# Some Results on the Asymptotic Downlink Capacity of MIMO Multi-user Networks

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***Abstract*— In this contribution[1], some results on the downlink capacity of MIMO Multi-user Networks are provided when only the scattering environment (and not the channel realization) is known at the transmitter. Considering a multi-user system where each terminal employs multiple antennas (including the Base Station), asymptotic (in the number of antennas) analytical expressions of the system capacity are presented and studied in three different cases: equal time sharing, equal power sharing and optimum time-power scheduling. Interestingly, based only on the knowledge at the base station of the number of scatterers at each receiver link and not the full instantaneous channel realization (which can be fed back with limited overhead), one is able to derive optimal power and time scheduling policies which enhance the performance with respect to uniform time and power allocation. Simulations provided in the case of a small number of antennas confirm the asymptotic claims.**

***Index Terms*— MIMO multi-user, analytical capacity, capacity optimization, time-sharing, power allocation, time allocation.**

## I. INTRODUCTION

During the last years, many studies were developed to investigate the capacity offered by Multiple-Input Multiple-Output (MIMO) systems [1]–[3]. By exploiting the multipath propagation channel, multiple antenna systems were shown to significantly increase the performance of single antenna systems. As a consequence, single user MIMO systems have gained more and more attention. However, MIMO multi-user networks are still an open issue and strategies are still not clear on how to increase the capacity from a system point of view (see the effect of channel hardening for MIMO multi-user communications [4], [5] and other related works [6] for example) as the results depend heavily on the knowledge of the channel state.

Recent works have provided some partial solutions about the optimal approach to maximize the capacity, depending on the avaiable knowledge of the channel state. In [3], for example, the authors provide achievable rates in a Gaussian broadcast channel with multiple antennas in the case of perfect Channel State Information (CSI) at the transmitter. However, the extension to partial CSI knowledge or knowledge of the environment structure is still in its infancy [7].

In this contribution, we derive analytical expressions of the capacity and show how time-power allocation within a time-frame can improve the system performance when only the number of scatterers at each receiving antenna link is known at the base station. This is in contrast with available algorithms in the literature which exploit the full knowledge of the channel at the base station and require a heavy feedback load for transmitter design. The idea presented in this work is based on an adaptive TDMA-based system, where each user time slot and power can be adapted according to the channel environment structure (and not realization). In particular, four strategies are studied in this paper: : 1) equal time-power sharing; 2) equal time sharing; 3) equal power sharing; 4) adaptive time-power sharing. The results are provided in the asymptotic regime and proved to hold for a low number of antennas (4 to 10). We also compare the loss with respect to the knowledge of the channel realization as well as the theoretical bound of the multi-user MIMO capacity.

This paper is organized as follows. In Section II we present the system model of MIMO multi-user networks. The scheduling policy is discussed in Section III, where we derive through Lagrangian multipliers the equations for the three different cases: equal time sharing system, equal power sharing system, joint optimization. Some simulation results on the asymptotic downlink capacity on MIMO multi-user networks are provided in Section IV. Finally, in Section V, conclusions and perspectives of our work are provided.

## II. MULTI-USER MIMO MODEL

The multi-user MIMO model considered in this paper combines two models: the multi-user model and the MIMO model. A brief description of each model is presented in the following.

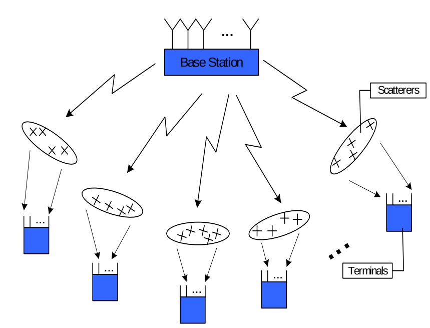


Fig. 1. Multiuser MIMO scheme.

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### A. Multi-user Model

The multi-user model is shown in Fig. 1, and consists of a base station (BS) with multiple terminals. Each terminal is surrounded by a set of scatterers. We assume that each link BS-terminal has independent scatterers, and for simplicity sake, we focus exclusively on the downlink where all the terminals and the BS have the same number of antennas ($n_r = n_t = N$). The extension of the analytical results to the case with different configurations of antennas is straightforward.

### B. MIMO model

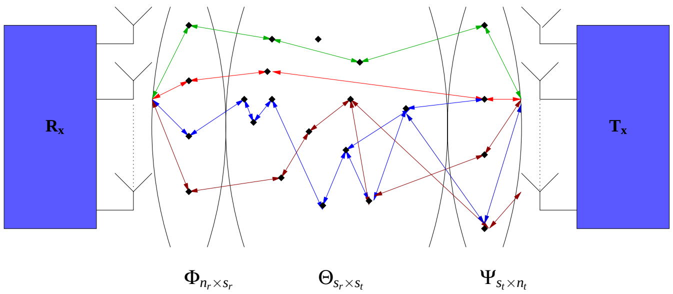


Fig. 2. Double Directional model.

In the following framework, the double directional model of Fig. 2 is considered with $s_r$ scatterers at the receiving side and $s_t$ scatterers at the transmitting side. The scatterers are defined by their (angular) position and their attenuation. Using maximum entropy considerations (see [8] for more details), the MIMO matrix was shown to have the following form

$$\mathbf{H} = \frac{1}{\sqrt{s_r s_t}} \mathbf{\Phi}_{n_r \times s_r} \mathbf{P^r}^{\frac{1}{2}} \mathbf{\Theta}_{s_r \times s_t} \mathbf{P^t}^{\frac{1}{2}} \mathbf{\Psi}_{s_t \times n_t}, \tag{1}$$

where $\mathbf{\Theta}_{s_r \times s_t}$ is an $s_r \times s_t$ i.i.d. Gaussian matrix with unit variance components. This general model has been shown to include the Kronecker model, Sayeed's virtual representation and the keyhole channel as particular cases [8]. The steering matrices $\mathbf{\Phi}_{n_r \times s_r}$ and $\mathbf{\Psi}_{s_t \times n_t}$ represent respectively the directions of arrival (DoAs) from scatterers near the receiver to the receiving antennas and the directions of departure (DoDs) from the transmitting antennas to scatterers near the transmitter:

$$\mathbf{\Phi}_{n_r \times s_r} = \begin{pmatrix} 1 & \dots & 1 \\ \vdots & \ddots & \vdots \\ e^{j2\pi \frac{d(n_r-1)\sin(\phi_1)}{\lambda}} & \dots & e^{j2\pi \frac{d(n_r-1)\sin(\phi_{s_r})}{\lambda}} \end{pmatrix}$$

and

$$\mathbf{\Psi}_{s_t \times n_t} = \begin{pmatrix} 1 & \dots & e^{j2\pi \frac{d(n_t-1)\sin(\psi_1)}{\lambda}} \\ \vdots & \ddots & \vdots \\ 1 & \dots & e^{j2\pi \frac{d(n_t-1)\sin(\psi_{s_t})}{\lambda}} \end{pmatrix}. \tag{2}$$

For the sake of simplicity, we focus on the case where the scatterers are maximally distant from each other, and have equal power. Therefore (see [8]), the angles are distributed according to the Fourier directions ($\psi_k = \frac{2k\pi}{n_t}$, $k = 1 \dots s_t$, and $\phi_k = \frac{2k\pi}{n_r}$, $k = 1 \dots s_r$ , where $s_t \leq n_t$ and $s_r \leq n_t$), and all steering directions have equal gain ($\mathbf{P^r} = \mathbf{I}_{s_r}$ and $\mathbf{P^t} = \mathbf{I}_{s_t}$). Moreover, we consider a rich scattering environment at the transmitter and therefore, one can show that this is equivalent to $\mathbf{\Psi}_{s_t \times n_t} = \mathbf{I}_{n_t}$. The case of limited diversity at the transmitter can be treated with the same methodology as proposed in this paper.

## III. Scheduling Policy

Consider a system with $L$ users, where $N$ antennas are employed at the transmitter and at the receiver. The scheduling policy is based on the restriction on time and power, i.e., $\sum_{i=1}^{L} \theta_i = T$ and $\sum_{i=1}^{L} P_i = P$, where $\theta_i$ and $P_i$ are respectively the time slot and the transmission power of the $i$-th user, and $P$ and $T$ are respectively the maximum power and maximum time slot of the whole system. We suppose that the same power is transmitted across all the antennas as the eigenvector structure is not known at the transmitter.

We are interested to find the vectors $\mathbf{\Theta} = [\theta_1, ..., \theta_L]$ and $\mathbf{P} = [P_1, ...P_L]$ that maximize the instantaneous capacity. The capacity per antenna is given by:

$$C = \frac{1}{N} \sum_{i=1}^{L} \theta_i T \log\det(\mathbf{I} + \frac{\rho_i}{N} T \frac{P_i}{\theta_i} \mathbf{H_i H_i}^H) \tag{3}$$

$$= \frac{1}{N} \sum_{i=1}^{L} \theta_i T \sum_{k=1}^{N} \log(1 + \rho_i T \frac{P_i}{\theta_i} \rho T \frac{P_i}{\theta_i} \lambda_k^i) \tag{4}$$

where $\mathbf{I}$ is the identity matrix, $\rho_i$ represents the instantaneous link level budget and $\lambda_k^i$ is the $k$-th eigenvalue related to the instantaneous MIMO matrix of the $i$-th user. The capacity formula is given in nats/s/Hz/Ant. For notation purposes, we will take in the following $T = 1$ and $P = 1$.

In the case of a high number of antennas and scatterers and using results from random matrix theory [9], the analytical capacity per antenna is given by:

$$C = \sum_{i=1}^{L} \theta_i \Big[ \frac{s_i}{N} \log\left(1 + \rho_i \frac{N P_i}{s_i \theta_i}(1 - \alpha_i)\right) + \log\left(1 + \rho_i \frac{P_i}{\theta_i}(1 - \frac{N}{s_i}\alpha_i)\right) - \alpha_i \Big] \tag{5}$$

with

$$\alpha_i = \frac{1}{2} + \frac{s_i}{2N}(1 + \frac{\theta_i}{\rho_i P_i}) - \frac{1}{2}\sqrt{(1 + \frac{s_i}{N}(1 + \frac{\theta_i}{\rho_i P_i}))^2 - 4\frac{s_i}{N}}. \tag{6}$$

Interestingly, the result shows that the capacity does not depend on the instantaneous channel realization but only on the ratio of the number of scatterers to the number of antennas as well as the instantaneous SNR. Hence, one can optimize the time and power allocation of each user knowing only the scattering environment (highly or poorly scattered) and the path loss (which determines the instantaneous SNR). Therefore, for each user, a water-filling policy is obtained (through Lagrangian multipliers) depending only on the number of scatterers.

Define $W$ as:

$$W = C - \lambda(\sum_{i=1}^{L} P_i - 1) - \mu(\sum_{i=1}^{L} \theta_i - 1) \quad (7)$$

To maximize the capacity with respect to the power allocation, the following condition must be met:

$$\begin{aligned} \frac{dW}{dP_i} &= \frac{\theta_i}{N}(\sum_{k=1}^{N} \frac{\frac{\rho_i \lambda_k^i}{\theta_i}}{1+\frac{\rho_i \lambda_k^i P_i}{\theta_i}}) - \lambda = 0 \quad (8) \\ \Leftrightarrow & \quad \lambda = \frac{\theta_i}{N}(\sum_{k=1}^{N} \frac{\frac{\rho_i \lambda_k^i}{\theta_i}}{1+\frac{\rho_i \lambda_k^i P_i}{\theta_i}}) \\ & \quad \lambda = \frac{\theta_i}{P_i}\frac{1}{N}(\sum_{k=1}^{N} \frac{\frac{\rho_i \lambda_k^i P_i}{\theta_i}+1-1}{1+\frac{\rho_i \lambda_k^i P_i}{\theta_i}}) \\ & \quad \lambda = \frac{\theta_i}{P_i}\left(1 - \frac{\theta_i}{\rho_i P_i}\frac{1}{N}(\sum_{k=1}^{N} \frac{1}{\frac{\theta_i}{\rho_i P_i}+\lambda_k^i})\right) \\ & \quad \lambda = \frac{\theta_i}{P_i}\left(1 - \frac{\theta_i}{\rho_i P_i} m_i(-\frac{\theta_i}{\rho_i P_i})\right) \quad (9) \end{aligned}$$

with $m_i(x)$ is the *Steltjes transform* [10], defined as:

$$m_i(x) = \frac{1}{2x}[\frac{s_i}{N} - 1 - x + \sqrt{(1+x-\frac{s_i}{N})^2 - 4x}]. \quad (10)$$

In the other hand, to maximize the capacity with respect to the time allocation, we have that:

$$\begin{aligned} \frac{dW}{d\theta_i} &= \frac{1}{N}\left(\sum_{k=1}^{N} \log(1+\frac{\rho_i P_i \lambda_k^i}{\theta_i})\right) + \\ & \quad \frac{\theta_i}{N}(\sum_{k=1}^{N} \frac{-\frac{\rho_i P_i \lambda_k^i}{\theta_i^2}}{1+\frac{\rho_i P_i \lambda_k^i}{\theta_i}}) - \mu = 0 \quad (11) \\ &= \frac{s_i}{N} \log\left(1+\rho_i \frac{N P_i}{s_i \theta_i}(1-\alpha_i)\right) + \\ & \quad \log\left(1+\rho_i \frac{P_i}{\theta_i}(1-\frac{N}{s_i}\alpha_i)\right) - \alpha_i + \\ & \quad \frac{\theta_i}{N}(\sum_{k=1}^{N} \frac{-\frac{\rho_i P_i \lambda_k^i}{\theta_i^2}}{1+\frac{\rho_i P_i \lambda_k^i}{\theta_i}}) - \mu = 0 \quad (12) \\ \Leftrightarrow & \quad \mu = \frac{s_i}{N} \log\left(1+\rho_i \frac{N P_i}{s_i \theta_i}(1-\alpha_i)\right) + \\ & \quad \log\left(1+\rho_i \frac{P_i}{\theta_i}(1-\frac{N}{s_i}\alpha_i)\right) - \alpha_i + \\ & \quad \frac{\theta_i}{N}(\sum_{k=1}^{N} \frac{-\frac{\rho_i P_i \lambda_k^i}{\theta_i^2}}{1+\frac{\rho_i P_i \lambda_k^i}{\theta_i}}). \quad (13) \end{aligned}$$

**Remark:** If all the users have the same number of scatterers ($s_i = s = cte$) and the same link budget ($\rho_i = cte$), we have that $m_i(x) = m(x)$, which implies by Eq. (9) that $\frac{\theta_i}{P_i}$ is a constant $\forall i$. Furthermore, as long as Eq. (13) should be satisfied on the optimization process, we can conclude that $\theta_i$ is a constant too, which results in having $\theta_i = \frac{1}{L}$ and $P_i = \frac{1}{L}$ for all users.

Interestingly, using Eqs. (9) and (13), one can optimize the capacity. Both equations are monotones, and the water-filling algorithm can be employed to find the optimal resource allocation. In the following, we restrict our analysis to some particular cases:

### A. Equal Time Sharing Case

In the following, the equal time sharing framework corresponds to a constant values for all $\theta_i$s, which means that each user has the same time slot length ($\theta_i = \frac{1}{L}$) to transmit. For this case, we employ an adaptive power allocation with the restriction that the total transmit power is constant ($\sum_{i=1}^{L} P_i = 1$). In this case, the optimal power allocation is given by the vector $\mathbf{P}$ which satisfies the following equation:

$$\lambda = \frac{1}{LP_i}\left(1 - \frac{1}{L\rho_i P_i} m_i(\frac{1}{L\rho_i P_i})\right), \quad \forall i. \quad (14)$$

For a finite number of users and antennas, the capacity is given by

$$\begin{aligned} \overline{C} &= \frac{1}{L}\sum_{i=1}^{L}\Big[\frac{s_i}{N}\log\left(1+\rho_i P_i L \frac{N}{s_i}(1-\alpha_i)\right) \\ & \quad + \log\left(1+\rho_i P_i L(1-\frac{N}{s_i}\alpha_i)\right) - \alpha_i\Big], \quad (15) \end{aligned}$$

with

$$\begin{aligned} \alpha_i &= \frac{1}{2} + \frac{s_i}{2N}(1+\frac{1}{L\rho_i P_i}) - \\ & \quad \frac{1}{2}\sqrt{(1+\frac{s_i}{N}(1+\frac{1}{L\rho_i P_i}))^2 - 4\frac{s_i}{N}}. \quad (16) \end{aligned}$$

### B. Equal Power Sharing Case

The equal power sharing famework is such that $P_i = \frac{1}{L}$ for all users. As a consequence, we optimize the capacity by adapting the time slot length of each user ($\theta_i$). Hence, Eq. (13) simplifies to

$$\begin{aligned} \mu &= \frac{s_i}{N}\log\left(1+\rho_i \frac{N}{L s_i \theta_i}(1-\alpha_i)\right) + \\ & \quad \log\left(1+\rho_i \frac{1}{L\theta_i}(1-\frac{N}{s_i}\alpha_i)\right) - \alpha_i + \\ & \quad \frac{\theta_i}{N}(\sum_{k=1}^{N} \frac{-\frac{\rho_i \lambda_k^i}{L\theta_i^2}}{1+\frac{\rho_i \lambda_k^i}{L\theta_i}}). \quad (17) \end{aligned}$$

For a finite number of users and antennas, the capacity is given by

$$\begin{aligned} \overline{C} &= \sum_{i=1}^{L}\theta_i\Big[\frac{s_i}{N}\log\left(1+\rho_i \frac{N}{L s_i \theta_i}(1-\alpha_i)\right) + \\ & \quad \log\left(1+\frac{\rho_i}{L\theta_i}(1-\frac{N}{s_i}\alpha_i)\right) - \alpha_i\Big], \quad (18) \end{aligned}$$

with

$$\alpha_i = \frac{1}{2} + \frac{s_i}{2N}(1 + \frac{L\theta_i}{\rho_i}) - \frac{1}{2}\sqrt{(1 + \frac{s_i}{N}(1 + \frac{L\theta_i}{\rho_i}))^2 - 4\frac{s_i}{N}}. \quad (19)$$

### C. Joint Optimization (Time-Power)

We suppose in this section that there is no restriction in terms of time and power allocation, which means that the only constraint on the model are $\sum_{i=1}^{L}\theta_i = 1$ and $\sum_{i=1}^{L} P_i = 1$. We optimize the capacity by adapting in a joint way the time slot length ($\theta_i$) and the power transmission ($P_i$) of each user $i$. We will show that the best policy is to allocate all the time length to the best user. We first derive a capacity upper-bound based on the fact that if all the users had the same number of scatterers as the user that has the highest one ($s_i = s_{max}$) the capacity will be equal or greater than the actual one.

$$\begin{aligned} C &= \sum_{i=1}^{L}\theta_i[\frac{s_i}{N}\log\left(1+\rho_i\frac{NP_i}{s_i\theta_i}(1-\alpha_i)\right) \\ &\quad +\log\left(1+\rho_i\frac{P_i}{\theta_i}(1-\frac{N}{s_i}\alpha_i)\right)-\alpha_i] \\ &\leq \sum_{i=1}^{L}\theta_i[\frac{s_{max}}{N}\log\left(1+\rho_i\frac{NP_i}{s_{max}\theta_i}(1-\alpha_{i,s_{max}})\right) \\ &\quad +\log\left(1+\rho_i\frac{P_i}{\theta_i}(1-\frac{N}{s_{max}}\alpha_{i,s_{max}})\right)-\alpha_{i,s_{max}}] \quad (20) \end{aligned}$$

Eq. 20 shows that the capacity is maximized by allocating all the time-slots for users that have the maximum number of scatterers, which means by Eq. (9) that the power needs to be uniformly spread on the time ($\frac{\theta_i}{P_i} = cte$). Hence, we can conclude that we can obtain the upper-bound capacity by allocating the slots only for the users that have the highest number of scatterers in such a way that all users have the same slot length (fairness assuming optimal capacity), which gives that

$$\begin{aligned} C &\leq \sum_{i=1}^{L_{s_{max}}}\frac{1}{L_{s_{max}}}[\frac{s_{max}}{N}\log\left(1+\rho_i\frac{N}{s_{max}}(1-\alpha_i)\right) \\ &\quad +\log\left(1+\rho_i(1-\frac{N}{s_{max}}\alpha_i)\right)-\alpha_i] \quad (21) \end{aligned}$$

where $L_{s_{max}}$ is the number of users that has the maximum number of scatterers and $P_i = \theta_i = \frac{1}{L_{s_{max}}}$.

**Remark:** It is important to note that in this work we are assuming that the number of scatterers is always inferior than the number of employed antennas. For number of scatterers higher than number of antennas, in the equations, we need to change the term $s_i$ as the minimum value between number of antennas and number of scatterers $\min(n_t, s_i)$.

## IV. Some Results

In this section, we present some results on the asymptotic downlink capacity of MIMO multi-user networks. In Fig. 3, we assume a single user system which employs 10 antennas at both sides (TX and RX), an environment with 10 scatterers and an SNR of 10dB. In Fig. 4, we assume a multi-user system (5 users) which employs 10 antennas at both sides, an SNR of 10dB and a different number of scatterers for each user (s={10,8,6,4,2}), and where the resources ($\theta_i$ and $P_i$) are equally allocated. As we can observe, the results of the simulations shown in Figs. 3 and 4 confirm the asymptotic claims of Section III.

In Fig. 5, Fig. 6 and Fig. 7, we respectively evaluate the impact of the SNR, the number of antennas $(N)$ and the maximum number of scatterers by using Monte Carlo simulations. For sake of simplicity, we consider a multi-user system with only 2 users, where the number of scatterers of each user is randomly generated and $\rho_i$ is a random variable with mean SNR. In Fig. 5, we assumed 10 antennas in both sides (TX and RX) and the maximum number of scatterers $(s_{max})$ equal to 10. As expected, the figure shows that asymptotic capacity is proportional to the SNR. Furthermore, as long as the SNR increases, the effect of resource allocations plays a role in terms of capacity optimization. Contrary to

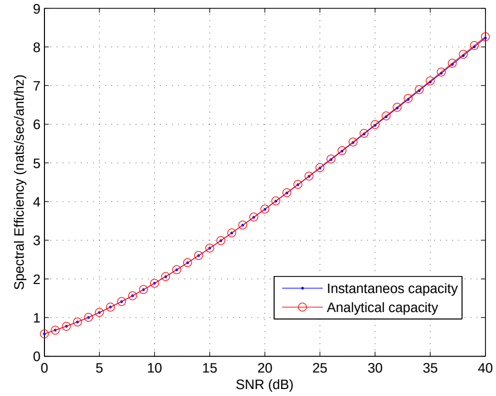


Fig. 3. Comparison between the asymptotic and non-asymptotic capacity for a mono-user system with $N = 10$, $s = 10$ and SNR=10dB.

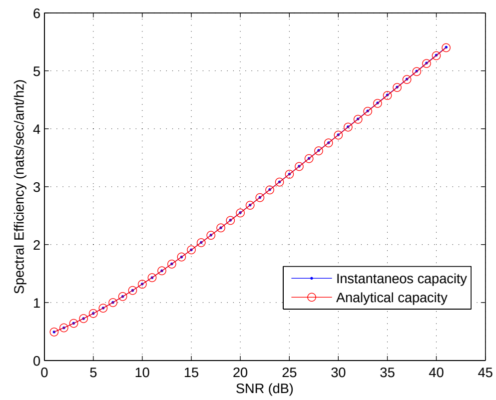


Fig. 4. Comparison between the asymptotic and non-asymptotic capacity for 5 users system with equal resources allocation ($N = 10$,SNR=10dB, $s = \{10, 8, 6, 4, 2\}$).

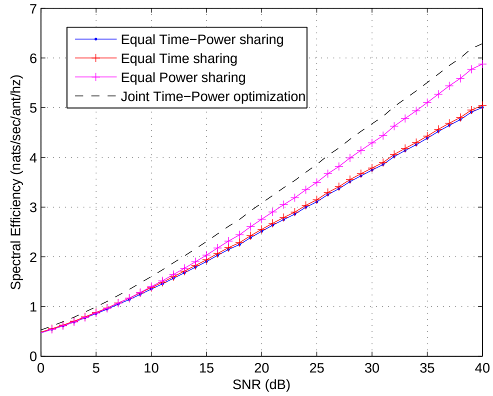


Fig. 5. Comparison between the asymptotic capacity for different SNR values. 2 users, $N = 10$ and $s_{max} = 10$.

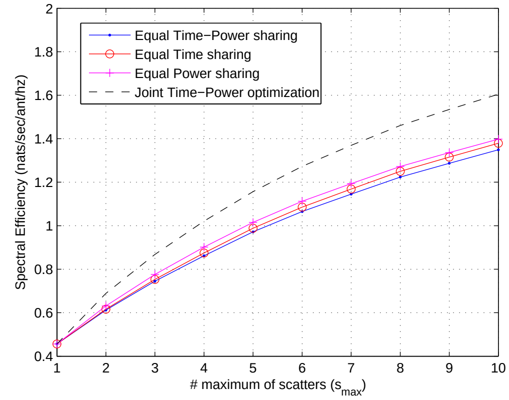


Fig. 7. Comparison between the asymptotic capacity for different number of scatterers. 2 users, $N = 10$ and SNR$= 10$.

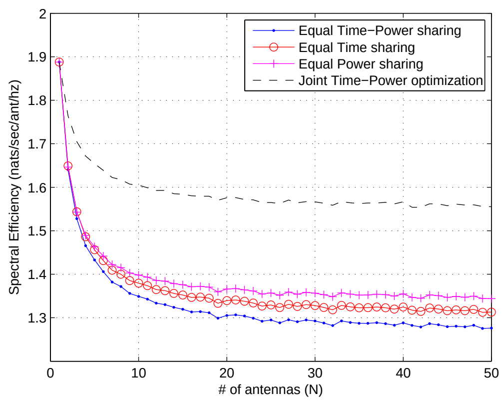


Fig. 6. Comparison between the asymptotic capacity for different number of antennas. 2 users, SNR$= 10$ and $s_{max} = N$.

expectations, the equal time sharing scheme does not permit a significant capacity gain by dynamic power allocation. On the other hand, dynamic time sharing scheme can significantly increase the system capacity, even when power constraints are imposed (as presented by the equal power sharing scheme). In Fig. 6, we assumed an SNR of 10dB and the maximum number of scatterers is given by the number of considered antennas $(s_{max} = N)$. As expected, when we increase the number of antennas, the effect of the dynamic resource allocation plays a role, even when equal time share scheme is employed. As in Fig. 5, the optimal capacity is given by the joint optimization, followed by equal power share scheme (time optimization), equal time share scheme (power optimization) and the worst performance is given by the equal time-power scheme. In Fig. 7, we can verify that the number of scatterers is directly related with the capacity. As long as the channel diversity is strongly related with the number of scatterers, the increase in the number of scatterers means an increase in the system capacity.

## V. Conclusions

In this contribution, some results on the downlink capacity of MIMO multi-user networks are provided when only the number of scatterers per user and the instantaneous SNR are know at the transmitter (not the channel realization). Analytical expressions of the downlink capacity of a MIMO multi-user system were derived and simulated. We show in particular that dynamic power-time allocation can improve the total capacity when only the number of scatterers per terminal is known at the base station. Furthermore, we verified that the dynamic time-slot length allocation introduces a better improvement than the dynamic power allocation.

For future works, we will explore the impact on the optimization for high number of users. Furthermore, we will compare the performance of the optimized capacity when only the number of scatterers are known at the transmitter with the capacity given when full CSI is known.

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