SPATIO-TEMPORAL ARRAY PROCESSING FOR CDMA/SDMA DOWNLINK TRANSMISSION

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ABSTRACT

We address the problem of performing optimum spatio-temporal processing when using adaptive antenna arrays at base stations for multiuser downlink transmission in CDMA systems, using periodic spreading sequences and assuming the knowledge of the channel of all the users. This assumption typically holds in TDD based mobile communication systems. We consider the SDMA strategy for using antenna arrays to gain system capacity. In that case the number of interfering users located in the same cell, namely d, can be higher than the spreading factor. The goal is to design FIR transmission filters at the base station in order to maximize the minimum matched filter bound among the d users. Several approaches, namely the zero-forcing, linear minimum mean square error, minimum output energy and the pre-rake are considered to solve the problem.

1. INTRODUCTION

The use of adaptive antenna arrays at the base station can increase the capacity of a mobile radio network allowing an increase in the number of users. In the downlink however, the possibility of spatial diversity reception by Multiple Antennas (MA) is limited due to complexity and space limitations. Furthermore, third generation systems like the UMTS TDMA/CDMA [2] envision operation in TDD mode. Note that in TDD based systems the uplink and the downlink channels can be considered to be practically the same, assuming the mobile velocity low enough and the receiver and transmitter appropriately calibrated. Under these circumstances, since the channel is known (or estimated) from the uplink, efficient spatio-temporal processing can be performed at the base station during transmission as well as during reception. On the contrary, the lack of channel knowledge represents a major hurdle in FDD based systems. In spite of that complication, solutions to perform optimum transmit array processing have been previously proposed only for FDD based systems, and only purely spatial filtering (i.e. beamforming) has been considered (see [3]–[5] and references therein). Only recently, in [1], the problem of performing optimum spatio-temporal processing at base stations for multiuser downlink transmission was addressed, in the context of TDD/TDMA based mobile communication systems. Here, we consider the same problem but in the CDMA case. In CDMA systems for low rate users, the length of the channel is fairly short compared to the symbol duration resulting in very little Inter-Symbol Interference (ISI). In typical downlink transmission (e.g., IS-95), the multiuser channel is synchronous and the users are assigned orthogonal Walsh-Hadamard spreading sequences. It must be noted, however, that the orthogonality is destroyed by the multipath propagation phenomenon and the actual capacity is much lower than the theoretical one. The orthogonality of the codes can however be restored through proper prefiltering at the base station, exploiting the knowledge of the downlink channels, which corresponds to Zero-Forcing (ZF) the Inter-User Interference (IUI). More appropriate Minimum Mean Square Error (MMSE) cost functions can be formulated when ZF cannot be obtained. Neither proper formulations nor solutions have been provided for the problem of performing optimum transmit spatio-temporal processing for CDMA systems. In our treatment the cost function results from the formulation of the Matched Filter Bound (MFB) optimization problem. We assume a TDD/CDMA mobile communication system employing Periodic Spreading Sequences (PSS), operating with Spatial Division Multiple Access (SDMA) frequency reuse technique to gain system capacity. Then more interfering users may be located in the same cell than the spreading factor (interference coming from other cells is neglected, except for the users in soft-handover mode). We point out that the framework can be easily extended to also include interferers from other cells. The maximization of the MFB leads to the minimum probability of error for an optimal receiver. We also assume that the reciprocity between up-link and downlink channels holds, i.e, the channel remains the same within successive uplink and downlink time slots. The base station performs transmission through m channels resulting from the inherent Over-Sampling (OS) due to the spreading factor, from the use of MA and/or from additional OS w.r.t. the chip rate, towards d users. Each one of the d mobile receivers is assumed to have one antenna, to sample at the chip rate (i.e., no additional OS is provided at the receivers) and to use a correlator. The goal is to design the $m \times d$ FIR transmission filters in order to maximize the minimum MFB among the d users.

2. MFB OPTIMIZATION PROBLEM FORMULATION

We assume a CDMA based system employing periodic spreading sequences, with a period equal to one symbol. Assuming the channels time-invariant for the observation time, because of such periodicity, the cascade of the code filter, the transmit filter, the channel and the receive filter results in a time-invariant system. Since the overall system is time-invariant we attempt to maximize the minimum MFB among all the d users. Actually, the ith user discrete-time received signal, for $i = 1\ \ldots,\ d$, is

$$y_i(k) = \mathbf{c}_i^H(q)\mathbf{H}_i^T(q)\sum_{j=1}^{d}\mathbf{F}_j(q)a_j(k) + v_i(k) \qquad (1)$$

where the $a_j(k)$ are the transmitted symbols intended for the jth user, q^{-1} is the unit sample delay operator (i.e., $q^{-1}y_i(k) = y_i(k-1)$), $\mathbf{H}_i^T(z)$ is the channel transfer function between the base station and the ith user, $\mathbf{c}_i^H(z)$ is the combiner matched to the code for the ith user, $\mathbf{c}_i$, $\mathbf{F}_j(z) = \mathbf{F}'_j(z)\mathbf{c}_j$ is the spatio-temporal filter for the transmitted symbols, accounting for both the actual transmit filter $\mathbf{F}'_j(z)$ to be optimized and the spreading code for the jth user, and $v_i(k)$ is the additive noise at the ith receiver. The superscripts ^{T} and ^{H} denote transpose and Hermitian transpose respectively. Assuming we have m_c chips per symbol period, each transmission filter $\mathbf{F}_i(z)$ will perform sampling at least at the chip rate, i.e., it will be at least a $m_c \times 1$ column vector. If no additional OS or MA are provided, the optimization problem for all the $\mathbf{F}_i(z)$'s reduces to one of spreading codes optimization at the transmitter in the presence of multiuser multipath channels. Moreover, in general $\mathbf{F}_j(z)$ will be a $m \times 1$ column vector, with $m = m_c m_{\mathrm{ma}} m_{\mathrm{os}}$, where m_{ma} is the number of MA and m_{os} is the additional OS rate.
We denote $\mathbf{G}_i^T(q) = \mathbf{c}_i^H\mathbf{H}_i^T(q)$ the overall channel associated to the ith user as seen from the base station. Note that since the receiver is assumed to sample at the chip rate, $\mathbf{H}_i^T(z)$ is a $m_c \times m$ matrix, $\mathbf{c}_i^H$ is a $1 \times m_c$ row vector, so that $\mathbf{G}_i^T(z)$ is a $1 \times m$ row vector, and $\mathbf{F}_j(z)$ is a $m \times 1$ column vector. Note that $\mathbf{G}_i(z)$ is the $m \times 1$ channel in the uplink from the ith user to the m base station channels.

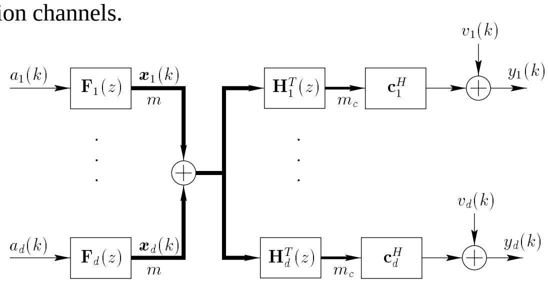


Figure 1: Transmission filters and channels for d users

2.1. Frequency Domain Problem Formulation

The frequency domain MFB definition for the ith user, considering interferers as Gaussian noise, is

$$\mathrm{MFB}_i = \frac{1}{2\pi \mathrm{j}}\oint \frac{\sigma_a^2\mathbf{T}_{ii}^{\dagger}(z)\mathbf{T}_{ii}(z)}{\sigma_a^2\sum_{j\neq i}\mathbf{T}_{ji}(z)\mathbf{T}_{ji}^{\dagger}(z) + \sigma_{v_i}^2}\frac{\mathrm{d}z}{z} \qquad (2)$$

where $\mathbf{T}_{ji}(z) = \mathbf{G}_i^T(z)\mathbf{F}_j(z)$, $\sigma_a^2 = \mathrm{E}\{|a_i(k)|^2\}$, $\sigma_{v_i}^2$ is the variance of the additive noise $v_i(k)$, assumed temporally and spatially white hereafter, for $i,\ j = 1,\ \ldots,\ d$, and, in general $\mathbf{T}^{\dagger}(z) = \mathbf{T}^H(1/z^*)$, where the superscript * denotes conjugate. The symbols are assumed to be i.i.d. and the symbol constellation is assumed to be circular (for a real constellation, the complex signals should be split into in phase and in quadrature components). The cost function is given by

$$\max_{\{\mathbf{F}_j(z)\}}\min_i\{\mathrm{MFB}_i\} \qquad (3)$$

2.2. Burst Processing Time Domain Problem Formulation

Consider the ith user I/O transmission chain (see fig. 1) regardless of the contributions intended for the other users. The channel $\boldsymbol{g}_i^T(t) = \boldsymbol{c}_i^H\boldsymbol{H}_i^T(t)$ and the transmission filter $\boldsymbol{f}_i(t) = \boldsymbol{F}'_i(t)\boldsymbol{c}_i$ are assumed to be FIR filters with duration N_iT and LT respectively (approximately), where $T = m_cT_c$ is the symbol period and T_c is the chip period. In discrete-time representation we have

$$\begin{aligned}
&\boldsymbol{x}_i(k) = \textstyle\sum_{l=0}^{L-1}\boldsymbol{f}_i(l)a_i(k-l) = \boldsymbol{F}_iA_{i,\,L}(k)\\
&y_i(k) = \textstyle\sum_{n=0}^{N_i-1}\boldsymbol{g}_i^T(n)\boldsymbol{x}_i(k-n) + v_i(k)\\
&\qquad = \boldsymbol{G}_i^t\boldsymbol{X}_{i,\,N_i}(k) + v_i(k)\\
&\boldsymbol{G}_i^t = [\boldsymbol{g}_i^T(N_i-1)\ \ldots\ \boldsymbol{g}_i^T(0)],\ \ \boldsymbol{F}_i = [\boldsymbol{f}_i(L-1)\ \ldots\ \boldsymbol{f}_i(0)]\\
&\boldsymbol{X}_{i,\,N_i}(k) = [\boldsymbol{x}_i^H(k-N_i+1)\ \ldots\ \boldsymbol{x}_i^H(k)]^H\\
&A_{i,\,L}(k) = [a_i^H(k-L+1)\ \ldots\ a_i^H(k)]^H
\end{aligned} \qquad (4)$$

where superscript ^{t} denotes transposition of the blocks in a block matrix. If we accumulate M consecutive symbol periods

$$Y_{i,\,M}(k) = \mathcal{T}_M(\boldsymbol{G}_i^t)\mathcal{T}_{M+N_i-1}(\boldsymbol{F}_i)A_{i,\,M+N_i+L-2}(k) + V_{i,\,M}(k)$$

where $Y_{i,\,M}(k) = [y_i^H(k-M+1)\ \ldots\ y_i^H(k)]^H$ and similarly for $V_{i,\,M}(k)$. $\mathcal{T}_M(\boldsymbol{A})$ is in general a block Toeplitz matrix with M block rows and $[\boldsymbol{A}\ \mathbf{0}_{p\times q(M-1)}]$ as first block row, where $\boldsymbol{A}$ is a matrix with $p \times q$ block entries.
Then, introducing also the contributions of all the other users, for the ith user we have

$$\begin{aligned}
Y_{i,\,M}(k) = &\sum_{j=1}^{d}\mathcal{T}_M(\boldsymbol{G}_i^t)\mathcal{T}_{M+N_i-1}(\boldsymbol{F}_j)A_{j,\,M+N_i+L-2}(k) +\\
&+ V_{i,\,M}(k)
\end{aligned} \qquad (5)$$

and in the corresponding burst covariance matrix

$$\boldsymbol{R}_i^{(M)} = \sum_{j=1}^{d}\boldsymbol{R}_{ji}^{(M)} + \sigma_{v_i}^2\mathbf{I}_M$$

we can distinguish the following contributions

$$\begin{aligned}
\boldsymbol{R}_{ii}^{(M)} &= \sigma_a^2\mathcal{T}_M(\boldsymbol{G}_i^t)\mathcal{T}_{M+N_i-1}(\boldsymbol{F}_i)\mathcal{T}_{M+N_i-1}^H(\boldsymbol{F}_i)\mathcal{T}_M^H(\boldsymbol{G}_i^t)\\
\boldsymbol{R}_{ji}^{(M)} &= \sigma_a^2\mathcal{T}_M(\boldsymbol{G}_i^t)\mathcal{T}_{M+N_i-1}(\boldsymbol{F}_j)\mathcal{T}_{M+N_i-1}^H(\boldsymbol{F}_j)\mathcal{T}_M^H(\boldsymbol{G}_i^t)
\end{aligned} \qquad (6)$$

where $\boldsymbol{R}_{ii}^{(M)}$ and $\boldsymbol{R}_{ji}^{(M)}$ are the contributions of the ith and jth transmitted signals respectively at the ith receiver, for $j \neq i$. Note that $\sum_{j\neq i}\boldsymbol{R}_{ji}^{(M)}$ represents the burst covariance matrix of the whole IUI at the ith receiver. Then the burst processing MFB is defined as

$$\mathrm{MFB}_i^{(M)} = \frac{1}{M}\,\mathrm{tr}\{\boldsymbol{R}_{ii}^{(M)}[\sum_{j\neq i}\boldsymbol{R}_{ji}^{(M)} + \sigma_{v_i}^2\mathbf{I}_M]^{-1}\} \qquad (7)$$

where $\mathrm{tr}\{\cdot\}$ denotes the trace operator. Remark that as $M \to \infty$, $\mathrm{MFB}_i^{(M)} \to \mathrm{MFB}_i$ in (2).
In a similar fashion to the frequency domain formulation (3) the optimization criterion results in

$$\max_{\{\boldsymbol{F}_j\}}\min_i\{\mathrm{MFB}_i^{(M)}\} \qquad (8)$$

Both problem formulations (3), (8) are too complicated to allow any analytical approach to find the optimum solution. Nevertheless analytical solutions can be found under the following assumption that the optimal solution corresponds to a low Interference-to-Noise Ratio (INR) for all the users, i.e.,

$$\mathrm{INR}_i = \frac{\sigma_a^2}{2\pi \mathrm{j} \sigma_{v_i}^2} \sum_{j \neq i} \oint \mathbf{T}_{ji}^{\dagger}(z) \mathbf{T}_{ji}(z) \frac{\mathrm{d}z}{z} \ll 1 \qquad \forall i \tag{9}$$

In that case, it is easy to see that maximizing the MFB is approximately equivalent to maximizing the Signal-to-Interference-plus-Noise Ratio (SINR) and vice versa.
Hence, referring to the burst processing problem formulation, the SINR definition for the ith user is

$$\mathrm{SINR}_i = \frac{\mathrm{tr}\{\boldsymbol{R}_{ii}^{(M)}\}}{\mathrm{tr}\{\sum_{j \neq i} \boldsymbol{R}_{ji}^{(M)} + \sigma_{v_i}^2 \mathbf{I}_M\}} \tag{10}$$

By introducing $\boldsymbol{F}_i^t = [\boldsymbol{f}_i^T(L-1)\ \dots\ \boldsymbol{f}_i^T(0)]$, it can be written as

$$\mathrm{SINR}_i = \frac{\sigma_a^2 \boldsymbol{F}_i^t \boldsymbol{R}_i \boldsymbol{F}_i^{tH}}{\sigma_a^2 \sum_{j \neq i} \boldsymbol{F}_j^t \boldsymbol{R}_i \boldsymbol{F}_j^{tH} + \sigma_{v_i}^2} \tag{11}$$

where $\boldsymbol{R}_i$ is a properly defined covariance matrix related to the channel $\boldsymbol{G}_i^t$, whose derivation is straightforward. In the continuous-processing case, we have $\boldsymbol{R}_i = \mathcal{T}_L(\boldsymbol{G}_i)\mathcal{T}_L^H(\boldsymbol{G}_i)$, where $\boldsymbol{G}_i = [\boldsymbol{g}(N_i - 1)\ \dots\ \boldsymbol{g}(0)]$. According to the definition (11) we denote $\mathrm{SINR}_i = \gamma_i$ in the sequel. Then let $\boldsymbol{F}_i^t = \sqrt{p_i}\boldsymbol{U}_i^t$, where $\boldsymbol{U}_i^t$ is a vector with unit norm (e.g., $\|\boldsymbol{U}_i^t\|_2 = 1$ or $\boldsymbol{U}_i^t \boldsymbol{R}_i \boldsymbol{U}_i^{tH} = 1$), the vector of the inverse SINR's $\boldsymbol{\gamma}^{-1} = [\gamma_1^{-1}\ \dots\ \gamma_d^{-1}]^T$ and the vector of the transmit powers $\boldsymbol{p} = [p_1, \dots, p_d]^T$. In addition we need to constrain the overall power transmitted by the base station to be less than or equal to $p_{\max}$. Hence the optimization criterion is

$$\min_{\boldsymbol{p},\,\{\boldsymbol{U}_i\}} \|\boldsymbol{\gamma}^{-1}\|_\infty \quad \text{s.t.} \quad \boldsymbol{\zeta}^T \boldsymbol{p} \le p_{\max} \tag{12}$$

where[1] $\boldsymbol{\zeta} = [\|\boldsymbol{U}_1^t\|_2^2\ \dots\ \|\boldsymbol{U}_d^t\|_2^2]^T$. In the rest of this paper we shall consider the SINR optimization criterion (12), regardless of its relationship to the MFB criterion in (3). In that case $\sigma_{v_i}^2$ can account for the variance of the inter-cell interference also. Then we define the normalized power delivered by the jth transmission filter $\boldsymbol{F}_j$ to the ith user as $c_{ji} = \boldsymbol{U}_j^t \boldsymbol{R}_i \boldsymbol{U}_j^{tH}$. For any i we have

$$\gamma_i^{-1} p_i c_{ii} = \sum_{j \neq i} p_j c_{ji} + \nu_i \tag{13}$$

where we introduced $\nu_i = \sigma_{v_i}^2 / \sigma_a^2$ for all the i's. In order to account for all the users we introduce the matrix $\boldsymbol{C}^T$ defined as

$$[\boldsymbol{C}^T]_{ij} = \begin{cases} c_{ji} & \text{for } j \neq i \\ 0 & \text{for } j = i \end{cases}$$

the matrix $\boldsymbol{D}_c = \mathrm{diag}\{[c_{11}\ \dots\ c_{dd}]\}$, the vector $\boldsymbol{\nu} = [\nu_1\ \dots\ \nu_d]^T$ and the matrix $\boldsymbol{P} = \mathrm{diag}(\boldsymbol{p})$. Then we have the following equation

$$\boldsymbol{\gamma}^{-1} = \boldsymbol{D}_c^{-1} \boldsymbol{P}^{-1} (\boldsymbol{C}^T \boldsymbol{p} + \boldsymbol{\nu}) \ . \tag{14}$$

So the criterion (12) generally leads to a set of coupled problems which cannot be solved analytically. It can be shown however that the optimum (12) leads to the same γ for all the users. Indeed if some γ_i's are not the same, then we can scale the $\{p_i\}$ to improve $\gamma_{\min}$ (refer to [4] for a detailed proof).

[1] Actually, the proper norm for the $\boldsymbol{U}_i^t$'s in $\boldsymbol{\zeta}$ is $\boldsymbol{U}_i^t \boldsymbol{W} \boldsymbol{U}_i^{tH}$, where $\boldsymbol{W}$ depends on the transmit pulse shape filter, but we shall ignore this issue in this paper.

3. MFB OPTIMIZATION PROBLEM SOLUTIONS

Generally the optimization problem cannot be solved analytically for both $\boldsymbol{p}$ and $\{\boldsymbol{U}_i^t\}$ at the same time. Nevertheless under certain assumptions the optimization can be carried out in a decoupled way for $\boldsymbol{p}$ and $\{\boldsymbol{U}_i^t\}$ allowing analytical approaches to find the optimum.

3.1. Zero-Forcing (ZF) Solution

In the noiseless case or assuming the assumption (9) holds, the MFB optimization becomes

$$\max_{\|\boldsymbol{U}_i^t\|_2 = 1} \{\boldsymbol{U}_i^t \boldsymbol{R}_i \boldsymbol{U}_i^{tH}\} \qquad \text{s.t.} \qquad \sum_{j \neq i} p_j \boldsymbol{U}_j^t \boldsymbol{R}_i \boldsymbol{U}_j^{tH} = 0 \tag{15}$$

Note that the condition $\sum_{j \neq i} p_j \boldsymbol{U}_j^{tH} \boldsymbol{R}_i \boldsymbol{U}_j^t = 0$ is equivalent to a set of ZF conditions in the form $\boldsymbol{U}_i^t \boldsymbol{R}_j \boldsymbol{U}_i^{tH} = 0$, for $j \neq i$. Then the optimization problem reduces to

$$\max_{\|\boldsymbol{U}_i^t\|_2 = 1} \|\boldsymbol{U}_i^t \mathcal{T}_L(\boldsymbol{G}_i)\|_2^2 \quad \text{s.t. } \boldsymbol{U}_i^t \mathcal{T}_L(\boldsymbol{G}_j) = \boldsymbol{0} \text{ for } j \neq i \tag{16}$$

Defining $\boldsymbol{B}_i = [\mathcal{T}_L(\boldsymbol{G}_j)]_{j \neq i}$, which is a block Toeplitz matrix accounting for all the channels but the channel $\boldsymbol{G}_i$, the solution of the problem (16) is $\boldsymbol{U}_i^{tH} = V_{\max}(\mathbf{P}_{\mathbf{B}_i}^{\perp} \boldsymbol{R}_i \mathbf{P}_{\mathbf{B}_i}^{\perp})$. In order for a non trivial solution to this problem to exist, we need $m > d-1$, which is easily achievable when MA and/or additional OS are employed, and the constraints should not fix all the available degrees of freedom and we require

$$L > \frac{\sum_{j \neq i} N_j - (d-1)}{m_{\text{eff}} - (d-1)} \tag{17}$$

where m_{eff} denotes the effective number of channels and is given by the row rank of $\boldsymbol{G}_N = [\boldsymbol{G}_1\ \dots\ \boldsymbol{G}_d]$. Note that $m_{\text{eff}} = \min\{N - d + \Delta_s,\ N,\ m\}$, where $\Delta_s = \mathrm{rank}\{[\boldsymbol{g}_1(N_1 - 1)\ \dots\ \boldsymbol{g}_d(N_d - 1)]\}$. We also assumed $\boldsymbol{B}_i$ to be full column rank $\forall i$. The constraints present in the optimization problem (16) lead to perfect IUI cancellation. This is obtained at the expense of increased ISI at the receiver. In order to consider the ISI as well as the IUI rejection in the optimization problem we rely on the ZF pre-equalization conditions.

3.1.1. *ZF Conditions for IUI and ISI Rejection*

In order to ensure ZF conditions for IUI and ISI for the ith user the set of constraints to be considered is

$$\boldsymbol{U}_i^t \mathcal{T}_L(\boldsymbol{G}_N) = [0\ \dots\ 0\ \dots\ \overbrace{|\,0\ \dots 0\, \alpha\, 0 \dots\ 0\,|}^{i\text{th user}}\ \dots\ 0\ \dots\ 0] \tag{18}$$

where $\mathcal{T}_L(\boldsymbol{G}_N) = [\mathcal{T}_L(\boldsymbol{G}_1) \dots \mathcal{T}_L(\boldsymbol{G}_d)]$, $N = \sum_{j=1}^d N_j$ and $\alpha \neq 0$ is an arbitrary constant to be fixed in order to satisfy the constraint on the norm of $\boldsymbol{U}_i^t$. When IUI and ISI are zero-forced we have $\mathrm{SINR}_i = \mathrm{SNR}_i = \mathrm{MFB}_i$ for any i. Assuming $m > d$ and $\mathcal{T}_L(\boldsymbol{G}_N)$ to be full column rank, to be able to satisfy all the constraints (18) we need to choose the length of each filter $\boldsymbol{U}_i^t$, L, such that the previous system is exactly or underdetermined. Hence

$$L \ge \underline{L} = \left\lceil \frac{N - d - 1}{m_{\text{eff}} - d} \right\rceil \tag{19}$$

Then assuming $L \ge \underline{L}$ we can consider two limiting set of constraints:

- IUI rejection, no ISI rejection, as in section 3.1.
- both IUI and ISI rejection: in this latter case the set of constraints is (18), i.e., we have $N_i + L - 1$ more constraints.

The goal is to maximize the MFB which, in the absence of IUI (equal to zero due to ZF), is proportional to the energy in the prefilter-channel cascade. Then, the MFB decreases if all the energy is constrained in one tap. Hence if no ISI rejection is provided the best performance will be achieved, for a specified L, due to the larger number of degrees of freedom. However, in that case the ith receiver needs to equalize a delay spread of up to $N_i + L - 1$ symbol periods, corresponding to the whole delay spread due to the convolution between the channel and the transmission filter. We may prefer that the introduction of the prefilter does not increase the delay spread, or we may want to limit the delay spread seen by the mobile to limit the complexity for the equalization task in the mobile. In those cases additional constraints in order to obtain at least partial ISI rejection, i.e., limited delay spread, can be added, leading to intermediate solutions between the previous two limiting cases. In general to have complete IUI and partial ISI rejection we add $(N_i+L-1)-L_{\mathrm{ISI}}$ constraints (coefficients of the prefilter-channel cascade being zero), with $1 \leq L_{\mathrm{ISI}} \leq (N_i + L - 1)$, where L_{ISI} corresponds to the residual delay spread, i.e., residual ISI. This optimization problem has to be carried out for all possible positions of the nonzero part of length L_{ISI} of the prefilter-channel cascade, and the best position should be chosen. Finally, note that as L increases the MFB increases as well. So, we shall choose the actual length of the transmission filters L according to a trade-off between performance and transmitter complexity.
Finally one may note that ZF here corresponds to the design of a bi-orthogonal perfect-reconstruction transmultiplexer in which the $\boldsymbol{F}_i$'s and $\boldsymbol{G}_i$'s are synthesis and analysis filter banks respectively.

3.2. Downlink Synchronous and Asynchronous Transmission

The downlink transmission can be performed in a synchronous or asynchronous fashion. In the asynchronous transmission mode the base station transmits maintaining the same asynchronous channel model for PSS-CDMA from the uplink, according to the TDD assumption of perfect channel reciprocity. On the contrary, the synchronous transmission mode corresponds to lining up all the user channels $\boldsymbol{g}_j(i)$'s in a synchronous fashion. In this case we have a *wide sense* TDD channel reciprocity, and the matrices $\boldsymbol{R}_{ji}^{(M)}$ in (6) have to be built *by hand* from the uplink channel estimates. In the previous developments we considered ZF-FIR conditions on both IUI and ISI, yielding an expression for the minimum transmission filter length $\underline{L}$. The condition for the ZF-FIR filter for IUI and ISI cancellation to exist is that the channel matrix $\mathcal{T}_L(\boldsymbol{G}_N)$ must have full column rank for a certain filter length $L \geq \underline{L}$. This assumption holds with probability close to one in the asynchronous mode for smaller $\underline{L}$ than in the synchronous mode. Indeed in the asynchronous mode $\Delta_s = d$ with probability close to one. On the contrary, in the synchronous mode Δ_s can almost surely decrease when the channel delay spreads for all users are smaller than the number of users. That results in a smaller m_{eff} which in turn results in a larger $\underline{L}$. Finally, note that $\mathcal{T}_L(\boldsymbol{G}_N)$ is full column rank with probability one for $L \geq N - d$.

3.3. Minimum Mean Square Error (MMSE) Solution

The MMSE criterion is given by

$$\min_{\{\boldsymbol{F}_j\}} \max_i \mathrm{E}\|y_i(k) - a_i(k-n)\|_2^2 \tag{20}$$

where n is a properly chosen delay to minimize the MMSE and

$$y_i(k) = \sum_{j=1}^{d} \boldsymbol{F}_j^t \mathcal{T}_L(\boldsymbol{G}_i) A_{j,\,N_i+L-1}(k) + v_i(k)$$

Then the criterion (20) can be written as

$$\begin{aligned} \min_{\boldsymbol{p},\,\|\boldsymbol{U}_j^t\|_2=1} \max_i \{ & \mathrm{E}\|p_i \boldsymbol{U}_i^t \mathcal{T}_L(\boldsymbol{G}_i) A_{i,\,N_i+L-1}(k) - a_i(k-n)\|_2^2 \\ & + \sigma_a^2 \sum_{j\neq i} p_j \boldsymbol{U}_j^t \mathcal{T}_L(\boldsymbol{G}_i) \mathcal{T}_L^H(\boldsymbol{G}_i) \boldsymbol{U}_j^{tH} + \sigma_{v_i}^2 \} \end{aligned} \tag{21}$$

where the first term corresponds to the ISI and the second one to the IUI. Hence it is straightforward to see that the MMSE corresponds to ZF on ISI and IUI when ZF conditions (18) can be applied.

3.4. Minimum Output Energy (MOE) Solution

Applying the MOE criterion leads to

$$\min_{\boldsymbol{p},\,\{\boldsymbol{U}_j^t\}} \max_i \{\sum_j p_j \boldsymbol{U}_j^t \boldsymbol{R}_i \boldsymbol{U}_j^{tH} + \sigma_{v_i}^2\} \quad \text{s.t. } p_i \boldsymbol{U}_i^t \mathcal{T}_L(\boldsymbol{G}_i) = \boldsymbol{q}_i \tag{22}$$

for any i, where $\boldsymbol{q}_i$ denotes a vector of constraints on the ith user prefilter-channel cascade. It is straightforward to see that for $m_{\mathrm{eff}} > d$ the MOE criterion leads to ZF conditions on IUI while the residual ISI depends on the constraint vector $\boldsymbol{q}_i$.

3.5. The Pre-Rake Scheme

The pre-rake solution consists of an independent pre-distortion of the downlink signal of each user by setting $\boldsymbol{U}_i^{tH} = \boldsymbol{G}_i^t / \|\boldsymbol{G}_i^t\|_2^2$. The mobile receiver then needs to tune to the largest peak of the pre-distorted signal. Although the pre-rake solution involves a low complexity, it is inherently sub-optimal in the presence of multiuser transmission, since it does not account for the IUI and the ISI. The aim is to avoid coherent combination of interfering signals while reducing the mobile receiver complexity.

3.6. Power Assignment Optimization

Assuming a given set $\{\boldsymbol{U}_i\}$, since the optimum involves all the γ_i's to be the same, expression (14) can be arranged in order to include the constraint on the transmitted power $\boldsymbol{\zeta}^T \boldsymbol{p} = p_{\max}$. By defining $\tilde{\boldsymbol{p}} = [\boldsymbol{p}^T\, 1]^T$, $\boldsymbol{\mu} = \boldsymbol{D}_c^{-1}\boldsymbol{\nu}$, $\boldsymbol{A}^T = \boldsymbol{D}_c^{-1}\boldsymbol{C}^T$, and $\boldsymbol{\zeta}^T\boldsymbol{p} = p_{\max}$, (14) reduces to the following problem (see [4] for details)

$$\boldsymbol{E}\tilde{\boldsymbol{p}} = \gamma^{-1}\tilde{\boldsymbol{p}}, \qquad \boldsymbol{E} = \begin{bmatrix} \boldsymbol{A}^T & \boldsymbol{\mu} \\ \frac{\boldsymbol{\zeta}^T \boldsymbol{A}^T}{p_{\max}} & \frac{\boldsymbol{\zeta}^T \boldsymbol{\mu}}{p_{\max}} \end{bmatrix} \tag{23}$$

Since $\boldsymbol{E}$ is a non-negative matrix its maximum eigenvalue is non-negative and the corresponding eigenvector is non-negative as well [6]. Hence the solution to the problem (23) is unique and it is given by $\gamma^{-1} = \lambda_{\max}(\boldsymbol{E})$ and $\tilde{\boldsymbol{p}} = V_{\max}(\boldsymbol{E})$. Note that we can always re-scale $\tilde{\boldsymbol{p}}$ in order to make its last element equal to one.

3.7. Implementation Issues

The presence of the noise makes the optimization of the filters $\{\boldsymbol{U}_i^t\}$ involve a set of coupled problems that does not allow any analytical approach to find a solution. Therefore, we suggest to compute the vectors $\{\boldsymbol{U}_i^t\}$ applying ZF conditions (16), (18), MMSE criterion (20) or MOE criterion (22) assuming $m_{\text{eff}} > d$, which is always the case in practice. Then, given $\{\boldsymbol{U}_i^t\}$, we optimize the power assignment according to the criterion (23).

When the noise is present, since the base station cannot estimate the noise variance $\sigma_{v_i}^2$ at each receiver, unless such an estimate is provided by the mobile, the vector $\boldsymbol{\nu}$ cannot be estimated. To remedy this drawback we shall properly define the SNR at the receiver. A possible definition is given by

$$\mathrm{SNR}_i = \frac{p_i}{\nu_i}\lambda_{\max}(\boldsymbol{R}_i), \quad \forall i$$

In practice we need

$$\min_i\{\mathrm{SNR}_i\} \geq \mathrm{SNR}_{\min} \tag{24}$$

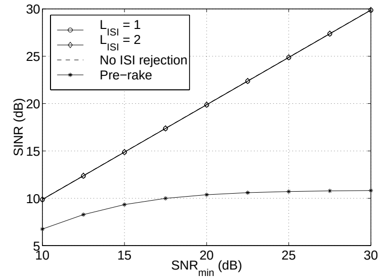
where $\mathrm{SNR}_{\min}$ is a value necessary for the mobile receiver to work with an outage probability below a specified maximum. Assuming all the users using the same receiver the worst case for the ith user occurs when $p_i = p_{\max}$ while $\nu_i = \nu_{\max} = \|\boldsymbol{\nu}\|_\infty$. Therefore a sufficient condition to satisfy the requirement (24) is given by setting

$$\mathrm{SNR}_{\min} = \frac{p_{\max}}{\nu_{\max}}\min_i\{\lambda_{\max}(\boldsymbol{R}_i)\} \tag{25}$$

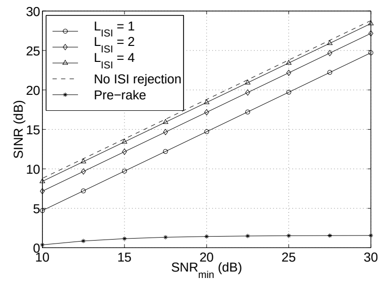
Given $\mathrm{SNR}_{\min}$ and $p_{\max}$, $\nu_{\max}$ can be derived. Then setting $\nu_i = \nu_{\max}$ for all the i's the condition (24) is satisfied. Finally, note that for $p_{\max} \to \infty$ the optimum solution is the one in the absence of noise, for any $\nu_{\max} > 0$.

4. SIMULATIONS

The following simulations are provided to illustrate a practical implementation of the proposed solutions. Here we consider an CDMA/SDMA scenario in the presence of $d = 3$ users which receive signals transmitted from a base station. The channels $\boldsymbol{G}_i$'s are known (or estimated from the uplink). In the first simulation we assumed $m_c = 16$ chips per symbol, $m_{\text{ma}} = 2$ antennas and $m_{\text{os}} = 2$ OS factor w.r.t. the chip rate, so that $m = 64$. The channel delay spreads were $N_1 = N_2 = N_3 = 2$ symbol periods while $m_{\text{eff}} = 6$. Since $m_{\text{eff}} > d$ ZF conditions (18) can be applied. By setting the length of all the transmit filters equal to $L = 2$ symbol periods we obtain the performances plotted in figure 2(a), in terms of SINR at each receiver versus the minimum SNR. Note that due to the large processing gain, m_c, w.r.t. the number of users and to the small delay spreads introduced by the channels the performances are insensitive to the residual delay spread L_{ISI} introduced by the prefilter-channel cascade. Furthermore, extensive simulations have shown that larger values of L do not yield significant improvement of performances in that case. In the second simulation we considered a *saturated* system configuration assuming $d = 3$, $m_c = 4$ and $m_{\text{ma}} = m_{\text{os}} = 1$. The channel delay spreads were $N_1 = N_2 = 3$ and $N_3 = 4$ symbol periods respectively. Also in this case $m_{\text{eff}} > d$ ($m_{\text{eff}} = 4$) and ZF conditions (18) can still be applied, but a larger filter length L will be needed. We fixed $L = 8$ symbol periods to achieve (18), whereas $L = 4$ suffices for the pre-rake. The resulting performances are plotted in fig. 2(b) where significant differences arise for different values of L_{ISI}. Note that the pre-rake, even power controlled and assuming an ideal receiver, like in this case, performs always worse than the proposed solution, since it does not provide IUI cancellation. The effect of IUI can become catastrophic when working close to the system saturation, namely when the number of users approaches m.



(a) $m = 64$, $m_{\text{eff}} = 6$ and $d = 3$



(b) $m = m_c = m_{\text{eff}} = 4$ and $d = 3$

Figure 2: Optimum SINR vs. $\mathrm{SNR}_{\min}$, pre-rake and ZF solution for different values of L_{ISI}

5. CONCLUSIONS

We addressed the problem of the optimization of the MFB with respect to the transmit filters at a base station performing spatio-temporal processing. A general problem formulation yielded the proper cost function to be minimized. We showed that the ZF solution allows analytical approach to the optimization problem and, under certain assumptions, it is optimal for the MFB maximization. We showed that both MMSE and MOE criteria lead to the same solution as ZF conditions in cases where ZF conditions (18) apply. The pre-rake scheme was also considered and it was shown that it performs always much worse than our ZF solution. We also discussed the effects of different values of the transmit filter length and different delay spreads introduced by the prefilter-channel cascade. Finally, we observe that the PSS-CDMA without any further array processing represents a particular case of the presented framework.

6. REFERENCES

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