-1}, & (15)\end{aligned}$$

where $d_t = L_t/(n-1)$ is the element spacing for n antennas over a length L_t. The channel matrices therefore read as

- for the first system,

$$\mathbf{H} = \mathbf{\Theta}_r^{1/2}\mathbf{H}_w\mathbf{\Theta}_t^{1/2}, \tag{16}$$

- for the second system,

$$\mathbf{H}_\times = \underbrace{\tilde{\mathbf{\Theta}}_r^{1/2}\tilde{\mathbf{H}}_w\tilde{\mathbf{\Theta}}_t^{1/2}}_{\tilde{\mathbf{H}}} \otimes \mathbf{X}, \tag{17}$$

Note that $\mathbf{\Theta}_r$, $\mathbf{H}_w$ and $\mathbf{\Theta}_t$ are $n \times n$ matrices, while $\tilde{\mathbf{\Theta}}_r$, $\tilde{\mathbf{H}}_w$ and $\tilde{\mathbf{\Theta}}_t$ are $n/2 \times n/2$. It is also interesting to note that

$$\mathbf{H}_\times\mathbf{H}_\times^H = \tilde{\mathbf{H}}\tilde{\mathbf{H}}^H \otimes \mathbf{X}\mathbf{X}^H \tag{18}$$

B. High SNR Analysis

The mutual information with identity transmit covariance reads as

$$\mathcal{I} = \log_2\det\left[\mathbf{I}_n + \frac{\rho}{n}\mathbf{H}\mathbf{H}^H\right], \tag{19}$$

where ρ is the SNR. At high SNR, a good approximation is given by

$$\mathcal{I} \approx \log_2\det\left[\frac{\rho}{n}\mathbf{H}\mathbf{H}^H\right]. \tag{20}$$

1) Uni-Polarized Systems: In this case, (20) can be developed as follows:

$$\begin{aligned} \mathcal{I} &\approx \log_2\left\{\left(\frac{\rho}{n}\right)^n \det\left[\mathbf{H}\mathbf{H}^H\right]\right\}, & (21)\\ &= n\log_2\left(\frac{\rho}{n}\right) + \log_2\det\mathbf{\Theta}_r + \log_2\det\mathbf{\Theta}_t \\ &+ \log_2\det\left[\mathbf{H}_w\mathbf{H}_w^H\right] & (22)\\ &= n\log_2\left(\frac{\rho}{n}\right) + (n-1)\log_2\left[1 - e^{-\frac{2L_t}{(n-1)\Delta_t}}\right] \\ &+ (n-1)\log_2\left[1 - e^{-\frac{2L_r}{(n-1)\Delta_r}}\right] \\ &+ \log_2\det\left[\mathbf{H}_w\mathbf{H}_w^H\right] & (23)\end{aligned}$$

The ergodic mutual information is then given by

$$\begin{aligned} \bar{\mathcal{I}} &= \mathcal{E}\{\mathcal{I}\} = n\log_2\left(\frac{\rho}{n}\right) + (n-1)\log_2\left[1 - e^{-\frac{2L_t}{(n-1)\Delta_t}}\right] \\ &+ (n-1)\log_2\left[1 - e^{-\frac{2L_r}{(n-1)\Delta_r}}\right] \\ &+ \frac{1}{\log 2}\left(\sum_{k=1}^{n}\sum_{l=1}^{n-k}\frac{1}{l} - n\gamma\right), & (24)\end{aligned}$$

where $\gamma \approx 0.57721566$ is Euler's constant.

2) Multi-Polarized Systems: In the high SNR regime, we have:

$$\mathcal{I}_\times \approx \log_2\left\{\left(\frac{\rho}{n}\right)^n \det\left[\mathbf{H}_\times\mathbf{H}_\times^H\right]\right\}, \tag{25}$$

which yields $\mathcal{I}_\times$

$$\begin{aligned}
&= n\log_2\left(\frac{\rho}{n}\right)+\frac{n}{2}\log_2\left(\eta_1\eta_2\right)+2\log_2\det\left[\tilde{\mathbf{H}}_w\tilde{\mathbf{H}}_w^H\right]\\
&+ 2\left(\frac{n}{2}-1\right)\log_2\left[1-e^{-\frac{4L_t}{(n-2)\Delta_t}}\right]\\
&+ 2\left(\frac{n}{2}-1\right)\log_2\left[1-e^{-\frac{4L_r}{(n-2)\Delta_r}}\right] \qquad (26)
\end{aligned}$$

The ergodic mutual information is then given by

$$\begin{aligned}
\bar{\mathcal{I}}_\times=\mathcal{E}\{\mathcal{I}_\times\} &= n\log_2\left(\frac{\rho}{n}\right)+\frac{n}{2}\,\mathcal{E}\Big\{\log_2\left(\eta_1\eta_2\right)\Big\}\\
&+ \frac{2}{\log 2}\left(\sum_{k=1}^{n/2}\sum_{l=1}^{n/2-k}\frac{1}{l}-\frac{n}{2}\gamma\right)\\
&+ 2\left(\frac{n}{2}-1\right)\log_2\left[1-e^{-\frac{4L_t}{(n-2)\Delta_t}}\right]\\
&+ 2\left(\frac{n}{2}-1\right)\log_2\left[1-e^{-\frac{4L_r}{(n-2)\Delta_r}}\right]. \qquad (27)
\end{aligned}$$

We are now able to calculate the normalized difference $\Delta\bar{\mathcal{I}}/n=(\bar{\mathcal{I}}_\times-\bar{\mathcal{I}})/n$ assuming that n is large, and that $L_t/\Delta_t=L_r/\Delta_r$. In this case, let us define $\xi=n\Delta_t/L_t=n\Delta_r/L_r$, which can be thought of as a normalized antenna density. This yields

$$\begin{aligned}
\Delta\bar{\mathcal{I}}/n &\approx 1+2\log_2\left[\frac{1-e^{-4/\xi}}{1-e^{-2/\xi}}\right]+\frac{1}{2}\,\mathcal{E}\Big\{\log_2\left(\eta_1\eta_2\right)\Big\}\\
&\approx 1+2\log_2\left[\frac{1-e^{-4/\xi}}{1-e^{-2/\xi}}\right] \qquad (28)
\end{aligned}$$

where the simplification in (28) is derived from the observation (through simulations) that $\mathcal{E}\Big\{\log_2\left(\eta_1\eta_2\right)\Big\}/2$ is small, irrespective of χ, if δ_1 and δ_2 are sufficiently small (say, below 0.25, which is usually the case). Interestingly, the MI difference only depends of ξ. Therefore, dual-polarized schemes offer higher ergodic MI at high SNR when $\Delta\bar{\mathcal{I}}/n\geq 0$, i.e. when $\xi\geq 2.27$.

C. Arbitrary SNR Analysis

At arbitrary SNR, the asymptotic mutual information of uni-polarized spatially correlated channels is well-known, and can be calculated using the Stieltjes transform [14]. Alternative methods can also be used (see [13] as an example). We eventually obtain that the asymptotic average mutual information per receive antenna $\bar{\mathcal{I}}/n$ is given by

$$\frac{\bar{\mathcal{I}}}{n}=\frac{1}{n}\log_2\det(\mathbf{I}_n+\beta_t\mathbf{\Theta}_r)+\frac{1}{n}\log_2\det(\mathbf{I}_n+\beta_r\mathbf{\Theta}_t)-\frac{1}{\rho}\beta_t\beta_r, \qquad (29)$$

where β_t and β_r are the solutions of

$$\left\{\begin{array}{l}\beta_t=\frac{\rho}{n}\mathrm{Tr}\left[\mathbf{\Lambda}_{\mathbf{\Theta}_t}\left(\mathbf{I}_n+\beta_r\mathbf{\Lambda}_{\mathbf{\Theta}_t}\right)^{-1}\right]\\ \beta_r=\frac{\rho}{n}\mathrm{Tr}\left[\mathbf{\Lambda}_{\mathbf{\Theta}_r}\left(\mathbf{I}_n+\beta_t\mathbf{\Lambda}_{\mathbf{\Theta}_r}\right)^{-1}\right]\end{array}\right. \qquad (30)$$

and $\mathbf{\Lambda}_{\mathbf{\Theta}_t}$ and $\mathbf{\Lambda}_{\mathbf{\Theta}_r}$ are diagonal matrices containing the eigenvalues of $\mathbf{\Theta}_t$ and $\mathbf{\Theta}_r$. Both correlation matrices have the form

$$\mathbf{\Theta}=\begin{pmatrix}1&\rho&\dots&\rho^{n-1}\\ \rho&1&\dots&\rho^{n-2}\\ \vdots&\vdots&\ddots&\vdots\\ \rho^{n-1}&\rho^{n-2}&\dots&1\end{pmatrix}, \qquad (31)$$

and it is known (see [15, p. 38]) that the eigenvalue distribution function of $\mathbf{\Theta}$ converges uniformly (as $n\to+\infty$) to

$$\begin{aligned}
f(\lambda_{\mathbf{\Theta}}) &= \sum_{k=0}^{\infty}\rho^k e^{jk\lambda_{\mathbf{\Theta}}}+\sum_{k=1}^{\infty}\rho^k e^{-jk\lambda_{\mathbf{\Theta}}} \qquad (32)\\
&= \frac{1}{1-\rho e^{j\lambda_{\mathbf{\Theta}}}}+\frac{\rho}{1-\rho e^{-j\lambda_{\mathbf{\Theta}}}} \qquad (33)
\end{aligned}$$

for $\lambda_{\mathbf{\Theta}}\in[0,2\pi]$. Therefore, the asymptotic per-antenna MI at fixed SNR is given by $\lim_{n\to+\infty}\bar{\mathcal{I}}/n$

$$\begin{aligned}
&= \int_0^{2\pi}\log_2(1+\frac{\alpha_t}{1-e^{-\frac{d_r}{\Delta_r}}e^{j\lambda_{\mathbf{\Theta}}}}+\frac{e^{-\frac{d_r}{\Delta_r}}\alpha_t}{1-e^{-\frac{d_r}{\Delta_r}}e^{-j\lambda_{\mathbf{\Theta}}}})d\lambda_{\mathbf{\Theta}}\\
&+ \int_0^{2\pi}\log_2(1+\frac{\alpha_r}{1-e^{-\frac{d_t}{\Delta_t}}e^{j\lambda_{\mathbf{\Theta}}}}+\frac{e^{-\frac{d_t}{\Delta_t}}\alpha_r}{1-e^{-\frac{d_t}{\Delta_t}}e^{-j\lambda_{\mathbf{\Theta}}}})d\lambda_{\mathbf{\Theta}}\\
&- \frac{1}{\rho}\alpha_t\alpha_r, \qquad (34)
\end{aligned}$$

where

$$\alpha_t=\frac{4\pi\rho e^{-\frac{d_t}{\Delta_t}}}{1+\sqrt{1+8e^{-\frac{d_t}{\Delta_t}}\pi\rho e^{-\frac{d_r}{\Delta_r}}}}$$

and

$$\alpha_r=\frac{\sqrt{1+8e^{-\frac{d_t}{\Delta_t}}\pi\rho e^{-\frac{d_r}{\Delta_r}}}-1}{2e^{-\frac{d_t}{\Delta_t}}}$$

are the solutions of

$$\left\{\begin{array}{rcl}
\alpha_t &=& \rho\displaystyle\int_0^{2\pi}\frac{\frac{1}{1-e^{-\frac{d_t}{\Delta_t}}e^{j\lambda_{\mathbf{\Theta}}}}+\frac{e^{-\frac{d_t}{\Delta_t}}}{1-e^{-\frac{d_t}{\Delta_t}}e^{-j\lambda_{\mathbf{\Theta}}}}}{1+\frac{\alpha_r}{1-e^{-\frac{d_t}{\Delta_t}}e^{j\lambda_{\mathbf{\Theta}}}}+\frac{\alpha_r e^{-\frac{d_t}{\Delta_t}}}{1-e^{-\frac{d_t}{\Delta_t}}e^{-j\lambda_{\mathbf{\Theta}}}}}d\lambda_{\mathbf{\Theta}}\\
\alpha_r &=& \rho\displaystyle\int_0^{2\pi}\frac{\frac{1}{1-e^{-\frac{d_r}{\Delta_r}}e^{j\lambda_{\mathbf{\Theta}}}}+\frac{e^{-\frac{d_r}{\Delta_r}}}{1-e^{-\frac{d_r}{\Delta_r}}e^{-j\lambda_{\mathbf{\Theta}}}}}{1+\frac{\alpha_t}{1-e^{-\frac{d_r}{\Delta_r}}e^{j\lambda_{\mathbf{\Theta}}}}+\frac{\alpha_t e^{-\frac{d_r}{\Delta_r}}}{1-e^{-\frac{d_r}{\Delta_r}}e^{-j\lambda_{\mathbf{\Theta}}}}}d\lambda_{\mathbf{\Theta}}.
\end{array}\right. \qquad (35)$$

For dual-polarized schemes, assume first that the eigenvalues of $\mathbf{X}\mathbf{X}^H$ are fixed. In this case, the n eigenvalues of $\mathbf{H}_\times\mathbf{H}_\times{}^H$ can be expressed as the product of the $n/2$ eigenvalues of $\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H$ by η_1 and η_2 respectively. Hence, we may decompose the conditional MI per antenna as

$$\left.\frac{\bar{\mathcal{I}_\times}}{n}\right|_{\eta_1,\eta_2}=$$
$$\frac{1}{2}\int\log_2\left[1+\rho\eta_1\lambda\right]p_\lambda(\lambda)d\lambda+\frac{1}{2}\int\log_2\left[1+\rho\eta_2\lambda\right]p_\lambda(\lambda)d\lambda, \qquad (36)$$

where λ designates the eigenvalues of $\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H/n$ and $p_\lambda(\lambda)$ is

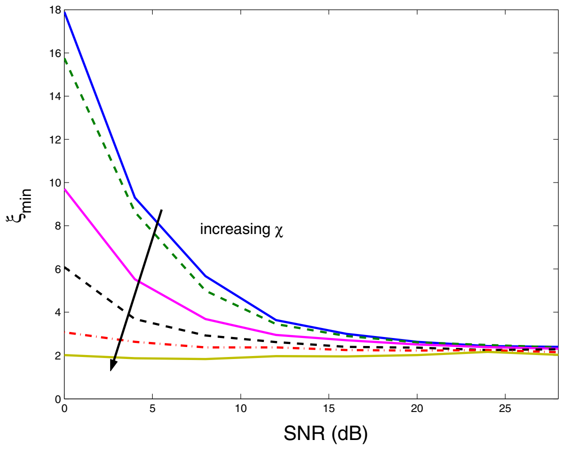

Fig. 1. Normalized antenna density $\xi_{\min}$ above which dual-polarization should be favored as a function of the SNR (χ is the scattering XPD, $\delta_1 = \delta_2 = 0$).

the limit probability density of λ when $n \to \infty$. The latter can be quite easily evaluated, e.g. as described in [13]. When η_1 and η_2 are random, the quantities $\rho\eta_1$ and $\rho\eta_2$ can be thought of as randomly varying effective SNRs. The randomness is represented by the four phase-shifts ϕ_k, $k = 1, \ldots, 4$ in (9) and (10), which are uniformly distributed over $[0, 2\pi)$. The ergodic MI per antenna is finally given by

$$\frac{\bar{\mathcal{I}}_{\times}}{n} = \frac{1}{32\pi^4} \int_0^{2\pi} \cdots \int_0^{2\pi} \sum_{k=1}^{2} \left\{ \int \log_2 \left[1 + \rho\eta_k\lambda \right] p_\lambda(\lambda)\, d\lambda \right\} d\phi_1\, d\phi_2\, d\phi_3\, d\phi_4 \tag{37}$$

Simulation results are illustrated in Figure 1. The minimum normalized antenna density $\xi_{\min}$ for which $\Delta\bar{\mathcal{I}}/n \geq 0$ is plotted for various values of χ and $\delta_1 = \delta_2 = 0$. $\xi_{\min}$ decreases as the SNR increases, and reaches its asymptotic value of 2.27 at high SNR. The impact of χ is also pretty intuitive: for small XPD values, uni-polarized schemes remain attractive for larger densities, as the dual-polarized transmissions are heavily penalized by the energy loss, especially at low SNR levels. At low SNR, it is indeed well known that the mutual information is essentially linked to the channel energy [9].

IV. Conclusions

This paper has presented a simple model of multi-polarized MIMO transmissions in Rayleigh fading channels, which combines spatial correlation effects and polarization-related mechanisms. The model has then been exploited to analyze the mutual information offered by MIMO schemes using either single, or multiple polarizations. In the high SNR regime, a closed-form criterion has been derived, which depends on a normalized antenna density. The latter is the product of the array antenna density by the spatial coherence distance of the channel. At arbitrary SNR levels, we have proposed a method to estimate the mutual information, and shown that for small depolarization, uni-polarized schemes remain attractive for large normalized densities, especially in the low SNR regime.

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