Capacity of Linear Decorrelating Detector for QS-CDMA *

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Abstract

We study the performance and capacity of a *quasi-synchronous* code division multiple access (QS-CDMA) cellular system employing the decorrelating detector. It is assumed that each of N users has a GPS generated local clock, and attempts to transmit in synchrony with the other users in its cell. The detector makes use of the small timing uncertainties to build the interference signal space and perfectly cancel the interference. Results are compared with those of an adaptive MSE receiver. It is shown that when the LMS adaptive algorithm is employed the mean-squared-error misadjustment significantly affects the receiver performance. Finally, system capacities in terms of users/cell are evaluated.

1 Introduction

The optimum multiuser detector proposed in [1] has exponential complexity in the number of users. Several sub-optimal receivers were subsequently proposed e.g., [2] whose complexity is only linear in the number of users. The cyclostationary behavior of multiple access interference (MAI) has been exploited in adaptive interference cancellation schemes based on the minimum mean-squared error (MMSE) criterion[3][4]. For such systems to converge to steady state, a training sequence is required. A new version of the adaptive detector was proposed in [5] which converged for any initialization to the MMSE detector. However, a constraint on the MMSE adaptive receivers is that the spreading pseudo-noise (PN) sequence has to stay the same from symbol to symbol making this kind of receiver unsuitable for *IS-95* applications. A new form of non-adaptive decorrelating detector was presented [6] which is specifically suited to a quasi-synchronous CDMA environment.

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In this scheme, all users attempt to transmit in synchrony using local GPS derived clocks. The delay uncertainty can, therefore, be kept to ± 1.3 chips [7] in a 1.2 MHz system (*IS-95* standard).

In this work, performance of the decorrelating detector for QS-CDMA receiver is compared with that of an adaptive receiver based on the MMSE criterion. In section 2 we review the system models for the two types of receivers and state the error-probability equations that will be used in section 3 for determining system capacities. Section 4 presents simulation results and gives numerical values for system capacities.

2 Signal Models and Receiver Structures

For the direct sequence CDMA system, assuming N users sharing the channel, the received low-pass signal is of the form

$$r(t) = \sum_{n=1}^{N} \sum_{m=-\infty}^{+\infty} d_n(m) a_n s_n(t - mT_b - T_n) + n(t), \tag{1}$$

where $s_n(t)$ is the signature waveform of the n-th user, and T_n is the corresponding signal arrival delay. If T_c and T_b are the chip and bit durations respectively, then $L = T_b/T_c$ gives the processing gain. Upon reception, the signal is passed through a chip duration integrator. The i-th duration received and sampled signal in vector form is therefore given as

$$\mathbf{r}_i(\mathbf{T}) = a_1 d_1 \mathbf{s}_1(i) + \sum_{n=2}^{N} \{a_n d_n(i) \mathbf{s}_n(T_n) + a_n d_n(i + sgn(T_n)) \mathbf{s}_n(T_n - sgn(T_n)T_b)\} + \mathbf{n}_i, \tag{2}$$

where $T_n = (p_n + \delta_n)T_c$ is the n-th user delay with p_n the integer part and δ_n the fractional part of the delay. $\mathbf{T} = [T_2, T_3, \ldots, T_N]^T$ is the vector of undesired user delays, $d_n(l) \in \{-1, +1\}$ is the l-th information symbol for the n-th user, a_n are the user amplitudes which are independent circular Gaussian random variables with zero mean and variance $P_n = E\{\mid a_n \mid^2\}$. Without loss of generality, user 1 is taken as the desired user. The first term in (2) is the desired user's signal considered to have undergone zero delay. We further assume that $a_1 = 1$. The summation term takes into account the interference both due to the delayed undesired signals and the next (or previous) bit of undesired signals of all users, i.e., the intersymbol interference (ISI).

The delays T_n are independent and uniformly distributed over the interval $[-\alpha T_b, +\alpha T_b]$ where $\alpha \ll 1/2$. The last term, $\mathbf{n}_i$, is the sampled noise vector whose elements are i.i.d. complex gaussian zero mean samples having variance $2N_oT_c$. The vectors $\mathbf{s_n}(T_n) \in \mathcal{R}^L$ and are given by [6]

$$\mathbf{s_n}(T_n) = [s_{n,1}(T_n), s_{n,2}(T_n), \ldots, s_{n,L}(T_n)]^T. \tag{3}$$

Assuming rectangular pulses $P_{T_c}(t)$, the samples $s_{n,k}(T_n)$ are given by

$$s_{n,k}(T_n) = \sqrt{\frac{2E_b}{T_b}}\,(c_{n,k-p_n}(1-\delta_n)T_c + c_{n,k-p_n-1}\delta_n T_c)\,, \tag{4}$$

where $c_{n,k} \in \{\pm 1\}$ is the pseudo-noise sequence for user n, and E_b is the energy per bit. Taking $\alpha T_b = MT_c$, where $M \ll L$ is an integer signifying the number of chips to which undesired users' signature waveforms are misaligned with respect to the desired user, the interference matrix $\mathbf{S_1'} \in \mathcal{C}^{L\times 2(N-1)(2M+1)}$ for the detector is given by [6]

$$\begin{aligned}\mathbf{S_1'} &= [\mathbf{s_2}(-MT_c), \mathbf{s_2}(-MT_c+T_b), \mathbf{s_2}(-(M-1)T_c), \mathbf{s_2}(-(M-1)T_c+T_b), \ldots, \\ &\quad \mathbf{s_2}(MT_c), \mathbf{s_2}(MT_c-T_b), \ldots, \mathbf{s_N}(-MT_c), \mathbf{s_N}(-MT_c+T_b), \mathbf{s_N}(-(M-1)T_c), \\ &\quad \mathbf{s_N}(-(M-1)T_c+T_b), \ldots, \mathbf{s_N}(MT_c), \mathbf{s_N}(MT_c-T_b)].\end{aligned} \tag{5}$$

The columns of $\mathbf{S_1'}$ make up the interference subspace. If $\mathbf{P}_{\mathbf{S}_1'}$ is the projection matrix [8] associated with $\mathbf{S_1'}$, then the bit-error rate for a BPSK modulated signal has been shown in [6] to be,

$$P_e = \frac{1}{2}\,\mathrm{erfc}\,\left(\sqrt{\frac{\mathbf{s}_1{}^T[\mathbf{I}-\mathbf{P}_{S_1}{}']\mathbf{s}_1}{2N_oT_c}}\right). \tag{6}$$

The adaptive receiver is implemented as a transversal delay line filter with the number of taps equal to the processing gain [9]. Since the system is asynchronous, the user delays, T_n, in (1) are independent random variables uniformly distributed over the whole symbol interval $[0, T_b)$. Signature waveforms are assumed fixed for successive symbol emissions from the transmitter. The contents of the transversal filter at instant i are given by (2). The delay and phase offset of the desired user are assumed to be zero. The signal covariance matrix $\mathbf{R}$ is given by

$$\mathbf{R} = E\{\mathbf{r}_i\mathbf{r}_i^H\} = \mathbf{s}_1\mathbf{s}_1^T + \sum_{n=2}^{N} P_n\mathbf{s}_n(T_n)\mathbf{s}_n(T_n)^T + \mathbf{s}_1\mathbf{s}_1^T + \sum_{n=2}^{N} P_n\mathbf{s}_n(T_n-T_b)\mathbf{s}_n(T_n-T_b)^T + \sigma^2\mathbf{I}_L, \tag{7}$$

where $\mathbf{I}_L$ is the $L\times L$ identity matrix and $\sigma^2 = 2N_oT_c$. Note that P_n is now the undesired-to-desired signal power ratio, with $P_1 = 1$.

We need to detect the symbol $d_1(i)$ which is the desired response for this filter. The filter design criterion is minimization of the mean-squared error J_{min} defined in [8] to be $J_{min} = \min_{\hat{d}_1(i)} E\{\mid \hat{d}_1(i) - d_1(i)\mid^2\}$, where $\hat{d}_1(i) = sgn(\mathbf{w}^T\mathbf{r})$ is the estimate of the i-th symbol of the desired user, and $\mathbf{w}$ is the tap-weight vector from the Wiener-Hopf equations [8], $\mathbf{w} = \mathbf{R}^{-1}\mathbf{s}_1$. Then, J_{min} is written in terms of the signal and tap-weight vectors as $J_{min} = 1 - \mathbf{s}_1^T\mathbf{w}$.

In [9], perfect knowledge of $\mathbf{R}$ is assumed. As $\mathbf{R}$ in practical cases is not known, we base our analysis on the LMS algorithm [8]. Since the LMS algorithm relies on noisy estimates of the gradient, the mean-squared error (MSE) does not quite converge to J_{min}, i.e., there is a certain *misadjustment* term, $\mathcal{M}$ even when the LMS algorithm attains steady-state [8]. The mean-squared error of the LMS algorithm at steady-state is given by [8]

$$J(\infty) = \frac{J_{min}}{1 - \sum_{i=1}^{L} \left(\frac{\mu\lambda_i}{2-\mu\lambda_i} \right)}, \tag{8}$$

where μ is the step-size parameter of the LMS algorithm, and its value is constrained by two conditions which are (i) $0 < \mu < \frac{2}{\lambda_{max}}$, and (ii) $\sum_{i=1}^{L} \frac{\mu\lambda_i}{2-\mu\lambda_i} < 1$, in order to allow for convergence to the optimum tap-weight vector $\mathbf{w}_o$, obtained from the Weiner-Hopf equations. In the above, λ_i and λ_{max} are the i-th and maximum eigenvalues of $\mathbf{R}$ respectively, and $L = T_b/T_c$ is the number of filter taps.

The bit error-rate of an adaptive receiver is difficult to calculate [3] since there is no closed form solution. However, one method [9] is to approximate the resultant interference and noise at the filter output as Gaussian. The BER is then given by

$$P_e = \frac{1}{2} \operatorname{erfc} \left(\sqrt{\frac{1}{2J(\infty)}} \right). \tag{9}$$

3 System Capacities

It is assumed [9] that the number of interferers m, present in a cell is Poisson distributed with ρ users/cell as the mean of the distribution. A measure of capacity defined in [9] is the blocking probability, P_B defined as the probability of the event that the actual BER exceeds a preset value P_{max}. Using this notion, we compute the capacities as presented in the sequel.

We define N_{max} as the maximum number of users for which the bit-error probability, $P_e < P_{max}$. It is seen from (6) that $\{\operatorname{erfc}^{-1}(2P_{max})\}^2 \leq \frac{\mathbf{s}_1{}^T[\mathbf{I}-\mathbf{P}_{\mathbf{s}_1'}(N_{max})]\mathbf{s}_1}{2N_oT_c}$, where the projection matrix, $\mathbf{P}_{\mathbf{S}_1'}(N)$, is a deterministic function of the number of users, N, when the codes are assigned in a fixed order. As soon as N increases beyond N_{max}, blocking occurs. Then, for a Poisson distribution of interferers in the cell, the blocking probability is approximately given by

$$P_B = Pr(P_e \geq P_{max}) \approx \sum_{m=N_{max}+1}^{\infty} \frac{\rho^m}{m!} e^{-\rho} = 1 - \sum_{m=0}^{N_{max}} \frac{\rho^m}{m!} e^{-\rho}. \tag{10}$$

The capacity of the quasi-synchronous system is the maximum value of ρ for which the right hand side of (10) stays below the chosen value of blocking probability, P_B.

A similar approach is adopted for the adaptive receiver. The same set of Gold codes is taken as the signal spreading sequences. Following (9), the blocking probability with N_s users in the cell and conditioned on the set of delays $\mathbf{T} = [T_2, T_3, \ldots, T_{N_s}]$ is given by

$$Pr(Block \mid N_s \; users, \; \mathbf{T}) = Pr\left(J(\infty, \; N_s, \; \mathbf{T}) \geq \frac{1}{2erfc^{-1}[(2P_{max})]^2}\right). \tag{11}$$

The matrix $\mathbf{R}$ is constructed according to (7) for a particular set of delays and J_{min} is determined. A value of μ is carefully chosen as 1/20 of the upper limit. For a particular realization of $\mathbf{R}$, the steady state error is calculated according to (8). A number of simulations are then carried out [9] and the mean and variance of $J(\infty)$ are determined. Finally, $J(\infty, N_s, \mathbf{T})$ is approximated as a Gaussian random variable [9] to give

$$P_B \approx \sum_{m=1}^{\infty} \left(\frac{\rho^m}{m!}e^{-\rho}\right) \frac{1}{2} erfc\left(\sqrt{\frac{[(erfc^{-1}(2P_{max}))^{-2}/2 - E(J(\infty, \; m, \; \mathbf{T}))]^2}{2 \cdot Var[J(\infty, \; m, \; \mathbf{T})]}}\right). \tag{12}$$

Once again, the maximum value of ρ is the user/cell capacity of the system.

4 Numerical Results

A few numerical examples are presented to illustate the comparative performances of these receivers. Note that the non-adaptive MMSE receiver is not practical since it requires estimation of undesired user delays and amplitudes. On the other hand it is fair to compare the LMS based adaptive receiver and the QS decorrelator since both do not require knowledge of these parameters.

Fig. 1 shows the error-probability curves of the two receivers. Note that as the size of the signal space grows, the QS receiver BER improves for a fixed number of users since there are more degrees of freedom and the subspace orthogonal to the interferers has greater correlation with the desired user.

Fig. 2 illustrates the performance degradation of the adaptive receiver due to the misadjustment $\mathcal{M}$ of the LMS algorithm. As expected, the steady-state value of the mean-squared error, $J(\infty)$ increases with an increase in the number of taps of the adaptive filter. This effect is visible in Fig. 2 where for a processing gain of L=255, the underlying performance loss is 1.7 dB as opposed to 1 dB at L=31.

Table 1 shows the the user/cell capacity. Note that for the adaptive receiver, the capacity-gap between the with and without misadjustment cases increases drastically as we increase the number of taps of the FIR filter, despite the fact that the adaptation step μ is very small (of the order of

10^{-5}). The QS decorrelator, on the other hand, promises a much better capacity as long as the delay uncertainty, M is kept to a single chip. As soon as M increases to two chips and beyond, the interference sub-space becomes too large for the desired signal to be projected on its orthogonal space and more and more of the desired signal cancels out along with the interference, thus affecting performance.

5 Conclusions

The adaptive receivers of [4] and [9] suggest a promising solution to the near-far problem. However, as shown, the excess MSE causes a severe reduction in capacity for large processing gains. We showed that if the timing offset for the QS-CDMA system is made small enough, a larger capacity than for the adaptive receiver can be achieved. Hence, for applications where quasi-synchronous operation is feasible, the fixed QS decorrelator receiver may offer greater capacity than the adaptive MMSE detectors.

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Table 1: System capacities in terms of users/cell, $P_{max} = 10^{-3}$, $E_b/N_o = 10$ dB, $P_B = 10^{-2}$

	Adaptive Receiver		QS Receiver		
L	No misadjustment	with misadjustment	M=1	M=2	M=3
31	4.50	4.15	3.50	1.27	1.27
63	11.40	5.86	7.47	3.55	2.35
127	33.80	9.70	14.08	7.47	5.42
255	70.90	12.10	38.38	15.62	11.08

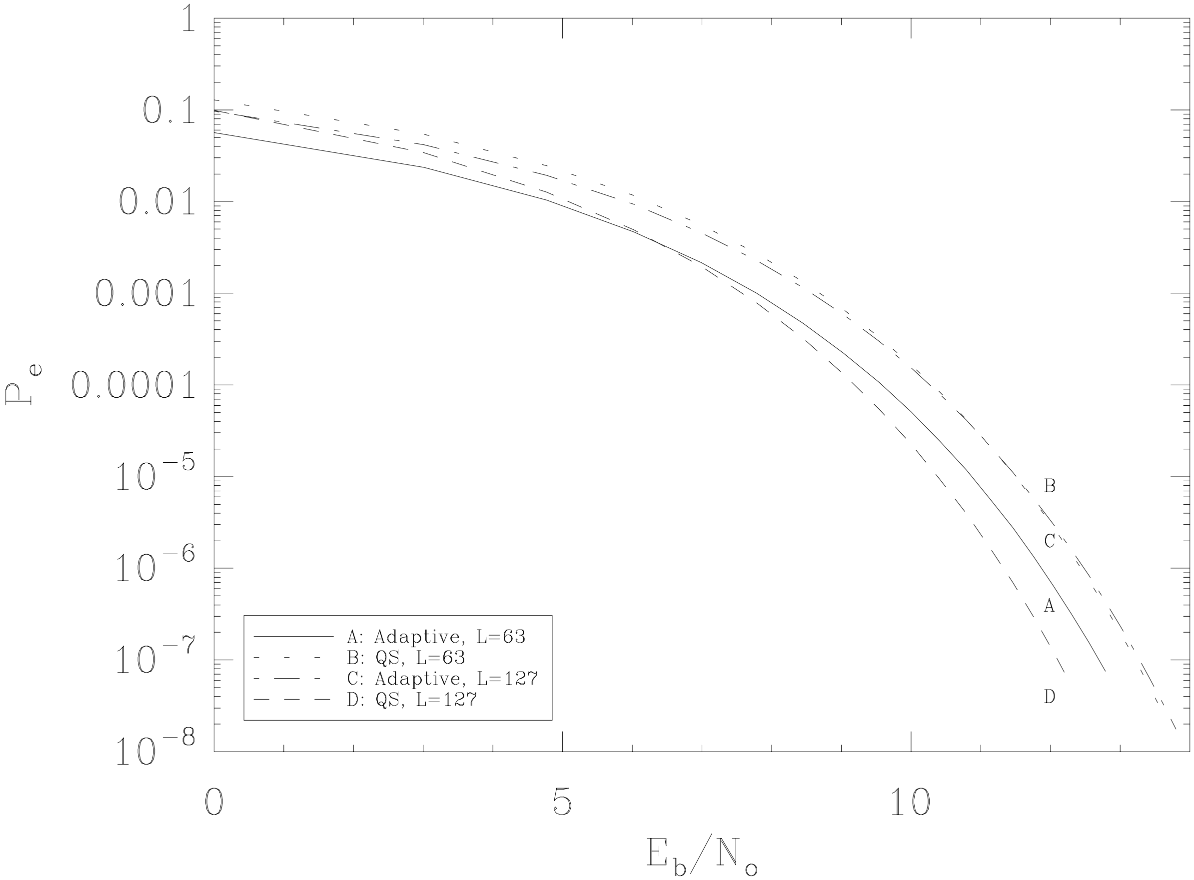

Figure 1: Comparison of Adaptive and Decorrelator detectors, M=1, J/S=20 dB, N=10

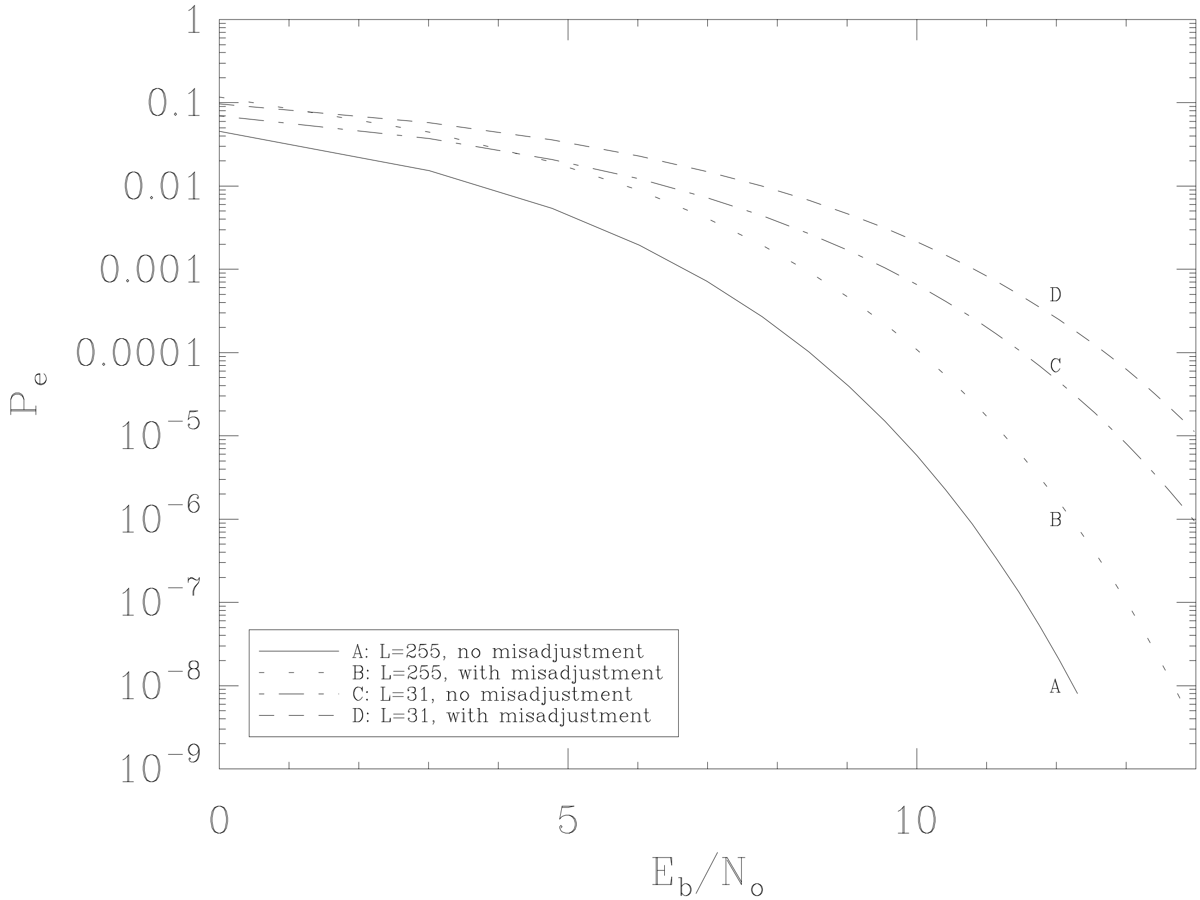

Figure 2: Performance degradation of the adaptive receiver due to LMS misadjustment M=1, J/S=20 dB, N=10