BLIND MMSE-ZF RECEIVER AND CHANNEL IDENTIFICATION FOR ASYNCHRONOUS CDMA IN MULTIPATH CHANNELS*

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ABSTRACT

We consider an asynchronous DS-CDMA system operating in a multipath environment. The received cyclostationary spread signal is oversampled with respect to the chip rate (and/or is received at multisensors) and is converted to a stationary vector signal, leading to a linear multichannel model. The blind MMSE-ZF receiver is derived and an estimate of the channel is obtained using the spreading sequence properties and second-order statistics. The receiver turns out to be the exact extension to the general asynchronous multipath case of the constrained minimum-output energy (MOE) receiver originally proposed in [1]. Dimensional requirements for the determination of the receiver and channel identification are derived and extension to the case of sparse channels is made. It is observed that especially for the sparse scenario, an estimate of the desired user channel is obtained in a small number of data samples, despite the fact that the processing gain is quite large and that severe near-far conditions are prevalent. This makes the algorithm particularly suitable for the case of slow fading in the UMTS WCMDA, where, multiuser algorithms can be easily implemented once accurate channel estimates are available.

I. INTRODUCTION AND PREVIOUS WORK

Blind solutions for DS-CDMA systems have received considerable attention since the pioneering work of [1], which is based upon a *constrained* minimum output energy (MOE) criterion. The constrained MOE receiver constrains the inner product of the received signal with the spreading sequence to be fixed, thus restricting the optimization problem to within the constrained space. The desirable feature of such a scheme is that its informational complexity is the same as that of a matched filter detector, i.e., only the desired user signature waveform and timing information are required for its operation. Besides, the algorithm is *near-far* resistant.

The problem addressed in [1] was that of DS-CDMA communications over a channel without multipath. A constrained optimization scheme was proposed in [2] for multipath channels where the receiver's output energy is minimized subject to a general distortionless constraint. Connections with the *Capon* philosophy were drawn in that paper. The above mentioned receivers can be shown to converge asymptotically (SNR$\rightarrow \infty$) to the zero-forcing (ZF) or decorrelating solution. It was shown in [3] that in order to accommodate a number of users approaching code space dimensions, longer receivers are required for the ZF solution to be achievable. Moreover, we presented in [3] the optimal MMSE receiver for multipath channels and asynchronous conditions, obtained by applying multichannel linear prediction to the received cyclostationary signal. Direct estimation of the MMSE receiver was introduced in [4] following the observation that the MMSE receiver lies in the signal subspace. MMSE receiver constrained to the signal subspace in the case of channels longer than a symbol period was investigated in [5], where a singular-value decomposition (SVD) was used to determine the orthogonal subspaces. The channel estimate in this work was obtained as a generalization to longer delay spreads of the subspace technique originally proposed in [6]. The MMSE-ZF receiver, which corresponds to the unbiased MOE in the noiseless case, was derived in its full generality in [7]. Moreover, the above mentioned schemes have high complexity since an estimate of subspaces is required. A further drawback of all these schemes is that a large number of data samples are required to converge to the MMSE solution. This makes them quite unfeasible for mobile cellular applications, especially like the ETSI UMTS WCDMA [8], where the entire frame of 10 ms. contains only 160 data samples. Such approaches can therefore find their application in long duration stationary environments with small spreading gains. In order to alleviate this problem, we presented in [7], a semi-blind method that results in an improved performance with the use of a small number of training symbols. Further improvements based upon decision re-use were also explored in that paper.

Delay estimation, in principle, is much less of a problem in CDMA systems as compared to the estimation of phase and amplitudes of the channel. However, the coherent RAKE performs much better than the non-coherent one thus necessitating the estimation of the latter quantities. This is typically done on the uplink by the insertion of dedicated pilot channels (or symbols) for each user. Such is the case in UMTS WCDMA, where each user essentially contributes as two users, thus engendering an overhead of almost fifty percent. Furthermore, the pilot channels are 6 dB weaker than the data channels upon transmission. This makes the channel estimation by classical single-user methods all the more difficult, especially in *near-far* situations. On the other hand, blind estimation methods are typically based on subspace decomposition and are much too complex to be considered of practical interest. It is worth mentioning, however, that in the blind context, CDMA systems possess the most desirable characterstics of all existing multiuser systems with the necessary (extra) bandwidth and integrated *a priori* knowledge in terms of distinct spreading sequences. Furthermore, quite severe near-far situations are handled with ease by such algorithms.

We extend, in this work, the blind MMSE zero-forcing receiver for asynchronous DS-CDMA systems presented in [7] to the case of sparse channels. This receiver exploits spreading sequence properties to estimate the desired user channel at a low cost (a non-SVD based approach). Identification issues for sparse channels are discussed and it is seen that the complexity of the algorithm can be significantly reduced by exploiting the sparseness property.

II. MULTIUSER DATA MODEL

The p users are assumed to transmit linearly modulated signals over a linear multipath channel with additive Gaussian noise. It is assumed that the receiver employs L sensors to receive the mixture

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of signals from all users. Oversampling with respect to the symbol rate is already inherent to DS-CDMA systems due to the large (extra) bandwidth and the need to resolve chip pulses. The signal received at the lth sensor can be written in baseband notation as

$$y^l(t) = \sum_{j=1}^{p} \sum_{k} a_j(k) g_j^l(t - kT_s) + v^l(t), \tag{1}$$

where $a_j(k)$ are the transmitted symbols from the user j, T_s is the common symbol period, $g_j(t)$ is the overall channel impulse response for the jth user. Assuming the $\{a_j(k)\}$ and $\{v^l(t)\}$ to be jointly wide-sense stationary, the process $\{y(t)\}$ is wide-sense cyclostationary with period T_s. The overall channel impulse response for jth user's signal at the lth sensor, $g_j^l(t)$, is the convolution of the spreading code c_j and $h_j^l(t)$, itself the convolution of the chip pulse shape and the actual channel (assumed to be FIR) representing the multipath fading environment. This can be expressed as

$$g_j^l(t) = \sum_{s=0}^{m-1} c_j(s) h_j^l(t - sT), \tag{2}$$

where T is the chip duration. The symbol and chip periods are related through the processing gain m: $T_s = mT$. Sampling the received signal R times the chip rate, we obtain the wide-sense stationary $mR \times 1$ vector signal $\boldsymbol{y}^l(k)$ at the symbol rate. We consider the overall channel delay spread between the jth user and all of the L antennas to be of length $m_j T$. Let k_j be the chip-delay index for the jth user: $\boldsymbol{h}_j^l(k_j T)$ is the first non-zero $R \times 1$ chip-rate sample of $h_j^l(t)$. The parameter N_j is the duration of $g_j^l(t)$ in symbol periods. It is a function of m_j and k_j. We consider user 1 as the user of interest and assume that $k_1 = 0$ (synchronization to user 1). Let $N = \sum_{j=1}^{p} N_j$. The vectorized oversampled signals at L sensors lead to a discrete-time $mLR \times 1$ vector signal at the symbol rate that can be expressed as

$$\begin{aligned} \boldsymbol{y}(k) &= \sum_{j=1}^{p} \sum_{i=0}^{N_j - 1} \boldsymbol{g}_j(i) a_j(k-i) + \boldsymbol{v}(k) \\ &= \sum_{j=1}^{p} \boldsymbol{G}_{j,N_j} A_{j,N_j}(k) + \boldsymbol{v}(k) = \boldsymbol{G}_N \boldsymbol{A}_N(k) + \boldsymbol{v}(k), \end{aligned} \tag{3}$$

$$\boldsymbol{y}(k) = \begin{bmatrix} \boldsymbol{y}_1(k) \\ \vdots \\ \boldsymbol{y}_m(k) \end{bmatrix}, \boldsymbol{y}_s(k) = \begin{bmatrix} \boldsymbol{y}_s^1(k) \\ \vdots \\ \boldsymbol{y}_s^L(k) \end{bmatrix}, \boldsymbol{y}_s^l(k) = \begin{bmatrix} y_{s,1}^l(k) \\ \vdots \\ y_{s,R}^l(k) \end{bmatrix}$$

$$\begin{aligned} \boldsymbol{G}_{j,N_j} &= \left[\boldsymbol{g}_j(N_j - 1) \ldots \boldsymbol{g}_j(0)\right], \ \boldsymbol{G}_N = \left[\boldsymbol{G}_{1,N_1} \ldots \boldsymbol{G}_{p,N_p}\right] \\ A_{j,N_j}(k) &= \left[a_j^H(k - N_j + 1) \ldots a_j^H(k)\right]^H \\ \boldsymbol{A}_N(k) &= \left[A_{1,N_1}^H(k) \ldots A_{p,N_p}^H(k)\right]^H, \end{aligned} \tag{4}$$

and the superscript ^{H} denotes Hermitian transpose. For the user of interest (user 1), $\boldsymbol{g}_1(i) = (\boldsymbol{C}_1(i) \otimes I_{LR})\, \boldsymbol{h}_1$, where $\boldsymbol{h}_1 = \left[\boldsymbol{h}_1^{1H} \cdots \boldsymbol{h}_1^{LH}\right]^H$ is the $m_1 LR \times 1$ channel vector, and the matrices $\boldsymbol{C}_1(i)$ are shown in figure 1, where the band consists of the spreading code $(c_0^H \cdots c_{m-1}^H)^H$ shifted successively to the right and down by one position. For the interfering users, we have a similar setup except that owing to asynchrony, the band in fig. 1 is shifted down k_j chip periods and is no longer conincident with the top left edge of the box. We denote by $\boldsymbol{C}_1$, the concatenation of the code matrices given above for user 1: $\boldsymbol{C}_1 = [\boldsymbol{C}_1^H(0) \cdots \boldsymbol{C}_1^H(N_1 - 1)]^H$.

It is clear that the signal model above addresses a multiuser setup with a possiblity of joint interference cancellation for all

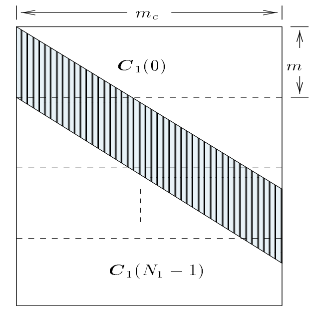

Figure 1. The Code Matrix $\boldsymbol{C}_1$

sources simultaneously [9] provided the timing information and spreading codes of all of them are available. As we shall see later, it is possible to decompose the problem into single user ones thus making the implementation suitable for applications such as at mobile terminals or as suboptimal processing stage at the base station.

III. BLIND MMSE ZERO FORCING RECEIVER

We stack M successive $\boldsymbol{y}(k)$ vectors in a super vector

$$\boldsymbol{Y}_M(k) = \mathcal{T}_M(\boldsymbol{G}_N) \boldsymbol{A}_{N+p(M-1)}(k) + \boldsymbol{V}_M(k), \tag{5}$$

where, $\mathcal{T}_M(\boldsymbol{G}_N) = \left[\mathcal{T}_{M,1}(\boldsymbol{G}_{1,N_1}) \cdots \mathcal{T}_{M,p}(\boldsymbol{G}_{p,N_p})\right]$ and $\mathcal{T}_M(\boldsymbol{x})$ is a banded block Toeplitz matrix with M block rows and $\left[\boldsymbol{x} \quad \boldsymbol{0}_{n \times (M-1)}\right]$ as first block row (n is the number or rows in $\boldsymbol{x}$), and $\boldsymbol{A}_{N+p(M-1)}(k)$ is the concatenation of user data vectors ordered as $\left[A_{1,N_1+M-1}^H(k), A_{2,N_2+M-1}^H(k) \cdots A_{p,N_p+M-1}^H(k)\right]^H$. Consider the scenario depicted in fig. 2 for a single user. Due

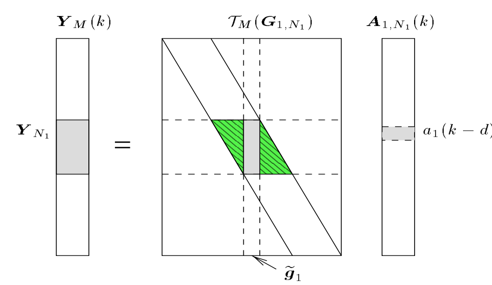

Figure 2. The ISI and MAI for Desired Symbol

to the limited delay spread, the effect of a particular symbol, $a_1(k-d)$, propagates to the next $N_1 - 1$ symbol periods, rendering the channel a moving average process of order $N_1 - 1$ [9]. For the other users, the matrices $\mathcal{T}_M(\boldsymbol{G}_{N_j})$, where $i = 2 \cdots p$, have a similar structure and can be viewed as being superimposed over the channel matrix $\mathcal{T}_M(\boldsymbol{G}_{N_1})$ in fig. 2. Same applies for the data vectors $A_{j,N_j+M-1}(k)$, $\forall j \neq i$. The overall effect of the ISI and the MUI is therefore that of engendering the shaded triangles in the figure, which need to be removed from $\boldsymbol{Y}_{N_1}$. To this end, let us introduce the following orthogonal transformation:

$$\boldsymbol{T}_1 = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{C}_1^H & \boldsymbol{0} \end{bmatrix} \otimes I_{LR}, \ \boldsymbol{T}_2 = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{C}_1^{\perp} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \otimes I_{LR}, \tag{6}$$

where, $\boldsymbol{C}_1^H$ is the spreading code correlator matched to the delayed versions of the spreading code, and $\boldsymbol{C}_1^{\perp H}$ is the orthogonal complement of $\boldsymbol{C}_1$, the tall code matrix given in section II. ($\boldsymbol{C}_1^{\perp} \boldsymbol{C}_1 = \boldsymbol{0}$). Then, $\boldsymbol{C}_1^H \boldsymbol{Y}_{N_1} = \boldsymbol{T}_1 \boldsymbol{Y}_M$ and the middle (block) row of the matrix $\boldsymbol{T}_2$ acts as a blocking transformation for the signal of interest. Note that $P_{T_1^H} + P_{T_2^H} = \boldsymbol{I}$, where, $P_X = X(X^H X)^{-1} X^H$ is the projection operator. This gives us a possiblity of estimating the $a_1(k-d)$ contribution in $\boldsymbol{Y}_{N_1}$ blindly. We have,

$$\boldsymbol{Z}(k) = [\boldsymbol{T}_1 - \boldsymbol{Q}\boldsymbol{T}_2]\, \boldsymbol{Y}_M(k), \tag{7}$$

and the interference cancellation problem settles down to minimization of the trace of the matrix $\boldsymbol{R}_{ZZ}$ for a matrix $\boldsymbol{Q}$, which results in

$$\boldsymbol{Q} = \left(\boldsymbol{T}_1 \boldsymbol{R}^d \boldsymbol{T}_2^H\right) \left(\boldsymbol{T}_2 \boldsymbol{R}^d \boldsymbol{T}_2^H\right)^{-1}, \tag{8}$$

and where, $\boldsymbol{R}^d$ is the noiseless (denoised) data covariance matrix, $\boldsymbol{R}_{YY}$, with the subscript removed for convenience. The output $\boldsymbol{Z}(k)$ can directly be processed by a multichannel matched filter to get the symbol, $\hat{a}_1(k-d)$, the data for the user 1.

$$\hat{a}_1(k-d) = \boldsymbol{F}^H \boldsymbol{Y}_M(k) = \boldsymbol{h}_1^H \left(\boldsymbol{T}_1 - \boldsymbol{Q}\boldsymbol{T}_2\right) \boldsymbol{Y}_M(k) \tag{9}$$

An estimate of the channel $\mathbf{G}_1(z) = (\mathbf{C}_1(z) \otimes I_{LR})\mathbf{h}_1(z)$ can be

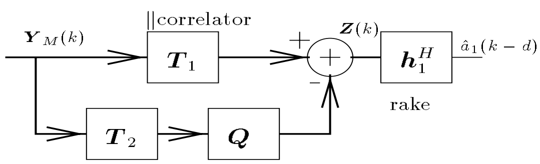

Figure 3. MMSE-ZF Receiver

obtained as a by product of the interference cancellation scheme. Notice that the interference canceler is analogous to a MMSE-ZF in the form of a smoother or two-sided linear predictor for the single user case [10] with $\boldsymbol{T}_1 = [\mathbf{0} \quad \boldsymbol{I} \quad \mathbf{0}]$ and $\boldsymbol{T}_2$ without the middle (block) row, which when employed in a multiuser scenario is no longer capable of MAI suppression coming from the middle block of $\boldsymbol{Y}_M(k)$ of fig. 2, unless a fair amount of data smoothing is introduced [10]. $\boldsymbol{Z}(k)$ corresponds to the vector of *prediction* errors, and the covariance matrix of the prediction errors is given by

$$\boldsymbol{R}_{ZZ} = \boldsymbol{T}_1 \boldsymbol{R}^d \boldsymbol{T}_1^H - \boldsymbol{T}_1 \boldsymbol{R}^d \boldsymbol{T}_2^H \left(\boldsymbol{T}_2 \boldsymbol{R}^d \boldsymbol{T}_2^H\right)^{-1} \boldsymbol{T}_2 \boldsymbol{R}^d \boldsymbol{T}_1^H, \tag{10}$$

From the above structure of the two-sided (or rather *full dimensional*) linear prediction problem, the key observation is that the matrix $\boldsymbol{R}_{ZZ}$ is rank-1 in the noiseless case! Using this fact, one can identify the composite channel as the maximum eigenvector of the matrix $\boldsymbol{R}_{ZZ}$, since $\boldsymbol{Z}(k) = (\boldsymbol{C}_1^H \boldsymbol{C}_1 \otimes I_{LR})\boldsymbol{h}_1 \tilde{a}_1(k-d)$.

IV. RELATION WITH OTHER APPRAOCHES

It can be seen that the derivation of the receiver structure in fig. 3 is in the spirit of the Generalized Sidelobe Canceller (GSC). The main branch ($\boldsymbol{T}_1$) is a bank of desired user code sequence correlators, while the second branch contains a blocker for the signal of interest. It is interesting to observe that in the event of $\boldsymbol{T}_2 = \mathbf{0}$, the receiver is no more than a coherent RAKE receiver. However, several interesting interpretations of the above method can be obtained in terms of various existing approaches.

Proposition:
Suppose that $\boldsymbol{F}$ is a linear receiver vector applied to the received data $\boldsymbol{Y}_M(k)$. $\boldsymbol{F}$ is unbiased if $\boldsymbol{F}^H \widetilde{\boldsymbol{g}}_1 = 1$, where, $\widetilde{\boldsymbol{g}}_1 = \boldsymbol{T}_1^H \boldsymbol{h}_1$. Then the following relation holds.

$$\arg \min_{F:\, F^H \widetilde{g}_1 = 1} \mathrm{MSE}_{\mathrm{unbiased}} = \arg \min_{F:\, F^H \widetilde{g}_1 = 1} \mathrm{OE} = \arg \max_{F} \mathrm{SINR}, \tag{11}$$

This simply implies that the minimum mean-squared error (MMSE), and the minimum output energy (MOE)[1], are interchangeable criteria under the unbiased constraint, and are equivalent to the maximization of the output SINR.

We shall discuss these interpretations in some detail and show that (11) proves to be true in the following subsections.

A. The Unbiased MOE Approach

The unbiased MOE criterion proposed in [2], which is a generalization of the instantaneous channel case of [1], is in principle a max/min problem solved in two steps with,

step:1 *Unbiased MOE*

$$\min_{F:\, F^H \widetilde{g}_1 = 1} \boldsymbol{F}^H \boldsymbol{R}_{YY} \boldsymbol{F} \Rightarrow \boldsymbol{F} = \frac{1}{\widetilde{\boldsymbol{g}}_1^H \boldsymbol{R}_{YY}^{-1} \widetilde{\boldsymbol{g}}_1} \boldsymbol{R}_{YY}^{-1} \widetilde{\boldsymbol{g}}_1, \tag{12}$$

with $\mathrm{MOE}(\hat{\boldsymbol{h}}_1) = \frac{1}{\widetilde{\boldsymbol{g}}_1 \boldsymbol{R}_{YY}^{-1} \widetilde{\boldsymbol{g}}_1}$, followed by,

step:2 *Capon's Method*

$$\max_{\hat{\boldsymbol{h}}_1 : \|\hat{\boldsymbol{h}}_1\| = 1} \mathrm{MOE}(\hat{\boldsymbol{h}}_1) \Rightarrow \min_{\hat{\boldsymbol{h}}_1 : \|\hat{\boldsymbol{h}}_1\| = 1} \hat{\boldsymbol{h}}_1^H \left(\boldsymbol{T}_1 \boldsymbol{R}_{YY}^{-1} \boldsymbol{T}_1^H\right) \hat{\boldsymbol{h}}_1, \tag{13}$$

from where, $\hat{\boldsymbol{h}}_1 = V_{min}(\boldsymbol{T}_1 \boldsymbol{R}_{YY}^{-1} \boldsymbol{T}_1^H)$. It can be shown that if $\boldsymbol{T}_2 = \boldsymbol{T}_1^{\perp}$, then

$$\boldsymbol{T}_1 \boldsymbol{R}_{YY}^{-1} \boldsymbol{T}_1^H = \left(\boldsymbol{T}_1 \boldsymbol{T}_1^H\right) \boldsymbol{R}_{ZZ}^{-1} \left(\boldsymbol{T}_1 \boldsymbol{T}_1^H\right), \tag{14}$$

where, $\boldsymbol{R}_{ZZ}$ is given by (10), and $\boldsymbol{Q}$, given by (8), is optimized to minimize the prediction error variance. $\boldsymbol{R}^d$ replaces $\boldsymbol{R}_{YY}$ in the above developments. From this, we can obtain $\hat{\boldsymbol{h}}_1$ as $\hat{\boldsymbol{h}}_1 = V_{max}\{\left(\boldsymbol{T}_1 \boldsymbol{T}_1^H\right)^{-1} \boldsymbol{R}_{ZZ} \left(\boldsymbol{T}_1 \boldsymbol{T}_1^H\right)^{-1}\}$. In order to evaluate the quality of the blind receiver obtained from the above criterion, we consider the noiseless received signal ($v(t) \equiv 0$). We have the following two cases of interest.

1. Uncorrelated symbols

In the absence of noise, with *i.i.d.* symbols, the stochastic estimation of $\boldsymbol{T}_1 \boldsymbol{Y}$ from $\boldsymbol{T}_2 \boldsymbol{Y}$ is the stochastic estimation of $\boldsymbol{T}_1 \mathcal{T}_M(\boldsymbol{G}_{1:p})\boldsymbol{A}$ from $\boldsymbol{T}_2 \mathcal{T}_M(\boldsymbol{G}_{1:p})\boldsymbol{A}$ with $\boldsymbol{R}_A = \sigma_a^2 \boldsymbol{I}$. Hence, it is equivalent to the deterministic estimation of $\mathcal{T}_M^H(\boldsymbol{G}_{1:p})\boldsymbol{T}_1^H$ from $\mathcal{T}_M^H(\boldsymbol{G}_{1:p})\boldsymbol{T}_2^H$: $\|\mathcal{T}_M^H(\boldsymbol{G}_{1:p})\boldsymbol{T}_1^H - \mathcal{T}_M^H(\boldsymbol{G}_{1:p})\boldsymbol{T}_2^H \boldsymbol{Q}^H\|_2^2$. Then, given the condition

$$\begin{aligned}
&\mathsf{span}\{\boldsymbol{T}_1^H\} \cap \mathsf{span}\{\mathcal{T}_M(\boldsymbol{G}_{1:p})\} = \mathsf{span}\{\mathcal{T}_M(\boldsymbol{G}_{1:p})\boldsymbol{e}_d^{'}\} \\
&\Rightarrow \mathsf{span}\{\mathcal{T}_M(\boldsymbol{G}_{1:p})\} \subset \mathsf{span}\{\boldsymbol{T}_2^H\} \oplus \mathsf{span}\{\widetilde{\boldsymbol{g}}_1\} \\
&\because \mathcal{T}_M(\boldsymbol{G}_{1:p})\boldsymbol{e}_d^{'} = \mathcal{T}_M(\boldsymbol{G}_1)\boldsymbol{e}_d = \widetilde{\boldsymbol{g}}_1 = \boldsymbol{T}_1 \boldsymbol{h}_1,
\end{aligned} \tag{15}$$

and where, $\boldsymbol{e}_d^{'}$ and $\boldsymbol{e}_d$ are vectors of appropriate dimensions with all zeros and one 1 selecting the desired column in $\mathcal{T}_M(\boldsymbol{G}_{1:p})$ and $\mathcal{T}_M(\boldsymbol{G}_1)$ respectively. We can write the channel convolution matrix $\mathcal{T}_M(\boldsymbol{G}_{1:p})$ as

$$\mathcal{T}_M(\boldsymbol{G}_{1:p}) = \widetilde{\boldsymbol{g}}_1 \boldsymbol{e}_d^{'H} + \mathcal{T}_M(\boldsymbol{G}_{1:p}) P_{e_d^{'\perp}} = [\widetilde{\boldsymbol{g}}_1 \quad \boldsymbol{T}_2^H]\boldsymbol{A}, \tag{16}$$

[1] also known as minimum variance distortionless response (MVDR), a particular instance of the linearly constrained minimum-variance (LCMV) criterion.

for some $\boldsymbol{A}$. Then we can write,

$$\begin{aligned}
&\mathcal{T}_M^H(\boldsymbol{G}_{1:p})\left(\boldsymbol{T}_1^H - \boldsymbol{T}_2^H\boldsymbol{Q}^H\right) = \\
&\boldsymbol{e}_d^{'}\boldsymbol{h}_1^H\boldsymbol{T}_1\boldsymbol{T}_1^H + \boldsymbol{A}^H\begin{bmatrix}\widetilde{\boldsymbol{g}}_1\boldsymbol{T}_1^H\\ \boldsymbol{0}\end{bmatrix} - \boldsymbol{A}^H\begin{bmatrix}\boldsymbol{0}\\ \boldsymbol{T}_2\boldsymbol{T}_2^H\end{bmatrix}\boldsymbol{Q}^H \\
&= \boldsymbol{e}_d^{'}\boldsymbol{h}_1^H\boldsymbol{T}_1\boldsymbol{T}_1^H + \boldsymbol{A}_1^H\widetilde{\boldsymbol{g}}_1^H\boldsymbol{T}_1^H - \boldsymbol{A}_2^H\left(\boldsymbol{T}_2\boldsymbol{T}_2^H\right)\boldsymbol{Q}^H.
\end{aligned} \quad (17)$$

Note that $\boldsymbol{e}_d^{'H}\boldsymbol{A}_i^H = 0, i \in \{1,2\}$. This implies that the first term on the R.H.S. of (17) is not predictable from the third. Therefore, if the second term is perfectly predictable from the third, then the two terms cancel each other out and $\boldsymbol{R}_{ZZ}$ turns out to be rank-1, and $\hat{\boldsymbol{h}}_1 = \left(\boldsymbol{T}_1\boldsymbol{T}_1^H\right)^{-1}\mathcal{V}_{max}\left(\boldsymbol{R}_{ZZ}\right)$.

2. *Correlated symbols*

In the case of correlated symbols, with a finite amount of data, given the conditions in (15), it still holds that $\mathsf{span}\{\mathcal{T}_M^H(\boldsymbol{G}_{1:p})\boldsymbol{T}_2^H\} = \mathsf{span}\{P_{e_d^{'\perp}}\mathcal{T}_M(\boldsymbol{G}_{1:p})\}$. Now, we can write the received vector $\boldsymbol{Y}_M(k)$ as

$$\boldsymbol{Y}_M(k) = \mathcal{T}_M(\boldsymbol{G}_{1:p})\boldsymbol{A} = \mathcal{T}_M(\boldsymbol{G}_{1:p})\boldsymbol{e}_d^{'}a_1(k-d) + \overline{\mathcal{T}}_M\bar{\boldsymbol{A}}. \quad (18)$$

Now, the estimation of $\boldsymbol{T}_1\boldsymbol{Y}$ in terms of $\boldsymbol{T}_2\boldsymbol{Y} = \boldsymbol{T}_2\mathcal{T}_M(\boldsymbol{G}_{1:p})\boldsymbol{A} = \boldsymbol{T}_2\overline{\mathcal{T}}_M\bar{\boldsymbol{A}}$ is equivalent to estimation in terms of $\bar{\boldsymbol{A}}$.

$$\begin{aligned}
\widetilde{\boldsymbol{T}_1\boldsymbol{Y}}|_{\boldsymbol{T}_2\boldsymbol{Y}} &= \boldsymbol{T}_1\boldsymbol{Y} - \widehat{\boldsymbol{T}_1\boldsymbol{Y}} \\
&= \boldsymbol{T}_1\boldsymbol{Y} - \left(\boldsymbol{T}_1\boldsymbol{R}_{YY}^d\boldsymbol{T}_2^H\right)\left(\boldsymbol{T}_2\boldsymbol{R}_{YY}^d\boldsymbol{T}_2^H\right)^{-1}\boldsymbol{T}_2\boldsymbol{Y} \\
\widetilde{\boldsymbol{T}_1\boldsymbol{Y}}|_{\bar{\boldsymbol{A}}} &= \boldsymbol{T}_1\mathcal{T}_M(\boldsymbol{G}_{1:p})\boldsymbol{e}_d^{'}\tilde{a}_1(k-d) \\
&= \boldsymbol{T}_1\boldsymbol{T}_1^H\boldsymbol{h}_1\tilde{a}_1(k-d)|_{\bar{\boldsymbol{A}}}.
\end{aligned} \quad (19)$$

This results in,

$$\left(\boldsymbol{T}_1\boldsymbol{R}_{YY}^{-d}\boldsymbol{T}_1^H\right)^{-1} = \sigma^2_{\tilde{a}_1(k-d)|\bar{A}}\boldsymbol{h}_1\boldsymbol{h}_1^H, \quad (20)$$

The rank-1 results in a normalized estimate of the channel. It must however be noted that the estimation error variance of the desired symbol is now smaller ($\sigma^2_{\tilde{a}_1(k-d)} < \sigma_a^2$).

B. *The Unbiased MMSE Criterion*

The MSE can be written as follows:

$$\begin{aligned}
\mathrm{MSE} &= E|a_{1,k-d} - \hat{a}_1|^2 = E|a_{1,k-d} - \boldsymbol{F}^H\boldsymbol{Y}|^2 \\
&= \sigma_a^2 - \sigma_a^2\boldsymbol{F}^H\widetilde{\boldsymbol{g}}_1 - \sigma_a^2\widetilde{\boldsymbol{g}}_1^H\boldsymbol{F} + \underbrace{\boldsymbol{F}^H\boldsymbol{R}_{YY}\boldsymbol{F}}_{\text{output energy}} \\
\Rightarrow &\min_{F^H\tilde{g}_1=1}\mathrm{MSE} = \text{unbiased MOE}
\end{aligned} \quad (21)$$

which proves the first equality in 11.

C. *Maximization of SINR*

The signal part in $\boldsymbol{Y}_M(k)$ is $\boldsymbol{Y}_s = \widetilde{\boldsymbol{g}}_1 a_{1,k-d}$, whereas the interference (MAI & ISI) plus noise is $\boldsymbol{Y}_{\rm in} = \overline{\mathcal{T}}_M\bar{\boldsymbol{A}} + \boldsymbol{V}_M$, where, $\overline{\mathcal{T}}_M = \mathcal{T}_M(G_{1:p})$ except for the column $\widetilde{\boldsymbol{g}}_1$. Then, for an arbitrary $\boldsymbol{F}$, assuming uncorrelated symbols, we obtain,

$$\mathrm{SINR} = \frac{\boldsymbol{F}^H\boldsymbol{R}_s\boldsymbol{F}}{\boldsymbol{F}^H\boldsymbol{R}_{\rm in}\boldsymbol{F}} = \frac{\sigma_a^2\boldsymbol{F}^H\widetilde{\boldsymbol{g}}_1\widetilde{\boldsymbol{g}}_1^H\boldsymbol{F}}{\boldsymbol{F}^H\left(\boldsymbol{R}_{YY} - \sigma_a^2\widetilde{\boldsymbol{g}}_1\widetilde{\boldsymbol{g}}_1^H\right)\boldsymbol{F}}, \quad (22)$$

from where,

$$\begin{aligned}
&\max_F \mathrm{SINR} \leftrightarrow \min_F \mathrm{SINR}^{-1} \leftrightarrow \quad \min_F \frac{\boldsymbol{F}^H\boldsymbol{R}_{YY}\boldsymbol{F}}{\sigma_a^2|\boldsymbol{F}^H\widetilde{\boldsymbol{g}}_1|^2} \\
&\Rightarrow \min_{F:F^H\tilde{g}_1=1}\boldsymbol{F}^H\boldsymbol{R}_{YY}\boldsymbol{F},
\end{aligned} \quad (23)$$

Hence the maximization of SINR is the same as the unbiased MOE cost function of 12.

V. IDENTIFICATION ISSUES

Continuing with the noiseless case, or with the denoised version of $\boldsymbol{R}_{YY}$, i.e., $\boldsymbol{R}_{YY}^d = \sigma_a^2\mathcal{T}_M(\boldsymbol{G}_{1:p})\mathcal{T}_M^H(\boldsymbol{G}_{1:p})$,

$$\min_{F:F^H\tilde{g}_1=1}\boldsymbol{F}^H\boldsymbol{R}_{YY}^d\boldsymbol{F} = \sigma_a^2, \quad \mathit{iff} \quad \boldsymbol{F}^H\mathcal{T}_M(\boldsymbol{G}_{1:p}) = \boldsymbol{e}_d^{'H}, \quad (24)$$

i.e., the zero-forcing condition must be satisfied. Hence, the unbiased MOE criterion corresponds to ZF in the noiseless case. This implies that $\mathrm{MOE}(\hat{\tilde{\boldsymbol{g}}}_1) < \sigma_a^2$ if $\hat{\tilde{\boldsymbol{g}}}_1 \not\sim \widetilde{\boldsymbol{g}}_1$. We consider that:

(**i**). *FIR zero-forcing conditions are satisfied*, and
(**ii**). $\mathsf{span}\{\mathcal{T}_M(\boldsymbol{G}_{1:p})\} \cap \mathsf{span}\{\boldsymbol{T}_1^H\} = \mathsf{span}\{\boldsymbol{T}_1^H\boldsymbol{h}_1\}$.

The two step max/min problem boils down to

$$\max_{\hat{h}_1:\|\hat{h}_1\|=1}\hat{\boldsymbol{h}}_1^H\left(\boldsymbol{T}_1\boldsymbol{T}_1^H\right)^{-1}\boldsymbol{T}_1\mathcal{T}_M P^{\perp}_{\mathcal{T}_M^H T_2^H}\mathcal{T}_M^H\boldsymbol{T}_1^H\left(\boldsymbol{T}_1\boldsymbol{T}_1^H\right)^{-1}\hat{\boldsymbol{h}}_1, \quad (25)$$

where, $P_X^{\perp} = \boldsymbol{I} - \boldsymbol{X}(\boldsymbol{X}^H\boldsymbol{X})^{-1}\boldsymbol{X}^H$. Then identifiability implies that $\mathcal{T}_M P^{\perp}_{\mathcal{T}_M^H T_2^H}\mathcal{T}_M^H = \boldsymbol{T}_1^H\boldsymbol{h}_1\boldsymbol{h}_1^H\boldsymbol{T}_1 = \widetilde{\boldsymbol{g}}_1\widetilde{\boldsymbol{g}}_1^H$, or

$$P^{\perp}_{\mathcal{T}_M^H T_2^H}\mathcal{T}_M^H(\boldsymbol{G}_{1:p}) = P_{e_d^{'}}\mathcal{T}_M^H(\boldsymbol{G}_{1:p}), \quad (26)$$

Condition (i) above implies that $\boldsymbol{e}_d^{'} \in \mathsf{span}\{\mathcal{T}_M^H(\boldsymbol{G}_{1:p})\}$. From condition (ii), since $\boldsymbol{T}_1^H\boldsymbol{h}_1 = \mathcal{T}_M(\boldsymbol{G}_{1:p})\boldsymbol{e}_d^{'}$, we have

$$\begin{aligned}
&\mathsf{span}\{\mathcal{T}_M(\boldsymbol{G}_{1:p})\boldsymbol{T}_2^H\} = \mathsf{span}\{P^{\perp}_{e_d^{'}}\mathcal{T}_M^H(\boldsymbol{G}_{1:p})\} \\
&\mathsf{span}\{\mathcal{T}_M^H(\boldsymbol{G}_{1:p})\} = \mathsf{span}\{\mathcal{T}_M^H(\boldsymbol{G}_{1:p})\boldsymbol{T}_2^H\} \oplus \mathsf{span}\{\boldsymbol{e}_d^{'}\}
\end{aligned} \quad (27)$$

from which, $\mathcal{T}_M^H(\boldsymbol{G}_{1:p}) = P_{\mathcal{T}_M^H T_2^H}\mathcal{T}_M^H(\boldsymbol{G}_{1:p}) + P_{e_d^{'}}\mathcal{T}_M^H(\boldsymbol{G}_{1:p})$, which is the same as (26).

A. *A Note on Sufficiency of Conditions*

We consider first the conditions (**i**). Furthermore, in the following developments, we consider that $p < m$, which is easily achievable when multiple sensors (or oversampling) is employed. The effective number of channels is given by $m_{\rm eff} = \mathsf{rank}\{\boldsymbol{G}_N\}$, where $\boldsymbol{G}_N$ is given in (5). Let $\mathbf{G}_1(z) = \sum_{k=0}^{N_1-1}\boldsymbol{g}_1(k)z^{-k}$ be the channel transfer function for user 1, with $\mathbf{G}(z) = [\mathbf{G}_1(z)\cdots\mathbf{G}_p(z)]$. Then let us assume the following:

(**a**). $\mathbf{G}(z)$ *is irreducible*, i.e., $\mathsf{rank}\{\mathbf{G}(z)\} = p, \forall z$.
(**b**). $\mathbf{G}(z)$ *is column reduced*:

$$\mathsf{rank}\{[\boldsymbol{g}_1(N_1-1)\cdots\boldsymbol{g}_p(N_p-1)]\} = p.$$

Given that the above two conditions hold, the composite channel matrix $\mathcal{T}(\boldsymbol{G}_{1:p})$ is full rank with probability 1. Then, the FIR length M required is given by,

$$M \geq \overline{M} = \left\lceil\frac{N-p}{m_{\rm eff}-p}\right\rceil. \quad (28)$$

Note that condition (**a**) holds with probability 1 due to the quasi-orthogonality of spreading sequences. As for (**b**), it can be violated in certain limiting cases e.g., in the synchronous case where $\boldsymbol{g}_j(N_j-1)$'s contain very few non-zero elements. Under these circumstances, instantaneous (static) mixture of the sources can zero out some of the $\boldsymbol{g}_j(N_j-1)$ (more specifically, at most $p-1$ of them). Then N gets reduced by at most $p-1$. However, even then, M given by (28) remains sufficient.

The condition (**ii**) can be restated as the following dimensional requirement:

$$\mathsf{rank}\{\mathcal{T}_M(\boldsymbol{G}_{1:p})\}+\mathsf{rank}\{\boldsymbol{T}_1^H\}\leqslant \mathsf{row}\{\mathcal{T}_M(\boldsymbol{G}_{1:p})\}+1, \tag{29}$$

from where, under the irreducible channel and column reduced conditions,

$$M\geq \underline{M}=\left\lceil\frac{N-p+m_1LR-1}{(mLR)_{\text{eff}}-p}\right\rceil, \tag{30}$$

where, m_1 is the channel length for user 1 in chip periods. If (30) holds, then condition (**ii**) is fulfilled w.p. 1, regardless of the N_j's, i.e., the $\mathsf{span}\{\boldsymbol{T}_1^H\}$ does not intersect with all shifted versions of $\boldsymbol{g}_j$'s, $\forall j>1$, which further means that no confusion is possible between the channel of the user of interest and those of other users, whether the mixing is static (same lengths) or dynamic (different channel lengths), with lengths measured in symbol periods.

1. Violation of condition (*ii*)

If the channel length m_1 is over-estimated, such that N_1 gets over-estimated, then condition (**ii**) is violated w.p. 1. In that case, more than one shifted versions of $\boldsymbol{g}_1$ will fit in the column space of $\boldsymbol{T}_1^H$. The estimated channel in that case can be expressed as $\widehat{\mathbf{G}}_1(z)=\mathbf{G}_1(z)b(z)$, where, $b(z)$ is a scalar polynomial of the order equaling the amount by which the channel has been over-estimated. A solution to this would be to try all orders for N_1 and stop at the correct one.

VI. SPARSE CHANNELS

Let us now consider the scenario where the channels for all users are sparse. This is essentially the case of most mobile channels [8]. Then we can write the channel vector of the jth user as $\boldsymbol{h}_j=\left[0\cdots 0\ \boldsymbol{h}_j^H(1)\ 0\cdots 0\ \boldsymbol{h}_j^H(m_j-1)\right]^H$, where m_j now denotes the number of non-zero taps (per sensor per oversampling phase) for the channel of the jth user, and the co-efficients $\boldsymbol{h}_j(k)$ are $LR\times 1$ vectors (L being the number of sensors, and R the oversampling factor). We further consider that the positions of these taps have been determined by the initial acquisition procedure. Referring back to fig. 1, we can write $(\boldsymbol{C}_1\otimes I_{LR})\boldsymbol{h}_1=(\overline{\boldsymbol{C}}_1\otimes I_{LR})\overline{\boldsymbol{h}}_1$, and in which the code matrix $\overline{\boldsymbol{C}}_1$ will now have m_1 non-zero columns. For the implementation of the MMSE-ZF algorithm described in the previous section, the matrix $\boldsymbol{T}_1$ now contains m_1LR rows, representing m_1 shifted versions of the spreading code correlator for each sub-channel. This operation is the same as building the fingers of the non-coherent RAKE receiver, once the tap positions have been located. A coherent RAKE can be implemented by premultiplying the $\boldsymbol{T}_1\boldsymbol{Y}_M(k)$ by the complex co-efficient vector $\overline{\boldsymbol{h}}_1^H$. However, we shall proceed with the estimation of the channel co-efficients by the MMSE-ZF algorithm as presented in the preceding section. It must be noted that the principle of the algorithm is not affected by the new dimensions of the matrices $\boldsymbol{T}_1$ and $\boldsymbol{T}_2$. However, the number of parameters to be estimated blindly is much reduced, i.e., only the non-zero taps of the channel need to be estimated.

A. Identification Issues under the Sparse Channel Model

For the purpose of the following, let us consider $L=R=1$. Referring to (28), we observe that for an underloaded system with $m_{\text{eff}}=N$, a stacking factor of $M=1$ suffices for (**i**) to be satisfied. (29) can be written as $\widetilde{M}\geq N_{\text{eff}}+p(M-1)+m_1-1$, where $\widetilde{M}$ is the number of rows of $\mathcal{T}(\boldsymbol{G}_{1:p})$. Substituting a value of $M=1$ in this equation, we get

$$\widetilde{M}\geq N_{\text{eff}}+m_1-1. \tag{31}$$

where, N_{eff} is the number of linearly independent columns in the reduced $\mathcal{T}(\boldsymbol{G}_{1:p})$. $N_{\text{eff}}<N$, since due to the mutually asynchronous nature of the users, certain columns which contributed in $\boldsymbol{G}_N$ are present in the reduced version as all-zero columns. It is as if the contribution of symbols associated with these columns of $\boldsymbol{G}_N$ is no longer there in the system. Furthermore, given the fact that m_1 is a small number and when $N\ll m$, as is the case of strongly underloaded systems, $\widetilde{M}(<m)$ rows of $\mathcal{T}(\boldsymbol{G}_{1:p})$ suffice for rendering the matrix $\boldsymbol{R}_{ZZ}$ rank-1, thus resulting in a perfect blind estimate of the channel asymptotically (in SNR or amount of data). $\widetilde{M}<m$ means that the stacking factor ($M<1$) corresponds to less than one symbol period of data. Some performance loss will be incurred due to the reduced dimensions, but as seen, in terms of the eigenvalue spread of the matrix $\boldsymbol{R}_{ZZ}$, in fig.4, this effect is rather insignificant. A spreading gain of $m=256$ is used

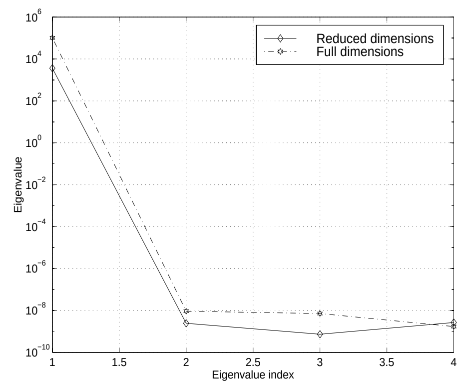

Figure 4. Eigenvalue spread for $\boldsymbol{R}_{ZZ}$, $m=256$, 50 users

with four non-zero taps in a channel length of 80 chips, which is the case of the vehicular B channel in the UMTS WCDMA specifications [8].

The equation (31) reveals the great practical interest of the algorithm in sparse channel environments. Due to the small size of the covariance matrix ($\widetilde{M}\times\widetilde{M}$ instead of $2\underline{M}\times 2\underline{M}$) to be estimated, relatively small data records will be required.

It must be mentioned that the reduced rank N_{eff} holds equally well for the case of non-sparse channels since it is a function of the delay spread. However, if m_1 is a large number, then a larger $\widetilde{M}$ is required to satisfy the rank condition, as can be seen from (31). So, it is of greater interest to take sparseness into account when m_1 is much smaller than the delay spread.

VII. NUMERICAL EXAMPLES

We consider 45 asynchronous users in the system with a spreading factor of $m=256$, which is the the spreading gain employed in the UMTS WCDMA. For the purpose of these examples, we consider no oversampling with respect to the chip rate ($R=1$). Furthermore, a single receiving antenna is employed for reception ($L=1$). For the first example, we consider the vehicular-B channel model [8], which represents a delay spread of approximately

80 chips with $4-5$ dominant taps per user channel. The channel delay spread is therefore much shorter than one symbol period. Near-far conditions prevail in that the interfering users are randomly (ranging from 10 to 15 dB.) stronger than the user of interest. The performance measure is the Normalized MSE (NMSE)[2].

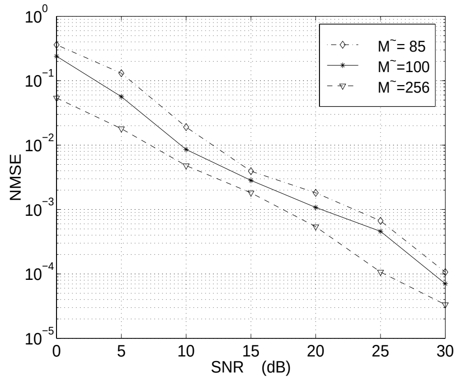

Figure 5. Channel estimation performance $m = 256$, $p = 45$, Veh.B channel

These results are averaged over $P = 200$ Monte-Carlo runs. Fig. 5 shows the quality of the channel estimate as a function of the input signal-to-noise ratio (SNR). $\widetilde{M}$ was always chosen to satisfy eq.(31). Blocks of 90, 110, and 260 data samples were considered for the estimation process correponding to a $\widetilde{M}$ of 85, 100, and 256 ($M = 1$) respectively. This corresponds to channel coherence times of 5.6, 7.0, and 16.25 ms. It can be seen that the performance for short $\widetilde{M}$ is slightly worse but would suffice for most practical purposes.

Also of interest is the case of the vehicular-A channel [8]. This is a channel of length 12 chip periods with $7-10$ significant taps. We shall consider 7 paths as opposed to the norm which proposes only 4 RAKE fingers. Due to the small delay spread, and due to the reduced dimension ($\widetilde{M}$) of $\mathcal{T}(G_N)$, most users contribute once to the rank most of the time. This reduces the required $\widetilde{M}$. Fig. 6 shows the NMSE of the channel estimates as a function of the input SNR. This time, we use a block of 70 symbols for the channel estimation algorithm with $\widetilde{M} = 65$. It can also be observed that there is practically no difference between the case with $\widetilde{M} = 100$ and $\widetilde{M} = 256$ ($M = 1$). It is to be noticed that since the channel delay spread and m_1 are both small this time, $\widetilde{M}$ will not be too large even for the non-sparse case.

VIII. CONCLUSIONS

The blind MMSE-ZF receiver for DS-CDMA was presented. This receiver turns out to be the exact extension to the general multipath case of the constrained MOE receiver derived in [1]. The equivalence to the unbiased MOE and the unbiased MMSE receiver was shown in terms of optimization criteria.

What is not shown in the numerical examples section is that the performance of the MMSE-ZF receiver itself is not acceptable with such short data records (see [7]). However, it was observed that sparse channels provide a framework for which the channel estimate obtained by the blind algorithm for underloaded systems is quite good despite the severe *near-far* conditions and short data records. It will perhaps be of practical interest in mobile cellular applications such as UMTS WCDMA, where a significant overhead is incurred by the training (pilot) data, to incorporate the channel estimates obtained by this method into multiuser receivers like the decorrelating detector or even for serial and parallel interference cancellation algorithms.

It was also shown that in the case where delay spread is large, the dimension of the problem can be reduced by taking into consideration the sparseness of the channel. For the desired user, it is the number of significant taps of the channel that counts, resulting in a reduced problem.

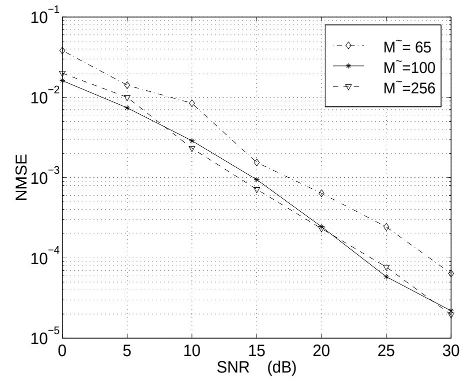

Figure 6. Channel estimation performance $m = 256$, $p = 45$, Veh.A channel

[2] NMSE $= E\frac{\|\boldsymbol{h}_1 - \hat{\boldsymbol{h}}_1\|^2}{\|\boldsymbol{h}_1\|^2} = \frac{1}{P}\sum_{i=1}^{P} \frac{\|\boldsymbol{h}_1 - \hat{\boldsymbol{h}}_1^{(i)}\|^2}{\|\boldsymbol{h}_1\|^2}$.

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