Hard Fairness versus Proportional Fairness in Wireless Communications: the Single-Cell Case (Extended Abstract)

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Motivation and outline of this work

We consider the uplink and the downlink of a multiuser wireless system with one base station and K user terminals (single-cell case). Each user is affected by a position-dependent path loss, fixed in time, and by a slowly time-varying frequency-selective fading channel modeled as M parallel block-fading channels. We study the system throughput (sum rate) versus E_b/N_0 under hard fairness and proportional fairness constraints.

By "hard fairness" we mean a system where each user transmits at its own desired rate in every channel condition. This corresponds to the so-called *delay-limited* capacity of fading multi-access channels [1]. When no fairness is imposed, the notion of *throughput* (or ergodic) capacity region [2] becomes relevant: this is the long-term average rate region achievable when the users adapt their rate and power according to the instantaneous channel conditions. [1] It is well-known that the maximum *long-term average* throughput is achieved by letting only the user with the best channel transmit on each time-frequency coding interval (referred to as 'slot" in the following) [3], [2]. However, in a cellular environment where users are at different distance from the base station, this strategy would result in a very unfair resource allocation: basically, only the users closest to the base station would be allowed to transmit. Hence, various scheduling algorithms aiming at maximizing the long-term average throughput subject to some fairness constraint have been proposed. Among these, the Proportional Fair Scheduling (PFS) algorithm [4] enjoys many desirable properties and was adopted in some evolutionary 3G wireless communication standards [5] for delay-tolerant data-oriented communications.

Our analysis allows us to quantify the effect of imposing "hard-fairness" (the delay-limited setting) versus "proportional-fairness" (the delay-tolerant setting) in a cellular environment, for given M, K and channel statistics. For finite K, we find simple iterative resource allocation algorithms that provably converge to the optimal delay-limited throughput. Also, in the limit of very large K and finite M we find closed-form expressions for the delay-limited throughput. We show that, for both optimal and orthogonal signaling, the optimal strategy in the limit of large K consists of letting the users transmit on their own best subchannel only, irrespectively of the other users. This result suggests a system where the users are able to "listen wideband", i.e., measure their channel gain on all the M subchannels, and "talk narrowband", i.e., they will transmit only on their best subchannel. This feature is sometimes referred to as "Cognitive Radio", a technology that is gaining an increasing interest also in the standardization environment.

In the case of PFS, we find a simple closed-form expression for the throughput in the considered cellular environment that holds for any K and M.

Finally, we carry out a closed-form analysis of the throughput versus system E_b/N_0 in the high and low spectral efficiency regions, for all systems under consideration. Our analysis shows that, in the high spectral efficiency (high-SNR) region, the penalty incurred by imposing hard-fairness is generally small. Furthermore, in some cases of practical interest (with reasonably large but finite K), the optimal delay-limited system may outperform PFS for high spectral efficiency. On the contrary, the gain of PFS over *any* delay-limited system can be significant in the low spectral efficiency region (low SNR).

Due to the strict pages limitations of this extended abstract submission, our results shall be only outlined and all proofs and many formulas shall be omitted. They can be found in [6], available from the authors upon request.

Summary of the results

Because of uplink-downlink duality [7], our results apply to both uplink (multi-access channel) and downlink (broadcast channel). This statement is proved formally in [6]. Here, we focus on the notation for the MAC without loss of generality. We consider the channel model

$$Y^m = \sum_{k=1}^{K} \sqrt{d_k^m} X_k^m + N^m, \qquad m = 1, \dots, M \tag{1}$$

where m is the sub-band index, and $N^m \sim \mathcal{CN}(0, N_0)$. This is an accurate model for a slowly-varying MAC with

[1] When coding over an arbitrarily large number of fading blocks is allowed, the same ergodic capacity region can be achieved by fixed-rate variable-power transmission. However, due to our assumption of block-fading channel, in this work we assume that a coding interval spans a single fading realization. Hence, the variable rate and power scheme is in place.

fading where the system bandwidth is M times larger than the channel coherence bandwidth. We study the system *spectral efficiency*, is given by

$$\mathsf{C} = \frac{\Gamma}{M} \tag{2}$$

and expressed in bit/s/Hz, versus the system E_b/N_0, defined as

$$\left(\frac{E_b}{N_0}\right)_{\rm sys} = \frac{\sum_{k=1}^{K} \overline{E}_k}{N_0 \sum_{k=1}^{K} R_k} \tag{3}$$

where E_k is the total transmit energy of user k, and R_k denotes the total number of bit per one use of the M parallel channels (bits per M dimensions). The quantity $\left(\frac{E_b}{N_0}\right)_{\rm sys}$ is relevant in the case where there is a total sum-power constraint (downlink), or when the users are statistically symmetric (uplink).

Channel statistics. In cellular communications, signal propagation is characterized by a frequency flat factor that depends on the distance between the user terminal and the base station (path loss), and by a frequency selective "small scale" fading that depends on the local scattering environment around the user terminal. The path loss varies so slowly in time with respect to the signal bandwidth that it can be considered constant forever. This corresponds to the realistic assumption that users do not change significantly their distance from the base station during a large number of consecutive slots. On the contrary, the small-scale fading changes in time depending on the channel Doppler bandwidth. In practice, its coherence time is such that it can be considered constant on each slot, but changing according to some stationary ergodic (possibly correlated) process from slot to slot. This model is referred to as block-fading.

We take into account these two effects by letting $d_k^m = s_k f_k^m$, where s_k denotes the path loss of user k (symbol s stands for "slow") and f_k^m is the frequency-selective block fading of user k in channel m (symbol f stands for "fast"). Clearly, s_k and f_k^m are mutually statistically independent, as they are due to completely different propagation effects.

We shall assume that the users are uniformly distributed in the unit-circle cell, but for a forbidden circular region of radius δ centered around the base station, where $0 < \delta < 1$ is a fixed system constant. We consider a normalization such that the loss at the cell border is equal to 1 (clearly, all results can be scaled by a constant factor equal to the loss at the cell border, for a given cell size and loss exponent). It will appear in the following that a key role in performance analysis is played by the cdf of the random variable $s \max\{f_1, \ldots, f_n\}$, where s is distributed as the path loss of a random user, and f_i are i.i.d. small-scale fading variables, distributed as $\exp(-z), z \geq 0$, (i.e., chi-squared with 2-degrees of freedom). The relevant cdf, denoted by $F_{s\max\{f\},n}(x)$, is given by

$$F_{s\max\{f\},n}(x) = \frac{1}{1-\delta^2}\frac{1}{x^{2/\alpha}}\int_{x^{2/\alpha}\delta^2}^{x^{2/\alpha}} \left(1 - {\rm e}^{-y^{\alpha/2}}\right)^n \, {\rm d}y \tag{4}$$

where $\alpha \geq 2$ is referred to as "path-loss exponent". Remarkably, $F_{s\max\{f\},n}(x)$ can be given in closed form in several cases of interest (see [6]).

Results for finite K and M. The aggregate rate and aggregate energy per symbol of user k are given by

$$R_k = \sum_{m=1}^{M} R_k^m, \qquad k = 1, \ldots, K \tag{5}$$

$$E_k = \sum_{m=1}^{M} E_k^m, \qquad k = 1, \ldots, K \tag{6}$$

respectively, where R_k^m and E_k^m denote the rate and the energy per symbol allocated by user k on subchannel m. For orthogonal multiple-access, we let $\boldsymbol{\Theta}^m = (\Theta_1^m, \ldots, \Theta_K^m)$, where Θ_k^m denotes the resource-sharing fraction of user k over channel m.

In a "hard-fainess" (delay-limited) situation, the rates R_k are fixed a priori, and the system has to allocate transmit energies in order to let the rate K-tuple inside the achievable rate region. We wish to find the partial rates allocation (and the resource-sharing fractions in the case of orthogonal signaling) in order to minimize the required $(E_b/N_0)_{\rm sys}$ to maintain a given rate K-tuple.

For optimal signaling, letting π^m denote the permutation that sorts the gains d_k^m in increasing order, the required transmit energy per symbol of user π_k^m in channel m is given by

$$E_{\pi_k^m}^m = \frac{N_0}{d_{\pi_k^m}^m}\left[\exp\left(\sum_{i\leq k} R_{\pi_i^m}^m\right) - \exp\left(\sum_{i<k} R_{\pi_i^m}^m\right)\right] \tag{7}$$

Optimizing the partial rates R_k^m in order to minimize $(E_b/N_0)_{\rm sys}$ is a convex minimization problem that can be solved with standard tools. In particular, in [6] we give a simple iterative block-coordinate descent algorithm that provably converges to the optimum.

For orthogonal signaling, the transmit energy per symbol of user k in subchannel m is given by

$$E_k^m = \Theta_k^m \frac{N_0}{d_k^m}\left(\exp(R_k^m/\Theta_k^m) - 1\right) \tag{8}$$

The minimization of $(E_b/N_0)_{\rm sys}$ with respect to $\{\boldsymbol{\Theta}^m, \mathbf{R}^m : m = 1, \ldots, M\}$ is also a convex optimization problem. In fact, it can be checked that the function $g(x,y) = x\exp(x/y)$ is convex. Since the constraints

$$\sum_{m=1}^{M} R_k^m \geq R_k, \quad k = 1, \ldots, K$$

and

$$\sum_{k=1}^{K} \Theta_k^m \leq 1, \quad m = 1, \ldots, M$$

with $R_k^m \geq 0$ and $\Theta_k^m > 0$ are also separable, we can use again a block-coordinate descent algorithm and have a convergent iterative optimization.

In a delay-tolerant situation, the user rates can be adapted according to their instantaneous channel conditions. We let $\mathsf{SNR} = E_{\rm tot}/N_0$ denote the transmit SNR in each slot, where $E_{\rm tot}$ is the total transmit energy. We assume that the channel gains are independent but not necessarily identically distributed across the users, and symmetrically distributed across the channels, that is, for any permutation π of $\{1,\ldots,M\}$, the joint cdf of the channel gains satisfies $F(d_k^1,\ldots,d_k^M) = F(d_k^{\pi_1},\ldots,d_k^{\pi_M})$, for all k. This means that no subchannel is statistically worse or better than any other. However, the users might have different channel gain distributions, i.e., our analysis is not restricted to the case of symmetric users, as in [3], [4].

In order to cope with users in different propagation conditions (here, specifically, at different distance from the base station) the PFS algorithm has been proposed [4]. In the limit of large fairness interval (i.e., when the interval over which fairness is imposed grows to infinity) we have the following result:

Theorem 1: For any given K and fixed path loss components $\mathbf{s} = (s_1,\ldots,s_K)$, under the channel gain statistics defined above, the long-term average throughput T achieved by PFS is given by

$$T = \frac{M}{K}\sum_{k=1}^{K}\int_0^\infty \log(1+s_k x\mathsf{SNR})\mathrm{d}F_{\max\{f\},K}(x) \qquad (9)$$

where $F_{\max\{f\},K}(x)$ is the cdf of $\max\{f_1^m,\ldots,f_K^m\}$, independent of m by the symmetry assumption. $\square$

As a corollary, it follows that the average spectral efficiency C as a function of $(E_b/N_0)_{\rm sys}$, where expectation is taken also with respect to the (random) path loss, is given implicitly by

$$\begin{aligned} \mathsf{C} &= \int_0^\infty \log_2(1+x\mathsf{SNR})\mathrm{d}F_{s\max\{f\},K}(x) \\ \left(\frac{E_b}{N_0}\right)_{\rm sys}^{\rm PFS} &= \frac{\mathsf{SNR}}{\mathsf{C}} \end{aligned} \qquad (10)$$

Figs. 1 and 2 compare the spectral efficiency achieved by PFS and delay-limited systems for finite number of users $K = 10, 20, 30, 50, 100$ (in Fig.2 we show only the case $K = 10$ and $K = \infty$ for the optimized-orthogonal system for the sake of clarity). In all cases, spectral efficiency improves with K. This effect is known as *multiuser diversity*. However, the effect of multiuser diversity is quite different in the delay-limited and delay-tolerant setting. While for the delay-tolerant systems increasing K yields a gain in terms of $(E_b/N_0)_{\rm sys}$ for all spectral efficiencies (roughly, an horizontal shift of the C vs. $(E_b/N_0)_{\rm sys}$ curve), for the delay-limited systems increasing K yields a change only for large spectral efficiency. This effect will be analyzed in depth in the asymptotic case of $K \to \infty$, as seen next.

Results for $K \to \infty$ and finite M. Under mild conditions (given in [6]) on the fading and path-loss distributions, satisfied by the model introduced before, the performance of delay-limited systems in the limit of large number of users is given by the following results.

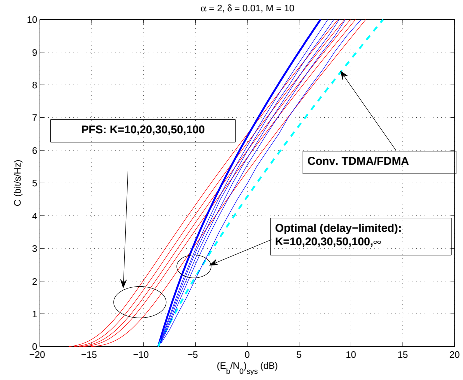

Fig. 1. Spectral efficiency vs. system E_b/N_0 for PFS and optimal delay-limited signaling. The curve for conventional TDMA/FDMA and $K = \infty$ users is shown for comparison. The channel parameters are $M = 10$, path loss exponent $\alpha = 2$ and radius of the forbidden region $\delta = 0.01$.

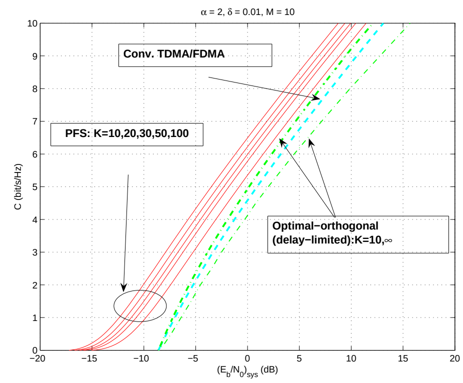

Fig. 2. Spectral efficiency vs. system E_b/N_0 for PFS and optimized-orthogonal delay-limited signaling. The curve for conventional TDMA/FDMA and $K = \infty$ users is shown for comparison. The channel parameters are $M = 10$, path loss exponent $\alpha = 2$ and radius of the forbidden region $\delta = 0.01$.

Theorem 2: As $K \to \infty$ the minimum $(E_b/N_0)_{\rm sys}$ for given system spectral efficiency C is given by

$$\left(\frac{E_b}{N_0}\right)_{\rm sys} = \log(2)\int_0^\infty 2^{\mathsf{C}F_{s\max\{f\},M}(x)}\frac{\mathrm{d}F_{s\max\{f\},M}(x)}{x} \qquad (11)$$

This is achieved by letting each user transmit on its best subchannel only, and by using superposition coding and successive decoding on each subchannel. $\square$

Theorem 3: As $K \to \infty$ the minimum $(E_b/N_0)_{\rm sys}$ for given spectral efficiency C achieved by orthogonal signaling is given by

$$\left(\frac{E_b}{N_0}\right)_{\rm sys} = \log(2)\int_0^\infty \frac{\exp\left(1+W\left(\frac{\mu x-1}{e}\right)\right)-1}{1+W\left(\frac{\mu x-1}{e}\right)}\frac{\mathrm{d}F_{s\max\{f\},M}(x)}{x} \qquad (12)$$

where $W(x)$ is Lambert's W function and where μ is the solution of

$$\int_0^\infty \frac{\mathrm{d}F_{s\max\{f\},M}(x)}{1+W\left(\frac{\mu x-1}{e}\right)} = \frac{1}{\mathsf{C}\log(2)} \qquad (13)$$

This is achieved by letting each user transmit on its own best subchannel only, and by using orthogonal signaling with optimized fractions on each subchannel. □

We shall compare the optimal and the optimized-orthogonal delay-limited systems of Theorems 2 and 3 with a conventional TDMA/FDMA system, where each user chooses its own best channel to transmit, but resource allocation (the fractions $\boldsymbol{\Theta}^m$) are proportional to the users' requested rates, disregarding the actual channel gains. The performance of conventional TDMA/FDMA is given by

Theorem 4: As $K \to \infty$ the $(E_b/N_0)_{\rm sys}$ for given system spectral efficiency C, achieved by letting each user transmit on its best subchannel only and allocating a fraction of channel uses proportional to its individual rate, is given by

$$\left(\frac{E_b}{N_0}\right)_{\rm sys} = \frac{2^{\mathsf{C}}-1}{\mathsf{C}} \int_0^\infty \frac{{\rm d}F_{s\max\{f\},M}(x)}{x} \tag{14}$$

□

Low and High spectral efficiency behaviors. Our comparison is based on the asymptotics of spectral efficiency C as a function of $(E_b/N_0)_{\rm sys}$. In general, the low spectral efficiency behavior ($\mathsf{C} \downarrow 0$) is characterized by the minimum system E_b/N_0, denoted by $(E_b/N_0)_{\rm min}$ and the *wideband slope* $\mathcal{S}_0$ (see definitions in [8]). The high spectral efficiency behavior ($\mathsf{C} \to \infty$) is characterized by the high-SNR slope $\mathcal{S}_\infty$ and by the horizontal dB penalty $\mathcal{L}_\infty$ (see definitions in [9]).

All the delay-limited systems achieve the same $(E_b/N_0)_{\rm min}$, given by

$$\left(\frac{E_b}{N_0}\right)_{\rm min} = \log(2) \int_0^\infty \frac{{\rm d}F_{s\max\{f\},M}(x)}{x} \tag{15}$$

The advantage of optimal over conventional delay-limited signaling is evidenced by the wideband slope, provided by

Theorem 5: As $K \to \infty$ the wideband slope $\mathcal{S}_0$ (in bit/dimension/3dB) of the spectral efficiency vs. $(E_b/N_0)_{\rm sys}$ curve for the delay-limited systems is given by

$$\mathcal{S}_0^{\rm optimal} = \frac{\int \frac{{\rm d}F_{s\max\{f\},M}(x)}{x}}{\int \frac{F_{s\max\{f\},M}(x)}{x} {\rm d}F_{s\max\{f\},M}(x)} \tag{16}$$

$$\mathcal{S}_0^{\rm opt.orthogonal} = \frac{2\int \frac{{\rm d}F_{s\max\{f\},M}(x)}{x}}{\left(\int \frac{1}{\sqrt{x}} {\rm d}F_{s\max\{f\},M}(x)\right)^2} \tag{17}$$

$$\mathcal{S}_0^{\rm conv.tdma/fdma} = 2 \tag{18}$$

□

The low spectral efficiency behavior of the PFS system is easily obtained and we have

$$\left(\frac{E_b}{N_0}\right)_{\rm min}^{\rm PFS} = \frac{\log(2)}{\int_0^\infty x \, {\rm d}F_{s\max\{f\},K}(x)} \tag{19}$$

Notice that, under mild conditions on the fading distribution, $(E_b/N_0)_{\rm min}^{\rm PFS}$ goes to zero as $K \to \infty$. Using extremal statistics theory, we can easily show that $\left(\frac{E_b}{N_0}\right)_{\rm min}^{\rm PFS}$ goes to zero as $O(\frac{1}{\log K})$.

As far as $\mathcal{S}_0$ is concerned, we have

$$\mathcal{S}_0^{\rm PFS} = \frac{2\left(\mathbb{E}[s]\mathbb{E}[\max\{f_1^m,\ldots,f_K^m\}]\right)^2}{\mathbb{E}[s^2]\mathbb{E}[(\max\{f_1^m,\ldots,f_K^m\})^2]} \tag{20}$$

Again, from extremal theory we get that

$$\lim_{K\to\infty} \mathcal{S}_0^{\rm PFS} = \frac{2\mathbb{E}[s]^2}{\mathbb{E}[s^2]}$$

By comparing (15) and (19) under the cellular channel statistics, we notice that for low specral efficiency the gain of the opportunistic scheme over the delay-limited scheme is two-fold: on one hand it achieves larger multiuser diversity as $K \gg M$, on the other hand it achieves a "Jensen's inequality" gain due to the convexity of $1/x$. We conclude that for low spectral efficiency the cost of imposing a strict constraint on rate and delay is very high. In fact, the optimal delay-limited system does not benefit in terms of $(E_b/N_0)_{\rm min}$ over a conventional orthogonal system (or a single-user system). In this regime, multiuser diversity appears only as a second-order effect, as a gain in the wideband slope.

Next, we focus on the high spectral efficiency regime. In this case, the high-spectral efficiency slope is given by $\mathcal{S}_\infty = 1$ for all systems under consideration. However, systems may differ significantly in their horizontal dB penalty. The conventional TDMA/FDMA system yields (calculation is immediate)

$$\mathcal{L}_\infty^{\rm conv.tdma/fdma} = \log_2\left(\int_0^\infty \frac{{\rm d}F_{s\max\{f\},M}(x)}{x}\right) \tag{21}$$

For the optimal delay-limited signaling, we have the following surprising behavior, already noticed in [10, Section 5.2.2] for the case of frequency-flat path-loss only:

Theorem 6: As $K \to \infty$ the horizontal dB penalty of the optimal delay-limited system is given by

$$\mathcal{L}_\infty^{\rm optimal} = -\log_2\left(F^{-1}_{s\max\{f\},M}\left(1-\frac{1}{\mathsf{C}}\right)\right) + O(1) \tag{22}$$

□

In general, in all cases where $F_{s\max\{f\},M}(x)$ is strictly increasing for all sufficiently large x, the horizontal dB "penalty" diverges to $-\infty$, indicating that optimal delay-limited signaling yields unbounded dB *gain* over the corresponding conventional TDMA/FDMA system. The following result provides the horizontal dB penalty of optimized-orthogonal delay-limited signaling.

Theorem 7: As $K \to \infty$ the horizontal dB penalty of the optimized-orthogonal delay-limited system is given by

$$\mathcal{L}_\infty^{\rm opt.orthogonal} = -\int_0^\infty \log_2(x){\rm d}F_{s\max\{f\},M}(x) \tag{23}$$

□

Finally, for the opportunistic PFS system we obtain, after simple direct calculation,

$$\mathcal{L}_\infty^{\rm PFS} = -\int_0^\infty \log_2(x)\,{\rm d}F_{s,\max\{f\},K}(x) \tag{24}$$

In the limit of large K, we have the behavior $\mathcal{L}_\infty^{\rm PFS} = O(\log\log K)$, typical of multiuser diversity systems [4].

Comparing (21), (23) and (24) we notice that $\mathfrak{L}_\infty^{\rm conv.tdma/fdma} \le \mathfrak{L}_\infty^{\rm opt.orthogonal}$ by Jensen's inequality. In the usual case where the number of users is much larger than the number of subchannels ($K \gg M$) we have that $\mathfrak{L}_\infty^{\rm opt.orthogonal} \le \mathfrak{L}_\infty^{\rm PFS}$. Note that the gain of PFS comes only from $K > M$ and the diversity associated with it. For M growing large, the gain vanishes. Thus, the wider the band, the less advantage for PFS in terms of spectral efficiency, despite the looser delay constraint.

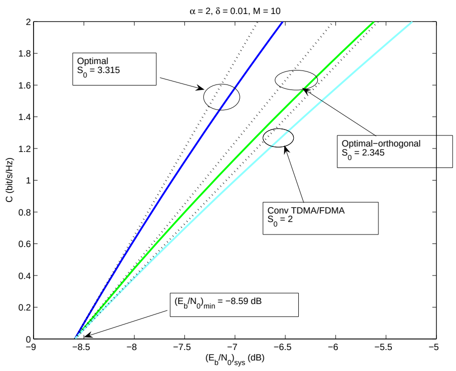

Fig. 3. Spectral efficiency vs. system E_b/N_0 for the optimal, optimized-orthogonal and conventional TDMA/FDMA delay-limited systems for $K = \infty$. The channel parameters are $M = 10$, path loss exponent $\alpha = 2$ and radius of the forbidden region $\delta = 0.01$. The dotted lines correspond to the low spectral efficiency approximation.

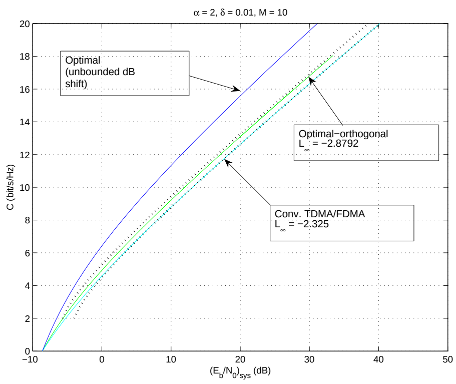

Fig. 4. Spectral efficiency vs. system E_b/N_0 for the optimal, optimized-orthogonal and conventional TDMA/FDMA delay-limited systems for $K = \infty$. The channel parameters are $M = 10$, path loss exponent $\alpha = 2$ and radius of the forbidden region $\delta = 0.01$. The dotted lines correspond to the high spectral efficiency approximation.

Figs. 3, 4, 5 and 6 show the low and high spectral efficiency behavior of all systems considered. In particular, we observe that, consistently with Theorem 6, the gain of the optimal delay-limited signaling over orthogonal signaling becomes unbounded as $\mathsf{C} \to \infty$.

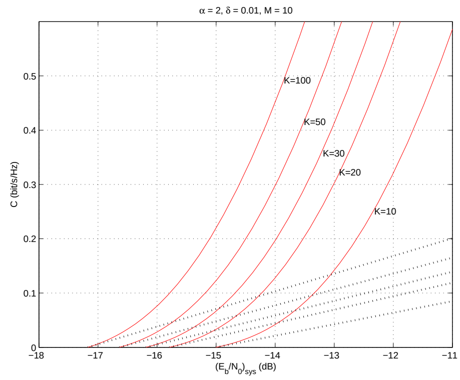

Fig. 5. Spectral efficiency vs. system E_b/N_0 for the PSF system for $K = 10, 20, 30, 50, 100$ (C is increasing with K). The channel parameters are $M = 10$, path loss exponent $\alpha = 2$ and radius of the forbidden region $\delta = 0.01$. The dotted lines correspond to the low spectral efficiency approximation.

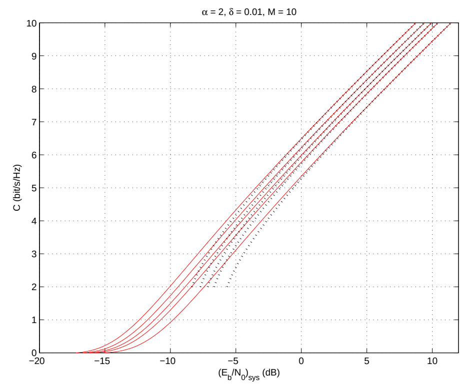

Fig. 6. Spectral efficiency vs. system E_b/N_0 for the PSF system for $K = 10, 20, 30, 50, 100$ (C is increasing with K). The channel parameters are $M = 10$, path loss exponent $\alpha = 2$ and radius of the forbidden region $\delta = 0.01$. The dotted lines correspond to the high spectral efficiency approximation.

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