COCHANNEL INTERFERENCE CANCELLATION WITHIN THE CURRENT GSM STANDARD

Hafedh Trigui *Dirk T.M. Slock*
Institut EURECOM, 2229 route des Crêtes, B.P. 193,
06904 Sophia Antipolis Cedex, FRANCE
Tel: +33 93002606 Fax: +33 93002627
email:{trigui,slock}@eurecom.fr

ABSTRACT

The performance of Maximum Likelihood Sequence Estimation (MLSE) is bounded (and often approximated well) by the Matched Filter Bound (MFB). In this paper, we first show how the GMSK modulation of GSM can be linearized and reformulated to have a real symbol constellation, leading to a two-channel aspect due to the in-phase and in-quadrature components of the received signal. More channels can be added by oversampling and/or the use of multiple antennas. We present the MFB for this model in the presence of colored noise (interferers) for MLSE that employs noise correlation information that in general may differ from the true one. Simulation results show that significant interference cancellation is possible, even with one antenna, if the correlation structure of the interference is taken into account properly in the MLSE.

1. LINEARIZATION OF A GMSK SIGNAL

We analyse first GMSK signals. Let $x(t) = e^{j\varphi(t)}$ be a baseband GMSK signal, where

$$\varphi(t) = \frac{\pi}{2}\sum_k a_k \int_{-\infty}^{t} rect\left(\frac{u-kT}{T}\right)*h(u)du = \sum_k a_k \phi(t-kT) \quad (1)$$

is the continuous modulated phase, $\phi(t)$ is the "phase impulse response", $\{a_k\}$ are the differentially encoded symbols from the original data $d_k \in \{-1,1\}$ and T is the symbol period. The phase impulse response is obtained by integrating a Gaussian filter $h(t) = \frac{1}{\sqrt{2\pi}\sigma T} e^{-\frac{t^2}{2\sigma^2 T^2}}$, so $\phi(t) = \frac{\pi}{2}\int_{-\infty}^{t} rect(\frac{u}{T}) * h(u)du = \frac{\pi}{2}\left[G(u+\frac{1}{2}) - G(u-\frac{1}{2})\right]$ where $\sigma = \frac{\sqrt{ln(2)}}{2\pi BT}$, B is the 3dB bandwith, $BT = 0.3$, and $G(u) = \sigma^2 T\ h(uT) + u\int_{-\infty}^{uT} h(t)dt$. It can be seen in figure 1a that $\phi(t)$ can be approximated by zero for $t < -\frac{3T}{2}$ and $\frac{\pi}{2}$ for $t > \frac{3T}{2}$. Interpreting this figure, we can conclude that one symbol a_k will have an influence on three symbol periods $(k-1, k, k+1)$.

Considering the data $\{d_k\}$, the differential encoder (see figure 2) yields $a_k = d_k d_{k-1} = \frac{d_k}{d_{k-1}}$. The relation between $\{d_k\}$ and $\{a_k\}$ is such that $a_k = 1$ if $d_k = d_{k-1}$ and $a_k = -1$ if $d_k = -d_{k-1}$. This implies that the phase increases by $\frac{\pi}{2}$ over three symbol periods if the symbols at instant k and $k-1$ are equal and decreases by the same quantity in the other case. We have a propagation with memory where some Inter-Symbol Interference (ISI) gets introduced (by the modulation itself).

A sampled GMSK signal can be approximated by a linear

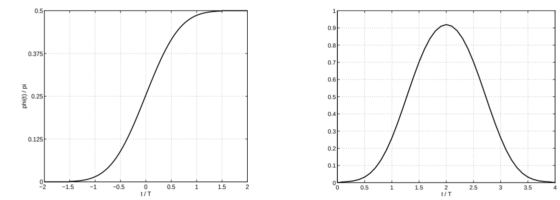

Figure 1. a) Phase impulse response of the GMSK modulation, b) GMSK pulse shape filter.

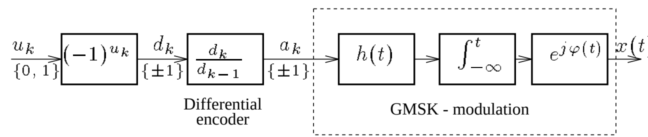

Figure 2. Signal flow diagram for the GMSK modulation.

filter with impulse response duration of about $L_f T = 4T$ (see figure 1b), fed by $b_k = j^k d_k$ ($j = \sqrt{-1}$), a modulated version of the transmitted symbols. Such a scheme (but with a shorter filter) holds exactly when sampled MSK signals are considered (see the Appendix). Exploiting the quasi-bandlimited character of the GMSK signal, we can even approximate the continuous-time GMSK signal by interpolating the output of a discrete-time linear system. To this end, we shall assume that sampling the GMSK signal at rate $\frac{q}{T}$ satisfies the Nyquist theorem. The linear system $\boldsymbol{F}$ that models the oversampled GMSK signal will be determined by least-squares estimation over a long stretch of signal, see figure 3.

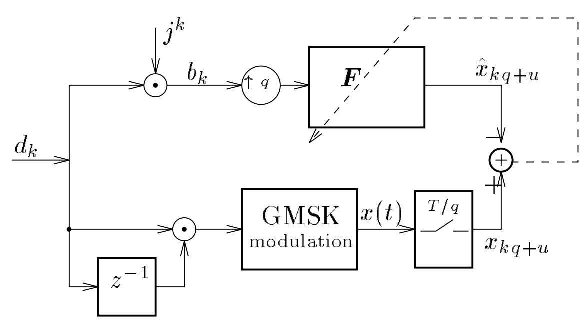

Figure 3. Identification of the GMSK modulation by least-squares.

Let $\{f_{qi+u}\}_{0\leq i\leq L_f-1\,;\,0\leq u\leq q-1}$ be the impulse response of the system $\boldsymbol{F}$ thus identified then

$$x(t_0+kT+u\frac{T}{q}) = x_{kq+u} \approx \hat{x}_{kq+u} = \sum_{i=0}^{L_f-1} f_{qi+u}\, b_{k-i} \quad (2)$$

and after interpolation

$$x(t) \approx \hat{x}(t) = \sum_{k=-\infty}^{+\infty} f(t-kT)b_k \quad (3)$$

where ($f(t)$ is shown in figure 1b)

$$f(t) = \sum_{i=0}^{L_f-1}\sum_{u=0}^{q-1} sinc(q\frac{t-t_0}{T} - qi - u) f_{qi+u}. \quad (4)$$

To have an idea of the quality of this estimation, we plot in figure 4 simultanously the real part of the baseband GMSK signal $x(t)$ and its estimate $\hat{x}(t)$ for $q=6$.

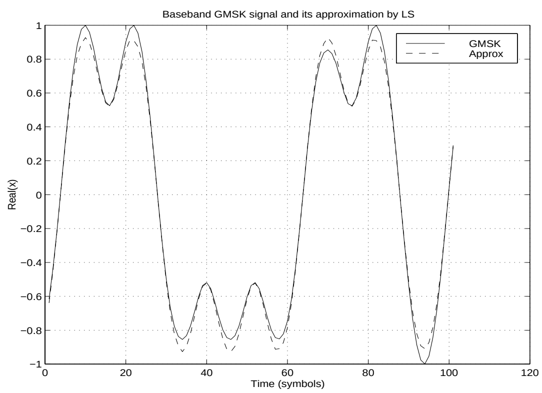

Figure 4. Linearization of a baseband GMSK signal.

It appears that the linearized model is accurate and does not affect the processing to be discussed next in which we assume (as usual) that the modulation is linear.

2. MULTICHANNEL DATA MODEL

Consider the linearized version of the GMSK modulation transmitted over a linear channel with additive noise. The cyclostationary received signal can be written as

$$y(t) = \sum_k h(t-kT)b_k + v(t) = \sum_k h(t-kT)j^k d_k + v(t) \quad (5)$$

where $h(t)$ is the combined impulse response of the modulation $f(t)$ and the channel $c(t)$: $h(t) = f(t) * c(t)$. The channel impulse response $h(t)$ is assumed to be FIR with duration NT. If K sensors are used and each sensor waveform is oversampled at the rate $\frac{p}{T}$, the discrete-time input-output relationship at the symbol rate can be written as:

$$\begin{aligned} \mathbf{y}_k &= \sum_{i=0}^{N-1}\mathbf{h}_i b_{k-i} + \mathbf{v}_k = \mathbf{H}_N B_N(k) + \mathbf{v}_k\,, \\ \mathbf{y}_k &= \begin{bmatrix} y_{1,k} \\ \vdots \\ y_{m,k}\end{bmatrix}, \mathbf{v}_k = \begin{bmatrix} v_{1,k} \\ \vdots \\ v_{m,k}\end{bmatrix}, \mathbf{h}_k = \begin{bmatrix} h_{1,k} \\ \vdots \\ h_{m,k}\end{bmatrix} \quad (6) \\ \mathbf{H}_N &= [\mathbf{h}_0 \cdots \mathbf{h}_{N-1}]\,, B_N(k) = \left[b_k^H \cdots b_{k-N+1}^H\right]^H \end{aligned}$$

where the first subscript i denotes the i^{th} channel, $m = pK$, and superscript ^{H} denotes Hermitian transpose. We have introduced the p phases of the K oversampled antenna signals: $y_{(n-1)p+l,k} = y_n(t_0 + (k+\frac{l}{p})T)$, $n = 1,\ldots,K$, $l = 1,\ldots,p$ where $y_n(t)$ is the signal received by antenna n.

The propagation environment is described by a channel $c(t) = \left[c_1^H(t)\cdots c_K^H(t)\right]^H$. In the simulations, we shall consider GSM channels models [1] which are taken as specular multipath channels with L_c paths of the form $c_n(t) = \sum_{r=1}^{L_c} a_{r,n}\delta(t-\tau_{r,n})$ for the n^{th} antenna. $a_{r,n}$ and $\tau_{r,n}$ are the amplitude and the delay of path r. The distribution of the amplitudes and the values of the delays depend on the propagation environment. We consider independent channel realizations for the K antennas. The received signal for antenna n can be written as

$$y_n(t) = \sum_{k=-\infty}^{+\infty} h_n(t-kT)b_k\,,\quad h_n(t) = \sum_{r=1}^{L_c} a_{r,n} f(t-\tau_{r,n}) \quad (7)$$

where $f(t)$ is specified in (4). The continuous-time channel $h_n(t)$ for the n^{th} antenna when sampled at the instant $t_0 + (k+\frac{l}{p})T$ yields the $((n-1)p+l)^{th}$ component of the vector $\mathbf{h}_k$. In order to obtain a causal discrete-time channel model, we choose q as a multiple of p.

The constellation for the symbols b_k, the inputs to the discrete-time multichannel, is complex whereas the constellation for the symbols d_k is real. It will be advantageous to express everything in terms of real quantities and in this way double the number of (fictitious) channels. To that end we downsample with a factor two and introduce another polyphase decomposition with two phases (even and odd):

$$\begin{cases} \mathbf{y}_{2k} = \sum_n \mathbf{h}_{2k-2n} j^{2n} d_{2n} + \sum_n \mathbf{h}_{2k-2n+1} j^{2n+1} d_{2n+1} \\ \mathbf{y}_{2k+1} = \sum_n \mathbf{h}_{2k+1-2n-1} j^{2n+1} d_{2n+1} + \sum_n \mathbf{h}_{2k+1-2n} j^{2n} d_{2n} \end{cases} \quad (8)$$

To differentiate the two fictitious channels, we introduce the following notation:

$$\begin{array}{lll} \mathbf{y}_{1,k} = \mathbf{y}_{2k} & \mathbf{h}_{1,k} = \mathbf{h}_{2k} & b_{1,k} = j^{2k} d_{2k} = (-1)^k d_{2k} \\ \mathbf{y}_{2,k} = \mathbf{y}_{2k+1} & \mathbf{h}_{2,k} = \mathbf{h}_{2k+1} & b_{2,k} = j^{2k} d_{2k+1} = (-1)^k d_{2k+1} \end{array} \quad (9)$$

Thus, we obtain the following two inputs $2m$ outputs system with real inputs and complex outputs:

$$\begin{cases} \mathbf{y}_{1,k} = \sum_n \mathbf{h}_{1,k-n} b_{1,n} + j\sum_n \mathbf{h}_{2,k-n-1} b_{2,n} \\ \mathbf{y}_{2,k} = \sum_n \mathbf{h}_{2,k-n} b_{1,n} + j\sum_n \mathbf{h}_{1,k-n} b_{2,n} \end{cases} \quad (10)$$

Decomposing the complex quantities into their real and imaginary parts like

$$\begin{cases} \mathbf{y}_{i,k} = \mathbf{y}_{i,k}^R + j\mathbf{y}_{i,k}^I \\ \mathbf{h}_{i,k} = \mathbf{h}_{i,k}^R + j\mathbf{h}_{i,k}^I \end{cases} \quad i = 1,2 \quad (11)$$

we obtain the data model

$$\begin{bmatrix} \mathbf{y}_{1,k}^R \\ \mathbf{y}_{1,k}^I \\ \mathbf{y}_{2,k}^R \\ \mathbf{y}_{2,k}^I \end{bmatrix} = \begin{bmatrix} H_1^R(q) & -q^{-1}H_2^I(q) \\ H_1^I(q) & q^{-1}H_2^R(q) \\ H_2^R(q) & -H_1^I(q) \\ H_2^I(q) & H_1^R(q) \end{bmatrix} \cdot \begin{bmatrix} b_{1,k} \\ b_{2,k} \end{bmatrix} + \begin{bmatrix} \mathbf{v}_{1,k}^R \\ \mathbf{v}_{1,k}^I \\ \mathbf{v}_{2,k}^R \\ \mathbf{v}_{2,k}^I \end{bmatrix} \tag{12}$$

where q is the advance operator: $q\,\mathbf{y}_{1,k} = \mathbf{y}_{1,k+1}$, and e.g. $H_1^R(z)$ is the z-transform of $\mathbf{h}_{1,k}^R$. This is a two inputs $4m$ outputs system with only real quantities, and a structurally constrained transfer matrix with impulse response of length $N' = \lceil \frac{N}{2} \rceil$. We can represent this system more conveniently in the following obvious notation

$$\mathbf{y}'_k = [\mathbf{H}_1(q)\;\; \mathbf{H}_2(q)] \cdot \begin{bmatrix} b_{1,k} \\ b_{2,k} \end{bmatrix} + \mathbf{v}'_k \,. \tag{13}$$

where $\mathbf{H}_i(z) = \sum_{k=0}^{N'-1} \mathbf{h}'_{i,k} z^{-k}$.

3. MATCHED FILTER BOUND

The MFB bounds the probability of error (P_e) in the sense that the P_e (of e.g. MLSE) is bounded below by the P_e of an AWGN channel with the MFB as SNR. MFBs in the presence of interferers are analyzed in [2]. The MFB expressions below will be derived in the case of burst (packet) transmission. Since in that case the MFB is symbol-dependent [3], we will consider the average MFB over the burst. Moreover, considering the data model (13) we have a MFB for each subsequence of the symbols. We will see, however, that the two MFBs are asymptotically equal. Assume we receive M samples:

$$\mathbf{Y}'_M \;=\; \mathcal{T}_M(\mathbf{H}_1)\, B_{1,M+N'-1} + \mathcal{T}_M(\mathbf{H}_2)\, B_{2,M+N'-1} + \mathbf{V}'_M \tag{14}$$

where $\mathbf{Y}'_M = [\mathbf{y}_M^{'H} \cdots \mathbf{y}_1^{'H}]^H$ and similarly for $\mathbf{V}'_M$, and $\mathcal{T}_M(\mathbf{H}_i)$ is a block Toeplitz matrix with M block rows and $[\mathbf{h}'_{i,0} \cdots \mathbf{h}'_{i,N'-1} \quad 0_{4m\times(M-1)}]$ as first block row. Let $R_{vv} = \mathrm{E}\, \mathbf{V}'_M \mathbf{V}_M^{'H}$. We shall consider the MFB for MLSE in which the noise covariance matrix is assumed to be $\widehat{R}_{vv}$ whereas its actual value is R_{vv}. It can be shown that this MFB for the subsequence i can be written as

$$\begin{aligned} MFB^{(i)} &= \frac{1}{M+N'-1} \sum_{j=1}^{M+N'-1} MFB_j^{(i)} \\ &= \frac{1}{M+N'-1} \sum_{j=1}^{M+N'-1} \frac{\sigma_b^2 (T_{i,j}^H \widehat{R}_{vv}^{-1} T_{i,j})^2}{T_{i,j}^H \widehat{R}_{vv}^{-1} R_{vv} \widehat{R}_{vv}^{-1} T_{i,j}} \end{aligned} \tag{15}$$

where $T_{i,j}$ is the j^{th} column of $\mathcal{T}_M(\mathbf{H}_i)$. Using Parseval's equality, one can show that the equivalent continuous processing MFB for the subsequence i can be written as (taking $\lim_{M\to\infty}$ of the previous expression)

$$MFB^{(i)} = \frac{\sigma_b^2 \left[\frac{1}{2\pi j} \oint \frac{dz}{z} \mathbf{H}_i^\dagger(z) \widehat{S}_{vv}^{-1} \mathbf{H}_i(z) \right]^2}{\frac{1}{2\pi j} \oint \frac{dz}{z} \mathbf{H}_i^\dagger(z) \widehat{S}_{vv}^{-1} S_{vv} \widehat{S}_{vv}^{-1} \mathbf{H}_i(z)} \tag{16}$$

where $S_{vv}(z)$ is the power spectral density matrix corresponding to (z-transform of) the correlation sequence of $\mathbf{v}'_k$ and $\mathbf{H}_i^\dagger(z) = \mathbf{H}_i^H(1/z^*)$. To show $MFB^{(1)} = MFB^{(2)}$, we shall consider the additive noise to consist of d interferers plus spatially and temporally white noise:

$$S_{vv}(z) = \sigma_b^2 \mathbf{G}(z)\mathbf{G}^\dagger(z) + \sigma_v^2 \mathbf{I} \tag{17}$$

where $\mathbf{G}(z) = [\mathbf{G}_1\;\mathbf{G}_2] = [\mathbf{G}_{1,1} \cdots \mathbf{G}_{1,d}\;\; \mathbf{G}_{2,1} \cdots \mathbf{G}_{2,d}]$ has dimensions $4m \times 2d$ and $\mathbf{G}_{i,k}$ has the same structure as $\mathbf{H}_i$. For MFB considerations, we shall treat the interferers as Gaussian colored noise. It is easy to see that $\mathbf{H}_2(z) = \mathbf{J}\mathbf{H}_1(z)$ where $\mathbf{J} = \begin{pmatrix} 0 & 0 & 0 & -z\mathbf{I}_m \\ 0 & 0 & z\mathbf{I}_m & 0 \\ 0 & -\mathbf{I}_m & 0 & 0 \\ \mathbf{I}_m & 0 & 0 & 0 \end{pmatrix}$. Since the $\mathbf{G}_i(z)$ have the same structure as the $\mathbf{H}_i(z)$ and $\mathbf{J}\mathbf{J}^\dagger = I_{4m}$, we have

$$S_{vv} = \mathbf{J} S_{vv} \mathbf{J}^\dagger \;, \quad \widehat{S}_{vv} = \mathbf{J} \widehat{S}_{vv} \mathbf{J}^\dagger \;. \tag{18}$$

It follows that $MFB^{(1)} = MFB^{(2)} = MFB(SNR, SIR)$.

4. INTERFERENCE CANCELLATION PERFORMANCE BOUNDS

In this section we consider $\widehat{S}_{vv}(z) = S_{vv}(z)$. We define a SNR w.r.t. the additive white noise in the data model (12) as the average SNR per channel: $SNR = \frac{1}{m} \frac{\sigma_b^2}{\sigma_v^2} \frac{2}{2\pi j} \oint \frac{dz}{z} \mathbf{H}_1^\dagger(z) \mathbf{H}_1(z)$ where we exploited the fact that $\oint \frac{dz}{z} \mathbf{H}_1^\dagger(z)\mathbf{H}_1(z) = \oint \frac{dz}{z} \mathbf{H}_2^\dagger(z)\mathbf{H}_2(z)$. We shall take the interferers to be of equal strength: $\oint \frac{dz}{z} \mathbf{G}_{i,k}^\dagger(z)\mathbf{G}_{i,k}(z) = \oint \frac{dz}{z} \mathbf{G}_{1,1}^\dagger(z)\mathbf{G}_{1,1}(z)$, $i = 1, 2$, $k = 1, \ldots, d$ and we can define the $SIR = \oint \frac{dz}{z} \mathbf{H}_1^\dagger(z)\mathbf{H}_1(z) / \oint \frac{dz}{z} \mathbf{G}_{i,k}^\dagger(z)\mathbf{G}_{i,k}(z)$. With $\widehat{S}_{vv}(z) = S_{vv}(z)$, we then get from (16) $MFB(SNR, SIR) = \frac{\sigma_b^2}{2\pi j} \oint \frac{dz}{z} \mathbf{H}_1^\dagger(z) S_{vv}^{-1} \mathbf{H}_1(z)$. If the multiple users would be detected jointly, a bound on the detection performance for the user of interest is obtained by assuming that the interferers are detected perfectly (in which case their signal contribution can be cancelled perfectly). This leads to $MFB_{JD} = MFB(SNR, \infty) = \frac{m}{2} SNR$. Here we consider single-user detection, treating the interferers as colored Gaussian noise. $MFB(SNR, SIR)$ is then the MFB for MLSE in which we take the correlation of the interferers and noise into account properly. By doing so, the multichannel aspect allows for some interference suppression. In order to get a feeling for how much interference suppression is possible, we compare to MFB_{JD} and introduce

$$MFB_{rel} = \frac{MFB(SNR, SIR)}{MFB(SNR, \infty)} = \frac{\sigma_v^2 \oint \frac{dz}{z} \mathbf{H}_i^\dagger(z) S_{vv}^{-1} \mathbf{H}_i(z)}{\oint \frac{dz}{z} \mathbf{H}_i^\dagger(z) \mathbf{H}_i(z)} \,. \tag{19}$$

It can be shown that in general

$$\sin^2\theta = 1 - \frac{\oint \frac{dz}{z} \mathbf{H}_i^\dagger(z) P_{\mathbf{G}(z)} \mathbf{H}_i(z)}{\oint \frac{dz}{z} \mathbf{H}_i^\dagger(z) \mathbf{H}_i(z)} \;\leq\; MFB_{rel} \;\leq\; 1 \tag{20}$$

where $P_{\mathbf{G}(z)} = \mathbf{G}(z) \left(\mathbf{G}^\dagger(z) \mathbf{G}(z) \right)^{-1} \mathbf{G}^\dagger(z)$, and θ is the angle between the (subspace spanned by the) interferers and the user of interest. The upper bound is attained as $\frac{SIR}{SNR} \to \infty$ (even if $\widehat{S}_{vv} \neq S_{vv}$, as long as $\widehat{S}_{vv}$ also converges to a multiple of the identity matrix) whereas the lower bound is attained as $\frac{SIR}{SNR} \to 0$. In this latter case, we can only cancel the part of the interferer which is orthogonal to the user of interest. Nevertheless, this shows that when the user of interest is roughly orthogonal to the strong interferers, something that is more likely to happen when more channels are available, then multichannel MLSE which takes the correlation of the noise and interferers into account leads to limited performance loss compared to joint detection, regardless of the strength of the interferers.

5. SIMULATIONS

We consider three typical GSM propagation environments: Rural Area (RU), Hilly Terrain (HI) and Typical Urban (TU). For each environment, we consider the 6-tap ($L_c = 6$) statistical channel impulse responses as specified in the ETSI standard [1] and this for both the user of interest and the interferers. Rural area (RU), Hilly Terrain (HI) and Typical Urban (TU). The channels for multiple antennas are taken independently. We show MFB curves as a function of SIR for a fixed $SNR = 40$dB. The MFB curves are averaged over 50 realizations of the channel impulse responses of the user of interest and the interferer(s), according to one of the three statistical channel models, with the channel response of the interferers being delayed independently w.r.t. to that of the user of interest by a delay that was taken uniform over $[0, T]$. Apart from the optimal way of taking the multichannel correlation into account, we also consider the performance of suboptimal MLSE receivers that only take "spatial" correlation into account and neglect the temporal correlation of the interferer after downsampling, or before downsampling, or even between in-phase and in-quadrature components ($\widehat{R}_{vv}$ only contains the $4m \times 4m$, $2m \times 2m$ or $m \times m$ diagonal blocks of R_{vv}), or that go as far as treating the interference plus noise as temporally and spatially white circular noise ($\widehat{R}_{vv}$ is a multiple of identity, the multiple being the average value of the diagonal elements of R_{vv}). We show in solid line the MFB when $\widehat{R}_{vv} = R_{vv}$ and respectively in dashline, dashdot, plus and star the case where $\widehat{R}_{vv}$ is diagonal or block-diagonal (with the $4m \times 4m$, $2m \times 2m$ or $m \times m$ diagonal blocks of R_{vv}). We consider one interferer and three combinations of multiple antennas and oversampling. We also consider the scenario of multiple interferers for a Typical Urbain channel and two antennas.

6. CONCLUSIONS

The simulations show that whenever the number of channels is larger than the number of users, then the suboptimal single-user detector which takes the (spatio-temporal) correlation structure of the interferers correctly into account suffers a bounded MFB loss (as the interferers get arbitrarily strong) w.r.t. to the more complex joint detection scheme. The simulations show that the exploitation of the in-phase and in-quadrature components allows to double the number of channels, with full diversity. Multiple channels obtained by oversampling also lead to bounded loss, be it much larger though. Due to the bandlimited nature of GMSK signals, the diversity obtained by oversampling is limited. The simulations show that, in the case of one interferer, one antenna and no oversampling, on the average the loss is bounded by 5dB. This interference reduction capacity is due exclusively to the two-channel aspect created by exploiting the real aspect of the symbol constellation after an appropriate reformulation of GMSK. This loss can be reduced to 2dB by using two antennas (with complete spatial diversity). The results also show that not taking the noise correlation into account properly (esp. $\widehat{R}_{vv} \sim I$, as is done in current GSM receivers) leads to the absence of robustness to interference, while taking only spatial correlation into account ($2m \times 2m$ blocks) leads to very limited robustness. It should be noted that the losses shown here are averaged over all possible user and interferer configurations. The actual loss can be quite a bit less most of the time.

REFERENCES

[1] European Telecommunications Standards Institute. European digital cellular telecommunications system (phase 2): Radio transmission and reception (GSM 05.05). Technical report, ETSI, Dec. 1995. Sophia Antipolis, France.

[2] D.T.M. Slock and H. Trigui. An Interference Cancelling Multichannel Matched Filter. In *Proc. Communication Theory Mini-Conference (Globecom)*, Nov. 1996. London, England.

[3] D.T.M. Slock and E. De Carvalho. Matched Filter Bounds for Reduced-Order Multichannel Models. In *Proc. Communication Theory Mini-Conference (Globecom)*, Nov. 1996. London, England.

APPENDIX: LINEARIZATION OF MSK SIGNALING

A CPFSK signal is $s(t) = \sqrt{\frac{2\mathcal{E}}{T}} \exp\left(j2\pi f_c t + j\varphi(t)\right)$ where $\mathcal{E}$ is the transmitted symbol energy, and T the symbol duration. The phase of $\varphi(t)$ of a CPFSK signal increases or decreases with time during each symbol duration of T seconds according to $\varphi(t) =$

$$2h \sum_k a_k \frac{\pi}{2} \frac{1}{T} \int_{\infty}^{t} rect\left(\frac{u - kT}{T}\right) du = 2h \sum_k a_k \phi(t - kT) \tag{21}$$

where $rect(x) = \begin{cases} 1 & , \ 0 \leq x \leq 1 \\ 0 & , \ \text{elsewhere} \end{cases}$ and

$$\phi(t) = \frac{\pi}{2T} \int_{-\infty}^{t} rect\left(\frac{u}{T}\right) du = \begin{cases} 0 & , \ t < 0 \\ \frac{\pi}{2}\frac{t}{T} & , \ 0 \leq t \leq T \\ \frac{\pi}{2} & , \ t > T \end{cases} \tag{22}$$

We analyze now the MSK modulation which is defined as CPFSK modulation with $h = 0.5$ as modulation index. Let's write the expression for the phase when $t \in [kT, (k+1)T]$. We get,

$$\varphi(t) = \frac{\pi}{2} \sum_{n=0}^{k-1} a_n + a_k \phi(t - kT) \tag{23}$$

which can be inserted into the expression for $s(t)$ to yield the MSK passband signal

$$y(t) = e^{j\varphi(t)} = j^k d_{k-1}\, e^{j a_k \phi(t-kT)}. \tag{24}$$

Sampling this signal at the instants $t = t_0 + kT + l\frac{T}{m}$ where $t_0 \in (0, T)$, $l = 0 \cdots m-1$ and m is the oversampling factor, we get

$$\begin{aligned} y_{mk+l} &= j^k d_{k-1} e^{j a_k \phi(t_0 + l\frac{T}{m})} = j^k d_{k-1} e^{j d_k d_{k-1} \phi(t_0 + l\frac{T}{m})} \\ &= j^k d_{k-1} (\cos\phi(t_0 + l\tfrac{T}{m}) + j \tfrac{d_k}{d_{k-1}} \sin\phi(t_0 + l\tfrac{T}{m})) \\ &= j^k d_{k-1} \cos\phi(t_0 + l\tfrac{T}{m}) + j^{k+1} d_k \sin\phi(t_0 + l\tfrac{T}{m}) = f_l(q) j^k d_k \end{aligned} \tag{25}$$

where, we used the fact that $d_k d_{k-1} \in \{\pm 1\}$, and

$$f_l(z) = j \sin\phi(t_0 + l\frac{T}{m}) + j\cos\phi(t_0 + l\frac{T}{m})\, z^{-1} \tag{26}$$

is a two-tap linear time invariant filter, and $q^{-1} j^k d_k = j^{k-1} d_{k-1}$.

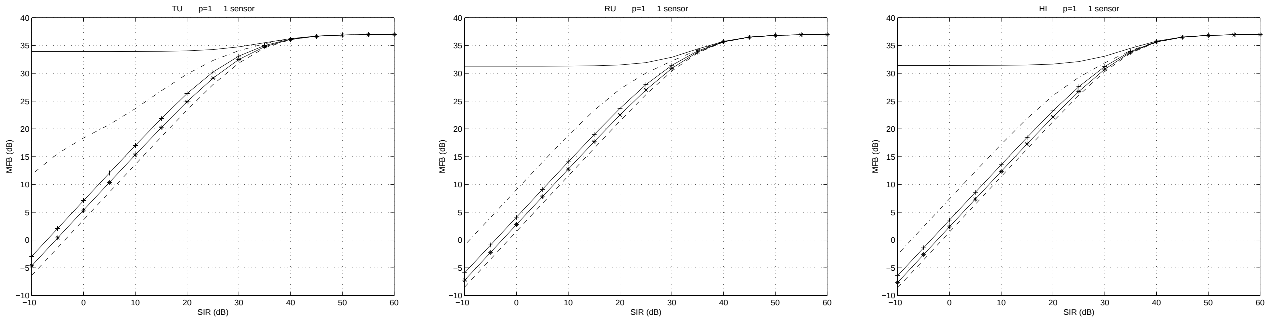

Figure 5. MFB vs SIR for SNR=40dB, one antenna, one interferer and no oversampling.

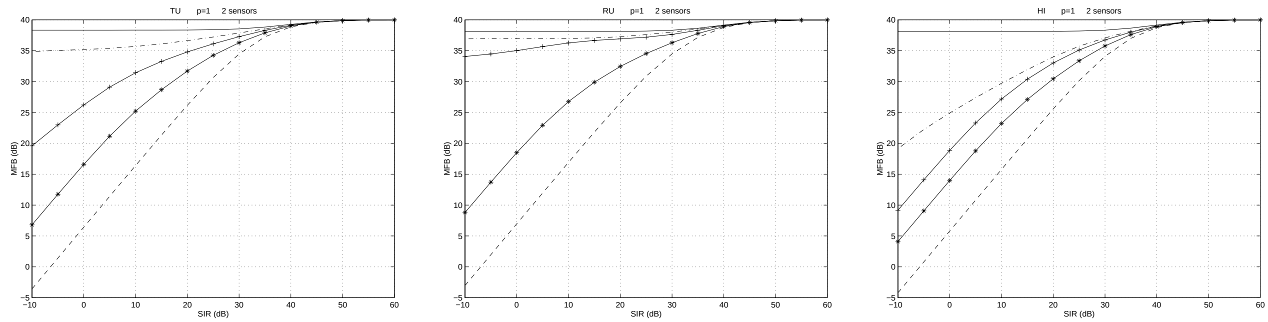

Figure 6. MFB vs SIR for SNR=40dB, two antennas, one interferer and no oversampling.

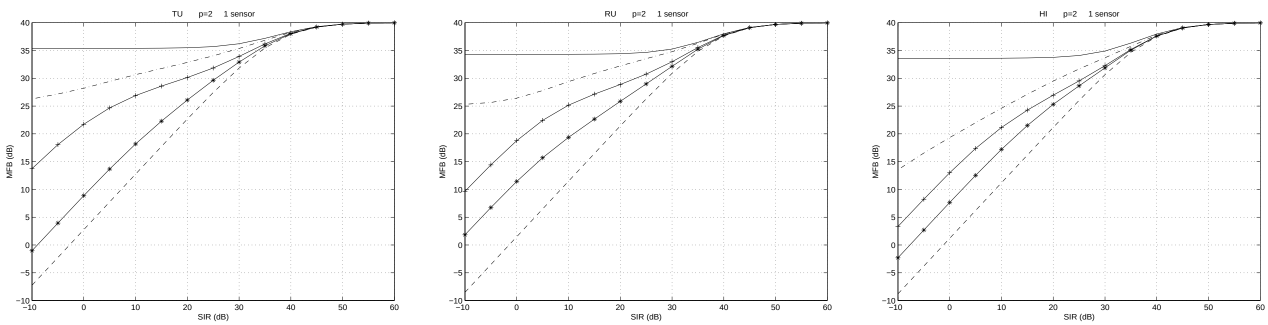

Figure 7. MFB vs SIR for SNR=40dB, one antenna, one interferer and twofold oversampling.

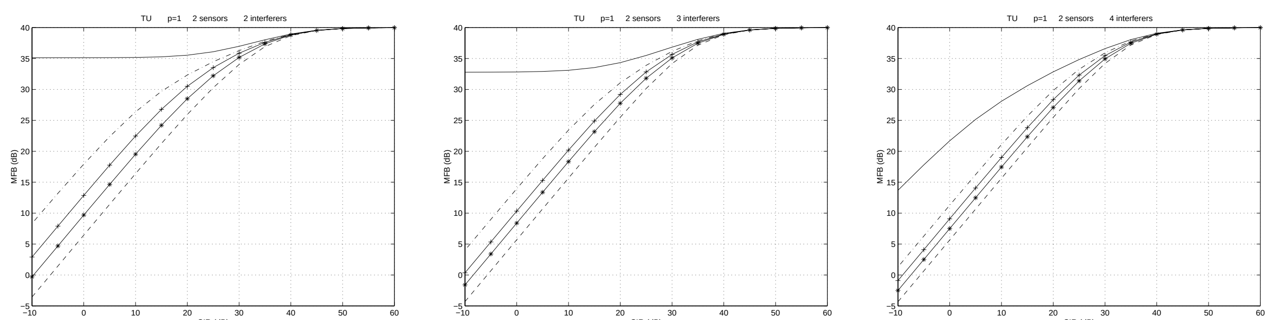

Figure 8. MFB vs SIR for SNR=40dB, two antennas, multiple interferers and no oversampling.