Available at
www.ElsevierComputerScience.com
POWERED BY SCIENCE DIRECT
Signal Processing 86 (2006) 1864–1878
SIGNAL PROCESSING
www.elsevier.com/locate/sigpro

Channel allocation algorithms for multi-carrier multiple-antenna systems

Issam Toufik*, Raymond Knopp

Institut Eurécom, 2229 route des Crêtes, 06904 Sophia Antipolis, France

Received 3 January 2005; received in revised form 15 September 2005; accepted 15 September 2005
Available online 27 December 2005

Abstract

Dynamic channel resource allocation (DCA) exploiting wideband multiuser diversity can provide data transmission with very high spectral efficiency by scheduling at each dimension (time, frequency, space) the user with the best channel conditions. The main issue arising from this allocation is fairness. Base station or users have to wait until their channel is most favorable to transmit. It is commonly considered that fairness comes at the cost of a significant system capacity penalty. In this paper we show that multiuser diversity, and thus an increase of aggregate data rates with the size of the user population, can still be successfully achieved even with deterministic channel use and that even under a hard fairness constraint we can achieve performance which comes close to those of the optimal unfair policy for K-user systems with K parallel sub-channels. We propose and compare different algorithms which perform channel allocation yielding variable-rate/constant-power (Max–Min allocation and maximum total rate allocation) and fixed-rate/variable-power (fixed rate allocation). We show the effect of system bandwidth (and thus sub-channel correlation) on wideband multiuser diversity. This paper also investigates the performance of combined orthogonal channel and antenna allocation algorithms in multiple-antenna multi-channel systems. We extend the proposed Max-Min allocation algorithm to the multiple-antenna systems and compare its performance in two different transmission scenarios (spatial multiplexing and space time coding). An extension of the Max–Min allocation algorithm to the general case of an arbitrary number of users is also given. The proposed techniques are applicable, for instance, in orthogonal frequency division multiple-access (OFDMA) systems with dynamic sub-carrier allocation.

Keywords: Multiuser diversity; Fairness; Channel allocation; Multiple antennas; Multi-carrier; OFDMA

1. Introduction

Current and future wireless communication systems are expected to provide a broad range of multimedia services. Customers are expecting high quality, reliability and easy access to high-speed communications. The use of multiple antennas at the transmitter and/or at the receiver (MIMO systems) and the use of dynamic channel allocation (DCA) had been identified as two of the major techniques promising significant improvement in terms of spectral efficiency and a combination of these techniques is surely a means to meet the future broadband service requirements.

*Corresponding author. Tel.: +33 493 002 917; fax: +33 493 002 627.
E-mail addresses: issam.toufik@eurecom.fr (I. Toufik), knopp@eurecom.fr (R. Knopp).

0165-1684/$ - see front matter
doi:10.1016/j.sigpro.2005.09.036

Multiple-input multiple-output (MIMO) systems have emerged as one of the most promising technical breakthroughs in modern wireless communications. The pioneering work by Foschini [1] and Telatar [2] predicted remarkable spectral efficiency for wireless systems using multiple antennas to increase data rates through multiplexing or to improve performance through diversity. There have been many studies of MIMO systems in a multiuser network environment including proposals for scheduling algorithms [3]. In [4] authors study optimal strategy of multiple access with multiple antennas at the base station. The proposed scheduling algorithm straddles both the physical and low level protocol layer. In [5] the authors study greedy scheduling in multiuser MIMO systems and show that it leads to lower average user experienced delays compared to single-user greedy scheduling.

DCA uses channel state information (CSI) available at the transmitter to schedule users across the available system bandwidth. In DCA strategies, one should employ the concept of multiuser diversity as introduced in [6–9] to guarantee high spectral efficiency. It has been shown in these studies that multiuser diversity yields an increase of the total throughput as a function of the number of users. The most remarkable result is that for multiuser systems significantly more information can be transmitted across a fading AWGN channel than a non-fading AWGN channel for the same average signal power at the receiver. Spectral efficiency can be increased by more than a factor of two for small signal-to-noise ratios (around 0 dB). This is due to the fact that at a given time and frequency, the channel gain is random and can be significantly higher than its average level. One can take advantage of this by using a proper dynamic time-frequency allocation based on the time/frequency varying characteristics of the channels. The success key of DCA is the use of CSI at the transmitter, but one may question the practicality of assuming that quasi-perfect CSI can be made available at the transmission end. This depends strongly on the considered system architecture. In systems such as HDR (also known as IS-865) the receiver estimates the CSI based on a common pilot and feeds the information back to the transmitter [10]. If we employ the same antenna array for transmission and reception in a time-division duplexing (TDD) system then channel reciprocity allows us to use our channel estimates obtained during reception for transmission, which is the case for instance in the DECT cordless telephone system and for power-control in UMTS-TDD. In practical TDD systems, amplitude information is reasonably simple to estimate from the opposite link, while for accurate phase information this is not the case, mainly due to the difficulty in calibrating the difference in phase response between the transmitter and receiver chains.

The system related main issue arising from channel-dependent resource allocation schemes is fairness. Users (or the base station) cannot always wait until their channel conditions are most favorable to transmit. This is particularly true for traditional circuit-switched services like voice or real-time video. In [11], the authors treat the fairness problem between users in the slow fading environment and discuss the implementation in the IS-865 system and propose methods to enhance fairness. Their approach consists in using multiple antennas to induce fast channel fluctuations combined with the proportionally fair allocation policy used in IS-865. In a similar vein for multi-cell systems, [12–14] study combined power control and base station assignment in multi-cell systems with fixed vector rate. This is also a form of fairness, since these algorithms allow users to transmit with their desired rates. Similar opportunistic techniques for multi-cell systems are briefly alluded to in [11]. In [15], the authors consider the sub-carrier assignment problem in orthogonal frequency division multiple-access (OFDMA) systems and compare the simplicity and fairness properties of different allocation algorithms. In [16], a fair allocation criterion yielding multiuser diversity with a deterministic channel use in a K-user system with K parallel sub-channels is proposed. We build our work on these results by addressing an algorithm that performs the allocation of users across sub-channels according to this criterion for OFDM-like systems on frequency selective channels. We analyze both fixed-power/variable-rate and variable-power/fixed-rate cases. The obtained results show that even with hard fairness constraint, multiuser diversity is still achievable and we can obtain performance approaching those of the optimal completely unfair scheme. The addressed results are very pertinent for slowly-varying channels since frequency selectivity is exploited. From a futuristic system point-of-view, an application of the ideas outlined in this work would be the allocation of users equipped with various radio interfaces in different parts of the radio spectrum, potentially using different radio-access technologies, based on link quality and quality-of-service (QoS) constraints. We could also envisage a centralized control of radio spectrum across very large bandwidths and co-localized radio-access technologies (e.g. GSM/UMTS-FDD/UMTS-TDD, 802.11, 802.16, DVB-T, etc.). We can also envisage an inter-operability between service providers, with a centralized access control which maps users over the available system resources independently on the users' service provider but only depending on the channel conditions.

The organization of this paper is as follows: Section 2 presents the underlying system model. In Section 3, we provide an algorithm to achieve the criterion in [16] and compare this algorithm to the one achieving the maximum total throughput. We also outline an algorithm for power control under a fixed rate vector constraint and present numerical results and outline the effect of bandwidth on multiuser diversity. We extend this study to the case of multiple transmitting antennas for the downlink in Section 4. We also compare different spatial combining methods when using multiple antennas at reception. Finally, in Section 5 we present our conclusions and outline ongoing extensions and future perspectives.

A note on notation: We use calligraphic letters (e.g. $\mathscr{H}$) to denote matrices and uppercase underlined letters (e.g. $\underline{X}$) to denote vectors. $E\{\cdot\}$ denotes the expectation. $\mathscr{A}^{-1}$ and $\mathscr{A}^{H}$ are, respectively, the inverse and conjugate transpose of matrix $\mathscr{A}$.

2. System models

We consider the downlink of a wireless system with K symmetric users. We assume that the base station is equipped with N_{t} antennas transmitting over M parallel channels and each user has N_{r} receiving antennas (For the case $N_{\mathrm{t}}=1$, the study in this paper is exactly the same for the uplink transmission). Fig. 1 shows a diagram of such a system: At the transmitter, using the CSI of all users, subcarriers and antennas are assigned to different users. At a receiver k, the subcarrier allocation information is then used to extract the data for user k from its assigned carrier. In this paper, for the multiple antenna at base station case we consider antenna allocation, but more generally the study and all related results can be applied to schemes where opportunistic beamforming or precoding is used. In such cases, the algorithm is applied in the same way to allocate beams or precoding vectors. The system model in the paper could represent any wideband OFDM system, such as mobile broadband wireless access (MBWA) systems, for instance the evolving IEEE 802.16 standard where an OFDMA technique is used. Another example of such a system could be the UTRAN HSDPA (high-speed data packet-access) 3GPP proposal using an OFDM(A) physical layer instead of WCDMA, proposed in [17] for the downlink channel. HSDPA also envisages multiple-antenna terminals. In the context of these systems, the algorithms proposed in this paper would be used to allocate the different frequency sub-bands and transmit antennas to users. We can also imagine the use of these techniques in extensions of IEEE802.11a/g, Hiperlan2 or multiband-OFDM for UWB systems.

We consider that each sub-channel is a fading AWGN channel with noise variance N_0. The $N_{\mathrm{r}}\times 1$ received signal vector for user k over a given sub-channel m is given by

$$\underline{Y}_{k,m}=\sum_{k'=1}^{K}\mathscr{H}_{k,m}.\mathscr{P}_{k'm}.\underline{X}_{k',m}+\underline{Z}_{k,m} \tag{1}$$

where $\underline{Z}_{k,m}$ is the noise vector in sub-channel m and $\underline{X}_{k',m}=[x_{k',m}(1),\dots,x_{k',m}(N_{\mathrm{t}})]^{\mathrm{T}}$ is the $N_{\mathrm{t}}\times 1$ signal vector for user k' on sub-channel m s.t. $x_{k',m}(n_{\mathrm{t}})$ is the signal transmitted for user k from antenna n_{t}. $\mathscr{P}_{k',m}$ is the $N_{\mathrm{t}}\times N_{\mathrm{t}}$ diagonal matrix $\mathrm{diag}(\sqrt{P_{k',m,1}},\dots,\sqrt{P_{k',m,N_{\mathrm{t}}}})$ s.t. $P_{k',m,n_{\mathrm{t}}}$ is the transmit power for user k' from antenna n_{t} on sub-channel m. We assume that $E\{x^2_{k,m}(n_{\mathrm{t}})\}=1$ and $E\{P_{k,m,n_{\mathrm{t}}}\}=P/MN_{\mathrm{t}}$ so that the average transmit power of user k across all channels and antennas is P.

$\mathscr{H}_{k,m}$ is the $N_{\mathrm{r}}\times N_{\mathrm{t}}$ channel gain matrix for user k on sub-channel m whose $n_{\mathrm{r}}n_{\mathrm{t}}$th element $H_{k,m}[n_{\mathrm{r}},n_{\mathrm{t}}]$ is the channel between the n_{r}th receive antenna of user k and n_{t}th transmit antenna. As has been mentioned previously, we assume that the amplitude of the channel responses for each antenna pair, for all users and over all sub-carriers are known at the transmitter. For reciprocal channels (for instance in TDD systems), the base station estimates the CSI for each user from received pilots which are known sequences transmitted by the users on each antenna and are *spread over the entire available bandwidth*. The estimated CSI is used to carry out the sub-channel allocation algorithm and a message is fed back to inform each user of its assigned sub-channel/transmit antenna (It is worth noting that for slowly-varying channels this is

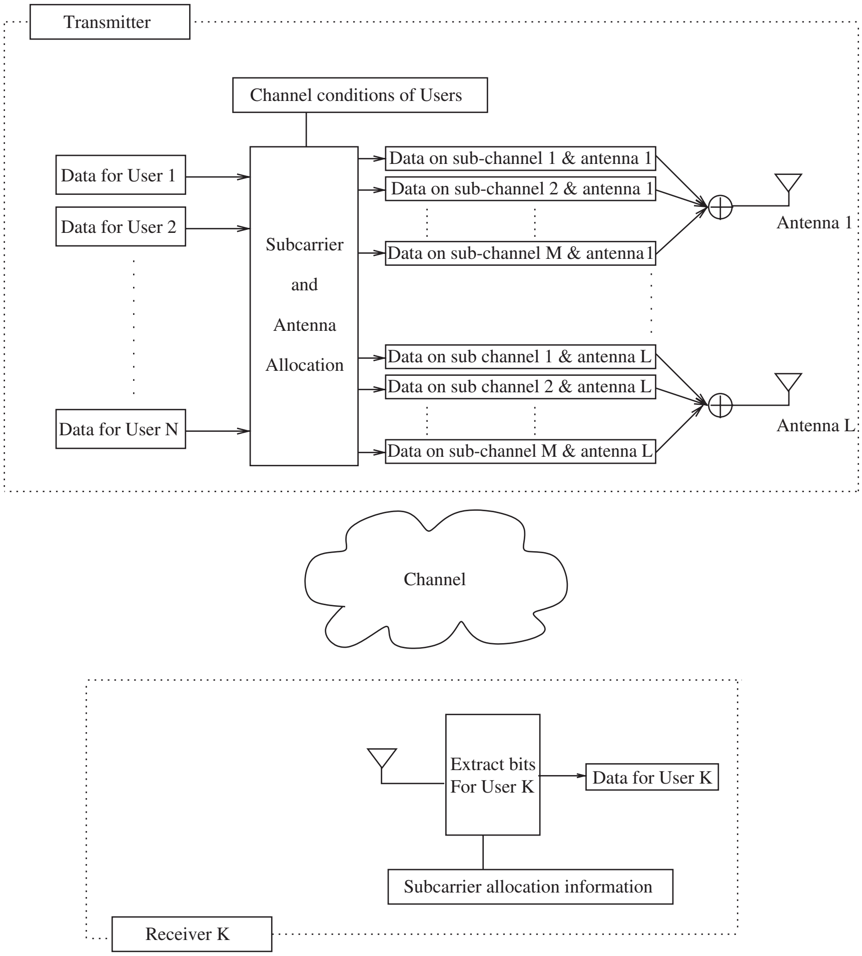

Fig. 1. System Model for a multiuser, multi-carrier and multiple antenna system.

reasonably simple to accomplish and consumes little signaling bandwidth since the allocation remains invariant for long period.) In the case of non-reciprocal channels, each user has to estimate its CSI over all available sub-channels and from all transmitting antennas based on known pilots and feeds this information back to the base station which carries out the antenna and sub-channel allocation algorithm.

In the spirit of OFDM-based systems, we model each channel gain $H_{k,m}[n_\mathrm{r}, n_\mathrm{t}]$ as a frequency sample of a discrete multipath channel having Γ significant uncorrelated paths with delays: $\tau_1, \tau_2, \ldots, \tau_\Gamma$, that is

$$h_{k,m}[n_\mathrm{r}, n_\mathrm{t}](t) = \sum_{l=0}^{\Gamma-1} \alpha_l \delta(t - \tau_l), \tag{2}$$

where the path gains α_l are zero mean Gaussian random variables with variance σ_l^2.

The channel is assumed stationary for the duration of coded transmission blocks, but may vary from block to block. The samples of the frequency response are given by

$$H_{k,m}[n_\mathrm{r}, n_\mathrm{t}] = \sum_{l=0}^{\Gamma-1} \alpha_l \mathrm{e}^{-\mathrm{j}(2\pi\tau_l f_m/M)} \tag{3}$$

and have covariance

$$\begin{aligned} E\{H_{k,m}[n_\mathrm{r}, n_\mathrm{t}] H^*_{k,m'}[n_\mathrm{r}, n_\mathrm{t}]\} &= \sum_{l=0}^{\Gamma-1} \sum_{l'=0}^{\Gamma-1} E\left\{\alpha_l \alpha_{l'} \mathrm{e}^{-\mathrm{j}(2\pi(\tau_l f_m - \tau_{l'} f'_m))/M}\right\} \quad (4)\\ &= \sum_{l=0}^{\Gamma-1} E\left\{|\alpha_l|^2\right\} \mathrm{e}^{-\mathrm{j}(2\pi\tau_l(f_m - f'_m))/M}, \quad (5) \end{aligned}$$

where f_m is the frequency corresponding to sub-carrier m. Channel gains for different antennas over the same sub-channel and for the same user are assumed to be uncorrelated.

The goal of the following sections will be to study allocation algorithms of users to sub-carriers according to optimization criteria based on mutual information.

3. Single antenna systems: orthogonal allocation algorithms with hard fairness

We first consider the single antenna case ($N_t = N_r = 1$). Thus, the antenna indices n_t and n_r drop from all equations for the moment and the received signal for user k over sub-channel m is

$$y_{k,m} = \sum_{k'=1}^{K} \sqrt{P_{k',m}} H_{k,m} x_{k',m} + z_{k,m}. \tag{6}$$

Let us impose a hard fairness constraint on the system, namely that each user is guaranteed one sub-channel at any given time instant and transmit with constant power P. We also assume that there is only one user per sub-channel (i.e. orthogonal multiaccess). The achievable rate for user k over sub-channel m, under the assumption of Gaussian transmit signal $x_{k,m}$ is

$$R_{k,m} = \log\left(1 + \frac{P|H_{k,m}|^2}{N_0}\right) \quad \text{bits/dim.} \tag{7}$$

Under this model, the results in this section are valid for both uplink and downlink transmissions. In this system we accommodate up to $K = M$ users. This K-user system with K parallel channels can be represented by a weighted bipartite graph $G = (X, Y, E)$ where the left-hand side (LHS) set of vertices X represents the users and the right-hand side (RHS) set Y represents the sub-channels (Fig. 2). E is the set of edges between X and Y. For notational simplicity, we denote the edge between vertex $x \in X$ and vertex $y \in Y$ by the tuple (x, y). Each edge (x, y) in the graph is weighted by the rate achieved by user x over sub-channel y given by Eq. (7) i.e. $w(x, y) = R_{x,y} = \log(1 + (P|H_{x,y}|^2/N_0))$.

Definition 1. A matching $\mathscr{M}at$ in G is a subset of E such that no two edges in $\mathscr{M}at$ have a vertex in common. Intuitively we can say that no vertex in $X \cup Y$ is incident to more than one edge in $\mathscr{M}at$.

Definition 2. A matching $\mathscr{M}at$ is said to be maximum if for any other matching $\mathscr{M}at'$, $card(\mathscr{M}at) \geqslant card(\mathscr{M}at')$. $\mathscr{M}at$ is the maximum sized matching (where $card(\mathscr{M}at)$ is the number of edges in the matching $\mathscr{M}at$).

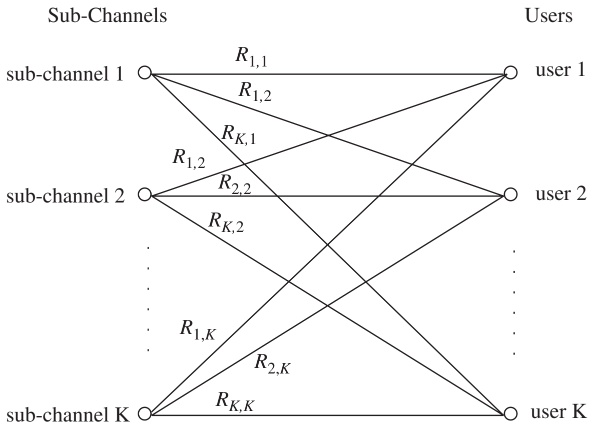

Fig. 2. Graph representation of the system.

Definition 3. A perfect matching is a maximum matching $\mathscr{M}at$ s.t. $card(\mathscr{M}at) = |X| = |Y|$. In other words, $\mathscr{M}at$ is a perfect matching if every vertex is incident to an edge in $\mathscr{M}at$. Thus, an allocation of users over sub-channels under our assumptions is equivalent to find a perfect matching in the system corresponding graph.

In the considered system all users can transmit on all sub-channels, thus the graph G is complete ($card(E) = K^2$). There exists $K!$ possible allocations of sub-channels to users each one represented by a perfect matching$\mathscr{M}at_i$ with $i = 1, \ldots, K!$. Let the vector $\mathbf{C}_{\mathscr{M}\mathbf{at_i}} = \{C_{\mathscr{M}at_i}(0), C_{\mathscr{M}at_i}(1), \ldots, C_{\mathscr{M}at_i}(K-1)\}$ represent the matching $\mathscr{M}at_i$ s.t. $C_{\mathscr{M}at_i}(k)$ is the sub-channel assigned to user k when matching $\mathscr{M}at_i$ is applied.

Assuming orthogonal multiplexing, an achievable ergodic sum rate is upper-bounded as

$$\sum_{k=0}^{K-1} R_k \leqslant E\left\{ \max_{i=1,\ldots,K!} \sum_{k=0}^{K-1} \log_2 \times \left(1 + \frac{P}{N_0} H_{k,C_{\mathscr{M}at_i}(k)}\right)\right\} \quad \text{bits/s} \tag{8}$$

when the transmitters (guided by the receivers) jointly select the best allocation vector given the instantaneous channel gains. This rate can be achieved either by adapting the data rate with the variation of the channels or by coding over many independent channel realizations.

3.1. Max–min allocation (MMA) policy

3.1.1. Allocation criterion

This policy consists of finding the matching $\mathcal{M}at_{i^*}$ s.t.

$$\begin{aligned} i^* &= \arg\max_{i=1,\dots,K!} \min_{k=0,\dots,K-1} H_{k,C_{\mathcal{M}at_i}(k)} \\ &= \arg\max_{i=1,\dots,K!} \min_{k=0,\dots,K-1} R_{k,C_{\mathcal{M}at_i}(k)}. \end{aligned} \tag{9}$$

This policy guarantees that at any given time instant the minimum channel gain allocated is the best possible among all allocations and thus maximizes the minimum of all user rates. It was shown in [16] that this criterion achieves multiuser diversity and provides a non-negligible gain with respect even to a non-fading channel. In the following we give a description of an algorithm that permits us to achieve this allocation criterion in polynomial time. In practice this policy (and similarly the one which follows) allows the instantaneous information rate to vary but is strictly non-zero.

3.1.2. Allocation algorithm description

The idea of the algorithm is that we remove the edges with the minimum weights (i.e eliminate the links with minimum channel gains) until no perfect matching could be found in the graph corresponding to the considered system. The last removed edge then corresponds to the link with the minimum allocated rate under MMA policy, since we cannot find a perfect matching with greater minimum rate. Once the first user is assigned a sub-channel, we remove the two nodes from the graph, we set the dimension of the graph to $(K-1)$ and operate in the same way to allocate a sub-channel to the second user and so on until all users are assigned one sub-channel. Before giving the details of the allocation algorithm let us first point out some general definitions that will help in the algorithm's description.

Definition 4. Given a matching $\mathcal{M}at$ in a bipartite graph G, an augmenting path for $\mathcal{M}at$ is a path that comprises edges in $\mathcal{M}at$ and edges not in $\mathcal{M}at$ alternately and which starts and ends at exposed vertices.

Theorem (*Berge's theorem [Berge [18]]*). *A matching $\mathcal{M}at$ is maximum iff it has no augmenting path.*

This theorem will be of great help in our algorithm. In fact, in some cases we will have to check the existence of perfect matching in a graph having a matching of cardinality equal to the graph dimension minus 1. Thus, if no augmenting path can be found for this matching this shows that the graph does not have any perfect matching. In Appendix A, we give an algorithm for the search of an augmenting path of a given matching $\mathcal{M}at$. We also describe how to augment a matching along an augmenting path.

Fig. 3 gives a descriptive diagram of the MMA algorithm that can be described as follows:

(1) *Initialization*
- We first begin by constructing the graph $G = (X, Y, E)$ corresponding to our system as described previously.
- The objective of our algorithm is to find the matching $\mathcal{M}at_{i^*}$ such that $i^* = \arg\max_{i=1,\dots,K!} \min_{k=0,\dots,K-1} H_{k,C_{\mathcal{M}at_i}(k)}$. If we consider the K^2 order statistics of the channel gains from the minimum to the maximum, then we have that

$$\mathrm{ord}\left(\min_{k=0,\dots,K-1} H_{k,C_{\mathcal{M}at_i}(k)}\right) \geqslant K$$

thus we remove the$K-1$ edges with the minimum weights from E,

$$E = E - \{(x,y)/\mathrm{ord}(w(x,y)) < K-1\}$$

- Then we construct an arbitrary perfect matching$\mathcal{M}at_\mathrm{p}$ corresponding to an arbitrary allocation.

(2) *Iteration*
 (a) Find, in graph$G$, the edge (x^*, y^*) with the minimum weight (i.e. the link with minimum channel gain)

$$(x^*, y^*) = \arg\min_{(x,y)\in X\times Y} \{w(x,y)\}$$

 (b)
- If$(x^*, y^*) \notin \mathcal{M}at_\mathrm{p}$, then remove the edge (x^*, y^*) from the graph (i.e. $E = E - (x^*, y^*)$) and go to (a).
- If $(x^*, y^*) \in \mathcal{M}at_\mathrm{p}$, then let $\mathcal{M}at'_\mathrm{p} = \mathcal{M}at_\mathrm{p} - (x^*, y^*)$ and
  - If $\mathcal{M}at'_\mathrm{p}$ admits an augmenting path P, then set $\mathcal{M}at_\mathrm{p}$ to the result of augmenting $\mathcal{M}at'_\mathrm{p}$ along$P$, and go to (a).
  - If $\mathcal{M}at'_\mathrm{p}$ admits no augmenting path, then we allocate sub-channel y^* to

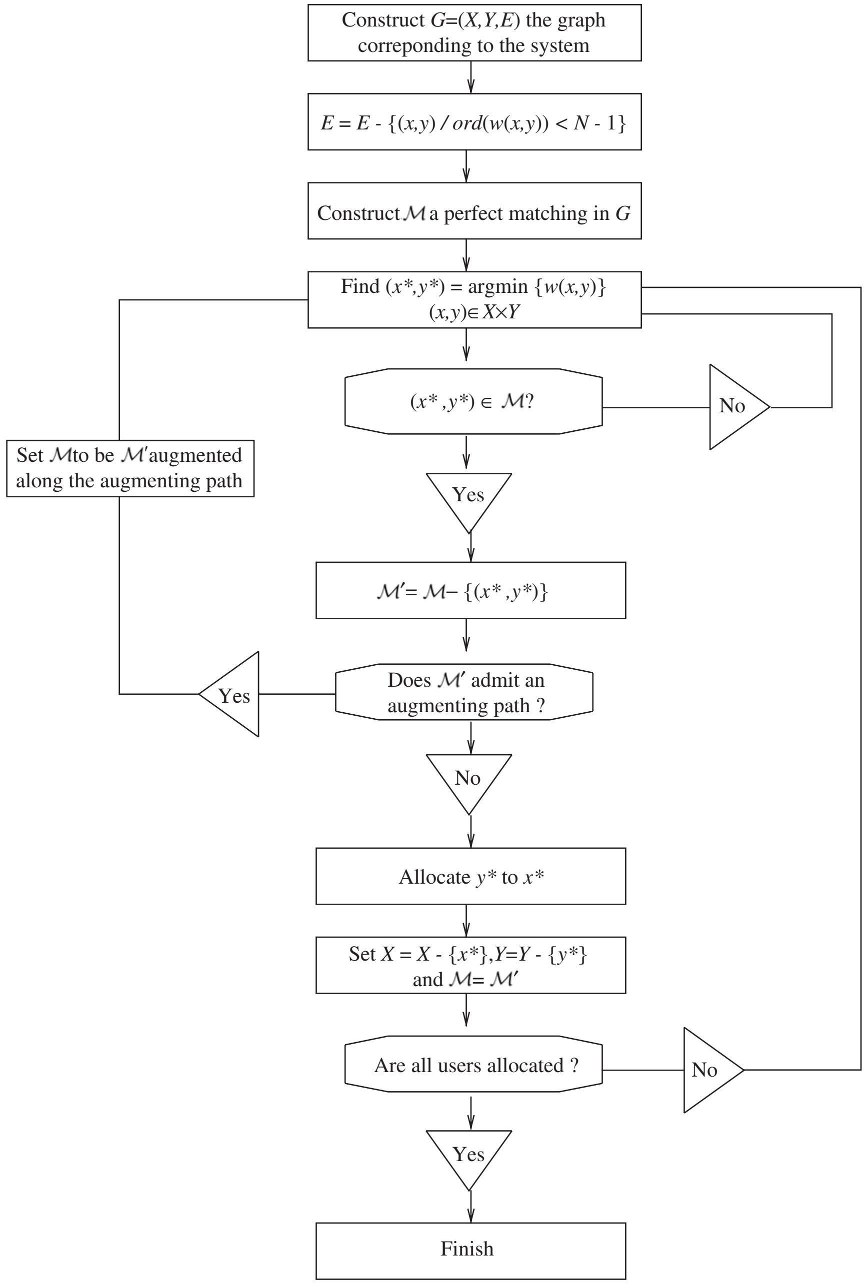

Fig. 3. Algorithm diagram.

user x^*, remove vertices x^* and y^* and all edges connected to them (after this the dimension of the graph is reduced by 1 and $\mathscr{M}at'_{\rm p}$ is a perfect matching in the new graph), set $\mathscr{M}at_{\rm p} = \mathscr{M}at'_{\rm p}$ and go to (a)

(c) Stop the algorithm when all user are assigned one sub-channel

3.2. *The maximum total rate allocation (MTRA) policy*

The MTRA policy is the strategy that achieves the maximum sum-rate given by Eq. (8). Here we permit to each user to have access to the channel but without any guarantee in the channel condition. The objective is to maximize the total throughput of the system under the constraint that each user has a sub-channel to transmit which, nevertheless, is a kind of fairness since in the optimal multiuser diversity based systems only the strongest user can transmit over each sub-channel.

As in the previous section, we can model the considered system by a bipartite graph. Finding the matching that maximizes the total weight is equivalent to find the allocation maximizing the total sum rate. Authors in [19] describe an algorithm that permits to find such a matching in $O(K^3)$ time based in the well-known Hungarian method.

3.3. Fixed rate allocation (FRA) policy

The objective here is to find the allocation of users to sub-channels minimizing the total transmit power while achieving some required rate-tuple $\mathbf{R} = (R_0, R_1, \ldots, R_{K-1})$, (i.e the SNR tuple $(\gamma_0, \gamma_1, \ldots, \gamma_{K-1})$) where R_k is the rate of user k. In this policy instantaneous power is allowed to vary in order to achieve a target per user information rate. This algorithm represents the most fair alternative since each user is given a guaranteed rate. The Hungarian method, presented in the previous section, performs the desired assignment with a small change in edge weights of the corresponding graph. For instance, the weight of the edge between user k and sub-channel m will be the negative of the power needed to achieve the desired SNR target γ_k if user k is assigned to sub-channel m, that is

$$w(k,m) = -P(R_k) = -\frac{\gamma_k N_0}{H_{k,m}}. \tag{10}$$

Finding the matching that maximizes the sum of weights permits us to find the desired allocation of users to sub-channels.

If a maximum power constraint is imposed, in some cases some users may not meet their desired rate and are allocated a smaller value.

3.4. Numerical results

Fig. 4 shows the average per user throughput as a function of the number of users with the MMA and MTRA fair allocation algorithms in a Rayleigh fading environment, which we compare to the unfair allocation where for each given sub-channel we choose the user with the best channel [6,7]. For these results we have assumed that the correlations between frequency channel gains in Eq. (5) are zero. Although unrealistic, this gives us an idea of the achievable rates as a function of the number of uncorrelated channels (or the approximate number of degrees of freedom of the propagation environment in the available system bandwidth). We first note in Fig. 4 that the per user average throughput increases with the number of users, in all cases, which is due to multiuser diversity. We can also see that even under a hard fairness constraint we can achieve performance which comes close to the optimal unfair policy. With a fixed rate (variable power) requirement we see that multiuser diversity can still be achieved and this additional constraint does not introduce any throughput degradation. This curve was computed for the same average SNR (0 dB) as in the variable cases.

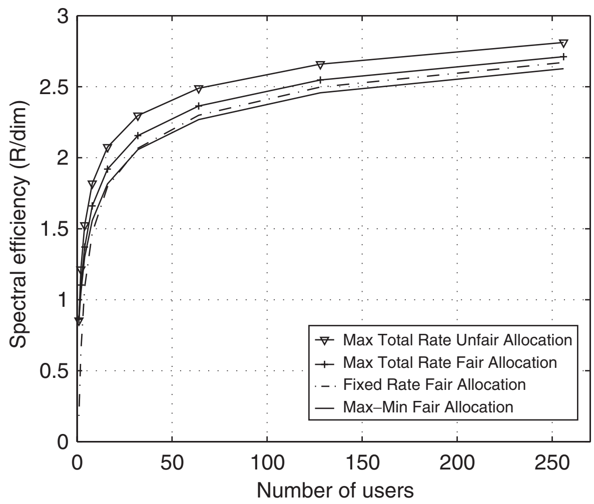

Fig. 4. Averaged throughput over Rayleigh fading at 0 dB with fair and unfair allocations.

Fig. 5 shows the spectral efficiency (SE) as a function of the number of users accommodated, using the proposed MMA algorithm on a frequency selective channel with correlated frequency channel gains, different values of the system bandwidth and with a fixed number of subcarriers equal to 64. Here the number of sub-carriers per user is M/K. For the correlated channel results, we assumed that the maximum path delay $\tau_{\max} = 2\,\mu\text{s}$ and an exponentially-decaying multipath intensity profile. The bandwidth of each of the 64 frequency beans is kept constant and only the spacing between different beans is changed, since our goal is to show the effect of frequency channel gains correlation. The performance of the algorithm with independent frequency channel gains is also given

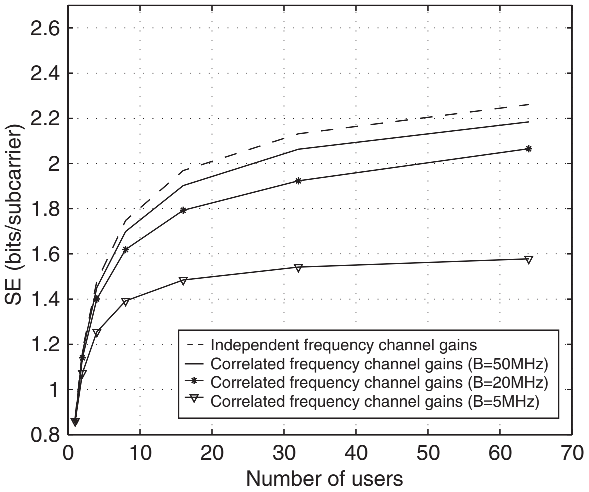

Fig. 5. SE variation with user population for a fixed number of sub-carriers.

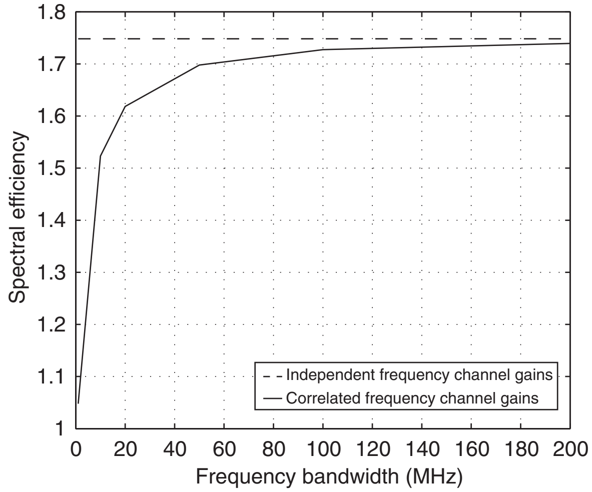

Fig. 6. SE variation with system bandwidth for fixed number of users and sub-carriers with the MMA policy.

for comparison. As expected, bandwidth plays an important role in how much scheduling users on sub-channels can increase spectral efficiency. We see that when the system bandwidth is appropriately chosen spectral efficiency can be increased by more than a factor of 2 for moderate user populations even with hard fairness constraints.

We also note that for sufficiently large bandwidth (here $B = 50\,\text{MHz}$) we approach the system performance of the independent frequency channel gains case. An other interesting result, given by the lower curve ($B = 5\,\text{MHz}$), is that for a number of users greater than the number of coherence bandwidths of the system, the increase of the number of users induces only a very slight improvement of the SE. This result is confirmed by Fig. 6 which shows the SE as a function of the system bandwidth for a fixed number of users ($K = 8$) and a fixed number of subcarriers ($M = 64$). This curve shows that an increase in the system bandwidth yields a rapid improvement of the system SE and slowly approaches the performances of the independent frequency channel gains case for system bandwidth greater than 50 MHz. This result is due to the increase in subcarrier spacing which becomes greater than the coherence bandwidth for $B > 50\,\text{MHz}$ and thus the frequency correlation of the channel becomes small. The value of 50 MHz is also the system bandwidth for which the number of coherence bandwidths is almost equal to the number of users accommodated in the system.

4. Multiple-antenna multiuser downlink transmission

Let us now consider the general case, presented in Section 2, where the base station side is equipped with $N_t > 1$ antennas transmitting in the down-link over M parallel sub-channels. MIMO techniques can be divided into two groups: *space time coding* which increases the performances of the communication system by coding the data over different transmitter branches and *space-division multiplexing*, which achieves a higher throughput by transmitting independent data over different transmit branches simultaneously. For a detailed study of MIMO systems, one can refer to [20] where different classes of techniques and algorithms, which attempt to realize the various benefits of MIMO including spatial-multiplexing and space-time coding, are presented.

In the following we compare the performance of these two transmission techniques using the MMA algorithm, by first considering the use of a single antenna at reception. We then consider a system with $N_r > 1$ receiving antennas and compare different spatial combining techniques of the received signals. The MMA in the multiple-antenna transmission system is the allocation that guarantees that the minimum SINR allocated is the best possible among all allocations. In all scenarios we consider the extreme case where we accommodate the maximum possible number of users, but the algorithm is easily generalized to the general case of an arbitrary number of users in the same way as in the single antenna case described in Appendix B.

4.1. Single antenna receivers

We assume in this section that $N_r = 1$. We consider two systems, the first attempts to achieve spatial-multiplexing whereas the second is a space-time coding approach.

4.1.1. System1

In the first system (*System1*) we assume that each user k is assigned one sub-channel and one antenna from which it receives its signal. In this system we accommodate up to $K = N_t M$ users and the allocation consists on the assignment of both sub-channel and antennas to users. We use the MMA algorithm described in the previous section to schedule users on antennas and sub-channels. The only difference with single antenna case is in the construction of the graph corresponding to the system (Fig. 7). Here the right-hand side set of vertices represents the tuples (antenna, sub-channel) to assign to users instead of only sub-channels in the single antenna case. The weight of the edge between each tuple (sub-channel, antenna)= (m, n_t) and user k is the SINR:

$$\gamma_k(m, n_t) = \frac{|H_{k,m}[n_t]|^2 (P/N_t)}{N_0 + \sum_{n'_t \neq n_t} |H_{k,m}[n'_t]|^2 (P/N_t)}. \tag{11}$$

We assume that each antenna transmits with power P/N_t over each sub-channel, thus the total transmitted power over each sub-channel is P. Under the assumption of Gaussian transmit signals, the achievable sum rate can be written as

$$\sum_{k=0}^{K-1} R_k = E\left\{ \sum_{k=0}^{K-1} \log_2(1 + \gamma_k(m_k^*, n_k^*)) \right\} \quad \text{(bits/ dim)}, \tag{12}$$

where m_k^* and n_k^* are, respectively, the sub-channel and antenna assigned to user k according to the MMA policy.

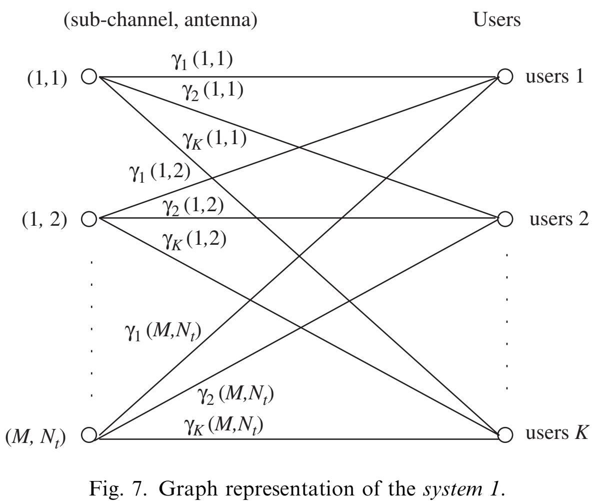

Fig. 7. Graph representation of the *system 1*.

4.1.2. System2

In the second system (*System2*), we assume that each user is assigned a single sub-channel and receives its signal from all antennas. This is known as transmit antenna diversity. *System2* can contain up to $K = M$ users. As for *System1* we assume that each antenna transmits with power P/N_t. This system can be seen as a K user system with K parallel sub-channels where the channel gain of user k over sub-channel m is $(\sum_{n_t=1}^{N_t} |H_{k,m}[n_t]|^2)/N_t$. This channel gains are the weights of the edges in the graph corresponding to the system. The sum rate of this system is

$$\sum_{k=0}^{K-1} R_k = E\left\{ \sum_{k=0}^{K-1} \log_2 \times \left(1 + \frac{\sum_{n_t=1}^{N_t} (P/N_t) |H_{k,m_k^*}[n_t]|^2}{N_0} \right) \right\} \quad \text{(bits/ dim)}, \tag{13}$$

where m_k^* is the sub-channel assigned to user k using MMA policy. In the following section, we compare the two transmission techniques of the multiple-antenna system in terms of the SE using the Max–Min fair allocation.

4.1.3. System comparison

Fig. 8 shows the spectral efficiency SE (averaged per sub-channel sum rate) as a function of the number of sub-channels for both *System1* and *System2* with 1, 2 and 4 antennas using the MMA algorithm. We assume a frequency selective channel with correlated frequency channel gains resulting from a maximum path delay $\tau_{max} = 2\,\mu s$ and an exponentially-decaying multipath intensity profile. The system bandwidth is assumed equal to $B = 20\,\text{MHz}$. We first note that SE increases with the number of sub-channels, in all cases, which is due to multiuser diversity. We can also note that *System1* permits transmission at a higher rate than *System2*. This is due to the "opportunistic" spatial-multiplexing offered by multiuser-diversity as

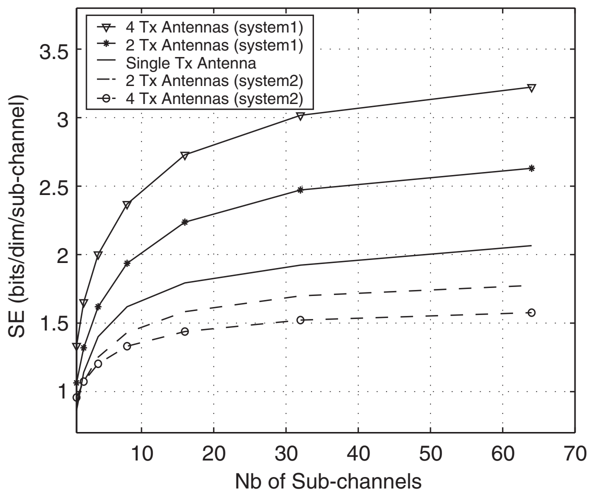

Fig. 8. The rate per sub-channel for both of systems as a function of the number of sub-channels with 1, 2 and 4 Tx antennas, 1 Rx antenna, SNR $= 0\,\mathrm{dB}$.

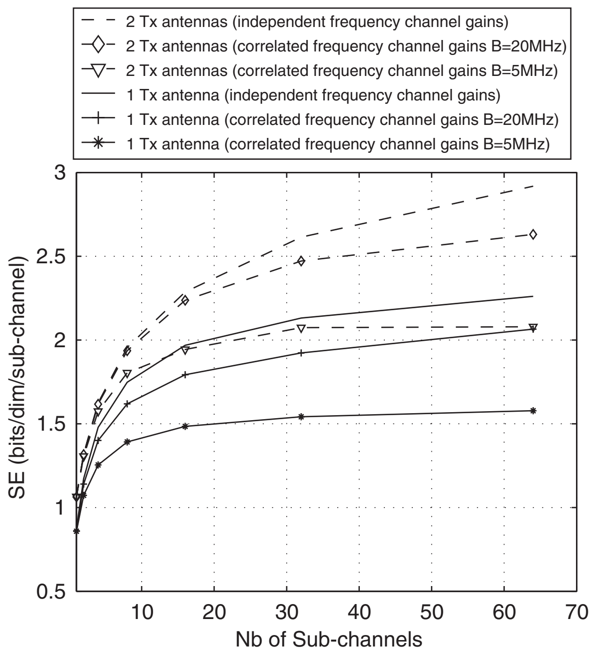

Fig. 9. *System1* SE for different bandwidth values with 1 and 2 Tx antennas, 1 Rx antenna, SNR $= 0\,\mathrm{dB}$.

described in [11,21]. Another interesting remark is that the throughput increases with number of antennas in *System1* but decreases in *System2* which is due to the fact that channel variation is reduced by antenna diversity (i.e. the benefits of multiuser diversity are reduced when less channel variation is present. This is due to the channel hardening by the use of multiple-antennas [22]).

The remainder of the paper will focus on *System1* since we are interested in increasing the system throughput. Fig. 9 shows the spectral efficiency of this system on a frequency selective channel with correlated frequency channel gains for different values of the system bandwidth and with 1 and 2 antennas at the transmission side. The SE with independent frequency channel gains is given for comparison. This figure confirms the results highlighted in the previous section for single antenna transmission. Bandwidth plays an important role on how much scheduling users on sub-channels can increase SE. We note that the benefit from using multiple antennas at transmission can be limited by the amount of bandwidth. For example, for a large number of sub-carriers, a single antenna 20 MHz system can outperform a double antenna 5 MHz system.

4.2. Multiple-antenna receivers

In this section we consider the use of multiple antennas at the receiver (mobile) side and we limit our study to *System1* (spatial multiplexing). We assume that each user has $N_\mathrm{r} > 1$ receiving antennas. The signal to interference plus noise ratio (SINR) corresponding to the signal received by user k from antenna n_t on sub-channel m is [23]

$$\gamma_k(m, n_\mathrm{t}) = \frac{(P/N_\mathrm{t})|\underline{w}_{k,m,n_\mathrm{t}}\underline{H}_{k,m,n_\mathrm{t}}|^2}{\underline{w}_{k,m,n_\mathrm{t}}(\sum_{n'_\mathrm{t}\neq n_\mathrm{t}}(P/N_\mathrm{t})\underline{H}_{k,m,n'_\mathrm{t}}\underline{H}^H_{k,m,n'_\mathrm{t}} + N_0.\mathscr{I})\underline{w}^H_{k,m,n_\mathrm{t}}},$$

where $\underline{H}_{k,m,n_\mathrm{t}} = [H_{k,m}[n_\mathrm{t}, 1], \ldots, H_{k,m}[n_\mathrm{t}, N_\mathrm{r}]]^\mathrm{T}$ is the n_tth column vector of the channel matrix $\mathscr{H}_{k,m}$, $\underline{w}_{k,m,n_\mathrm{t}}$ is a weight vector performing spatial combining and $\mathscr{I}$ is the identity matrix.

In the case of maximum ratio combining, the expression of the weight vector is given by

$$\underline{w}_{k,m,n_\mathrm{t}} = \underline{H}^H_{k,m,n_\mathrm{t}}.$$

For MMSE, the filter for the detection of x_k is given by

$$\underline{w}_{k,m,n_\mathrm{t}} = \left(\frac{\sqrt{P/N_\mathrm{t}}}{\mu_k}\Sigma_k^{-1}\underline{H}_{k,m,n_\mathrm{t}}\right)^H,$$

where $\Sigma_k^{-1} = N_0\mathscr{I} + (P/N_\mathrm{t})\sum_{n'_\mathrm{t}\neq n_\mathrm{t}}\underline{H}_{k,m,n'_\mathrm{t}}\underline{H}^H_{k,m,n'_\mathrm{t}}$ and where $\mu_n = (P/N_\mathrm{t})\underline{H}^H_{k,m,n_\mathrm{t}}\Sigma_k^{-1}\underline{H}_{k,m,n_\mathrm{t}}$.

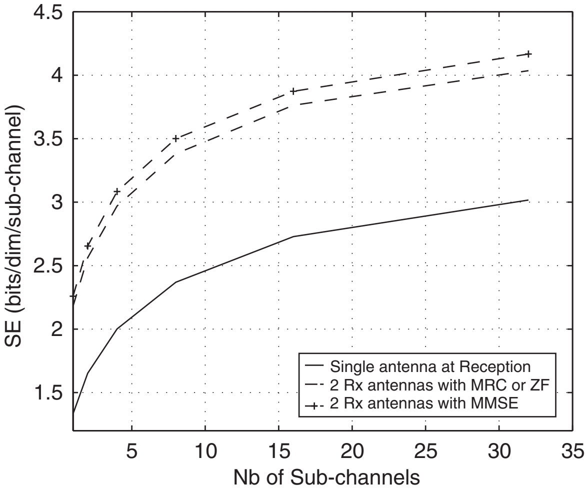

Fig. 10. Reception techniques comparison with MMA algorithm (4 Tx antennas, SNR $= 0$ dB, Correlated frequency channel gains).

The filter in the MMSE receiver requires the estimation of the channel gains from interfering antennas and takes advantage of this to mitigate interference.

Numerical results: Fig. 10 shows the system SE for a MIMO system using spatial multiplexing (*System1*) with different receivers (MMSE and MRC). We consider a system with 4 transmitting antennas at the base station and each receiver has $N_r = 2$ antennas. The allocation of sub-channels and transmitting antennas is made according to the MMA policy. We assume again a frequency-selective channel with correlated frequency channel gains resulting from a maximum path delay $\tau_{max} = 2\,\mu s$ and an exponentially-decaying multipath intensity profile. The system bandwidth is assumed equal to $B = 20$ MHz. This figure highlights the SE gain that can be reached by using multiple antennas at the reception. Concerning the reception techniques comparison, as expected, MMSE receiver takes advantage from the knowledge of the interferer channel gains and yields slightly better performance than MRC.

5. Conclusion

In this paper we treated multiuser allocation algorithms for multi-carrier systems. We proposed an algorithm that performs the Max–Min Fair allocation criterion described in [16] and have shown that even under hard fairness constraints, we can achieve performance close to those of optimal unfair allocation. These results are pertinent for any type of system for which bandwidth can be allocated to a large population of users in a centralized fashion. This could be, for instance for wideband OFDMA systems or potentially future systems allocating users with multiple radio-interfaces across large portions of radio spectrum using potentially different radio-access technologies. The results are generalized to MIMO transceivers. We proposed an extension of the MMA algorithm to the multi-carrier multiple-antenna downlink transmissions and showed that spatial multiplexing and interference mitigation in addition to multiuser-diversity can also be achieved through similar allocation algorithms. We showed also the gain of using multiple antennas at the receiver. In this paper, we assumed that all users have the same traffic load. An extension of this work would be to investigate inter-layer scheduling techniques taking into account the traffic load of different users [4,24].

Appendix A

A.1. Searching for an augmenting path

The following is a pseudo code for searching for an augmenting path:

```
assign all vertices as unvisited;
create an empty vertex queue Q;
AP_found:=false;
while an exposed u in U is unvisited
  andnotAP_foundloop
  add u to Q;
  visit u and record u as start vertex;
  while Q not empty loop
    remove v from (front of) Q;
    for each w adjacent to v loop
      if w unvisited then
        predecessor(w):=v;
        if w is exposed then
          record w as end vertex;
          AP_found:=true;
          exit;
        else
          visit w;
          add w's mate to (rear of) Q;
        end if;
      end if;
    end loop;
end loop;
end loop;
```

A.2. Augmenting the matching along an augmenting path

Let $G = (X, Y, E)$ be a bipartite graph and $\mathscr{M}at$ is a matching (not maximum) in G. Consider $\mathscr{P}at$ to be an augmenting path for $\mathscr{M}at$. The augmenting path must have $2k + 1$ edges for some k. We can form a matching $\mathscr{M}at'$ of size $card(\mathscr{M}at) + 1$ by augmenting along $\mathscr{P}at$ as follows:

- Initially let $\mathscr{M}at' = \mathscr{M}at$.
- Remove from $\mathscr{M}at'$ the k edges on the augmenting path $\mathscr{P}at$ belonging to $\mathscr{M}at$.
- Add to $\mathscr{M}at'$ the $k + 1$ edges on the augmenting path $\mathscr{P}at$ not belonging to $\mathscr{M}at$.

Appendix B

In this section we present the extension of the MMA algorithm to the general case of an arbitrary number of users. Let consider a system with K users sharing M parallel sub-channels (As in Section 3, all the reasoning and results are valid for both uplink and downlink). The Max–Min assignment problem can be formulated as

$$\max_{S_k} \min_{k} \quad \sum_{m \in S_k} \log\left(1 + \frac{P_{k,m}.H_{k,m}}{N_0}\right) \tag{14}$$

$$\text{s.t.} \quad \sum_{k=0}^{K-1} \sum_{m=1}^{M} P_{k,m} \leqslant P_{\max}, \tag{15}$$

$$P_{k,m} \geqslant 0 \quad \text{for all } k, m,$$

$$S_0 S_2, \ldots, S_{K-1} \quad \text{are disjoint},$$

$$S_0 \cup S_2 \cup \ldots \cup S_{K-1} = \{1, 2, \ldots, M\},$$

where $H_{k,m}$ is the channel gain of user k on sub-channel m, S_k is the set of indices of sub-channels assigned to user k, and $\mathscr{P}at_{k,m}$ is the power assigned to user k on sub-channel m. We assume that $\mathscr{P}at_{k,m}$ is either 0 or $\mathscr{P}at$ and that

$$\sum_{k=0}^{K-1} P_{k,m} = P \quad \text{for all } m \in \{1, 2, \ldots, M\}$$

which means that only one user can transmit in each sub-channel and that the total transmit power per sub-channel is the same.

Regarding the complexity of searching the optimal solution to Eq. (14), we present in the following a sub-optimal solution which has performance close to those of the optimal one obtained by an exhaustive search.

In this allocation, we proceed in two steps. We first allocate one sub-channel to each user and then use the remaining sub-channels to improve the amount of rate allocated to the users with the worst channel gains. As in Section 3.2, we begin by constructing the graph $G = (X, Y, E)$ corresponding to the system. We have now $|X| \leqslant |Y|$, so we call a perfect matching each maximum matching $\mathscr{M}at$ s.t. $card(\mathscr{M}at) = |X|$. Not all vertices of Y are incident to an edge of a perfect matching any more.

An important thing to notice is that, this algorithm is optimal for a K-user system with K parallel sub-channels.

B.1. Algorithm description

(1) *Initialization*
- We first begin by constructing the graph $G = (X, Y, E)$ corresponding to our system as described previously.
- The objective of our algorithm is to find the matching $\mathscr{M}at_{i^*}$ such that $i^* = \arg\max_{i=1,\ldots,K!} \min_{k=0,\ldots,K-1} H_{C_{\mathscr{M}at_i}(k),k}$. If we consider the K^2 order statistics of the channel gains from the minimum to the maximum, then we have that

  $$\text{ord}\left(\min_{k=0,\ldots,K-1} H_{k,C_{\mathscr{M}at_i}(k)}\right) \geqslant K,$$

  thus, we remove the $K - 1$ edges with the minimum weights from E,

  $$E = E - \{(x, y)/\text{ord}(w(x, y)) < K - 1\}.$$

- Then we construct an arbitrary perfect matching $\mathscr{M}at_{\text{p}}$ corresponding to an arbitrary allocation.
- set $R_x = 0$ for all $x = 0, \ldots, K - 1$ and $A = \{1, \ldots, M\}$.

(2) *Iteration*

(a) Find, in graph G, the edge (x^*, y^*) with the minimum weight (i.e. the link with minimum channel gain)

$$(x^*, y^*) = \arg\min_{(x,y) \in X \times Y} \{w(x, y)\}.$$

(b)
- If $(x^*, y^*) \notin \mathscr{M}at_{\text{p}}$, then remove the edge (x^*, y^*) from the graph (i.e. $E = E - (x^*, y^*)$) and go to (a).
- If $(x^*, y^*) \in \mathscr{M}at_{\text{p}}$, then let $\mathscr{M}at'_{\text{p}} = \mathscr{M}at_{\text{p}} - (x^*, y^*)$ and

- if $\mathscr{M}at'_{\mathrm{p}}$ admits an augmenting path $\mathscr{P}at$, then set $\mathscr{M}at_{\mathrm{p}}$ to the result of augmenting $\mathscr{M}at'_{\mathrm{p}}$along $\mathscr{P}at$, and go to (a).
- If $\mathscr{M}at'_{\mathrm{p}}$ admits no augmenting path, then we allocate sub-channel y^* to user x^*, set $R_{x^*} = w(x^*, y^*)$ and $A = A - y^*$, remove vertices x^* and y^* and all edges connected to them, set $\mathscr{M}at'_{\mathrm{p}} = \mathscr{M}at$ and go to (a).

(c) When each user is assigned one sub-channel go to (3).

(3) *Amelioration*
While $A \neq \emptyset$ {
- Findk^* satisfying $R_{k^*} \leqslant R_k$ for all $k \in 0, \ldots, K-1$.
- Find m^* satisfying $H_{k^*,m^*} \geqslant H_{k^*,m}$ for all $m \in A$.
- set $R_{k^*} = R_{k^*} + \log(1 + \frac{P.H_{k^*,m^*}}{N_0})$ and $A = A - m^*$.

}

(4) Stop the algorithm.

B.2. Numerical results

Fig. 11 shows the minimum allocated rate of all users as a function of the SNR for a system with 5 users and 8 sub-channels. This choice of parameters is completely arbitrary. This figure shows that our proposed algorithm performs almost as well as the exhaustive search based allocation.

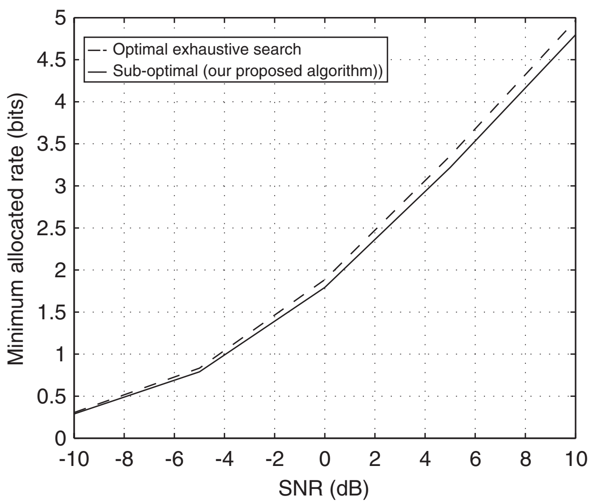

Fig. 11. The minimum of all user rates vs SNR with the optimal exhaustive search and our sub-optimal proposed allocation for system with 5 users and 8 sub-channels.

References

[1] G.J. Foschini, M.J. Gans, On limits of wireless communications in a fading environment when using multiple antennas, Wireless Pers. Commun. 6 (March 1998) 311–335.

[2] E. Telatar, Capacity of multiantenna Gaussian channels, AT&T Bell Laboratories, Tech. Memo., June 1995.

[3] G. Caire, S. Shamai, On the achievable throughput of a multi-antenna Gaussian broadcast channel, IEEE Trans. Inform. Theory 49 (7) (July 2003) 1691–1706.

[4] H. Boche, A. Jorswieck, T. Haustein, Channel aware scheduling for multiple antenna multiple access channels, in: Proceedings 2003. The Thirty-Seventh Asilomar Conference, vol. 1, Novenber 2003, pp. 992–996.

[5] M. Airy, S. Shakkottai, R.W. Heath, Spatially greedy scheduling in multi-user MIMO wireless systems, in: Proceedings 2003. The Thirty-Seventh Asilomar Conference, vol. 1, November 2003, pp. 982–986.

[6] R. Knopp, P.A. Humblet, Information capacity and power control in single-cell multiuser communications, in: Proceedings of the IEEE ICC'95, Seattle, WA, June 1995.

[7] R. Knopp, P.A. Humblet, Multiple-accessing over frequency-selective fading channels, in: Proceedings of the IEEE PIMRC'95, Toronto, Ontario, September 1995.

[8] D.N.C. Tse, S.D. Hanly, Multiaccess fading channels. I: Polymatroid structure, optimal resource allocation and throughput capacities, IEEE Trans. Inform. Theory 44 (November 1998) 2796–2815.

[9] S.D. Hanly, D.N.C. Tse, Multiaccess fading channels. II: delay-limited capacities, IEEE Trans. Inform. Theory 44 (November 1998) 2816–2831.

[10] D.N.C. Tse, Transmitter directed, multiple receiver system using path diversity to equitably maximize throughput, patent filed, May 24, 1999.

[11] P. Viswanath, D.N.C. Tse, R. Laroia, Opportunistic beamforming using dumb antennas, IEEE Trans. Inform. Theory 48 (June 2002) 1277–1294.

[12] S.D. Hanly, An algorithm for combined cell-site selection and power control to maximize cellular spread spectrum capacity, IEEE J. Select. Areas Commun. 13 (September 1995) 1332–1340.

[13] R. Yates, C.Y. Huang, Integrated power control and base station assignment, IEEE Trans. Vehicul. Technol. 44 (August 1995) 638–644.

[14] R. Yates, A framework for uplink power control in cellular radio systems, IEEE J. Select. Areas Commun. 13 (September 1995) 1341.

[15] M. Ergen, S. Coleri, P. Varaiya, QoS aware adaptive resource allocation techniques for fair scheduling in OFDMA based BWA systems, IEEE Trans. Broadcasting (December 2003).

[16] R. Knopp, Achieving multiuser diversity under hard fairness constraints, in: Proceedings of the 2002 IEEE International Symposium on Information Theory, 2002, p. 451.

[17] Feasibility Study For OFDM for UTRAN enhancement 3GPP TR 25.892 V1.1.0, 2004-03).

[18] C. Berge, Two theorems in graph theory, Proc. Nat. Acad. Sci. 43 (1957) 842–844.
[19] R.K. Ahuja, T.L. Magnanti, J.B. Orlin, Network Flows, Prentice-Hall, Englewood Cliffs, NJ, 1993.
[20] D. Gesbert, et al., From theory to practice: an overview of MIMO space-time coded wireless systems, IEEE J. Selected Areas Commun. 21 (3) (April 2003) 281–302.
[21] M. Sharif, B. Hassibi, On the capacity of MIMO broadcast channel with partial side information, IEEE Trans. Inform. Theory 51 (2) (February 2005) 506–522.
[22] M. Kobayashi, D. Gesbert, G. Caire, Antenna diversity versus multi-user diversity: quantifying the trade-offs, ISITA 2004, International Symposium on Information Theory and its Applications, October 10–13, 2004, Parma, Italy.
[23] Y. Akyildiz, B.D. Rao, Maximum ratio combining performance with imperfect channel estimates, in: Proceedings of the IEEE ICASSP, vol. 3, 2002, pp. 2485–2488.
[24] E.M. Yeh, An inter-layer view of multiaccess communications, in: Proceeding IEEE ISIT, 2002.