Precoding for Distributed Space-Time Codes in Cooperative Diversity-Based Downlink

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***Abstract*— In this paper, we investigate the cooperative diversity concept for use in MIMO multi-cell networks. We show that, in such networks, cooperative diversity processing must be optimized to account for the variability of channel conditions across the cooperative devices. This can be done via distributed precoding and, in mobile networks, it is based realistically on channel statistics. The cooperative MIMO correlation matrix admits a special structure which is used to optimize the precoder. We investigate algorithms for exact error-rate and low-complexity approximated optimization. Gains are evaluated in multi-cell scenarios with collaborating base stations.**

I. Introduction

MIMO systems yield their best in the case of uncorrelated channel matrix elements. Therefore, interest in MIMO networks has recently focused on scenarios that provide additional dimensions, yielding independent sources of diversity. This includes setups where some or all of the multiple antenna elements of the overall MIMO system are distributed over the network, instead of being localized on a unique device. Prominent examples of this are given by (i) the so-called multiuser multi-cell MIMO, where one [1] or more [2], [3], [4] access points address the data needs of multiple user terminals simultaneously and in a joint fashion and by (ii) the so-called cooperative diversity setup, where multiple devices collaborate to combat the detrimental effects of fading at any one particular device.

Most cooperative diversity scenarios investigated so far include single-antenna user terminals relaying data between a source terminal and the target destination [5], [6], [7]. This includes the use of Space-Time Block Codes (STBC), where the spatial elements of the codewords are distributed over the antennas of the collaborating devices [7], [8], [9].

In this paper, we focus on the cooperative processing using a distributed orthogonal STBC [10]. We ignore the issues associated with the relay protocol or consider that such a protocol is not needed. A practically relevant example is downlink cooperative diversity in a multi-cell network, where multiple base stations collaborate to serve one user terminal.

All transmitters and receivers may be equipped with multiple antennas. Because of the large-scale separation of the collaborating devices, the channel conditions to the common destination (path loss, correlation, etc.) are different. The cooperative diversity scheme ought to be optimized with respect to the channel conditions, and we handle this via distributed precoding, optimized based on channel statistics only (incurs less overhead than instantaneous channels). We make the following contributions:

- The collaborative MIMO network is recast as a point-to-point MIMO system, where the channel second-order statistics admit a special non-Kronecker form.
- Upon feedback of the statistics to the transmitters, a distributed linear precoder is optimized and applied prior to transmission.
- We show that, under realistic conditions, the optimal precoder takes the form of a diagonal precoder, boiling down to a power allocation scheme.
- We express the exact average error probability at the receiver, as function of the precoder and channel statistics.
- We investigate several optimization algorithms for the precoder, including one intuitive equivalent criterion coined the ”maximum diversity criterion”, and provide a justification in the form of a Gaussian approximation of the combined cooperative transmitter channel gain.

II. Signal and Channel Model

We consider a MIMO-system with L spatially distributed base stations (BS), engaged in downlink communication with a single, receiving mobile unit (MU). Each BS has M_{t_l} antennas, $l \in \{0, 1, \dots, L-1\}$, while the receiver is equipped with an array of M_r antennas. In total, there are $M_t = \sum_{l=0}^{L-1} M_{t_l}$ transmit antennas. Fig. 1 illustrates the described scenario.

The synchronized BSs are connected to a central unit (CU) via fast optical links. The CU and BSs only have access to long-term statistical knowledge of the channel conditions, whereas the MU knows the full, instantaneous downlink channel. The channel is assumed to be flat fading, but results are expected to carry over through an OFDM setting.

Thanks to the more generous antenna spacing at the base station side as well as the large inter-cell spacing, we assume that the effect of transmit correlation is negligible, i.e., the outgoing paths from all the M_t antennas are uncorrelated. MU antennas, however, will be correlated, and the amount of correlation is likely to depend on which BS the signal is coming from. Additionally, different BSs may see different

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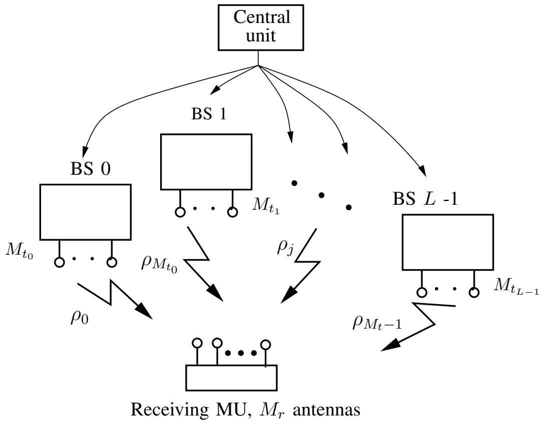

Fig. 1. Distributed MIMO-system, with L transmitters and one receiver.

path loss and slow fading coefficients to the same MU. We consider Rayleigh fading channel conditions.

Although several forms of cooperation schemes are available, including distributed spatial multiplexing, we limit ourselves to the cooperative diversity setup where a space-time code is applied in a distributed manner over the M_t antennas of the L collaborating BSs. We focus on the use of distributed Orthogonal Space-Time Block Codes (OSTBC), as they lend themselves to easier analysis.

To compensate/exploit the effect of unequal path loss and transmit correlation, we apply linear precoding over the cooperating transmit antennas before launching the codeword into the channel. We represent the linear precoder by the $M_t \times B$ matrix $\boldsymbol{F}$, so the $M_r \times N$ received signal $\boldsymbol{Y}$ at the user terminal writes as

$$\boldsymbol{Y} = \boldsymbol{H}\,\boldsymbol{F}\,\boldsymbol{C}(\boldsymbol{x}) + \boldsymbol{V}\,. \tag{1}$$

Here, $\boldsymbol{H}$ represents the overall $M_r \times M_t$ MIMO channel. The $M_r \times N$-sized matrix $\boldsymbol{V}$ represents the additive, complex Gaussian, circularly distributed noise, $v_{ij} \sim \mathcal{CN}(0, \sigma_v^2)$ and $\boldsymbol{x} = [x_0, x_1, \ldots, x_{K-1}]$ is the vector of M-ary modulated symbols. The $B \times N$ matrix $\boldsymbol{C}(\boldsymbol{x})$ is the result of applying a chosen orthogonal design to the vector $\boldsymbol{x}$, related to the chosen generalized complex design as $\boldsymbol{C}(\boldsymbol{x}) = (\mathcal{G}_c^B)^T$ [10]. A block model of the transmit/receive chain is shown in Fig. 2.

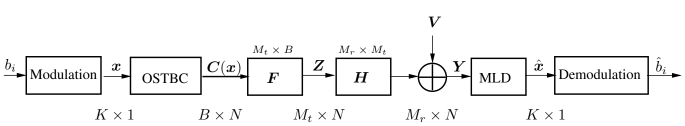

Fig. 2. Block model of a linearly precoded MIMO-system.

The large-scale attenuations associated with *each transmit antenna* are collected in

$$\boldsymbol{P} = \mathrm{diag}\Big(\big[\,\sqrt{\rho_0} \quad \sqrt{\rho_1} \quad \cdots \quad \sqrt{\rho_{M_t-1}}\,\big]\Big) \;:\; M_t \times M_t\,,$$

so that the total channel $\boldsymbol{H} = \boldsymbol{H}_c \boldsymbol{P}$ decomposes into

$$\boldsymbol{H} = \Big[\sqrt{\rho_0}\,\boldsymbol{h}_{c_0} \;\; \sqrt{\rho_1}\,\boldsymbol{h}_{c_1} \;\; \cdots \;\; \sqrt{\rho_{M_t-1}}\,\boldsymbol{h}_{c_{M_t-1}}\Big] \;:\; M_r \times M_t\,.$$

$\boldsymbol{h}_{c_i}$ is the $M_r \times 1$ channel from the i-th transmit antenna to the correlated receiver array. We define $\boldsymbol{R}_{r_i} = E[\boldsymbol{h}_{c_i}\boldsymbol{h}_{c_i}^H]$ as the receive correlation matrix for the i-th transmit path. The transmit antennas are assumed to be uncorrelated, so we write the full correlation of the path-loss normalized MIMO matrix as the $M_t M_r \times M_t M_r$-sized $\boldsymbol{R} = E[\mathrm{vec}(\boldsymbol{H_c})\,\mathrm{vec}^H(\boldsymbol{H_c})]$:

$$\boldsymbol{R} = \begin{bmatrix} \boldsymbol{R}_{r_0} & 0_{M_r\times M_r} & \ldots & \boldsymbol{0}_{M_r\times M_r} \\ \boldsymbol{0}_{M_r\times M_r} & \boldsymbol{R}_{r_1} & \ldots & \boldsymbol{0}_{M_r\times M_r} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{0}_{M_r\times M_r} & \boldsymbol{0}_{M_r\times M_r} & \ldots & \boldsymbol{R}_{r_{M_t-1}} \end{bmatrix}. \tag{2}$$

Importantly, the different transmit antennas experience different correlation matrices upon reception. That is because the signals from different base stations may see a very different angular spread at the terminal. This leads to a practical instance of the non-Kronecker correlation structure evoked recently in [11].

The receiver is assumed to know $\boldsymbol{H}$ perfectly, whereas the L transmitters only have access to long-term statistics in $\boldsymbol{P}$ and $\boldsymbol{R}$, consistent with a practical multi-cell signaling overhead.

For all generalized complex orthogonal designs $\mathcal{G}_c$, we know that [10]

$$(\mathcal{G}_c^{M_t})^T(\mathcal{G}_c^{M_t})^* = a\,(|x_0|^2 + |x_1|^2 + \cdots + |x_{K-1}|^2)\boldsymbol{I}_B\,, \tag{3}$$

where $\boldsymbol{I}_B$ is the identity matrix of size B and the scalar a depends on the choice of orthogonal design. In the following, we let $B = M_t$ and find that $\boldsymbol{C}(\boldsymbol{x})\boldsymbol{C}^H(\boldsymbol{x}) = a\sum_{l=0}^{K-1}|x_l|^2\boldsymbol{I}_{M_t}$.

The symbol-per-period rate of the code is $R = K/N$, and $R < 1$ for all $M_t > 2$.

Now, we define the scalar $\alpha \triangleq \|\boldsymbol{HF}\|_F^2$, where $\|\cdot\|_F$ is the Frobenius norm. From [12], we know that the OSTBC system with a full complex-valued precoder $\boldsymbol{F}$ and a full channel correlation matrix $\boldsymbol{R}$ has an equivalent Single-Input Single-Output (SISO) formulation where the output $y_k^{'}$ for an input $x_k,\ k \in \{0, 1, \ldots, K-1\}$ is

$$y_k^{'} = \sqrt{\alpha}x_k + v_k^{'}\,, \tag{4}$$

That is, every input symbol $x_k,\ \ k \in \{0, 1, \ldots, K-1\}$ experiences the same channel gain $\sqrt{\alpha}$ and independent, additive noise $v_k^{'} \sim \mathcal{CN}(0, \sigma_v^2/a)$.

III. SER Expressions

From the above SISO formulation of the OSTBC, we find that the instantaneous received SNR γ is expressed as

$$\gamma \;\triangleq\; \frac{a\,\sigma_x^2\,\alpha}{\sigma_v^2} = \delta\,\alpha \; = \delta\|\boldsymbol{HF}\|_F^2\,, \;\text{ where }\; \delta = \frac{a\,\sigma_x^2}{\sigma_v^2}\,. \tag{5}$$

Given that all the symbols x_k, where $k \in \{0, 1, \ldots, K-1\}$, go through the same channel, the average symbol error rate (SER) of the MIMO system with OSTBC is [12]

$$\begin{aligned} \mathrm{SER} \triangleq \Pr\{\mathrm{Error}\} &= \int_0^\infty \Pr\{\mathrm{Error}|\gamma\}p_\gamma(\gamma)d\gamma \\ &= \int_0^\infty \mathrm{SER}_\gamma p_\gamma(\gamma)d\gamma\,, \end{aligned} \tag{6}$$

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where $p_\gamma(\gamma)$ is the probability density function (pdf) of γ, while SER_γ is the symbol error probability for a chosen bit-to-symbol mapping, e.g. M-PAM, M-PSK or M-QAM, and a given SNR-value γ as shown per (5). As an example, the SNR_γ for M-PSK is

$$\text{SER}_\gamma = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} e^{-\frac{g_{\text{PSK}}\gamma}{\sin^2(\theta)}}d\theta\,, \quad g_{\text{PSK}} = \sin^2\big(\frac{\pi}{M}\big)\,. \tag{7}$$

IV. Optimal Precoding

We want to find the matrix $\boldsymbol{F}$ such that the exact SER is minimized, under a global peak power constraint for the transmitted block $\boldsymbol{Z} \triangleq \boldsymbol{F}\boldsymbol{C}(\boldsymbol{x})$. Note that this sum power constraint across multiple base stations makes engineering sense. Since, in practice, there will be many users and those are likely to be symmetrically distributed across the cells, each base station should distribute its total power optimally across the cell users.

We normalize with $a\sum_{k=0}^{K-1}\sigma^2_{\boldsymbol{x}_k} = 1$, where $\sigma^2_{\boldsymbol{x}_l}$ is the variance of the k-th symbol in the OSTBC. The peak power constraint is expressed as

$$\text{Tr}\{\boldsymbol{F}\boldsymbol{F}^H\} = P\,, \tag{8}$$

where P is a measure of the average power used. Now, the problem of minimizing the SER can be expressed as

Problem 1:

$$\min_{\{\boldsymbol{F}\in\mathbb{C}^{M_t\times M_t}\,|\,\text{Tr}\{\boldsymbol{F}\boldsymbol{F}^H\}=P\}} \text{SER} \tag{9}$$

Because the transmit antennas are assumed to be uncorrelated (unlike the receive antennas), it can be shown that the search for the optimal precoder $\boldsymbol{F}$ can be limited to a diagonal precoder:

Theorem 1: If (2) holds, the optimal $\boldsymbol{F}$ can be chosen diagonal, with real and non-negative diagonal elements.

Proof: This is an application of the theorem presented in [12].

Note that the authors of that paper did not consider different path loss specifically, but we observe that by building scaled correlation matrices such that $\boldsymbol{R}'_{r_i} = \rho_i\boldsymbol{R}_{r_i}$ their theorem can be made to hold here as well. In other words, the problem of optimal precoding for cooperative diversity is reduced to an optimal allocation of power between the M_t transmit antennas. More importantly, the implementation of this precoder is fully compatible with distributed processing over each cell (rather than multi-cell joint processing).

We now proceed to find the optimal power allocation, first by directly exploiting the SER expression, then, alternatively, by resorting to a modified, simpler criterion.

A. Optimal SER Precoder

A minimization of the SER as it is given in (6) involves the pdf of the instantaneous received SNR γ. To find this function, we begin by developing γ for the case of a diagonal precoder $\boldsymbol{F} = \text{diag}(\,[f_0,\ f_1,\ \ldots\ , f_{M_t-1}]\,)$. We assume a unit power constraint, such that $\sum_{i=0}^{M_t-1} f_i^2 = 1$, i.e., $P = 1$ in (8).

Now, the combined channel gain α may be written out as $\alpha = \sum_{i=0}^{M_t-1}\rho_i f_i^2\|\boldsymbol{h}_{c_j}\|^2$. The receive correlation matrices $\boldsymbol{R}_{r_i}$ have the following eigen-decomposition: $\boldsymbol{R}_{r_i} = \boldsymbol{V}_{r_i}\boldsymbol{\Lambda}_{r_i}\boldsymbol{V}^H_{r_i}$, so that $\boldsymbol{h}_{c_i} = \boldsymbol{R}^{1/2}_{r_i}\boldsymbol{h}_{w_i}$, where $\boldsymbol{h}_{w_i}$ is an M_r-sized vector of complex Gaussian distributed independent variables, $h_{w_{i_j}} \sim \mathcal{CN}(0,1)$. Now, α may be further developed into

$$\begin{aligned}\alpha &= \sum_{i=0}^{M_t-1}\rho_i f_i^2\boldsymbol{h}^H_{w_i}\boldsymbol{V}_{r_i}\boldsymbol{\Lambda}_{r_i}\boldsymbol{V}^H_{r_i}\boldsymbol{h}_{w_i} = \sum_{i=0}^{M_t-1}\rho_i f_i^2(\boldsymbol{h}'_{w_i})^H\boldsymbol{\Lambda}_{r_i}\boldsymbol{h}'_{w_i}\\ &= \sum_{i=0}^{M_t-1}\rho_i f_i^2\sum_{j=0}^{M_r-1}\lambda_{i_j}|h'_{w_{i_j}}|^2\,,\end{aligned} \tag{10}$$

where $\boldsymbol{h}'_{w_i} = \boldsymbol{V}^H_{r_i}\boldsymbol{h}_{w_i} \sim \mathcal{CN}(0,\boldsymbol{I}_{M_r})$ and λ_{i_j} is the j-th eigenvalue of $\boldsymbol{R}_{r_i}$. The SNR γ becomes

$$\gamma = \delta\sum_{i=0}^{M_t-1}\sum_{j=0}^{M_r-1}\rho_i f_i^2\lambda_{i_j}|h'_{w_{i_j}}|^2\,. \tag{11}$$

The instantaneous SNR γ is thus a random variable on the form $\gamma = \sum_{l=0}^{N-1}a_l|Z_l|^2$. When all the coefficients a_l, $l \in \{0,1,\ldots,N-1\}$, are different[1], we can find the exact pdf of γ, and the SER for this scenario. For details, see [13].

Lemma 1: The pdf of γ is

$$p_\gamma(\gamma) = \frac{1}{\delta}\sum_{i=0}^{M_t-1}\sum_{j=0}^{M_r-1}\frac{(\rho_i f_i^2\lambda_{i_j})^{M_tM_r-2}e^{(-\gamma/(\delta\rho_i f_i^2\lambda_{i_j}))}\,u(\gamma)}{\prod\limits_{k\in\{0,M_t-1\}}\prod\limits_{\substack{l\in\{0,M_r-1\}\\(k,l)\neq(i,j)}}(\rho_i f_i^2\lambda_{i_j}-\rho_k f_k^2\lambda_{k_l})}$$

Theorem 2: The exact SER is expressed as

$$\text{SER} = \int_0^\infty\frac{\text{SER}_\gamma}{\delta}\sum_{i=0}^{M_t-1}\sum_{j=0}^{M_r-1}\frac{(\rho_i f_i^2\lambda_{i_j})^{M_tM_r-2}\,e^{(-\gamma/(\delta\rho_i f_i^2\lambda_{i_j}))}}{\prod\limits_{k\in\{0,M_t-1\}}\prod\limits_{\substack{l\in\{0,M_r-1\}\\(k,l)\neq(i,j)}}(\rho_i f_i^2\lambda_{i_j}-\rho_k f_k^2\lambda_{k_l})}d\gamma \tag{12}$$

under the sum power constraint $\sum_{i=0}^{M_t-1} f_i^2 = 1$.

For M-PSK mapping, substitution of SER_γ allows for integration over γ.

$$\begin{aligned}\text{SER} = \frac{1}{\pi}\sum_{i=0}^{M_t-1}\sum_{j=0}^{M_r-1}&\frac{(\rho_i f_i^2\lambda_{i_j})^{M_tM_r-1}}{\prod\limits_{k\in\{0,M_t-1\}}\prod\limits_{\substack{l\in\{0,M_r-1\}\\(k,l)\neq(i,j)}}(\rho_i f_i^2\lambda_{i_j}-\rho_k f_k^2\lambda_{k_l})}\\ &\times\int_0^{\frac{(M-1)\pi}{M}}\frac{1}{1+\frac{g_{\text{PSK}}\delta\rho_i f_i^2\lambda_{i_j}}{\sin^2(\theta)}}d\theta\end{aligned}$$

We note that a *simple*, closed-form expression of the exact SER is not obtainable and also that minimization of the SER with respect to the power values f_i^2 (keeping θ constant), is a difficult task. Nonetheless, it is possible to optimize using the iterative algorithm developed in [12].

Note that the SER depends on the SNR-related coefficient $\delta = a\sigma_x^2/\sigma_v^2$. If we let $\delta \to \infty$, with *non-zero* path-loss ρ_i and

[1] This is the general case, if $\rho_i f_i^2\lambda_{i_j} == \rho_k f_k^2\lambda_{k_l}$, for $(i,j)\neq(k,l)$, the pdf needs to be modified. However, the case of repeated, equal factors can be derived from the general case, by the use of the limit operator.

eigenvalues λ_{i_j}, $\forall(i,j)$, then it can be shown, using the same arguments as in [14], that the optimal power allocation will approach the even distribution $f_i^2 = 1/M_t$.

In general, the optimal power allocation depends on both the receive correlation conditions, the path loss and the average SNR. We now attempt to find the power allocation via another optimality criterion.

B. Closed-Form Precoder for Equal Average SNR

We now assume that slow power control is activated at the transmitters. Then the path loss coefficients are set equal (say $\rho_i = 1,\ i \in \{0,1,\ldots,M_t-1\}$). The correlation matrices remain possibly different. In this case a closed-form precoder can be formulated by generalizing the *maximum diversity principle* introduced in [12], for the case of $M_t = 2$ transmit antennas.

1) The Maximum Diversity Principle: When $\rho_i = 1,\ i \in \{0,1,\ldots,M_t-1\}$, the SNR γ in (11) simplifies to

$$\gamma = \delta \sum_{i=0}^{M_t-1} f_i^2 \sum_{j=0}^{M_r-1} \lambda_{i_j} |h'_{w_{j_i}}|^2 . \tag{13}$$

The SNR γ is a sum of M_tM_r uncorrelated diversity branches, each weighted by $f_i^2\lambda_{i_j}$. In accordance with the proposed maximum diversity principle, we attempt to spread the sum energy of the signal evenly across *all* the diversity branches. This is done by making the weights as equal as possible under a sum power constraint. The mean m of the weights, under this constraint, is

$$m = \frac{\delta}{M_tM_r} \sum_{i=0}^{M_t-1} \sum_{j=0}^{M_r-1} f_i^2\lambda_{i_j} = \frac{\delta}{M_t} \sum_{i=0}^{M_t-1} f_i^2 = \frac{\delta}{M_t} ,$$

where we use that $\mathrm{Tr}\{\boldsymbol{R}_{r_i}\} = M_r,\ i \in \{0,1,\ldots,M_t-1\}$.

We propose that the weights be decided according to the following minimum variance problem.

Problem 2:

$$\min_{\left\{f_i \geq 0,\, i\in\{0,1,\ldots,M_t-1\} \,\middle|\, \sum_{i=0}^{M_t-1} f_i^2 = 1\right\}} \sum_{i=0}^{M_t-1} \sum_{j=0}^{M_r-1} \left(f_i^2\lambda_{i_j} - \frac{\delta}{M_t}\right)^2$$

This problem has a simple closed-form solution, stated in Lemma 2 below and proved in [13].

Lemma 2: Each power weight f_i is found as

$$f_i = \sqrt{\frac{\beta}{\sum_{j=0}^{M_r-1} \lambda_{i_j}^2}} ,\ i \in \{0,1,\ldots,M_t-1\} , \tag{14}$$

where the constant β can be computed by using the sum power constraint $\sum_{i=0}^{M_t-1} f_i^2 = 1$, such that

$$\sum_{i=0}^{M_t-1} \frac{\beta}{\sum_{j=0}^{M_r-1} \lambda_{i_j}^2} = 1 \quad \Longrightarrow \quad \beta = \frac{1}{\sum_{i=0}^{M_t-1} \frac{1}{\sum_{j=0}^{M_r-1} \lambda_{i_j}^2}} .$$

In words, this means that the power is distributed so that a transmit antenna i for which the receive correlation is negligible, i.e. uncorrelated, will receive more transmit power than another antenna that sees a higher level of correlation at the receiver.

From the results in Lemma 2, we notice that, unlike the exact SER, the power allocation after the maximum diversity principle is independent of the SNR. It turns out this criterion is good for low to moderate values of SNR and shows sub-optimality at high SNR.

2) Gaussian Approximation Based Precoder: In this section, we pursue another approach for the optimization of the power allocation. Since γ is sum of M_tM_r weighted, independent random variables, we are motivated by the central limit theorem in proposing to approximate γ's distribution by a truncated Gaussian, where the truncation follows from the known positivity of γ.

With this assumption, the symbol-error-rate in (6) is now

$$\mathrm{SER} \approx \frac{C}{\sigma_\gamma \pi \sqrt{2\pi}} \int_0^{\frac{(M-1)\pi}{M}} \int_0^{\infty} e^{-\frac{(\gamma-m_\gamma)^2}{2\sigma_\gamma^2}} e^{-\frac{g_{\mathrm{PSK}}}{\sin^2(\theta)}\gamma} \, d\gamma d\theta , \tag{15}$$

where C, given as

$$\frac{1}{C} = \int_0^{\infty} \frac{1}{\sigma_\gamma\sqrt{2\pi}} e^{-\frac{(\gamma'-m_\gamma)^2}{2\sigma_\gamma^2}} \, d\gamma' , \tag{16}$$

is the correction factor due to truncation.

The moment generating function (mgf) of a real Gaussian-distributed random variable $\eta \sim \mathcal{N}(m_\eta, \sigma_\eta^2)$ is defined as

$$M_\eta(t) = \mathrm{E}[e^{t\eta}] = \int_{-\infty}^{\infty} \frac{1}{\sigma_\eta\sqrt{2\pi}} \exp\Big(-\frac{(\eta-m_\eta)^2}{2\sigma_\eta^2} + t\eta\Big) d\eta , \tag{17}$$

or in an alternative form

$$M_\eta(t) = \exp\Big(m_\eta t + \sigma_\eta^2 \frac{t^2}{2}\Big) . \tag{18}$$

We observe that the inner integral of (15) resembles (17), with $t = -g_{\mathrm{PSK}}/\sin^2(\theta)$. Using (18), the SER is rewritten as

$$\mathrm{SER} \approx \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} \exp\Big(-m_\gamma \frac{g_{\mathrm{PSK}}}{\sin^2(\theta)} + \frac{1}{2}\sigma_\gamma^2 \Big(\frac{g_{\mathrm{PSK}}}{\sin^2(\theta)}\Big)^2\Big) d\theta . \tag{19}$$

Now, we propose to minimize the integrand $\exp\Big(-m_\gamma \frac{g_{\mathrm{PSK}}}{\sin 2(\theta)} + \frac{1}{2}\sigma_\gamma^2 \big(\frac{g_{\mathrm{PSK}}}{\sin^2(\theta)}\big)^2\Big)$, for any value of θ. This problem reduces to minimizing $\Big(-m_\gamma + \frac{1}{2}\sigma_\gamma^2 \frac{g_{\mathrm{PSK}}}{\sin^2(\theta)}\Big)$.

The mean m_γ and variance σ_γ^2 of γ, derived in [13], are given as

$$m_\gamma = \delta M_r , \qquad \text{and} \qquad \sigma_\gamma^2 = \delta^2 \sum_{i=0}^{M_t-1} (f_i^2)^2 \sum_{j=0}^{M_r-1} \lambda_{i_j}^2 . \tag{20}$$

Because m_γ is a constant and $\sin^2(\theta)$ is always non-negative, the approximate SER in (19) is minimum when the variance σ_γ^2 is minimum, so we optimize

$$\min_{\left\{f_i \geq 0, i\in\{0,M_t-1\} \,\middle|\, \sum_{i=0}^{M_t-1} f_i^2 = 1\right\}} \sigma_\gamma^2 ,$$

with the variance σ_γ^2 as given in (20).

The solution to this minimum variance problem is developed in [13]. For $i \in \{0, 1, \dots, M_t - 1\}$, we have

$$f_i = \sqrt{\frac{\beta}{\sum_{j=0}^{M_r-1} \lambda_{i_j}^2}}\,, \ \ \beta = \frac{1}{\sum_{i=0}^{M_t-1} \frac{1}{\sum_{j=0}^{M_r-1} \lambda_{i_j}^2}}\,.$$

Interestingly, this closed-form precoder is identical to that obtained from the maximum diversity principle and presented in Lemma 2. This gives insights into the justification of the use of the maximum diversity principle, beyond simple intuition.

C. Precoder for Unequal Average SNR

In the most general case, the different transmit antennas may see different average path loss. If these path losses are substantially different and slow power control is not activated, the Gaussian approximation above looses credibility and another approach should be resorted to. We propose to optimize the PEP-based metric shown in [15]

$$\max_{\{\boldsymbol{F} \in \mathbb{C}^{M_t \times B}\}} \det\Big(\boldsymbol{I}_{M_t M_r} + \frac{a d_{\min}^2}{4\sigma_v^2} \boldsymbol{R}^{1/2} \bar{\boldsymbol{P}} \big[(\boldsymbol{F}^* \boldsymbol{F}^T) \otimes \boldsymbol{I}_{M_r}\big] \bar{\boldsymbol{P}} \boldsymbol{R}^{1/2}\Big),$$

under the constraint $\mathrm{Tr}\{\boldsymbol{F}\boldsymbol{F}^H\} = 1$.

With uncorrelated transmitters, it still holds that the optimal $\boldsymbol{F}$ can be chosen as diagonal, real and non-negative, hence we must solve a power allocation problem [15]. We introduce the substitution $k = \frac{a d_{\min}^2}{4\sigma_v^2}$. When using the knowledge of $\boldsymbol{F}$ from Theorem 1, the optimization is reduced to the less complex problem

$$\max_{\{f_i^2 \geq 0, i \in \{0,1,\dots,M_t-1\}\}} \prod_{i=0}^{M_t-1} \prod_{j=0}^{M_r-1} \big(1 + k\rho_i f_i^2 \lambda_{i_j}\big)\,, \ \sum_{i=0}^{M_t-1} f_i^2 = 1\,,$$

which is equivalent to

$$\max_{\{f_i^2 \geq 0, i \in \{0,1,\dots,M_t-1\}\}} \sum_{i=0}^{M_t-1} \sum_{j=0}^{M_r-1} \log\big(1 + k\rho_i f_i^2 \lambda_{i_j}\big), \ \sum_{i=0}^{M_t-1} f_i^2 = 1\,. \tag{21}$$

Interestingly, the maximization of the above sum of logarithms resembles the well-known capacity maximization problem, for which the solution is the classical water-filling. Unfortunately, the above objective function is different in the fact that each power variable f_i^2 appears in M_r terms of the sum, making it a totally different problem to solve in general, except in special cases.

1) Solution for Uncorrelated Case: Assume as a special case that there is no correlation on either side, neither transmit nor receive. In this case, $\boldsymbol{R}_{r_i} = \boldsymbol{I}_{M_r}$ for all $i \in \{0, 1, \dots, M_t - 1\}$. Then, the optimization problem in (21) is simplified to

$$\max_{\{f_i^2 \geq 0, i \in \{0,1,\dots,M_t-1\}\}} M_r \sum_{i=0}^{M_t-1} \log\big(1 + k\rho_i f_i^2\big)\,, \ \sum_{i=0}^{M_t-1} f_i^2 = 1\,.$$

We recognize this problem as one for which the solution is the classical water-filling, such that

$$f_i^2 = \Big(\mu - \frac{1}{k\rho_i}\Big)_+\,, \quad i \in \{0, 1, \dots, M_t - 1\}\,, \ \sum_{i=0}^{M_t-1} f_i^2 = 1\,.$$

This special setting of unequal average SNR and no correlation can be seen as an equivalent to the optimization problem presented by Sampath and Paulraj in [16]. There, the authors assumed transmit correlation, no receive correlation and equal average SNR.

2) Modified Waterfilling Solution for Correlated Case: We now return to the most general case where each transmitter experiences a different path loss and receive correlation.

To find a solution to the optimization problem in (21), we differentiate with respect to f_i^2 $i \in \{0, 1, \dots, M_t - 1\}$, introduce a Lagrange multiplier μ with the power constraint $\mathrm{Tr}\{\boldsymbol{F}\boldsymbol{F}^H\} = 1$, and find M_t equations on the form

$$\sum_{j=0}^{M_r-1} \frac{k\rho_i \lambda_{i_j}}{1 + k\rho_i f_i^2 \lambda_{i_j}} + \mu = 0 \quad \text{for } i \in \{0, 1, \dots, M_t - 1\}\,.$$

For each transmit antenna $i \in \{0, 1, \dots, M_t - 1\}$, we optimize with respect to the leading term in the sum. We assume that the eigenvalues are ordered, so that $\lambda_{i_k} \geq \lambda_{i_l}$, for all $k \leq l$. The other $M_r - 1$ terms in the sum, we fix in a constant c_i.

$$c_i = \sum_{j=1}^{M_r-1} \frac{k\rho_i \lambda_{i_j}}{1 + k\rho_i f_i^2 \lambda_{i_j}} \quad \text{for } i \in \{0, 1, \dots, M_t - 1\}\,.$$

Now, the M_t independent equations can be rewritten as

$$\frac{k\rho_i \lambda_{i_0}}{1 + k\rho_i f_i^2 \lambda_{i_0}} = -(\mu + c_i) \quad \text{for } i \in \{0, 1, \dots, M_t - 1\}\,.$$

By imposing the power constraint $\sum_{i=0}^{M_t-1} f_i^2 = 1$, we attempt to find μ. This approach means solving an equation of degree M_t, analytically feasible for $M_t \leq 4$. We get the expression

$$\sum_{i=0}^{M_t-1} f_i^2 = -\sum_{i=0}^{M_t-1} \Big(\frac{1}{\mu + c_i} + \frac{1}{k\rho_1 \lambda_{i_0}}\Big) = 1\,.$$

Of the possible values found for μ, use the one that minimizes

$$\sum_{i=0}^{M_t-1} |f_i^2| = \sum_{i=0}^{M_t-1} \Big|\frac{1}{\mu + c_i} + \frac{1}{k\rho_i \lambda_{i_0}}\Big|\,,$$

and finally, we update the power values

$$f_i^2 = \frac{1}{\mu + c_i} + \frac{1}{k\rho_i \lambda_{i_0}} \quad \text{for } i \in \{0, 1, \dots, M_t - 1\}\,.$$

The above described procedure is repeated until convergence in a fixed-point is reached. No formal proof for the conditions of convergence is available yet, and from the theory on fixed-point iterations [17], we know that non-convergence may occur. However, using a slightly modified approach, detailed in [13], a fixed-point is still obtainable.

V. RESULTS

We present simulation results for selected scenarios, assuming both equal and unequal average received SNR. In the first case, we show the BER vs SNR for the PEP-based modified water-filling and the maximum diversity principle. For the case of unequal average SNR, we show the modified water-filling results. In both cases, comparisons are made against the trivial equal power allocation to evaluate the gain of precoding, and also against results using the minimum, exact SER precoding from [12].

Two single-antenna BSs transmit cooperatively to one MU with $M_r = 4$ antennas. The Monte Carlo simulations transmit $1.5 \cdot 10^6$ bits, over 1500 channel realizations. The receive correlation matrices are $\boldsymbol{R}_{r_0} = \mathbf{1}_{M_t}$ and $\boldsymbol{R}_{r_1} = \boldsymbol{I}_{M_t}$, where $\mathbf{1}_{M_t}$ is an $M_t \times M_t$ matrix of all ones. This corresponds to the extreme situation of one BS seeing a fully correlated link, while the other sees a fully decorrelated link.

The results for the equal average SNR case are shown in Fig. 3. For unequal SNR values, we use $[\rho_0, \rho_1] = [0.75, 1]$, obtaining the results of Fig. 4. The results show that the iterative algorithm minimizing the PEP gives the same performance as the maximum diversity-based algorithm, for the equal SNR case. In both figures, we observe that the equal power allocation is outperformed by almost 1 dB at BER 10^{-3}. Also, the Gaussian and PEP-based approaches obtain the same results as the iterative, minimum, exact SER precoding from [12].

VI. CONCLUSIONS

We address the problem of distributed space-time coding for the cooperative downlink cellular network. The channel correlation structure is highly non-Kronecker and can be compensated by a distributed power allocation.

Several approaches are presented to find the optimal precoder, including a closed form precoder based on the maximum diversity principle and iterative solutions based on modified waterfilling.

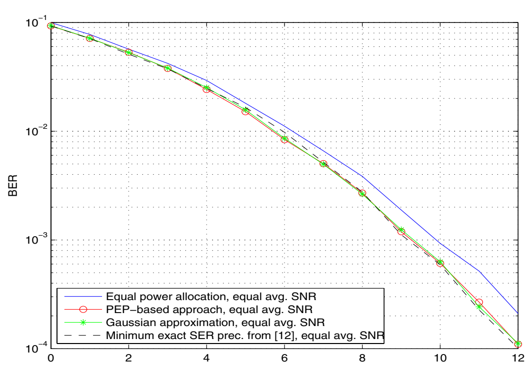

Fig. 3. Comparison of BER-performance versus SNR for $M_t = 2$, $M_r = 4$, in the case where $\boldsymbol{R}_{r_0} = \mathbf{1}_{M_t}$ and $\boldsymbol{R}_{r_1} = \boldsymbol{I}_{M_t}$ and for equal average SNR, $[\rho_0, \rho_1] = [1, 1]$. The power allocations based on the Gaussian approximation of γ and the PEP-measure have almost equal BER-results, outperforming the equal power allocation by close to 1 dB at BER 10^{-3}.

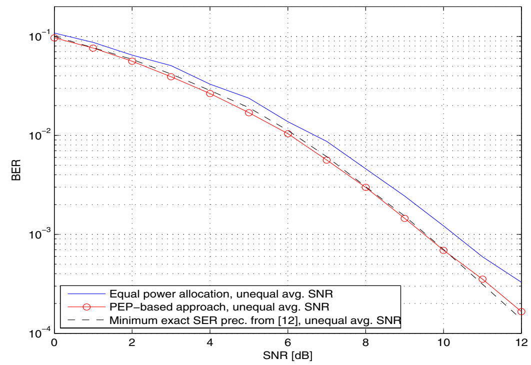

Fig. 4. Comparison of BER-performance versus SNR for $M_t = 2$, $M_r = 4$, in the case where $\boldsymbol{R}_{r_0} = \mathbf{1}_{M_t}$ and $\boldsymbol{R}_{r_1} = \boldsymbol{I}_{M_t}$ and for non-equal average SNR, realized by setting the path losses as $[\rho_0, \rho_1] = [0.75, 1]$. The power allocation based on the PEP-measure outperforms the equal power allocation by close to 1 dB at BER 10^{-3}.

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