BLIND CHANNEL AND LINEAR MMSE RECEIVER DETERMINATION IN DS-CDMA SYSTEMS

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ABSTRACT

We consider p users in a DS-CDMA system operating asynchronously in a multipath environment. Oversampling w.r.t. the chip rate is applied to the cyclostationary received signal and multi-antenna reception is considered, leading to a linear multichannel model. Channels for different users are considered to be FIR and of possibly different lengths. We consider an individualized linear MMSE receiver for a given user, exploiting its spreading sequence and timing information. The blind determination of the receiver boils down to the blind channel identification. We explore blind channel identifiability requirements. Sufficiency of these requirements is established and it is shown that if zero-forcing conditions can be satisfied, then the CDMA channel (and hence the receiver) is identifiable with probability 1. It is also shown that linear MMSE receivers obtained by different criteria (including a new one) have the same identifiability requirements asymptotically in SNR.

1. INTRODUCTION AND PREVIOUS WORK

The search for blind receivers for DS-CDMA systems kicked off with the pioneering work of [1], which is based upon a constrained minimum output energy (MOE) criterion. The constrained MOE receiver constrains its inner product with the spreading sequence matched filter to be fixed, thus restricting the optimization problem to within the constrained space. A constrained optimization scheme was proposed in [2] which turns out to be the extension to multipath channels of [1]. In this scheme, the receiver's output energy is minimized subject to the distortionless constraint. Connections with the *Capon* philosophy were drawn in that paper. The receivers mentioned above can be shown to converge asymptotically (SNR$\rightarrow\infty$) to the zero-forcing (ZF) or decorrelating solution. Direct estimation of the MMSE receiver was introduced in [3] following the observation that the MMSE receiver lies in the signal subspace. The MMSE receiver constrained to signal subspace (which is essentially the same as the MMSE) in the case of channels longer than a symbol period was investigated in [4], where a singular-value decomposition (SVD) was used to determine the orthogonal subspaces. The channel estimate in this work was obtained as a generalization to longer delay spreads of the subspace technique originally proposed in [5]. Identifiability issues under long delay spread conditions were however not elaborated upon. Moreover, the schemes mentioned above have high complexity since an estimate of subspaces is required.

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The purpose of this paper is to give the FIR channel identifiability conditions for a DS-CDMA system with users having possibly unequal channel lengths (even longer than a symbol period), and to show the sufficiency of these conditions. It is also shown that linear receiver algorithms related to Capon's method [2] [6], and the directly estimated MMSE receiver [3] have the same identifiability requirements.

2. MULTIUSER DATA MODEL

The p users are assumed to transmit linearly modulated signals over a linear multipath channel with additive Gaussian noise. It is assumed that the receiver employs L antennas to receive the mixture of signals from all users. The signal received at the lth sensor can be written in baseband notation as

$$y^l(t) = \sum_{j=1}^{p} \sum_{k} a_j(k) g_j^l(t - kT_s) + v^l(t), \tag{1}$$

where $a_j(k)$ are the transmitted symbols from the user j, T_s is the common symbol period, $g_j^l(t)$ is the overall channel impulse response between jth user and the lth antenna. Assuming the $\{a_j(k)\}$ and $\{v^l(t)\}$ to be jointly wide-sense stationary, the process $\{y^l(t)\}$ is wide-sense cyclostationary with period T_s. The overall channel impulse response for the jth user's signal at the lth antenna, $g_j^l(t)$, is the convolution of the spreading code $c_j(s)$ and $h_j^l(t)$, itself the convolution of the chip pulse shape, the receiver filter and the actual channel (assumed to be FIR) representing the multipath fading environment. This can be expressed as

$$g_j^l(t) = \sum_{s=0}^{m-1} c_j(s) h_j^l(t - sT), \tag{2}$$

where T is the chip duration. The symbol and chip periods are related through the processing gain m: $T_s = mT$. Sampling the received signal at R times the chip rate, we obtain the wide-sense stationary $mR \times 1$ vector signal $\boldsymbol{y}^l(k)$ at the symbol rate. We consider the overall channel delay spread between the jth user and all of the L antennas to of length m_jT. Let k_j be the chip-delay index for the jth user: $\boldsymbol{h}_j^l(k_jT)$ is the first non-zero $R \times 1$ chip-rate sample of $h_j^l(t)$. The parameter N_j is the duration of $g_j^l(t)$ in symbol periods. It is a function of m_j and k_j. We consider user 1 as the user of interest and assume that $k_1 = 0$ (synchronization to user 1). Let $N = \sum_{j=1}^{p} N_j$. The vectorized oversampled signals at all L sensors lead to a discrete-time $mLR \times 1$ vector signal at the symbol rate that can be expressed as

$$
\begin{aligned}
\boldsymbol{y}(k) &= \sum_{j=1}^{p}\sum_{i=0}^{N_j-1} \boldsymbol{g}_j(i)a_j(k-i)+\boldsymbol{v}(k)\\
&=\sum_{j=1}^{p}\boldsymbol{G}_{j,N_j}A_{j,N_j}(k)+\boldsymbol{v}(k)=\boldsymbol{G}_N\boldsymbol{A}_N(k)+\boldsymbol{v}(k),
\end{aligned} \quad (3)
$$

$$
\boldsymbol{y}(k)=\begin{bmatrix}\boldsymbol{y}_1(k)\\ \vdots \\ \boldsymbol{y}_m(k)\end{bmatrix}, \boldsymbol{y}_s(k)=\begin{bmatrix}\boldsymbol{y}_s^1(k)\\ \vdots \\ \boldsymbol{y}_s^L(k)\end{bmatrix}, \boldsymbol{y}_s^l(k)=\begin{bmatrix}y_{s,1}^l(k)\\ \vdots \\ y_{s,R}^l(k)\end{bmatrix}
$$

$$
\begin{aligned}
\boldsymbol{G}_{j,N_j} &= \left[\boldsymbol{g}_j(N_j-1)\ldots\boldsymbol{g}_j(0)\right] \ , \boldsymbol{G}_N = \left[\boldsymbol{G}_{1,N_1}\ldots\boldsymbol{G}_{p,N_p}\right]\\
A_{j,N_j}(k) &= \left[a_j^H(k-N_j+1)\ldots a_j^H(k)\right]^H\\
\boldsymbol{A}_N(k) &= \left[A_{1,N_1}^H(k)\ldots A_{p,N_p}^H(k)\right]^H,
\end{aligned} \quad (4)
$$

and the superscript ^{H} denotes Hermitian transpose. For the user of interest (user 1), $\boldsymbol{g}_1(i) = (\boldsymbol{C}_1(i)\otimes I_{LR})\,\boldsymbol{h}_1$, and the matrices $\boldsymbol{C}_1(i)$ are shown in figure 1, where the band consists of the spreading code $(c_0^H \cdots c_{m-1}^H)^H$ shifted successively to the right and down by one position. For interfering users, we

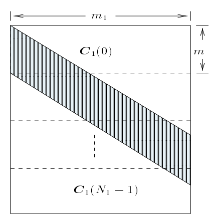

Figure 1: The Code Matrix $\boldsymbol{C}_1$

have a similar setup except that owing to asynchrony, the band in fig. 1 is shifted down k_j chip periods and is no longer conincident with the top left edge of the box. We denote by $\boldsymbol{C}_1$, the concatenation of the the code matrices given above for user $1: \boldsymbol{C}_1 = [\boldsymbol{C}_1^H(0)\cdots\boldsymbol{C}_1^H(N_1-1)]^H$.

3. CHANNEL IDENTIFICATION

3.1. MMSE-ZF Criterion

We stack M successive $\boldsymbol{x}(k)$ vectors to from the $mLRM \times 1$ supervector

$$
\boldsymbol{Y}_M(k)=\mathcal{T}_M(\boldsymbol{G}_N)\boldsymbol{A}_{N+p(M-1)}(k)+\boldsymbol{V}_M(k), \quad (5)
$$

where, $\mathcal{T}_M(\boldsymbol{G}_N) = \left[\mathcal{T}_{M,1}(\boldsymbol{G}_{1,N_1})\ \cdots\ \mathcal{T}_{M,p}(\boldsymbol{G}_{p,N_p})\right]$ and $\mathcal{T}_M(\boldsymbol{x})$ is a banded block Toeplitz matrix with M block rows and $\left[\boldsymbol{x}\quad \boldsymbol{0}_{n\times(M-1)}\right]$ as first block row (n is the number or rows in $\boldsymbol{x}$), and $\boldsymbol{A}_{N+p(M-1)}(k)$ is the concatenation of user data vectors ordered as $[A_{1,N_1+M-1}^H(k), A_{2,N_2+M-1}^H(k)\cdots A_{p,N_p+M-1}^H(k)]^H$. Let us introduce the following orthogonal transformation:

$$
\boldsymbol{T}_1=\left[\,\boldsymbol{0}\quad \boldsymbol{C}_1^H\quad \boldsymbol{0}\,\right]\otimes I_{LR},\ \boldsymbol{T}_2=\begin{bmatrix}\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{C}_1^{\perp} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}\end{bmatrix}\otimes I_{LR}, \quad (6)
$$

where, $\boldsymbol{C}_1^{\perp}$ spans the orthogonal complement of $\boldsymbol{C}_1$, the tall code matrix given in figure 1. Then, the middle (block) row of the block diagonal element of the matrix $\boldsymbol{T}_2$ acts as a blocking transformation for the signal of interest from all sensors. Note that $P_{T_1^H}+P_{T_2^H}=\boldsymbol{I}$, where, $P_X = \boldsymbol{X}(\boldsymbol{X}^H\boldsymbol{X})^{-1}\boldsymbol{X}^H$. This gives us a possiblity of estimating the $a_1(k-d)$ contribution in $\boldsymbol{T}_1\boldsymbol{Y}_M$ blindly. Consider the estimation error

$$
\widetilde{\boldsymbol{Y}}(k) = [\boldsymbol{T}_1 - \boldsymbol{Q}\boldsymbol{T}_2]\,\boldsymbol{Y}_M(k), \quad (7)
$$

and the (IS and MA) interference cancellation problem settles down to minimization of the trace of the matrix $\boldsymbol{R}_{\widetilde{Y}\widetilde{Y}}$ w.r.t. matrix $\boldsymbol{Q}$, which results in

$$
\boldsymbol{Q} = \left(\boldsymbol{T}_1\boldsymbol{R}^d\boldsymbol{T}_2^H\right)\left(\boldsymbol{T}_2\boldsymbol{R}^d\boldsymbol{T}_2^H\right)^{-1}, \quad (8)
$$

where $\boldsymbol{R}^d$ is the noiseless (denoised) data covariance matrix, $\boldsymbol{R}_{YY}$, with the subscript removed for convenience. The output $\widetilde{\boldsymbol{Y}}(k)$ can directly be processed by a multichannel matched filter to get the symbol estimate $\hat{a}_1(k-d)$, the data for user 1:

$$
\hat{a}_1(k-d) = \boldsymbol{F}^H\boldsymbol{Y}_M(k) = \boldsymbol{h}_1^H\left(\boldsymbol{T}_1 - \boldsymbol{Q}\boldsymbol{T}_2\right)\boldsymbol{Y}_M(k), \quad (9)
$$

where, $\boldsymbol{h}_1 = [\boldsymbol{h}_1^{1H}\cdots\boldsymbol{h}_1^{LH}]$. An estimate of the channels

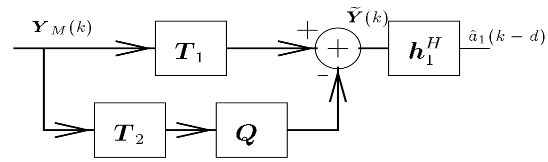

Figure 2: MMSE-ZF Receiver

$\mathbf{G}_1(z) = (\mathbf{C}_1(z)\otimes I_{LR})\,\mathbf{h}_1(z)$ can be obtained as a byproduct of the interference cancellation scheme. Notice that the above scheme is analogous to a MMSE-ZF equalizer for the single user case with $\boldsymbol{T}_1 = \mathrm{diag}\{[\boldsymbol{0}\quad \boldsymbol{I}\quad \boldsymbol{0}]\}$ and $\boldsymbol{T}_2$ block diagonal without the middle (block) row in (6), which when employed in a multiuser scenario is no longer capable of MAI suppression coming from the middle block of $\boldsymbol{Y}_M(k)$, unless a fair amount of data smoothing is introduced. This corresponds to the two-sided linear prediction approach of [7]. $\widetilde{\boldsymbol{Y}}(k)$ corresponds to the vector of *prediction* errors, and the covariance matrix of the prediction errors is given by

$$
\boldsymbol{R}_{\widetilde{Y}\widetilde{Y}}=\boldsymbol{T}_1\boldsymbol{R}^d\boldsymbol{T}_1^H-\boldsymbol{T}_1\boldsymbol{R}^d\boldsymbol{T}_2^H\left(\boldsymbol{T}_2\boldsymbol{R}^d\boldsymbol{T}_2^H\right)^{-1}\boldsymbol{T}_2\boldsymbol{R}^d\boldsymbol{T}_1^H. \quad (10)
$$

From the above structure of the two-sided (or rather *full-dimensional*) linear prediction problem, the key observation is that the matrix $\boldsymbol{R}_{\widetilde{Y}\widetilde{Y}}$ is rank-1 in the noiseless case, enabling identification of the composite channel as the maximum eigenvector of the matrix $\boldsymbol{R}_{\widetilde{Y}\widetilde{Y}}$, since $\widetilde{\boldsymbol{Y}}(k) = (\boldsymbol{C}_1^H\boldsymbol{C}_1\otimes I_{LR})\boldsymbol{h}_1\tilde{a}_1(k-d)$.

3.2. Relation with the Unbiased MOE Criterion

The unbiased MOE criterion proposed in [2] [1], which is a generalization of the instantaneous channel case of [1], is in principle a max/min problem solved in two steps with the minimization of the unbiased MOE,

$$
\min_{F:\,F^H\widetilde{g}_1=1}\boldsymbol{F}^H\boldsymbol{R}_{YY}\boldsymbol{F}\Rightarrow\boldsymbol{F}=\frac{1}{\widetilde{\boldsymbol{g}}_1^H\boldsymbol{R}_{YY}^{-1}\widetilde{\boldsymbol{g}}_1}\boldsymbol{R}_{YY}^{-1}\widetilde{\boldsymbol{g}}_1, \quad (11)
$$

where, $\mathsf{MOE}(\hat{\boldsymbol{h}}_1) = \left(\widetilde{\boldsymbol{g}}_1\boldsymbol{R}_{YY}^{-1}\widetilde{\boldsymbol{g}}_1\right)^{-1}$, and $\widetilde{\boldsymbol{g}}_1 = \boldsymbol{T}_1^H\widetilde{\boldsymbol{h}}_1$, followed by channel estimation by Capon's method,

$$
\max_{\hat{h}_1:\|\hat{h}_1\|=1}\mathsf{MOE}(\hat{\boldsymbol{h}}_1)\Rightarrow\min_{\hat{h}_1:\|\hat{h}_1\|=1}\hat{\boldsymbol{h}}_1^H\left(\boldsymbol{T}_1\boldsymbol{R}_{YY}^{-1}\boldsymbol{T}_1^H\right)\hat{\boldsymbol{h}}_1, \quad (12)
$$

[1] also known as the minimum variance distortionless response (MVDR) approach: a particular instance of the linearly constrained minimum-variance (LCMV) criterion.

from which, $\hat{\boldsymbol{h}}_1 = V_{\min}(\boldsymbol{T}_1\boldsymbol{R}_{YY}^{-1}\boldsymbol{T}_1^H)$. It can be shown that if $\boldsymbol{T}_2 = \boldsymbol{T}_1^{\perp}$, then

$$\boldsymbol{T}_1\boldsymbol{R}_{YY}^{-1}\boldsymbol{T}_1^H = \left(\boldsymbol{T}_1\boldsymbol{T}_1^H\right)\boldsymbol{R}_{\tilde{Y}\tilde{Y}}^{-1}\left(\boldsymbol{T}_1\boldsymbol{T}_1^H\right), \tag{13}$$

where, $\boldsymbol{R}_{\tilde{Y}\tilde{Y}}$ is given by (10), and $\boldsymbol{Q}$, given by (8), is optimized to minimize the estimation error variance. From this, we can obtain $\hat{\boldsymbol{h}}_1$ as $\hat{\boldsymbol{h}}_1 = V_{\max}\{\left(\boldsymbol{T}_1\boldsymbol{T}_1^H\right)^{-1}\boldsymbol{R}_{\tilde{Y}\tilde{Y}}\left(\boldsymbol{T}_1\boldsymbol{T}_1^H\right)^{-1}\}$. In order to evaluate the quality of the blind receiver obtained from the above criterion, we consider the noiseless received signal ($v(t) \equiv 0$). There are two cases of interest:

3.2.1. *Uncorrelated Symbols*

In the absence of noise, with *i.i.d.* symbols, the stochastic estimation of $\boldsymbol{T}_1\boldsymbol{Y}$ from $\boldsymbol{T}_2\boldsymbol{Y}$ is the stochastic estimation of $\boldsymbol{T}_1\mathcal{T}_M(\boldsymbol{G}_N)\boldsymbol{A}$ from $\boldsymbol{T}_2\mathcal{T}_M(\boldsymbol{G}_N)\boldsymbol{A}$ with $\boldsymbol{R}_{AA} = \sigma_a^2\boldsymbol{I}$. Hence, it is equivalent to the deterministic estimation of $\mathcal{T}_M^H(\boldsymbol{G}_N)\boldsymbol{T}_1^H$ from $\mathcal{T}_M^H(\boldsymbol{G}_N)\boldsymbol{T}_2^H$: $\|\mathcal{T}_M^H(\boldsymbol{G}_N)\boldsymbol{T}_1^H - \mathcal{T}_M^H(\boldsymbol{G}_N)\boldsymbol{T}_2^H\boldsymbol{Q}^H\|_2^2$. Then, given the condition

$$\begin{aligned}
&\mathsf{span}\{\boldsymbol{T}_1^H\} \cap \mathsf{span}\{\mathcal{T}_M(\boldsymbol{G}_N)\} = \mathsf{span}\{\mathcal{T}_M(\boldsymbol{G}_N)\boldsymbol{e}_d^{'}\}\\
&\Rightarrow \mathsf{span}\{\mathcal{T}_M(\boldsymbol{G}_N)\} \subset \mathsf{span}\{\boldsymbol{T}_2^H\} \oplus \mathsf{span}\{\widetilde{\boldsymbol{g}}_1\}\\
&\because \mathcal{T}_M(\boldsymbol{G}_N)\boldsymbol{e}_d^{'} = \mathcal{T}_M(\boldsymbol{G}_1)\boldsymbol{e}_d = \widetilde{\boldsymbol{g}}_1 = \boldsymbol{T}_1^H\boldsymbol{h}_1,
\end{aligned} \tag{14}$$

and where, $\boldsymbol{e}_d^{'}$ and $\boldsymbol{e}_d$ are vectors of appropriate dimensions with all zeros and one 1 selecting the desired column in $\mathcal{T}_M(\boldsymbol{G}_N)$ and $\mathcal{T}_M(\boldsymbol{G}_1)$ respectively, we can write the channel convolution matrix $\mathcal{T}_M(\boldsymbol{G}_N)$ as

$$\mathcal{T}_M(\boldsymbol{G}_N) = \widetilde{\boldsymbol{g}}_1\boldsymbol{e}_d^{'H} + \mathcal{T}_M(\boldsymbol{G}_N)P_{\boldsymbol{e}_d^{'\perp}} = [\widetilde{\boldsymbol{g}}_1 \quad \boldsymbol{T}_2^H]\boldsymbol{A}, \tag{15}$$

for some $\boldsymbol{A}$. Then we can write

$$\begin{aligned}
&\mathcal{T}_M^H(\boldsymbol{G}_N)\left(\boldsymbol{T}_1^H - \boldsymbol{T}_2^H\boldsymbol{Q}^H\right) =\\
&\boldsymbol{e}_d^{'}\boldsymbol{h}_1^H\boldsymbol{T}_1\boldsymbol{T}_1^H + \boldsymbol{A}^H\begin{bmatrix}\widetilde{\boldsymbol{g}}_1\boldsymbol{T}_1^H\\ \boldsymbol{0}\end{bmatrix} - \boldsymbol{A}^H\begin{bmatrix}\boldsymbol{0}\\ \boldsymbol{T}_2\boldsymbol{T}_2^H\end{bmatrix}\boldsymbol{Q}^H\\
&= \boldsymbol{e}_d^{'}\boldsymbol{h}_1^H\boldsymbol{T}_1\boldsymbol{T}_1^H + \boldsymbol{A}_1^H\widetilde{\boldsymbol{g}}_1^H\boldsymbol{T}_1^H - \boldsymbol{A}_2^H\left(\boldsymbol{T}_2\boldsymbol{T}_2^H\right)\boldsymbol{Q}^H.
\end{aligned} \tag{16}$$

Note that $\boldsymbol{e}_d^{'H}\boldsymbol{A}_i^H = 0, i \in \{1, 2\}$. This implies that the first term on the R.H.S. of (16) is not predictable from the third. Therefore, if the second term is perfectly predictable from the third, then the two terms cancel each other out and $\boldsymbol{R}_{\tilde{Y}\tilde{Y}}$ turns out to be rank-1, and $\hat{\boldsymbol{h}}_1 = \left(\boldsymbol{T}_1\boldsymbol{T}_1^H\right)^{-1}V_{\max}\left(\boldsymbol{R}_{\tilde{Y}\tilde{Y}}\right)$.

3.2.2. *Correlated Symbols*

Given the conditions in (14), it still holds that $\mathsf{span}\{\mathcal{T}_M^H(\boldsymbol{G}_N)\boldsymbol{T}_2^H\} = \mathsf{span}\{P_{\boldsymbol{e}_d^{'\perp}}\mathcal{T}_M(\boldsymbol{G}_N)\}$. We can write the received vector $\boldsymbol{Y}_M(k)$, in the noiseless case, as

$$\boldsymbol{Y}_M(k) = \mathcal{T}_M(\boldsymbol{G}_N)\boldsymbol{A} = \mathcal{T}_M(\boldsymbol{G}_N)\boldsymbol{e}_d^{'}a_1(k-d) + \overline{\mathcal{T}}_M\bar{\boldsymbol{A}}. \tag{17}$$

Now, the estimation of $\boldsymbol{T}_1\boldsymbol{Y}$ in terms of $\boldsymbol{T}_2\boldsymbol{Y} = \boldsymbol{T}_2\mathcal{T}_M(\boldsymbol{G}_N)\boldsymbol{A} = \boldsymbol{T}_2\overline{\mathcal{T}}_M\bar{\boldsymbol{A}}$ is equivalent to estimation in terms of $\bar{\boldsymbol{A}}$.

$$\begin{aligned}
\widetilde{\boldsymbol{T}_1\boldsymbol{Y}}|_{\boldsymbol{T}_2\boldsymbol{Y}} &= \boldsymbol{T}_1\boldsymbol{Y} - \widehat{\boldsymbol{T}_1\boldsymbol{Y}}\\
&= \boldsymbol{T}_1\boldsymbol{Y} - \left(\boldsymbol{T}_1\boldsymbol{R}_{YY}^d\boldsymbol{T}_2^H\right)\left(\boldsymbol{T}_2\boldsymbol{R}_{YY}^d\boldsymbol{T}_2^H\right)^{-1}\boldsymbol{T}_2\boldsymbol{Y}\\
\widetilde{\boldsymbol{T}_1\boldsymbol{Y}}|_{\bar{\boldsymbol{A}}} &= \boldsymbol{T}_1\mathcal{T}_M(\boldsymbol{G}_N)\boldsymbol{e}_d^{'}\tilde{a}_1(k-d)\\
&= \boldsymbol{T}_1\boldsymbol{T}_1^H\boldsymbol{h}_1\tilde{a}_1(k-d)|_{\bar{\boldsymbol{A}}}.
\end{aligned} \tag{18}$$

This results in,

$$\left(\boldsymbol{T}_1\boldsymbol{R}_{YY}^{-d}\boldsymbol{T}_1^H\right)^{-1} = \sigma^2_{\tilde{a}_1(k-d)|\bar{A}}\boldsymbol{h}_1\boldsymbol{h}_1^H, \tag{19}$$

The rank-1 results in a normalized estimate of the channel. It can however be noted that the estimation error variance of the desired symbol is now smaller ($\sigma^2_{\tilde{a}_1(k-d)} < \sigma_a^2$).

3.3. Channel Identification by Subspace Fitting

An extension of [5] to the longer (than a symbol period) delay spread case can be made to identify the channel from the following subspace fitting criterion:

$$\min_{\widetilde{g}_1}\|\mathcal{T}_M^H(\boldsymbol{G}_1)\mathcal{V}_N\|_F^2 = \min_{\widetilde{g}_1}\mathsf{tr}\left\{\mathcal{T}_M^H(\boldsymbol{G}_1)\mathcal{V}_N\mathcal{V}_N^H\mathcal{T}_M(\boldsymbol{G}_1)\right\}, \tag{20}$$

where, $\mathcal{V}_N$ denotes the noise subspace, or, by an SVD-free approach as

$$\min_{\widetilde{g}_1}\mathsf{tr}\Big\{\mathcal{T}_M^H(\boldsymbol{G}_1)\boldsymbol{R}^{-d}\mathcal{T}_M(\boldsymbol{G}_1)\Big\} = \min_{\|h_1\|=1}\sum_s\boldsymbol{h}_1^H\Big(\boldsymbol{T}_{1,s}\boldsymbol{R}^{-d}\boldsymbol{T}_{1,s}^H\Big)\boldsymbol{h}_1, \tag{21}$$

from where, $\hat{\boldsymbol{h}}_1 = \mathcal{V}_{\min}\left(\sum_s\boldsymbol{T}_{1,s}\boldsymbol{R}^{-d}\boldsymbol{T}_{1,s}^H\right)$, s is a delay and $\boldsymbol{R}^d$ is the denoised and appropriately regularized $\boldsymbol{R}_{YY}$. The above method always yields a unique channel estimate $\hat{\boldsymbol{h}}_1$ regardless of the $\{N_j\}$,$\forall j$, once the $\mathsf{span}\{\boldsymbol{T}_{1,s}\}$ does not intersect with all shifted versions of $\widetilde{\boldsymbol{g}}_j$,$\forall j \neq 1$.

4. IDENTIFIABILITY CONDITIONS

4.1. MMSE-ZF and Subspace Method

Continuing with the noiseless case, or with the denoised version of $\boldsymbol{R}_{YY}$, i.e., $\boldsymbol{R}^d = \sigma_a^2\mathcal{T}_M(\boldsymbol{G}_N)\mathcal{T}_M^H(\boldsymbol{G}_N)$,

$$\min_{F:F^H\widetilde{g}_1=1}\boldsymbol{F}^H\boldsymbol{R}_{YY}^d\boldsymbol{F} = \sigma_a^2, \quad \textit{iff} \quad \boldsymbol{F}^H\mathcal{T}_M(\boldsymbol{G}_N) = \boldsymbol{e}_d^{'H}, \tag{22}$$

i.e., the zero-forcing condition must be satisfied. Hence, the unbiased MOE criterion corresponds to ZF in the noiseless case. This implies that $\mathsf{MOE}(\hat{\widetilde{\boldsymbol{g}}}_1) < \sigma_a^2$ *if* $\hat{\widetilde{\boldsymbol{g}}}_1 \not\sim \widetilde{\boldsymbol{g}}_1$. We consider that:

(i). *FIR zero-forcing conditions are satisfied*, and
(ii). $\mathsf{span}\{\mathcal{T}_M(\boldsymbol{G}_N)\} \cap \mathsf{span}\{\boldsymbol{T}_1^H\} = \mathsf{span}\{\boldsymbol{T}_1^H\boldsymbol{h}_1\}$.

The two step max/min problem boils down to

$$\max_{\hat{h}_1:\|\hat{h}_1\|=1}\hat{\boldsymbol{h}}_1^H\left(\boldsymbol{T}_1\boldsymbol{T}_1^H\right)^{-1}\boldsymbol{T}_1\mathcal{T}_M P^{\perp}_{\mathcal{T}_M^H\boldsymbol{T}_2^H}\mathcal{T}_M^H\boldsymbol{T}_1^H\left(\boldsymbol{T}_1\boldsymbol{T}_1^H\right)^{-1}\hat{\boldsymbol{h}}_1, \tag{23}$$

where, $P_X^{\perp} = \boldsymbol{I} - \boldsymbol{X}(\boldsymbol{X}^H\boldsymbol{X})^{-1}\boldsymbol{X}^H$. Then identifiability implies that $\mathcal{T}_M P^{\perp}_{\mathcal{T}_M^H\boldsymbol{T}_2^H}\mathcal{T}_M^H = \boldsymbol{T}_1^H\boldsymbol{h}_1\boldsymbol{h}_1^H\boldsymbol{T}_1 = \widetilde{\boldsymbol{g}}_1\widetilde{\boldsymbol{g}}_1^H$, or

$$P^{\perp}_{\mathcal{T}_M^H\boldsymbol{T}_2^H}\mathcal{T}_M^H(\boldsymbol{G}_N) = P_{\boldsymbol{e}_d^{'}}\mathcal{T}_M^H(\boldsymbol{G}_N), \tag{24}$$

Condition (i) above implies that $\boldsymbol{e}_d^{'} \in \mathsf{span}\{\mathcal{T}_M^H(\boldsymbol{G}_N)\}$. From condition (ii), since $\boldsymbol{T}_1^H\boldsymbol{h}_1 = \mathcal{T}_M(\boldsymbol{G}_N)\boldsymbol{e}_d^{'}$, we have

$$\begin{aligned}
&\mathsf{span}\{\mathcal{T}_M(\boldsymbol{G}_N)\boldsymbol{T}_2^H\} = \mathsf{span}\{P^{\perp}_{\boldsymbol{e}_d^{'}}\mathcal{T}_M^H(\boldsymbol{G}_N)\}\\
&\mathsf{span}\{\mathcal{T}_M^H(\boldsymbol{G}_N)\} = \mathsf{span}\{\mathcal{T}_M^H(\boldsymbol{G}_N)\boldsymbol{T}_2^H\} \oplus \mathsf{span}\{\boldsymbol{e}_d^{'}\}
\end{aligned} \tag{25}$$

from which, $\mathcal{T}_M^H(\boldsymbol{G}_N) = P_{\mathcal{T}_M^H T_2^H}\mathcal{T}_M^H(\boldsymbol{G}_N) + P_{e_d'}\mathcal{T}_M^H(\boldsymbol{G}_N)$, which is the same as (24). The smoothing factor M required to satisfy condition (i) is given by

$$M \geq M_{\mathrm{ZF}} = \left\lceil \frac{N-p}{(mLR)_{\mathrm{eff}} - p} \right\rceil, \tag{26}$$

while the condition (ii) can be restated as the following dimensional requirement:

$$\mathsf{rank}\{\mathcal{T}_M(\boldsymbol{G}_{1:p})\} + \mathsf{rank}\{\boldsymbol{T}_1^H\} \leqslant \mathsf{row}\{\mathcal{T}_M(\boldsymbol{G}_{1:p})\} + 1, \tag{27}$$

from where, under the irreducible channel and column reduced conditions [6], it can be shown that the receiver length M is related to other parameters as

$$M \geq \underline{M} = \left\lceil \frac{N-p+m_1LR-1}{(mLR)_{\mathrm{eff}} - p} \right\rceil, \tag{28}$$

where, $(mLR)_{\mathrm{eff}} = \mathsf{rank}\{\boldsymbol{G}_N\}$ is the effective number of channels.

4.1.1. *Relation with Twosided Linear Prediction*

In the case where the middle block row of $\boldsymbol{T}_2$ is removed (6), the problem reduces to twosided linear prediction [7], and the matrix $\boldsymbol{R}_{\tilde{Y}\tilde{Y}}$ is still rank-1 for $M \geq 2\underline{M} + N_1$, where $\underline{M}$ is given by (26). This holds if $N_1 < N_j, \forall j \neq 1$. However, a mixture of channels of different users' channels is obtained if N_1 is not the smallest N_j.

4.2. Direct Estimation of MMSE Receiver

It can be noted that the MMSE and MOE approaches are equivalent under the unbiased constraint. And the unbiased MMSE receiver is the best among linear receivers in that it results in the minimum error probability. Following the observation that the MMSE receiver lies in the signal subspace [3], the MMSE receiver, $\boldsymbol{F}$, can be estimated from

$$\min_{F} \boldsymbol{F}^H \left(\alpha \mathcal{V}_N \mathcal{V}_N^H + \boldsymbol{R}_{YY}\boldsymbol{T}_2\boldsymbol{T}_2^H\boldsymbol{R}_{YY} \right) \boldsymbol{F}, \tag{29}$$

where, $\mathcal{V}_N$ is the noise subspace and α is an arbitrary scalar. The first term in parentheses can also be replaced by $\boldsymbol{R}_{YY}^{-d}$ resulting in an SVD-free approach. The above relation can be interpreted as the MMSE receiver constrained to the intersection of the signal subspace with $\boldsymbol{T}_1^H$, which being of dimension 1, gives the MMSE receiver uniquely, once the identifiability conditions of section 4.1 are satisfied.

Consider the noiseless case ($v(t) \equiv 0$). The second term in (29) is nulled out if the zero-forcing constraint $F^H\mathcal{T}(G_N) = e_d'^H$ is satisfied, while the first translates into the condition (ii) of section 4.1. Hence, the same set of identifiability conditions hold for the MMSE receiver obtained by this method.

5. SIMULATIONS AND CONCLUDING REMARKS

We presented relationships between different asymptotically equivalent receiver algorithms related to the unbiased MMSE criterion and derived identifiability conditions for these schemes. Although these receivers are obtained from different optimization criteria, they can all be shown to be equivalent to the unbiased MOE approach. Furthermore, they share the same identifiability conditions. Once zero forcing conditions can be satisfied in the noiseless (denoised) case, the receivers can be identified with probability 1. Channel identification by subspace fitting requires the same smoothing factors as the MMSE-ZF scheme. Output SINR per-

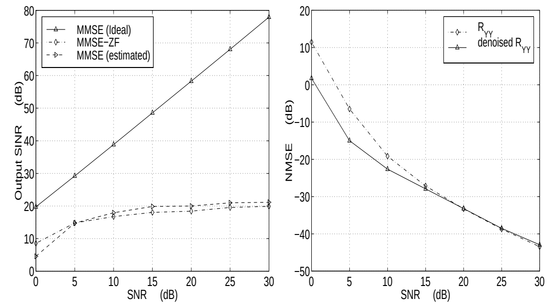

Figure 3: Output SINR and Channel Estimation Performance

formance of different receiver algorithms in a near far scenario is compared in fig. 3(a) for 5 users with a processing gain of 16, where it can be seen that the results are comparable. A single antenna ($L = 1$) and no oversampling ($R = 1$) are employed in these simulations with 200 data samples. The directly estimated MMSE receiver performs slightly better than the MMSE-ZF receiver. A flooring effect exists for both due to the fact that the length of the data record is not enough to estimate $\widehat{\boldsymbol{R}}_{YY}$ accurately. Semi-blind techniques along with exploitation of the finite symbol alphabet can be used to alleviate this effect [6]. In fig. 3(b) we show the quality of channel estimate obtained from the MMSE-ZF algorithm from the denoised and the non-denoised data covariance matrices. It can be seen that denoising improves significantly, the quality of the channel estimate.

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