

# Power Allocation and Feedback Reduction for MIMO-OFDMA Opportunistic Beamforming

Issam Toufik  
Eurécom Institute  
Mobile Communications Department  
BP 193, 06904 Sophia-Antipolis, France  
Email: issam.toufik@eurecom.fr

Marios Kountouris  
France Telecom R&D  
38 - 40 rue du Général Leclerc  
92794 Issy-Moulineaux cedex 9, France  
Email: marios.kountouris@francetelecom.com

**Abstract**—MIMO-OFDMA systems using opportunistic beamforming are a promising solution to satisfy the increasing demand in terms of data rate and Quality-of-Service (QoS). An important practical issue in MIMO-OFDMA systems is the feedback load. As a large number of carriers (e.g. 2048 for WiMax) is usually used in such systems, feeding back full Channel State Information at the transmitter (CSIT) for each carrier is prohibitive. In this paper, the problem of feedback reduction in MIMO-OFDMA opportunistic beamforming is addressed. We present different partial CSIT schemes that reduce significantly the feedback overload at little expense of system throughput. We additionally investigate different power control strategies that show significant capacity gain for low to moderate number of users over standard opportunistic beamforming approaches.

**Index Terms**—MIMO-OFDMA, Opportunistic Beamforming, Scheduling, Partial CSIT, Feedback, Power Control.

## I. INTRODUCTION

Current and future wireless communication systems are expected to provide a broad range of multimedia services with different delay and Quality-of-Service requirements. The use of multiple antennas at both transmitter and receiver provides enhanced performance in terms of diversity and data rate without increasing the transmit power or bandwidth. A great deal of research work has been devoted to the area of combining this spatial scheme with Orthogonal Frequency Division Multiplexing (OFDM) systems. Such systems combine the advantages of both techniques, providing simultaneously increased data rate and robustness against channel delay spread.

Orthogonal Frequency Division Multiple Access (OFDMA) is an emerging multiple access technology that converts a frequency-selective fading channel into several flat-fading subchannels, exploiting the fact that different users experience different amount of fading at a particular instant of time and scheduling efficiently the data tones to the users. A very important feature of OFDMA is its capability of exploiting the Multiuser Diversity [1], which, combined with dynamical resource allocation, can increase significantly the system throughput, even in the case where hard fairness between active users is required [2], [3].

Multiple-Input Multiple-Output (MIMO) systems have emerged as one of the most promising technologies in modern wireless communications, motivated by the pioneering work of

Foschini [4] and Telatar [5]. The interested reader is referred to [6]. In multiuser MIMO systems, one way to exploit the multiuser diversity gain is through opportunistic scheduling [7]. In [8], the authors propose a partial feedback scheme exploiting opportunistic multiuser beamforming as a multiuser extension of the opportunistic beamforming initially introduced in [7]. Previous work on opportunistic scheduling has been mainly focused on frequency-flat fading channels. However, in an OFDMA network, only few works have utilized opportunistic schemes to enhance the system throughput. One of the major problems in employing an opportunistic scheme in MIMO-OFDMA systems is the large amount of feedback required to feedback to the transmitter. In [9], the authors proposed an opportunistic scheme in which adjacent subchannels are clustered into groups and then information on the best clusters is fed back to the base station.

The objective of our work is to propose practical feedback reduction schemes that are more efficient than the obvious extension of the narrowband strategies. In essence, our goal is to reduce the feedback rate without significantly compromising the sum rate performance. In this paper we propose different partial channel state information (CSI) schemes for MIMO-OFDMA combined with opportunistic beamforming. Our method is distinct from that of [9] as we place ourselves in an SDMA context and the best carriers within a cluster are fed back. In [8], it is shown that random beamforming followed by intelligent scheduling is asymptotically optimal for large number of users. However, for sparse networks (i.e. low to moderate number of users) random beamforming yields severely degraded performance. Different power allocation strategies that shows substantial gain over standard opportunistic beamforming are presented in order to compensate this performance degradation.

The organization of this paper is as follows: Section II presents the underlying system model and formulates the problem. In Section III, we present three partial feedback schemes for the MIMO-OFDMA system, which are combined with two power techniques presented in Section IV. Simulation results are provided in Section V, and Section VI concludes the paper.

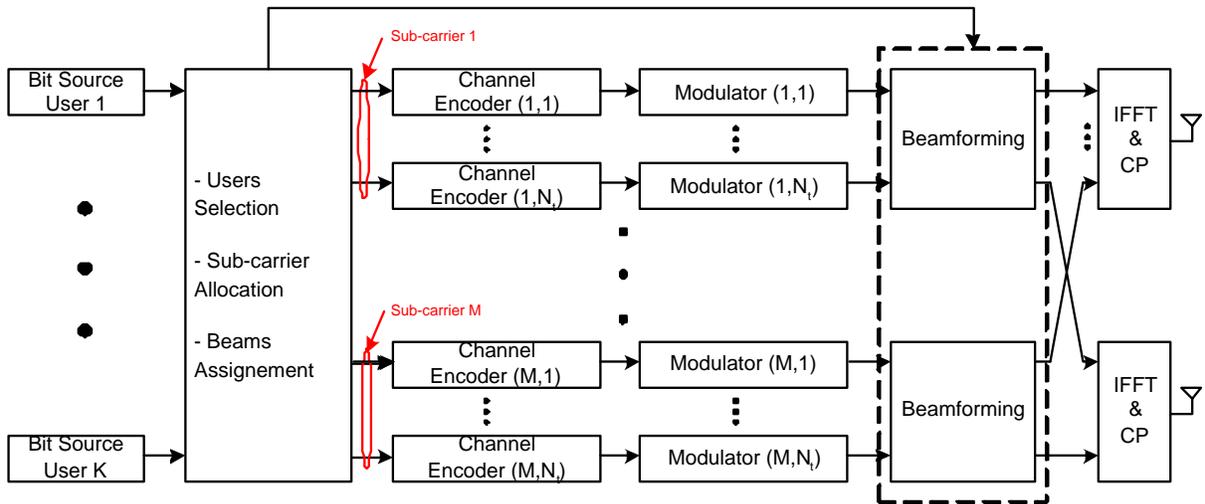


Fig. 1. MIMO-OFDMA Transmitter model

## II. SYSTEM MODEL

We consider the downlink of a multiuser MIMO-OFDMA system as shown in Fig. 1. The base station (BS) is equipped with  $N_t$  transmit antennas and each receiver has  $N_r$  receive antennas. Let  $K$  denote the number of users and  $M$  the number of subcarriers. A frequency-selective channel is characterized by  $L$  significant delayed paths. Let  $\mathbf{x}[t]$  be the  $N_t \times 1$  complex transmitted signal vector and  $\mathbf{y}[t]$  the  $N_r \times 1$  received signal in the baseband during the  $t$ -th signaling interval. Then a discrete-time baseband model can be mathematically described as

$$\mathbf{y}[t] = \sum_{l=0}^{\mathcal{L}-1} \mathcal{H}_l \mathbf{x}[t-l] + \mathbf{n}[t] \quad (1)$$

where  $\mathcal{H}_l$  is an  $N_r \times N_t$  matrix representing the  $l$ -th tap of the discrete-time MIMO channel response, and  $\mathbf{n}[t]$  is an additive Gaussian noise with zero mean and unit variance. Without loss of generality, we assume that  $N_r = 1$  for the remainder of the paper.

Let  $\mathbf{H}_{k,m} = [H_{k,m}[1], \dots, H_{k,m}[N_t]]^T$  be the  $N_t \times 1$  vector of channel gains between each transmit antenna and the receive antenna of user  $k$  on subcarrier  $m$ . The  $H_{k,m}[i]$  denotes the channel gain from transmit antenna  $i$  to receiver  $k$  and corresponds to the frequency sample, at the frequency corresponding to subcarrier  $m$ , of the multipath time domain channel impulse response given by

$$h_k[i](t) = \sum_{\gamma=0}^{\mathcal{L}-1} \alpha_\gamma \delta(t - \tau_\gamma) \quad (2)$$

where  $\alpha_\gamma$  is the path gain following zero-mean Gaussian distribution with variance  $\sigma_\gamma^2$ ,  $\tau_\gamma$  is the delay corresponding to path  $\gamma$ , and  $\mathcal{L}$  is the maximum channel order.

We assume that the channel is invariant during each coded block, but is allowed to vary independently from block to

block. The samples of the frequency response are given by

$$H_{k,m}[i] = \sum_{\gamma=0}^{\mathcal{L}-1} \alpha_\gamma e^{-j \frac{2\pi \tau_\gamma f_m}{M}} \quad (3)$$

where  $f_m$  is the frequency corresponding to subcarrier  $m$ .

As in [8], random beamforming is used for transmission, i.e.,  $N_t$  users can be simultaneously scheduled in each subcarrier. The BS constructs  $N_t$  random orthonormal beams  $\mathbf{q}_i \in \mathbb{C}^{N_t \times 1}$  for  $i = 1, \dots, N_t$ , and the user selection and beam allocation on each carrier can be made jointly depending on the users' feedback. After that, each user's data is mapped to its allocated subcarriers and bits are coded and modulated. Let  $\mathcal{K}_m = \{k_1^m, k_2^m, \dots, k_{N_t}^m\}$  be a set of  $N_t$  scheduled users on subcarrier  $m$ , such that user  $k_i^m$  is assigned the beamforming vector  $\mathbf{q}_i$ . The transmitted signal on subcarrier  $m$  is then given by

$$\mathbf{x}_m = \sum_{i=1}^{N_t} \mathbf{q}_i s_{k_i^m} \quad (4)$$

where  $s_{k_i^m}$  is the modulated symbol of user  $k_i^m$  in sub-carrier  $m$ .

Assuming that each user can estimate its channel with no error, the signal to interference plus noise ratio (SINR) at receiver  $k$  on  $i$ -th beam and  $m$ -th subcarrier can be calculated as

$$\text{SINR}_{i,k,m} = \frac{|\mathbf{H}_{k,m} \mathbf{q}_i|^2}{N_t / \rho + \sum_{j=1, j \neq i}^{N_t} |\mathbf{H}_{k,m} \mathbf{q}_j|^2} \quad (5)$$

where  $\rho$  is the signal-to-noise ratio (SNR), assumed to be the same for each user. The achievable rate of  $k$ -th user on  $i$ -th beam over subcarrier  $m$  is given by

$$C_{i,k,m} = \log_2 (1 + \text{SINR}_{i,k,m}) \quad (6)$$

One of the main problems in MIMO-OFDMA systems is the large amount of feedback required for optimal joint subcarrier/beam allocation. Since different users can be assigned on different subcarriers, full channel state information (CSI) on each subcarrier is needed, which leads to prohibitive feedback load. In the following section, we present different feedback scenarios where each user feeds back only partial CSI for a group of neighboring subcarriers.

### III. FEEDBACK REDUCTION AND SCHEDULING

We assume that the feedback channel is error and delay free and that each receiver has perfect CSI for all subcarriers and antennas. However, only partial channel state information is available at the transmitter.

We divide the set of available subcarriers into  $G$  groups, each one containing  $L$  neighboring carriers. Without loss of generality, we assume that  $M$  is a multiple of  $G$  so that each group has the same number of subcarriers, i.e.,  $L = \frac{M}{G}$ .

Let  $\{m_l^g\}_{l=1,\dots,L}$  be the set of  $L$  subcarriers of group  $g$  and let  $SINR_{i,k,m_l^g}$  and  $C_{i,k,m_l^g}$  denote the SINR and the throughput of  $k$ -th user on subcarrier  $m_l^g$  and beam  $\mathbf{q}_i$  given by (5) and (6) respectively.

#### A. Maximum SINR Feedback Strategy (MSFS)

Here we assume that for each group  $g$ , user  $k$  feeds back the highest SINR computed as

$$SINR_g(k) = \max_{\substack{1 \leq l \leq L \\ 1 \leq i \leq N_t}} SINR_{i,k,m_l^g}$$

This value represents the maximum SINR achieved by user  $k$  on group  $g$  over all carriers and all beams. The user also informs the base station of the indices  $i^*$  and  $m_{i^*}^g$  where

$$(i^*, m_{i^*}^g) = \max_{\substack{1 \leq l \leq L \\ 1 \leq i \leq N_t}} SINR_{i,k,m_l^g} \quad (7)$$

For each carrier, the BS assigns each beam to the user with the highest corresponding SINR as in [8]. This scheme is asymptotically optimal in terms of sum rate, however, for low number of users it is evident that a large number of carriers are not chosen by the users, especially when the number of carriers per group  $L$  is large. This intuitive result calls to investigate more sophisticated feedback schemes where each user has the option to use a larger set of carriers at each group. For that, we propose the following two schemes.

For each group  $g$  and beam  $\mathbf{q}_i$ , each user  $k$  computes a representative rate  $\overline{C}_g(i, k)$  for the set  $\{C_{i,k,m_l^g}\}_{l=1,\dots,L}$  given by

$$\overline{C}_g(i, k) = \mathcal{F} \left( \left\{ C_{i,k,m_l^g} \right\}_{l=1,\dots,L} \right) \quad (8)$$

where  $\mathcal{F}(\cdot)$  is a multi-variable function that depends on the feedback scenario considered.

Let beam  $\mathbf{q}_i$  be assigned to user  $k$  in subcarriers of group  $g$ , then the BS can transmit at a rate equal to  $\overline{C}_g(i, k)$  in all subcarriers for which the user's capacity is greater or equal to  $\overline{C}_g(i, k)$  (i.e., subcarriers such that  $C_{i,k,m_l^g} \geq \overline{C}_g(i, k)$ ). No

transmission will be scheduled on the remaining subcarriers of the group (i.e., subcarriers where the user's capacity is less than  $\overline{C}_g(i, k)$ ) as this will lead to an outage event. Evidently, when the user feeds back the representative capacity, it should also inform the base station about the subcarriers that can support this rate. Under this scheme, the sum rate achieved by user  $k$  on beam  $\mathbf{q}_i$  over the subcarriers of group  $g$  is

$$R_g(i, k) = \mathcal{A}(\overline{C}_g(i, k)) \cdot \overline{C}_g(i, k) \quad (9)$$

where  $\mathcal{A}(\overline{C}_g(i, k))$  denotes the number of subcarriers in the group  $g$  where user  $k$  can support rate greater or equal to  $\overline{C}_g(i, k)$ .

The number of representative values computed by each user for each group is equal to the number of beams  $N_t$ . Depending on the feedback reduction scheme, one or a set of these values is fed back to the transmitter. In the following sections, we present two different reduction schemes having different CSIT.

#### B. All beams Max-Rate representative

In this scheme, the representative capacity is chosen such that the sum of achievable rates by user  $k$  on beam  $\mathbf{q}_i$  over the subcarriers of group  $g$  is maximized

$$\begin{aligned} \overline{C}_g(i, k) &= \arg \max R_g(i, k) \\ &= \arg \max_{C_{i,k,m_l^g}} \mathcal{A}(C_{i,k,m_l^g}) \cdot C_{i,k,m_l^g} \end{aligned}$$

The user determines the set  $\{C_{i,k,m_l^g}\}_{l=1,\dots,L}$  and sorts its values in increasing order. Let  $C'_1, C'_2, \dots, C'_L$  be the sorted values, then the representative is given by

$$\overline{C}_g(i, k) = \arg \max_{C'_j} (L - j) C'_j \quad (10)$$

For each group  $g$ , each user feeds back the  $N_t$  values  $\overline{C}_g(i, k)$ ,  $i = 1, \dots, N_t$  and informs the BS on the carriers that can support this rate. Within one group, the users may not use all subcarriers and can ask for a different set of 'preferred' subcarriers. Hence, the scheduling must be performed independently for each subcarrier. Note also that for each subcarrier, the transmitter assigns the beams to the users that support the highest SINR on these beams as in [8].

#### C. Best beams Max-Rate representative

In this scheme, each user computes the representative capacities, for each group  $g$  and beams  $\mathbf{q}_i$ , in the same manner as in the previous scheme. In the spirit of [8], instead of feeding back the representatives for all beams, each user feeds back only the representative value for its best beam (i.e., the beam with the highest sum rate over the frequencies of the group). For that, the user determines the beam vector  $\mathbf{q}_{i^*}$  achieving,

$$i^* = \arg \max_{i=1,\dots,N_t} R_g(i, k) \quad (11)$$

where  $R_g(i, k)$  is given by (9).

The index  $i^*$  and the corresponding value  $\overline{C}_g(i^*, k)$  are fed back to the transmitter. Additionally, each user  $k$  informs

the BS about the subcarriers that exceed the representative capacity. As in the previous case, for each beam we pick the user that achieves the maximum throughput on that beam.

*Remark:* Additional feedback reduction can be achieved if each user chooses the  $G'$  best groups ( $G' < G$ ) and feeds back their CSI instead of feeding back information for all groups. The comparison between groups is made in terms of the users' achievable sum rate over the subcarriers of the group.

#### IV. POWER ALLOCATION BASED ON PARTIAL CSIT

In this section, we present two general classes of power allocation algorithms for sum rate maximization on each subcarrier. Our objective is to optimize the sum rate of scheduled users based on their partial CSIT subject to a fixed amount of power available at the transmitter.

Let us denote  $\mathcal{S}_m$  the scheduling set containing the indices of users selected via the above mentioned schemes on the  $m$ -th subcarrier ( $|\mathcal{S}_m| \leq N_t$ ). On each subcarrier  $m$ , once the group of scheduled user is defined, the transmitter allocates different power levels to the randomly generated beams to improve the system overall performance.

Let  $\mathbf{p} = [P_1, \dots, P_{N_t}]$  denote the set of transmission powers of beams  $\{\mathbf{q}_i\}_{i=1}^{N_t}$ , and the SINR of  $k$ -th user is given by

$$SINR_{i,k,m} = \frac{P_i |\mathbf{H}_{k,m} \mathbf{q}_i|^2}{N_t/\rho + \sum_{j=1, j \neq i}^{N_t} P_j |\mathbf{H}_{k,m} \mathbf{q}_j|^2} \quad (12)$$

The optimization problem of the power vector  $\mathbf{p}$  that maximizes the throughput can be formulated as

$$\begin{aligned} \max_{\mathbf{p}} \quad & \sum_{\substack{i=1, \dots, N_t \\ k \in \mathcal{S}_m}} \log(1 + SINR_{i,k,m}) \\ \text{subject to} \quad & \sum_{i=1}^{N_t} P_i = P \end{aligned}$$

The intuition behind the beam power allocation is the fact that for low - yet practical - number of users, it becomes more and more unlikely that  $N_t$  randomly generated, equipowered beams will match well the channels of any set of  $N_t$  users in the network. This performance degradation can be compensated by redistributing the power to the beams. Two power allocation methods for multiuser opportunistic beamforming based on different amount of feedback have been proposed [10]. Below we provide a brief description of the extension of these methods to the OFDMA case.

##### A. SIR-based power allocation (SPA)

We consider the Maximum SINR Feedback Strategy (MSFS) where for each group, each user feeds back the maximum SINR and the indices of the corresponding carrier and beam. Based on this information, the BS determines the set  $\mathcal{S}_m$  of scheduled user on each carrier  $m$ . In order to improve performance a second step where additional CSIT feedback is provided, yet involving the scheduled users. Each user feeds back for the carrier it had been assigned the values of the

*effective channel gain*  $\gamma_{i,k,m} = |\mathbf{H}_{k,m} \mathbf{q}_i|^2$ . The total power is equally divided between active carriers and power allocation is performed in each carrier depending on the number of scheduled users:

- For  $|\mathcal{S}_m| > 2$ , the power is allocated using the iterative power allocation proposed in [10].
- For  $|\mathcal{S}_m| = 2$ , the power is allocated according the optimal closed-form solution as in [10].
- For  $|\mathcal{S}_m| = 1$ , all power is allocated to the scheduled user.

##### B. Greedy power allocation (GPA)

Here we consider the case where only one user is assigned power in each carrier, i.e., only one beam per carrier is turned on. In such a scheme, users feed back the *effective channel gain* values.

1) *Opportunistic Greedy Power Allocation:* For each group  $g$ , user  $k$  feeds back the highest  $\gamma_{i^*,k,m_i^g}$ , given by

$$\gamma_{i^*,k,m_i^g} = \max_{\substack{1 \leq l \leq L \\ 1 \leq i \leq N_t}} \gamma_{i,k,m_i^g} \quad (13)$$

Each carrier is then allocated to the user with the highest *effective channel gain* and water filling power allocation is performed over the frequencies. The power allocated to subcarrier  $m$  is given by:

$$P_m = \left[ \frac{1}{\lambda} - \frac{1}{\rho \gamma_m} \right]^+ \quad (14)$$

where  $\gamma_m$  is the *effective channel gain* fed back by the selected user for subcarrier  $m$  and  $\lambda$  satisfies  $\sum_{i=1}^M P_m = M \cdot P$

2) *Representative Greedy Power Allocation:* Here each user computes a representative value for each beam as follows

$$\bar{\gamma}_g(i, k) = \arg \max \left[ \mathcal{A} \left( \bar{\gamma}_{i,k,m_i^g} \right) \cdot \log(1 + \rho \bar{\gamma}_{i,k,m_i^g}) \right] \quad (15)$$

The user feeds back the representative value and the index of the beam where the user achieves the maximum sum rate over the frequencies of group  $g$ . The user also informs the BS of the carriers in the group with an *effective channel gain* on the selected beam higher than the representative value. As in the previous scheme, each carrier is assigned to the user with the highest corresponding *effective channel gain*. The power is allocated using a frequency water filling algorithm.

#### V. NUMERICAL RESULTS

In all simulations we assume a system bandwidth of 1.25MHz with 128 equally spaced sub-carriers. The carrier spacing is the same as in IEEE802.16 and 3GPP Long Term Evolution specifications. We also consider a multipath channel model with ITU Vehicular B power delay profile. This channel model is the one with the smallest coherence bandwidth among all ITU channel models and thus is the one where we have the highest performance degradation in frequency grouping based algorithms. The plots are obtained through Monte-Carlo simulations.

In Figure 2, we compare the Spectral Efficiency (SE) performance of the proposed feedback and scheduling schemes as a function of the number of users for  $N_t = 4$ ,  $L = 16$  and  $\text{SNR} = 0\text{dB}$ . As we mentioned before, MSFS is asymptotically optimal, however for low number of users, the other two schemes show significant capacity gain. Figure 3 shows the system SE for the four proposed power allocation techniques as a function of the number of users. For low number of users, the greedy power allocation algorithm have the best performance exhibiting gain of more than 1bps/Hz and 2bps/Hz compared to SPA and MSFS strategies respectively. This gain however vanishes for moderate and large number of users as it cannot exploit the multiplexing gain available in the channel. Note also that SPA converges to MSFS for large number of users as equal power allocation is asymptotically optimal.

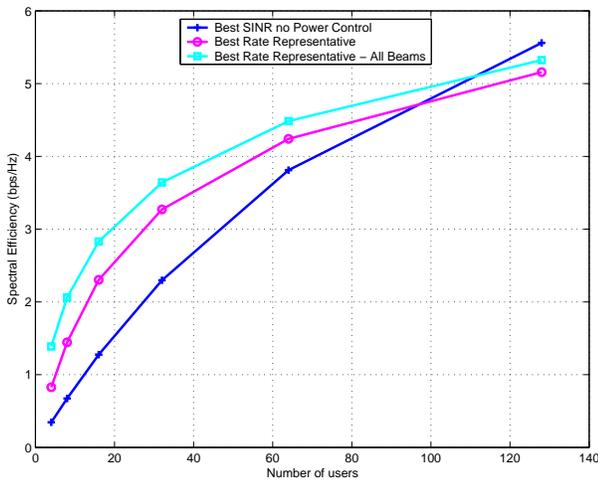


Fig. 2. Spectral efficiency as a function of the number of users for different feedback reduction methods,  $N_t = 4$  and  $L = 16$

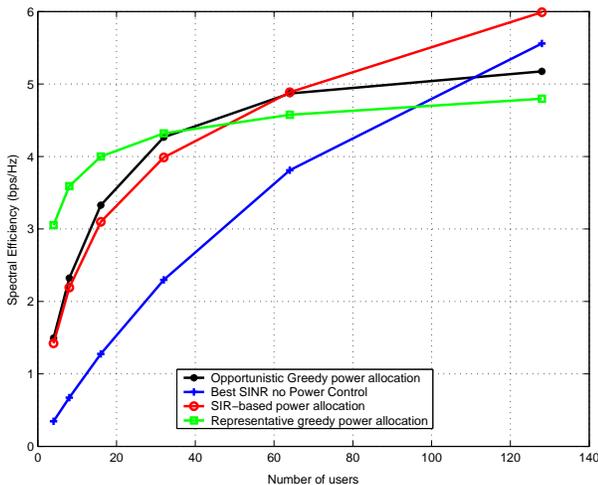


Fig. 3. Spectral efficiency as a function of the number of users for different feedback reduction/power allocation schemes,  $N_t = 4$  and  $L = 16$

## VI. CONCLUSIONS

The important issue of feedback reduction in MIMO-OFDMA networks using opportunistic beamforming was addressed here. We propose and evaluate three practical low rate feedback schemes that allow to reduce significantly the amount of required CSIT at little expense of throughput. Our results indicates that MIMO-OFDMA combined with opportunistic scheduling can be a very promising technology for future generation wireless systems.

## REFERENCES

- [1] R. Knopp and P. A. Humblet, "Information capacity and power control in single-cell multiuser communications," *In Proc. IEEE ICC'95, Seattle, WA.*, June 1995.
- [2] I. Toufik and R. Knopp, "Channel Allocation Algorithms For Multi-carrier Systems," *IEEE VTC Fall 2004, Los Angeles, USA* pp. 1129 - 1133, vol. 2, Sept. 2004.
- [3] I. Toufik and R. Knopp, "Multiuser Channel Allocation Algorithms Achieving Hard Fairness," *IEEE GLOBECOM 2004, Dallas, USA* pp. 146 - 150, vol.1. 29 Nov.-3 Dec. 2004.
- [4] G.J. Foschini and M.J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Pers. Commun.*, vol. 6, pp. 311 - 335, Mar. 1998.
- [5] E. Telatar, "Capacity of multi-antenna Gaussian channels," *AT&T Bell Laboratories, Tech. Memo.*, June 1995.
- [6] D. Gesbert et al, "From Theory to Practice: An Overview of MIMO Space-Time Coded Wireless Systems," *IEEE Journal Selected Areas in Communications*, pp. 281 - 302, vol.21, April 2003.
- [7] P. Viswanath, D. N. C. Tse and R. Laroia, "Opportunistic beamforming using dumb antennas," *IEEE Transactions on Information Theory*, vol. 48, pp. 1277 - 1294, June 2002.
- [8] M. Sharif and B. Hassibi, "On the Capacity of MIMO Broadcast Channel with Partial Side Information," *IEEE Transactions on Information Theory*, vol. 51, no.2, pp. 506 - 522, February 2005.
- [9] P. Svedman, S. K. Wilson, L. Cimini, and B. Ottersten, "A simplified feedback and scheduling scheme for OFDM," *in Proc. IEEE Veh. Tech. Conf. (VTC 2004)*, Milan, Italy, May 2004.
- [10] M. Kountouris and D. Gesbert, "Robust multi-user opportunistic beamforming for sparse networks," *Proc. IEEE Int. Work. on Sig. Proc. Adv. on Wirel. Comm. (SPAWC)*, New York, U.S.A, June 2005.