

Probabilistic Access Functions for Multi-Cell Wireless Schemes

Jan Egil Kirkebø*, David Gesbert†, Saad G. Kiani†

*Department of Informatics, University of Oslo,

P.O. Box 1080, N-0316 Oslo, Norway. E-mail: janki@ifi.uio.no

†Mobile Communications Department, Eurecom Institute,

06560 Sophia Antipolis, France. E-mail: {gesbert, kiani}@eurecom.fr

Abstract—In recently introduced “multi-cell access” schemes, cells (rather than users) compete for the spectral resource. In a previous paper [1], a framework for such a scheme was proposed, using only local channel information. In the framework each cell competes for access through a credit that is a function of the signal to noise ratio of its scheduled user. Access is then given to a cell with a probability dependent on the access function. In [1] an ad-hoc choice of function was formulated. In this work we investigate which access function actually optimizes the system capacity, utilizing a numerical optimization procedure. We obtain a surprisingly simple solution which also corroborates previous results. For a realistic path loss model we find that our multicell distributed access scheme gives a 19% gain compared to having all cells on, and more than a 50% increase in system capacity compared to keeping a traditional static spectral reuse scheme.

I. INTRODUCTION

Spectral reuse planning plays an important role in mitigating co-channel interference, which limits link capacity of every transmit-receiver pair. This enables links to operate at an acceptable signal to noise plus interference ratio (SINR). However, such static reuse schemes are not very efficient since they do not exploit information available on variations in the underlying physical layer to fully exploit the achievable capacity gains. Clearly, by exploiting some form of coordination between the different cells occupying the same spectral resource, significant capacity gains can be attained.

Coordination usually implies communication of information to a centralized entity. In this case, channel gain information of all users in all cells is collected by a centralized controller that schedules co-channel users on a given resource while incurring the least loss of capacity due to inter-cell interference. This gives an interesting trade-off between multi-user diversity [2] due to intra-cell gain maximization and co-channel interference

minimization. Various approaches for exploiting inter-cell coordination exist. In the downlink of CDMA data networks, using a binary power allocation is shown to provide gains over intermediate power allocation [3]. In [4], a centralized heuristic algorithm works by inserting co-channel users one by one, as long as the channel throughput increases. In [5] power-profile shaping based resource assignment allows the access point (AP) to transmit with varying power in different portions of the frame and users are allotted slots according to the amount of interference tolerated. Centralized, yet much simplified schemes, have also been proposed, see e.g. [6]. However, even there a central network controller needs to collect quite a lot of multi-cell channel state information for all users. The idea behind these approaches is that substantial gains can be achieved by switching off transmission in cells which do not contribute enough to the capacity to outweigh the interference degradation caused by them to the rest of the network. However, in a realistic network centralized multi-cell coordination is hard to realize, especially in fast fading environments.

In this paper we address the problem of *fully distributed coordination*, and use the concept of multi-cell access (MCA) schemes introduced in [1]. In MCA schemes, network cells (rather than the users) directly compete for access to the spectral resource. Thus, at any given scheduling period, only a subset of the total number of cells are active simultaneously. The other cells stay silent for that scheduling period, but can compete again during the next scheduling period. The difference between MCA schemes and traditional cellular networks is that the pattern of on-cells and off-cells is highly irregular (cfr. Figure 1) from one scheduling period to the next, depending on the channel state information of the users. This dynamic aspect of the reuse allows for significant capacity gains at network level.

MCA schemes are somewhat similar to the modified

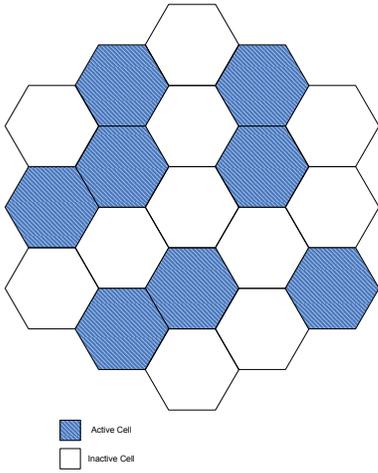


Fig. 1. Possible irregular reuse pattern at a given scheduling period due to multi-cell access.

ALOHA protocol proposed in [7], where the uplink transmit probability of users in the *ALOHA* protocol is adapted to exploit multi-user diversity to maximize the system throughput. Even though the goal of increased capacity is the same, the modified *ALOHA* protocol is for one common receiver (single-cell). MCA schemes, on the other hand, consider concurrent interfering transmissions (multi-cell), and do not only exploit multiuser diversity but also employ interference mitigation to obtain network capacity gains. Similarly, [8] proposes a framework for exploiting channel state information for *slotted ALOHA*. The goal is achieving stable throughput, again by optimizing the single-cell uplink access probability.

In [1], the proposed multi-cell access schemes works by comparing the signal to noise ratio (SNR) of the best user in each cell with a pre-given threshold. If the gain falls below the threshold, the cell stays silent for that period. Otherwise the APs transmits at full power. In this work, we attempt to generalize the MCA scheme by optimizing the probability of access as a function of the SNR. Surprisingly, our results tend to fully justify the use of the threshold function for the probability of access function. We compare system capacity for the new scheme with keeping all cells on, and fixed reuse patterns. We show capacity gains of almost 20 % compared to keeping all cells on, and over 50 % compared to fixed reuse patterns.

II. SYSTEM MODEL

Consider a multicell system in which a set of APs communicate with user terminals (UTs), all using the same spectral resource.

A. Signal Model

We consider the downlink of an N cell system with U_n users distributed randomly in each cell n . The received signal at user u_n is given by

$$Y_{u_n} = \sqrt{G_{u_n,n}}X_{u_n} + \sum_{\substack{i=1 \\ i \neq n}}^N \sqrt{G_{u_n,i}}X_{u_i} + Z_{u_n},$$

where $G_{u_n,i}$ is the channel gain between any arbitrary AP i and user u_n in cell n , and X_{u_n} is the signal from the serving AP. The noise Z_{u_n} is additive white Gaussian. The signal to interference-plus-noise ratio (SINR) is then given by

$$\Gamma_{u_n} = \frac{G_{u_n,n}P_{u_n}}{N_0 + \sum_{\substack{i=1 \\ i \neq n}}^N G_{u_n,i}P_{u_i}},$$

where $P_{u_i} = E[|X_{u_i}|^2]$ and $N_0 = E[|Z_{u_n}|^2]$ is the thermal noise power. Using the Shannon capacity, summing over all the cells, we can express the network capacity of the system (in bits/Hz/sec/cell) as

$$C = \frac{1}{N} \sum_{n=1}^N \log_2 (1 + \Gamma_{u_n}). \quad (1)$$

III. ACCESS SCHEME

We now proceed to present the MCA scheme based on a random access mechanism requiring only local channel information. This enables us to find the expected network capacity for such a system. Furthermore, we show how to optimize this expected network capacity, given only the fading statistics, through optimization of the probability of access function.

In the proposed MCA scheme, a cell obtains permission to transmit when it has enough credit. To keep the algorithm distributed, the credit is only based on the intra-cell channel state information. The credit measures how worthwhile to the overall network capacity a given cell is at a given instant of time.

In this paper, the credit measure for each cell is given by *the SNR of the scheduled user*. The scheduled user is picked at random, and the long-term fairness of the system is thus equivalent to round-robin scheduling.

A. Probabilistic access

In the proposed framework we further extend the random access protocol idea to the multi-cell scenario, i.e. a cell will be transmitting to its best user with a probability derived from the credit. Thus, the MCA

scheme here lets the access point transmit to its user with the best channel gain, with a given probability $P(g)$, where g is the channel gain of the user. The function $P(g)$ can take on any shape as long as it satisfies:

$$0 \leq P(g) \leq 1, \quad g > 0, \quad (2)$$

$$P(g_2) \geq P(g_1), \quad g_2 > g_1. \quad (3)$$

B. Expected Network Capacity

In view of the random access mechanism described above, we wish to evaluate the multi-cell network capacity. First, the expectation of a cell being activated is given by

$$F = \int_0^\infty P(g) f_G(g) dg,$$

where $P(g)$ is the probability of cell activation for a cell whose best user exhibits a channel gain g , and $f_G(g)$ is the corresponding pdf for the distribution of the channel gain of the cell users. For simplicity, and for creating user fairness, we assume that the scheduled user in each cell is picked at random. This gives an approximation to round-robin scheduling.

1) *Interference modeling*: To allow a decentralized algorithm, we model the interference so that it is independent of the individual realizations of the inter-cell channel gains.

To this end we use the *interference-ideal network* model, valid for large full reuse networks [9], which states that the total interference that the users receive is weakly dependent on its location in the cell if many sources of interference are present (dense network).

Thus we obtain, for large N ,

$$\sum_{\substack{i=1 \\ i \neq n}}^N G_{u_n, i} P_{u_i} \approx G_I \sum_{\substack{i=1 \\ i \neq n}}^N P_{u_i}, \quad (4)$$

where G_I is the average interference gain value.

Monte-Carlo simulations of the distribution of the out-of-cell interference in a finite network show this to be good approximation [9], where the variation in the interference from the boundary of the cell to the center is small. Note that this model is employed in this paper only to obtain a simplified algorithm, and not for numerical results: *The numerical results are based on a realistic channel model for all AP-UT links.*

Thus, for a network with a large number of cells, the expected value of the interference becomes

$$I = G_I(N - 1)FP_{\max}.$$

The expected network capacity of the system is then

$$E[\mathcal{C}] = \int_0^\infty P(\tilde{g}) f_G(\tilde{g}) \cdot \log_2 \left(1 + \frac{\tilde{g}P_{\max}}{N_0 + I} \right) d\tilde{g}. \quad (5)$$

IV. OPTIMIZATION OF NETWORK CAPACITY

The problem now is to find the function $P(g)$ satisfying criteria (2) and (3) which maximizes the expected network capacity, i.e. finding \mathcal{C}^* that solves the problem:

$$\mathcal{C}^* = \max_{P(\tilde{g})} \{E[\mathcal{C}]\}. \quad (6)$$

Although the problem in (5) is a difficult one in general, the advantage of this particular MCA scheme lies in the possibility of doing off-line optimization of the function $P(g)$. Once determined, this function can be used by each individual AP to determine if it will be active or not. The problem now is that optimizing the capacity with respect to the access probability distribution is very difficult to solve analytically. Differentiating (5), setting the derivative equal to zero, and solving for $P(g)$ can not be done for realistic $f_G(g)$, or without severely restricting $P(g)$.

Due to the difficulty in finding the optimal solution for many optimization problems, algorithms that can find near-optimal solutions have received a lot of attention. These heuristic algorithms do not guarantee the quality of the solution found. Though, in practice many have shown to be sufficient for a wide range of purposes, finding near-optimal solutions bounded by a polynomial time of low order. *Simulated annealing* (SA) is one such heuristic, addressing combinatorial optimization problems. It has found its way into many areas. Amongst these are the traveling salesman problem, antenna array optimization, image processing, and physical design of computers

A. Simlated Annealing

Simulated annealing is a method that came out of statistical physics [10], where it is desirable that melted solids reach low-energy states by lowering the temperature slowly through several stages. To simulate this process a simple Monte Carlo method was set forth in [11] to model the solidifying process as follows: The solid is characterized by the positions of its particles. A randomly chosen particle is given a small random displacement, and ΔE_{ij} is the difference in energy between the perturbed state j and the previous state i . If $\Delta E_{ij} < 0$ the perturbed state is kept. If not the

perturbed state is kept with probability $e^{-\Delta E_{ij}/T}$ (this term relates to the Boltzmann distribution). This is done by picking a random number from a uniform distribution on the interval $(0, 1)$, and comparing this to $P(\Delta E_{ij})$. If the number is less than $P(\Delta E_{ij})$ the perturbed state is kept, otherwise the original state is used. As opposed to iterative improvement, the probabilistic behavior of simulated annealing gives the algorithm a chance to escape local minima. However, it is important that the cooling is sufficiently slow so that the solid reaches thermal equilibrium at each stage. If not defects can be frozen into the solid, and it does not reach the minimum energy state. The rate at which the temperature is lowered, as well as how many random displacements are made at each temperature level, is given by the *cooling schedule*. The algorithm written in pseudo-code can be found in e.g. [12].

B. Adaptation to Multiple Access Scheme

We start by discretizing the probability of access function $P(g)$, transforming the problem given in (6) into a combinatorial one. Let the channel gains be discretized into the V values g_1, \dots, g_V , and the function values be discretized into the W values P_1, \dots, P_W so that $P(g_v) \in \{P_w\}$. Applying this to the constraints in (2) and (3) we get

$$0 \leq P(g_v) \leq 1 \quad (7)$$

$$P(g_v) \geq P(g_{v-1}). \quad (8)$$

For optimization by simulated annealing we let the capacity \mathcal{C} correspond to the energy function and $\{P_w\}$ represent the states of the system. A small random displacement is given as one where, picking one of the g_v values at random, the corresponding P_w value is changed randomly so that $\{P_w\}$ still satisfies the constraints given in (7) and (8).

The initial input to the SA algorithm should be a random function, which in practice is difficult to generate. A generating function must only satisfy the constraints given in (2) and (3). We found that a simple ad-hoc solution to this problem is the function $\{P(g_v)\}$ found by running Algorithm 1. The idea is for each choice of $P(g_v)$ to have an average derivative α_v which changes as v increases, with α_1 and α_V chosen from a uniform distribution \mathcal{U} , with added randomness to it.

In Algorithm 1 the function `linspace` generates a vector α of V linearly equally spaced numbers between α_1 and α_2 , `rand` is a function returning numbers uniformly distributed between zero and one, `round(R)` rounds R to the nearest integer, and $\mathbf{P} = [P(g_1), \dots, P(g_V)]$.

Algorithm 1 Generating Random Function

- 1: Initialize $\alpha_1, \alpha_V \sim \mathcal{U}(0.5 + \epsilon, 1)$
 - 2: $\alpha = \text{linspace}(\alpha_1, \alpha_V, V)$
 - 3: $P(g_1) = 1$
 - 4: **for** $v = 2$ **to** V **do**
 - 5: $R = \alpha_v \cdot \text{rand}$
 - 6: $P(g_v) = P(g_{v-1}) + \text{round}(R)$
 - 7: **end for**
 - 8: $\mathbf{P} = \lceil \mathbf{P}/P_V \rceil$
-

The function $\lceil \cdot \rceil$ denotes the ceiling function. Running Algorithm 1 several times showed that it served its purpose, yielding probability of access functions ranging from being nearly equivalent to having all cells on ($g_v = 1$ for $v = 1, \dots, V$) to those having most cells turned off ($g_v = 0$ for $v = 1, \dots, V$).

The SA algorithm cools down through K temperature levels. A cooling schedule with the temperature at the k th level being $T = T_0/k$ was found to ensure a “liquid” state in the beginning, but convergence to a solid state for the last part of the iterations. At each temperature level the algorithm goes through L permutations. As K becomes smaller the algorithm converges towards a solution for the probability of access function $P(g)$ yielding a “low energy” / high capacity state. This gives the SA algorithm for our optimization problem shown in Algorithm 2.

Algorithm 2 Simulated Annealing Algorithm

- 1: Initialize $\{P(g_v)\}_i$ and T_0
 - 2: **for** $k = 1$ **to** K **do**
 - 3: $T_k = T_0/k$
 - 4: **for** $l = 1$ **to** L **do**
 - 5: $\{P(g_v)\}_j = \mathcal{P}(\{P(g_v)\}_i)$
 - 6: $\Delta\mathcal{C} = \mathcal{C}(\{P(g_v)\}_j) - \mathcal{C}(\{P(g_v)\}_i)$
 - 7: **if** $\Delta\mathcal{C} > 0$ **or** $e^{-\Delta\mathcal{C}/T_k} > \text{rnd}(0, 1)$ **then**
 - 8: $\{P(g_v)\}_i = \{P(g_v)\}_j$
 - 9: **end if**
 - 10: **end for**
 - 11: **end for**
-

V. SIMULATIONS

A hexagonal cellular system (cfr. Figure 1) functioning at 1800 MHz is considered, consisting of $N = 19$ cells of 1 km radius. The users in each cell are randomly placed according to a uniform distribution. Channel gains for *both inter-cell and intra-cell* AP-UT links are based on a COST 231 path loss model [13] including log-normal

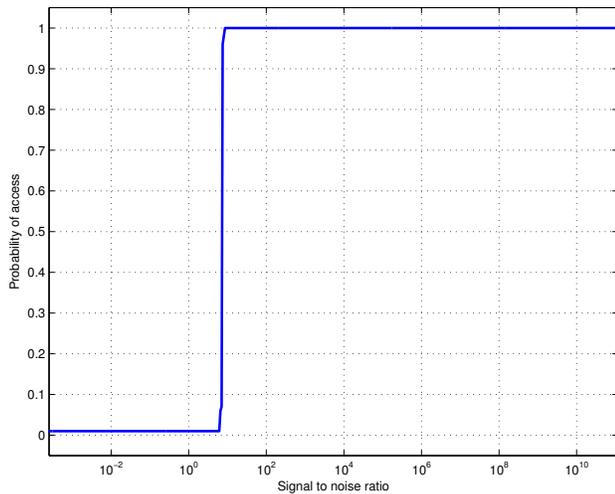


Fig. 2. Optimized probability of access function for $N = 19$ cells.

shadowing plus fast-fading. Log-normal shadowing is a zero mean Gaussian distributed random variable in dB with a standard deviation of 10 dB. Fast-fading is modeled by i.i.d. $\mathcal{CN}(0, 1)$ random variables and $P_{MAX} = 1$ W.

For the simulated annealing we choose an initial temperature of $T_0 = 1$. For the initial probability of access function a choice of $\epsilon = 0.05$ in Algorithm 1 gave a wide range of functions. The number of iterations in Algorithm 2 was $K = 250 \cdot 10^3$, and the number of permutations for each iteration was 10^3 . These choices ensure a liquid state in the beginning, and no new solutions being accepted at the end.

Surprisingly, the SA algorithm converged to a binary probability of access function $P(g) = u(g - T)$, where $u(g)$ is the unit step function and $T = 7.5$. This solution is shown in Figure 2. We then ran the simulations 10 more times, having a range of functions for the initial configuration, and the result was validated with convergence to the same solution every time. This is actually quite a powerful result, since off-line optimization of the probability of access function $P(g)$ can be done based only on the fading statistics. During operation each individual AP then can use $P(g)$ to determine if it will be active or not.

We compared this with simulations of the capacity of a realistic wireless system, using (1). The performance evaluation is based on Monte Carlo simulations running over 10 000 random channel realizations, using the same path loss model as for the expected capacity calculations. This resulted in a capacity of 3.95 bits/sec/Hz/cell. Compared to having all cells on, the gain of the MCA

scheme was 19 %. The gain over the static reuse schemes with factor three and four were a superior 55 % and 70 %, respectively.

VI. CONCLUSION

In this work we have studied the optimization of a fully decentralized framework for multi-cell access schemes. The path loss model used for the simulations is based on realistic wireless networks. Through numerical optimization it has been found that a binary probability of access function optimizes the system capacity, yielding a significant increase compared to traditional static reuse schemes, and corroborating previously reported results.

REFERENCES

- [1] J. E. Kirkebø, D. Gesbert, and S. G. Kiani, "Maximizing the capacity of wireless networks using multi-cell access schemes," in *Proc. IEEE SPAWC*, Cannes, France, July 2006.
- [2] R. Knopp and P. Humblet, "Information capacity and power control in single-cell multiuser communications," in *Proc. IEEE ICC*, vol. 1, Seattle, WA, June 1995, pp. 331–335.
- [3] A. Bedekar, S. Borst, K. Ramanan, P. Whiting, and E. Yeh, "Downlink scheduling in cdma data networks," in *Proc. IEEE Globecom*, vol. 5, Rio de Janeiro, Brazil, Dec. 1999, pp. 2653–2657.
- [4] I. Koutsopoulos and L. Tassiulas, "Channel state-adaptive techniques for throughput enhancement in wireless broadband networks," in *Proc. IEEE INFOCOM*, vol. 2, Anchorage, AK, Apr. 2001, pp. 757–766.
- [5] V. Tralli, R. Veronesi, and M. Zorzi, "Power-shaped advanced resource assignment for fixed broadband wireless access systems," *IEEE Trans. Wireless Commun.*, vol. 3, no. 6, pp. 2207–2220, Nov. 2004.
- [6] S. G. Kiani, D. Gesbert, J. E. Kirkebø, A. Gjendemsjø, and G. E. Øien, "A simple greedy scheme for multicell capacity maximization," in *Proc. IEEE ITS*, Fortaleza, Brazil, Sept. 2006.
- [7] X. Qin and R. Berry, "Exploiting multiuser diversity for medium access control in wireless networks," in *Proc. IEEE INFOCOM*, vol. 2, San Francisco, CA, Apr. 2003, pp. 1084–1094.
- [8] S. Adireddy and L. Tong, "Exploiting decentralized channel state information for random access," *IEEE Trans. Inform. Theory*, vol. 51, no. 2, pp. 537–561, Feb. 2005.
- [9] S. G. Kiani and D. Gesbert, "Optimal and distributed scheduling for multicell capacity maximization," *IEEE Trans. Wireless Commun.*, Submitted for publication 2006.
- [10] S. Kirkpatrick, C. D. Gelatt Jr., and M. P. Vecchi, "Optimization by simulated annealing," *Science*, vol. 220, no. 4598, pp. 671–680, May 1983.
- [11] N. Metropolis, A. Rosenbluth, M. Rosenbluth, A. Teller, and E. Teller, "Equation of state calculations by fast computing machines," *J. of Chem. Physics*, vol. 21, 1953.
- [12] P. J. M. van Laarhoven and E. H. L. Aarts, *Simulated Annealing: Theory and Applications*. Reidel Publishing Company, 1987.
- [13] E. Damosso and L. M. Correia, Eds., *Digital Mobile Radio Towards Future Generation Systems*. Brussels, Belgium: European Commission, 1999, COST 231 Final Report.