Low Complexity Scheduling and Beamforming for Multiuser MIMO Systems

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Abstract – The problem of joint scheduling and beamforming for a multiple antenna broadcast channel with partial channel state information at the transmitter (CSIT) is considered. We show how long-term statistical channel knowledge can be efficiently combined with instantaneous low-rate feedback for user selection and linear beamforming. A low complexity algorithm based on an estimate (bound) of the multiuser interference is proposed. Our scheme is shown to exhibit significant throughput gain over opportunistic techniques, approaching the sum rate of full CSIT for small angle spreads.

1. INTRODUCTION

Multiple-input multiple-output (MIMO) systems can significantly increase the spectral efficiency by exploiting the spatial degrees of freedom created by multiple antennas. In point-to-point MIMO systems, the capacity increases linearly with the minimum of the number of transmit/receive antennas, irrespective of the availability of channel state information (CSI). In the MIMO broadcast channel, it has recently been proven [1] that the sum capacity is achieved by dirty paper coding (DPC) [2]. However, the applicability of DPC is limited due to its computational complexity and the need for full channel state information at the transmitter (CSIT). Several downlink techniques based on Space Division Multiple Access (SDMA) have been proposed [3], achieving the same asymptotic sum rate as that of DPC.

The capacity gain of multiuser MIMO systems is highly dependent on the available CSIT. While having full CSI at the receiver can be assumed, this assumption is not reasonable at the transmitter side. In [4], a finite rate feedback model is proposed, in which each receiver quantizes its channel and feeds back a finite number of bits regarding its channel realization based on a codebook. An SDMA extension of opportunistic beamforming [5] using partial CSIT in the form of individual signal-to-interference-plus-noise ratio (SINR) is proposed in [6], achieving optimum capacity scaling for large number of users. However, the schemes proposed in [3]-[6] do not exploit long-term statistical knowledge of the channel. Combining the spatial correlation with the channel norm is addressed in [7], however it has not been exploited in an SDMA context. A means to combine instantaneous scalar feedback with statistical CSIT for the sole purpose of user selection is proposed in [8].

Here we make the following key points:

- Statistical CSIT, while causing almost negligible per-slot feedback overhead, can reveal information about the spatial separability of users.
- Scalar feedback can be used at the transmitter to evaluate the quality of the CSIT and estimate the multiuser interference.

Based on the above points we propose a practical low complexity scheme with joint scheduling and beamforming. Each user has a predefined beamforming vector, matched to the principal eigenvector of the channel correlation matrix. In order to achieve full multiuser diversity gain, we propose to feed back the following scalar values: 1) the alignment between the channel and each user's predefined beamforming vector 2) the channel norms. Our method shows a considerable gain over random opportunistic beamforming for angle spread less than 45 degrees, which makes it a practical approach especially for cellular outdoor systems.

2. SYSTEM MODEL

We consider a multiple antenna broadcast channel consisting of M transmit antennas and K single-antenna receivers. The signal received by the k-th user is mathematically described as

$$y_k = \mathbf{h}_k^H \mathbf{x} + n_k, \qquad k = 1, \ldots, K \tag{1}$$

where $\mathbf{x} \in \mathbb{C}^{M\times 1}$ is the transmitted signal, $\mathbf{h}_k \in \mathbb{C}^{M\times 1}$ is the channel vector, and n_k is additive white complex

Gaussian noise, which is assumed to be independent and identically distributed (i.i.d.) with zero mean and unit variance. We assume a block-fading channel with independent fading from block to block and perfectly known to the receiver.

We consider transmission in a downlink where insufficient scattering around the transmitter makes the MIMO channel spatially correlated. The channel vector is complex Gaussian distributed with zero mean and full-rank correlation matrix $\mathbf{R}_k = \mathbb{E}\{\mathbf{h}_k \mathbf{h}_k^H\}$. We assume that $\mathbf{R}_k$ is perfectly known at both ends of the link, which can be obtained from uplink measurements or using a low-rate feedback channel. The eigendecomposition of the transmit correlation matrix is $\mathbf{R}_k = \mathbf{V}_k \mathbf{\Lambda}_k \mathbf{V}_k^H$, where $\mathbf{\Lambda}_k$ is a diagonal matrix with the eigenvalues of $\mathbf{R}_k$ in descending order and $\mathbf{V}_k = [\mathbf{v}_k^1 \ \mathbf{v}_k^2 \cdots \mathbf{v}_k^M]$ is a unitary matrix with the eigenvectors of $\mathbf{R}_k$.

3. JOINT SCHEDULING AND BEAMFORMING

We limit here to the case of linear beamforming where exactly M spatially separated users access the channel simultaneously.

Let $\mathbf{w}_k$ and s_k be the (normalized) beamforming vector and data symbol of the k-th user, respectively. Define $\mathbf{H} \in \mathbb{C}^{K \times M}$ as the concatenation of all user channels, $\mathbf{H} = [\mathbf{h}_1 \ \mathbf{h}_2 \ldots \mathbf{h}_K]^H$. Let $\mathcal{Q}$ be the set of all possible subsets of cardinality M of disjoint indices among the complete set of user indices $\{1, \cdots, K\}$. Let $\mathcal{S} \in \mathcal{Q}$, be one such group of M users selected for transmission at a given time slot. Then $\mathbf{H}(\mathcal{S})$, $\mathbf{W}(\mathcal{S})$, $\mathbf{s}(\mathcal{S})$, $\mathbf{y}(\mathcal{S})$ are the concatenated channel vectors, beamforming vectors, uncorrelated data symbols and received signals respectively for the set of scheduled users. The signal model is

$$\mathbf{y}(\mathcal{S}) = \mathbf{H}(\mathcal{S})\mathbf{W}(\mathcal{S})\mathbf{s}(\mathcal{S}) + \mathbf{n} \tag{2}$$

At the k-th mobile the received signal is given by

$$y_k = \sum_{i \in \mathcal{S}} \mathbf{h}_k^H \mathbf{w}_i s_i + n_k, \qquad k = 1, \ldots, K \tag{3}$$

Assuming an average transmit power constraint P and equal power allocation on each of the data streams, the SINR at the k-th receiver is

$$SINR_k = \frac{\frac{P}{M} |\mathbf{h}_k^H \mathbf{w}_k|^2}{\sum_{i \in \mathcal{S}, i \neq k} \frac{P}{M} |\mathbf{h}_k^H \mathbf{w}_i|^2 + 1} \tag{4}$$

We focus on the ergodic sum rate (SR) which, assuming Gaussian inputs, is equal to

$$SR = \mathbb{E}\left\{ \sum_{k \in \mathcal{S}} \log\left[1 + SINR_k\right] \right\} \tag{5}$$

4. PROPOSED ALGORITHM

We propose an algorithm that performs joint scheduling and beamforming in the downlink, based on statistical channel information and limited instantaneous channel feedback. As our optimization criterion is to maximize the system throughput, it is desirable to schedule a set of M users with large channel gains and mutually orthogonal beamforming vectors. The proposed algorithm is outlined in Table 1.

4.1. Feedback Strategy

If the CSIT consists of channel gains and quantized channel directions, the multiuser interference cannot be completely eliminated, resulting to a bounded sum rate even for $K \to \infty$ [9]. Motivated by that, in order to achieve full multiuser diversity gain, we propose that each user feeds back the following scalar values: *1)* the alignment between its instantaneous normalized channel vector $\overline{\mathbf{h}}_k$ and a preset beamforming vector, $\rho_k = \left|\overline{\mathbf{h}}_k^H \mathbf{w}_k\right|$ and *2)* its channel norm $\|\mathbf{h}_k\|^2$.

4.2. User scheduling

If a perfectly orthogonal set of beamforming vectors can be found, the above mentioned instantaneous limited feedback is sufficient to achieve the same asymptotic sum rate as that of DPC. However, in practice, this cannot be fulfilled and the remaining interference cannot be calculated explicitly. Therefore, we derive a bound on the multiuser interference based on the available limited feedback, which is shown in details in the next section. For user k and index set $\mathcal{S}$, the multiuser interference can be expressed as $I_k(\mathcal{S}) = \sum_{i \in \mathcal{S}, i \neq k} \frac{P}{M} |\mathbf{h}_k^H \mathbf{w}_i|^2 = \frac{P}{M} \|\mathbf{h}_k\|^2 \, \overline{I}_k(\mathcal{S})$, where $\overline{I}_k(\mathcal{S})$ denotes the interference over the normalized channel $\overline{\mathbf{h}}_k$. Using $\overline{I}_{UB_k}(\mathcal{S})$, the upper bound on $\overline{I}_k(\mathcal{S})$, we have the following lower bound on the SINR:

$$SINR_k^{LB}(\mathcal{S}) = \frac{\frac{P}{M} \|\mathbf{h}_k\|^2 \rho_k^2}{\frac{P}{M} \|\mathbf{h}_k\|^2 \, \overline{I}_{UB_k}(\mathcal{S}) + 1} \tag{6}$$

The scheduler is optimized to select the set of users that maximize a lower bound on the SR as follows

$$\mathcal{S}^* = \arg\max_{\mathcal{S}} \sum_{k \in \mathcal{S}} \log\left[1 + SINR_k^{LB}(\mathcal{S})\right] \tag{7}$$

In order to reduce the complexity of the user search, we define a threshold μ_{th} and consider only users with product $\|\mathbf{h}_k\| \rho_k > \mu_{th}$. This threshold is defined by selecting the top μ percent values of $\|\mathbf{h}_k\| \rho_k$, and becomes necessary in dense networks where exhaustive search may be prohibitive, even though the computation of (7) entails low complexity.

Table 1: Outline of the Proposed Algorithm

INITIALIZATION	
MS	
Update & Feedback	$\mathbf{w}_k = \mathbf{v}_k^1 \to$ BS $\forall k = 1, \cdots, K$
BS	
Compute & Store	$\alpha_k(\mathcal{S})$, $\beta_k(\mathcal{S})$, $\gamma_k(\mathcal{S})$ $\forall k, \mathcal{S}$ (eq.10)
AT EACH TIME SLOT	
MS	
Compute & Feedback	$\|\mathbf{h}_k\| \to$ BS $\quad \forall k = 1, \cdots, K$ $\rho_k = \left\|\overline{\mathbf{h}}_k^H \mathbf{w}_k\right\| \to$ BS
BS	
User selection	
Step 0 Preselect users with $\|\mathbf{h}_k\|\rho_k > \mu_{th}$, $\mathcal{Q} \to \mathcal{Q}'$ Set $SR_{LB}^* = 0$ and $\mathcal{S}^* = \emptyset$	
For all $\mathcal{S} \in \mathcal{Q}'$ repeat	
Step 1 Compute $\overline{I}_{UB_k}(\mathcal{S}) = \rho_k^2 \alpha_k(\mathcal{S}) + (1-\rho_k^2)\,\beta_k(\mathcal{S}) + 2\rho_k\sqrt{1-\rho_k^2}\gamma_k(\mathcal{S})$	
Step 2 Compute $SINR_k^{LB}(\mathcal{S}) = \frac{\frac{P}{M}\|\mathbf{h}_k\|^2\rho_k^2}{\frac{P}{M}\|\mathbf{h}_k\|^2\overline{I}_{UB_k}(\mathcal{S})+1}$	
Step 3 Compute $SR_{LB} = \sum_{k\in S}\log\left[1 + SINR_k^{LB}(\mathcal{S})\right]$	
Step 4 If $SR_{LB} > SR_{LB}^*$, $SR_{LB} \to SR_{LB}^*$ and $\mathcal{S} \to \mathcal{S}^*$	
Beamforming	
Construct beamforming matrix $\mathbf{W}(\mathcal{S})$	

4.3. Beamforming Vectors

As a low complexity approach, we consider a system where each user has a preferred beamforming vector known both by the base station (BS) and mobile (MS). As shown in [10], for single-user MIMO communications, given a certain user k with correlation matrix $\mathbf{R}_k$ the average rate is maximized by matching the beamforming vector to the principal eigenvector of its correlation matrix, $\mathbf{w}_k = \mathbf{v}_k^1$. Hence, we design each user's beamforming vector according to this strategy. This is equivalent to a system where each user has a trivial - yet practical - codebook with a single codevector (eigen-codebook), which is updated at a very low rate when the transmit correlation changes in time.

5. BOUND ON THE MULTIUSER INTERFERENCE

First, we need to introduce the following lemma.

***Lemma* 1.** *Let $\mathbf{U}_k \in \mathbb{C}^{M\times(M-1)}$ be an orthonormal basis spanning the null space of $\mathbf{w}_k$. Then,*

$$\left\|\overline{\mathbf{h}}_k^H \mathbf{U}_k\right\|^2 = 1 - \rho_k^2 \tag{8}$$

Proof: See Appendix A.

Define the matrix $\Psi_k(\mathcal{S}) = \sum_{i\in S, i\neq k} \mathbf{w}_i\mathbf{w}_i^H$ and the operator $\lambda_{max}\{\cdot\}$, which returns the largest eigenvalue. We obtain the following result:

Theorem 1. *Given an arbitrary set of unit-norm beamforming vectors in $\mathbb{C}^M$: $\{\mathbf{w}_i, i \in \mathcal{S}\}$, the interference experienced by the k-th user can be bounded as follows*

$$\begin{aligned} \overline{I}_k(\mathcal{S}) \;\leq\; & \rho_k^2\, \mathbf{w}_k^H \Psi_k(\mathcal{S})\mathbf{w}_k \\ & + \left(1-\rho_k^2\right)\lambda_{max}\left\{\mathbf{U}_k^H \Psi_k(\mathcal{S})\mathbf{U}_k\right\} \\ & + 2\rho_k\sqrt{1-\rho_k^2}\left\|\mathbf{U}_k^H \Psi_k(\mathcal{S})\mathbf{w}_k\right\| \end{aligned} \tag{9}$$

Proof: See Appendix B.

Note that, in the proposed algorithm, when computing the interference bound from (9) for each user k and index set $\mathcal{S}$, the following values can be prestored at the BS

$$\begin{cases} \alpha_k(\mathcal{S}) = \mathbf{w}_k^H \Psi_k(\mathcal{S})\mathbf{w}_k \\ \beta_k(\mathcal{S}) = \lambda_{max}\left\{\mathbf{U}_k^H \Psi_k(\mathcal{S})\mathbf{U}_k\right\} \\ \gamma_k(\mathcal{S}) = \left\|\mathbf{U}_k^H \Psi_k(\mathcal{S})\mathbf{w}_k\right\| \end{cases} \tag{10}$$

hence not contributing to increase complexity during runtime. As shown in Table 1, the initialization procedure is performed before data transmission starts. Afterwards, the quantities $\alpha_k(\mathcal{S})$, $\beta_k(\mathcal{S})$, $\gamma_k(\mathcal{S})$ and $\mathbf{w}_k$ can be updated either when a new user enters the system or periodically, in order to account for variations in the users' long-term statistics. Thus, the upper bound on the multiuser interference involves low computational complexity and can be expressed as

$$\overline{I}_{UB_k}(\mathcal{S}) = \rho_k^2\alpha_k(\mathcal{S}) + \left(1-\rho_k^2\right)\beta_k(\mathcal{S}) + 2\rho_k\sqrt{1-\rho_k^2}\gamma_k(\mathcal{S}) \tag{11}$$

Consider the following particular cases to have a more intuitive idea of this bound: *(a)* $\rho_k \to 1$, then $\overline{I}_{UB_k}(\mathcal{S}) \to \rho_k^2\alpha_k(\mathcal{S})$. In this case, the interference is due to non-orthogonalities between $\mathbf{w}_k$ and the remaining beamforming vectors; *(b)* as the beamforming vectors become orthogonal, i.e. $\left|\mathbf{w}_i^H\mathbf{w}_j\right| \to 0\ \forall i \neq j \in \mathcal{S}$, then $\alpha_k(\mathcal{S}) \to 0$, $\beta_k(\mathcal{S}) \to 1$, $\gamma_k(\mathcal{S}) \to 0$ and hence $\overline{I}_{UB_k}(\mathcal{S}) \to 1-\rho_k^2$. Particularly, when perfect orthogonality exists, we can state the following

***Corollary* 1.** *Given an orthonormal set of beamforming vectors in $\mathbb{C}^M$, the interference experienced by user $k \in \mathcal{S}$ is given by*

$$\overline{I}_k(\mathcal{S}) = 1 - \rho_k^2 \tag{12}$$

Proof: Direct consequence of *Lemma 1* when the interfering beamforming vectors are basis vectors of the null space of $\mathbf{w}_k$.

Hence, the derived interference bound becomes tighter as the orthogonality between beamforming vectors increases.

5.1. Efficient metric for dense networks

Motivated by *Corollary 1*, an alternative SINR metric from equation (6) can be derived. Note that the alignment ρ_k corresponds to the cosine of the angle between $\overline{\mathbf{h}}_k$ and $\mathbf{w}_k$, i.e., $\rho_k = \cos\phi_k$. Assuming orthogonal beamforming vectors, the interference in (12) can be expressed as $\overline{I}_k(\mathcal{S}) = 1 - \rho_k^2 = 1 - \cos^2\phi_k = \sin^2\phi_k$. In that case, the achievable SINR becomes

$$SINR_k = \frac{\frac{P}{M}\|\mathbf{h}_k\|^2\cos^2\phi_k}{\frac{P}{M}\|\mathbf{h}_k\|^2\sin^2\phi_k + 1} \tag{13}$$

This reduced complexity metric has the advantage that it can be computed at the MS without user cooperation and only *one* scalar value needs to be fed back. At the BS, scheduling users with orthogonal beamforming vectors ensures they achieve the SINR in (13). Alternatively, in the case of quasi-orthogonal set of users, the above metric becomes an efficient estimate (upper bound) of the SINR which, combined with user selection, can achieve the optimal capacity scaling of $M \log\log K$ [9].

6. NUMERICAL RESULTS

At the transmitter side (BS), we consider a uniform linear antenna array (ULA) with antenna spacing $d = 0.4\lambda$, where λ is the wavelength (here for a 2GHz system). We assume that the channel evolves according to a specular model where the channel impulse response is a superposition of a finite number of paths. The path gains are assumed to be zero-mean, unit variance complex Gaussian distributed. Each of these paths have a Gaussian distributed angle of incidence with respect to the transmitter broadside.

We evaluate our scheme for $M = 2$ antennas and $\text{SNR} = 10\text{dB}$. Figures 1 and 2 show a performance comparison in terms of sum rate for several approaches, as a function of the number of users and the angle spread, respectively. Since the proposed scheme is based on matched filtering (MF), we give as a reference the performance of MF with perfect CSIT. In order to evaluate the proposed user selection metric based on reduced feedback, we also compare with a scheme that while having the same beamforming strategy (transmission along the principal eigenvector of $\mathbf{R}_k$), uses full CSIT for user scheduling. Our method shows a clear gain over random beamforming [6] for angle spread less than 45 degrees making it a practical approach for cellular outdoor systems.

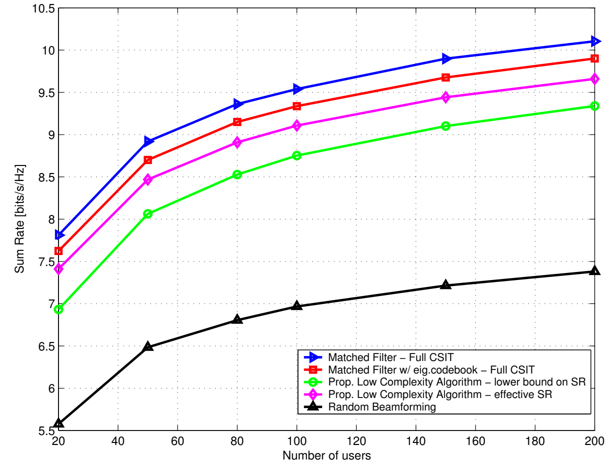

Figure 1: Sum rate as a function of the number of users for $M = 2$, and $\sigma_\theta = 0.1\pi$.

7. CONCLUSIONS

We presented a low complexity beamforming/scheduling algorithm for spatially correlated MIMO channels, exploiting statistical channel knowledge combined with limited instantaneous feedback. The proposed scheme exhibits performance close to that of full CSIT when the multipath angular spread at the transmitter is small enough, making this approach suitable to wireless systems with elevated base station such as outdoor cellular networks.

Appendix A.

Proof of Lemma 1:
Define the orthonormal basis $\mathbf{Z}_k$ of $\mathbb{C}^M$ obtained by stacking the column vectors of $\mathbf{U}_k$ and $\mathbf{w}_k$: $\mathbf{Z}_k = [\mathbf{U}_k \mathbf{w}_k]$. Since $\mathbf{Z}_k\mathbf{Z}_k^H = \mathbf{I}$ and $\overline{\mathbf{h}}_k$ has unit power

$$\left\|\overline{\mathbf{h}}_k^H \mathbf{Z}_k\right\|^2 = \overline{\mathbf{h}}_k^H \mathbf{Z}_k \mathbf{Z}_k^H \overline{\mathbf{h}}_k = \overline{\mathbf{h}}_k^H \overline{\mathbf{h}}_k = 1 \tag{14}$$

Then, by definition of $\mathbf{Z}_k$ we can separate the power of $\overline{\mathbf{h}}_k$ as follows

$$\left\|\overline{\mathbf{h}}_k^H \mathbf{Z}_k\right\|^2 = \left\|\overline{\mathbf{h}}_k^H [\mathbf{U}_k \mathbf{w}_k]\right\|^2 = \left\|\overline{\mathbf{h}}_k^H \mathbf{U}_k\right\|^2 + \left|\overline{\mathbf{h}}_k^H \mathbf{w}_k\right|^2 = 1 \tag{15}$$

Setting $\left|\overline{\mathbf{h}}_k^H \mathbf{w}_k\right|^2 = \rho_k^2$ and solving the above equation for $\left\|\overline{\mathbf{h}}_k^H \mathbf{U}_k\right\|^2$ we obtain the desired result.

Appendix B.

Proof of Theorem 1:
Using the definition of $\Psi_k(\mathcal{S})$, the interference for user k and index set $\mathcal{S}$ can be expressed as

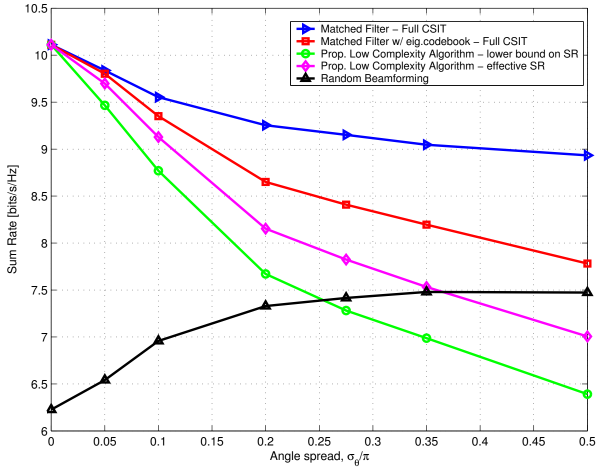

Figure 2: Sum rate as a function of angle spread for $M = 2$, antenna spacing $d = 0.4\lambda$ and $K = 100$ users.

$$\overline{I}_k(\mathcal{S}) = \sum_{i\in\mathcal{S}, i\neq k} \left|\overline{\mathbf{h}}_k^H \mathbf{w}_i\right|^2 = \sum_{i\in\mathcal{S}, i\neq k} \overline{\mathbf{h}}_k^H \mathbf{w}_i \mathbf{w}_i^H \overline{\mathbf{h}}_k = \overline{\mathbf{h}}_k^H \Psi_k(\mathcal{S}) \overline{\mathbf{h}}_k \quad (16)$$

The normalized channel $\overline{\mathbf{h}}_k$ can be expressed as a linear combination of orthonormal basis vectors. Using *Lemma 1*, all possible unit-norm $\overline{\mathbf{h}}_k$ vectors with $\left|\overline{\mathbf{h}}_k^H \mathbf{w}_k\right| = \rho_k$ can be written as follows

$$\overline{\mathbf{h}}_k = \rho_k e^{j\alpha_k} \mathbf{w}_k + \sqrt{1-\rho_k^2}\,\mathbf{U}_k \mathbf{B}_k \mathbf{e}_k \quad (17)$$

where $\mathbf{B}_k$ is a diagonal matrix with entries $e^{j\beta_i}, i = 1, \ldots, M-1$ and $\mathbf{e}_k$ is an arbitrary unit-norm vector in $\mathbb{C}^{M-1}$. The complex phases β_i and α_k are unknown and lie in $[0, 2\pi]$. Substituting (17) into (16) we get

$$\begin{aligned} \overline{I}_k(\mathcal{S}) &= \rho_k^2\ \mathbf{w}_k^H \Psi_k(\mathcal{S}) \mathbf{w}_k \\ (a) \qquad & + \left(1-\rho_k^2\right) \mathbf{e}_k^H \mathbf{B}_k^H \mathbf{U}_k^H \Psi_k(\mathcal{S}) \mathbf{U}_k \mathbf{B}_k \mathbf{e}_k \\ (b) \qquad & + \rho_k \sqrt{1-\rho_k^2}\, [e^{-j\alpha_k} \mathbf{w}_k^H \Psi_k(\mathcal{S}) \mathbf{U}_k \mathbf{B}_k \mathbf{e}_k \\ & \qquad + \mathbf{e}_k^H \mathbf{B}_k^H \mathbf{U}_k^H \Psi_k(\mathcal{S}) \mathbf{w}_k e^{j\alpha_k}] \end{aligned} \quad (18)$$

Since the first term in (18) is perfectly known, the upper bound on $\overline{I}_k(\mathcal{S})$ is found by joint maximization of the summands (a) and (b) with respect to α_k, $\mathbf{B}_k$ and $\mathbf{e}_k$. We use a simpler optimization method, which consists of bounding separately each term.

(a) Defining $\mathbf{A}_k(\mathcal{S}) = \mathbf{U}_k^H \Psi_k(\mathcal{S}) \mathbf{U}_k$ for clarity of exposition, the second term can be bounded as follows

$$\max_{\mathbf{B}_k, \mathbf{e}_k} \left(1-\rho_k^2\right) \mathbf{e}_k^H \mathbf{B}_k^H \mathbf{A}_k(\mathcal{S}) \mathbf{B}_k \mathbf{e}_k = \left(1-\rho_k^2\right) \lambda_{max}\{\mathbf{A}_k(\mathcal{S})\} \quad s.t.\ \|\mathbf{e}_k\| = 1 \quad (19)$$

where the operator $\lambda_{max}\{\cdot\}$ returns the largest eigenvalue. The maximum in (19) is obtained when the vector $\mathbf{B}_k \mathbf{e}_k$ equals the principal eigenvector of the matrix $\mathbf{A}_k(\mathcal{S})$.

(b) Defining $\mathbf{q}_k = \mathbf{B}_k^H \mathbf{U}_k^H \Psi_k(\mathcal{S}) \mathbf{w}_k e^{j\alpha_k}$ and noting that the matrix $\Psi_k(\mathcal{S})$ is Hermitian by construction, the bound on the third term in (18) can be written as follows

$$\max_{\mathbf{q}_k, \mathbf{e}_k} \rho_k \sqrt{1-\rho_k^2} \left[\mathbf{q}_k^H \mathbf{e}_k + \mathbf{e}_k^H \mathbf{q}_k\right] = \max_{\mathbf{q}_k} 2\rho_k \sqrt{1-\rho_k^2}\, \|\mathbf{q}_k\| \quad s.t.\ \|\mathbf{e}_k\| = 1 \quad (20)$$

The left hand side is maximized for $\mathbf{e}_k = \frac{\mathbf{q}_k}{\|\mathbf{q}_k\|}$, which satisfies the unit-norm constraint, yielding the modified bound in (20). The solution is given by

$$\begin{aligned} \max_{\mathbf{q}_k} 2\rho_k \sqrt{1-\rho_k^2}\, \|\mathbf{q}_k\| &= \max_{\mathbf{B}_k, \alpha_k} 2\rho_k \sqrt{1-\rho_k^2} \left\|\mathbf{B}_k^H \mathbf{U}_k^H \Psi_k(S) \mathbf{w}_k e^{j\alpha_k}\right\| \\ &= 2\rho_k \sqrt{1-\rho_k^2} \left\|\mathbf{U}_k^H \Psi_k(S) \mathbf{w}_k\right\| \end{aligned} \quad (21)$$

Finally, incorporating into (18) the bounds obtained in (19) and (21) we obtain the desired bound.

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