CDMA SYSTEM DESIGN THROUGH ASYMPTOTIC ANALYSIS*

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Abstract

An asymptotic-analysis tool, based on recent results of Tse and Hanly and allowing analysis and design of CDMA systems with limited computer simulation, is described. As an example of application, some aspects of system design for a UMTS personal-communication satellite system are presented. Specifically, the tradeoffs involved in the allocation of available bandwidth between spreading and coding, the selection of modulation schemes, and the selection of multi-rate formats for the uplink channel are considered.

Introduction

An important observation, and one which prompted the investigation reported in [9], is that the networking-level problems of resource allocation and power control are not as well understood with multiuserCDMA receiver as they are with TDMA, FDMA, and conventional CDMA receiver. For example, in a TDMA or FDMA system, the network resource is shared among users via disjoint frequency and time slots, and this sharing provides a simple abstraction for resource allocation problems at the networking layer. For CDMA with multiuser receivers the latter are less well understood, because clean separation between the networking and physical layers does not exist here. A first step in bridging between resource-allocation problems at the networking layer and multiuser techniques at the physical layer has been taken by Tse and Hanly in [9]. The main goal of this manuscript is to show how the theoretical results in [8, 9] can be directly used to develop useful design techniques for CDMA systems, requiring only a modicum of computer simulation. Although this study was motivated by a UMTS personal-communication satellite, and hence our examples are developed within this context, the results here are applicable to a multiplicity of different scenarios.

We examine a CDMA system with coding: thus, rather than looking at symbol-by-symbol detection, we consider demodulation, i.e., the extraction of good estimates of the coded symbols of each user. These are used as soft decisions by the decoder. The receiver consists of a linear front-end, viz., either a single-user matched filter (SUMF) detector or a linear minimum-mean-square error receiver (LMMSER) detector [11], followed by a single-user decoder.

The key performance measure here is the Signal-to-Interference Ratio (SIR) of those estimates: users' quality of service can be expressed in terms of a target SIR. The SIR cumulative distribution function (cdf) yields immediately the *outage probability*, i.e., the probability that the actual SIR, say β, is below the required SIR target, denoted β_0. In general, this probability depends on the coding scheme and on the channel fading statistics.

Our study is *asymptotic*, in the sense that we let the number of users grow without bound, while the ratio of number of users to spreading-sequence length is kept fixed. Asymptotic performance of CDMA under these conditions, and with different receivers, has been analyzed in a number of recent contributions besides those cited above [10, 11, 12].

In this work, after a description of the system model, we shall examine system capacity in Section 3 (specifically, the coding-vs.-spreading tradeoff), the selection of modulation formats in Section 4, and the selection of multi-rate formats in Section 5.

System model

We consider a single-cell, chip-synchronous DS-CDMA system, and assume perfect power control, so that shadowing and deterministic path attenuation are perfectly compensated for. This model applies for example to a mobile-satellite system operating on a channel with line-of-sight propagation and a large Rician factor. Moreover, our model involves:

(a) K users.

(b) Random spreading sequences.[1]

(c) Spreading-sequence length $L = T/T_c$, where $1/T$ is the symbol rate and $1/T_c$ the chip rate.

Finally, our analysis is *asymptotic*, in the sense that we assume a large number of users ($K \to \infty$) and $K/L \to \alpha$ (a constant "channel load" as the length of the spreading sequences increases to accommodate the K users).

Let the empirical distribution function of the received interfering power from all users converge, as $K \to \infty$, to $F_{\mathcal{P}}(x)$. From [8, 9], with single-user matched filter (SUMF) reception and under mild convergence conditions we have the asymptotic SIR:

$$\beta = \frac{\mathcal{P}}{N_0 + \alpha \int_0^\infty x\, dF_{\mathcal{P}}(x)}, \tag{1}$$

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[1] With a mobile-satellite system this assumption, which is crucial for the analysis that follows, restricts the validity of our results to the uplink, as it will be discussed below.

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where $\mathcal{P}$ is the useful power received by reference user, and N_0 is the variance of the additive white Gaussian noise samples.

With linear minimum mean-square error reception the asymptotic SIR β is the unique real non-negative solution of

$$\beta = \frac{\mathcal{P}}{N_0 + \alpha \int_0^\infty \frac{x\mathcal{P}}{\mathcal{P} + x\beta} dF_{\mathcal{P}}(x)}. \tag{2}$$

To show the convergence of the actual output SIR to the values predicted by (1) and (2), we generated random sets of spreading sequences, and computed the SIR cdf for increasing values of L. These results are not reported here due to space limitations but are included in [3].

System capacity: Coding vs. spreading

Consider again a perfectly power-controlled single-cell system, subject to constraints on the signal-to-noise ratio and on the signal-to-interference ratio: specifically, let SNR$< \eta_0$ and SIR$= \beta_0$, respectively. From [8, 9] we have the following results:

SUMF: All users must have at least SNR $= \beta_0/(1 - \alpha\beta_0)$, so that α (which is the number of users per degree of freedom, and hence a measure of system capacity) must satisfy

$$\alpha \le \frac{1}{\beta_0} - \frac{1}{\eta_0}.$$

LMMSER: All users must have at least SNR $= \beta_0/(1 - \alpha\beta_0/(1+\beta_0))$, so that α must satisfy

$$\alpha \le (1+\beta_0)\left(\frac{1}{\beta_0} - \frac{1}{\eta_0}\right)$$

Based on the above results, we can evaluate the maximum spectral efficiency in bit/s/Hz, defined as the product $\gamma = \alpha R_{\max}$, where $R_{\max}$ is the channel capacity $\log_2(1+\beta_0)$, as follows:

$$\gamma \le \begin{cases} \left(\frac{1}{\beta_0} - \frac{1}{\eta_0}\right)\log_2(1+\beta_0) & \text{SUMF} \\ (1+\beta_0)\left(\frac{1}{\beta_0} - \frac{1}{\eta_0}\right)\log_2(1+\beta_0) & \text{LMMSER} \end{cases}$$

Results are displayed in Fig. 1 for SNR=6 dB. We see that for the SUMF γ is maximum for $\beta_0 \to 0$, i.e., for $\alpha \to \infty$ and code rate $R \to 0$. Thus, the whole bandwidth expansion should be allocated to low-rate coding. For LMMSER, γ is maximum for $\beta_0 = \beta_{\text{opt}}$ (a function of SNR). If we examine the behavior of β_{opt} as SNR changes, we observe that for high SNR we should use high-rate codes (e.g., TCM with 256-QAM) and long L. This fact is interpreted (see, e.g., [4]) by saying that low-rate coding would rob "dimensions" from the interference-suppression filter. This is not valid for low SNR, though: in fact, in these conditions we should use low-rate codes and short L, which means that coding compensates for the residual interference.

Selection of modulation formats

Let us now turn to the selection of uplink modulation formats. We examine the following:

1. BPSK with Walsh-Hadamard (WH) + complex spreading, and pilot symbols inserted among the data.
2. Dual BPSK with WH + complex spreading, and pilot symbols inserted among the data.
3. BPSK with WH + complex spreading, and pilot symbols transmitted in quadrature with different power levels.

Due to space limitations, here we report only numerical results and omit some analytical details which can be found in [3]. For our simulations we examine systems with (uncoded) symbol rate 8 kbit/s. The pilot power over data power ratio (PDR) is -8 dB. The nominal SNR equals 6 dB. The SIR is obtained from the real part of the receiver FIR output. Hence, the maximum achievable SIR is 9 dB (this would actually be achieved by a single-user system.) The three modulation formats described above were simulated for $L = 64$ with different values of channel load $\alpha = K/L$. Fig. 2 shows the cdf of the SIR with $\alpha = 0.8$. Here, "B" denotes BPSK, "D" denotes dual BPSK and "P" denotes BPSK with quadrature pilot. "As." and "As. P" denote the asymptotic SIR of BPSK/dual BPSK (it takes on the same value) and of BPSK with quadrature pilot, respectively. The asymptotic SIR cdf is a unit-step function, since the SIR converges to a deterministic quantity.

Thick lines are used for the LMMSER and thin lines are used for the SUMF. Dual BPSK and BPSK have almost identical average SIR, but dual BPSK invariably yields a SIR distribution with shorter tails (corresponding to smaller outage probabilities). BPSK with quadrature pilot is always outperformed by the other two schemes, in terms of both average SIR and distribution tails. It is interesting to notice the behavior as the channel load increases. When α is small, the performance loss caused by the insertion of quadrature pilot is large for the SUMF, and reduced for the LMMSER. As α increases, the performance loss of the quadrature-pilot scheme decreases with the SUMF, while increases with the LMMSER. This effect is less visible when the PDR decreases. This fact can be interpreted as follows. The SUMF is optimum for white background interference. Thus, as α increases and all available system degrees of freedom are used, the effect of the low-power pilot signals is close to that of a white additive noise, and hence it is negligible. On the contrary, the LMMSER exploits the non-white structure of interference to filter it out and it is greatly impaired by white noise, which cannot be filtered by linear processing. Actually, as the noise power increases the LMMSER reduces to a SUMF. In conclusion, the effect of quadrature pilots is that of increasing this noise-like appearence of interference, thus accelerating the collapse of the LMMSER.

Observation. The results presented here show that, for ideal synchronous detection, the quadrature pilot approach is not efficient in terms of output SIR, especially with LMMSER. However, since timing and phase recovery may be considerably simpler and more robust with quadrature pilot signals. Thus, the ultimate comparative study for assessing the best modulation format should involve simulations including actual timing and phase recovery.

Selection of multi-rate formats

We turn now our attention to the selection of a multirate format. Specifically, among the methods recently proposed for implementing multirate CDMA (see, e.g., [1, 5]), we examine and compare the following:

1. Multi-modulation (MM).
2. Multi-code (MC).
3. Variable spreading (VS).

We assume that there are K' users with (low) rate R' bit/s, K'' users have rate $R'' > R'$ where $r = R''/R'$ is an integer. We define the channel loadings $\alpha' = K'/L$ and $\alpha'' = K''/L$. $T = 1/R'$ is the symbol interval of low-rate users and T_c is the common chip time. The spreading length is $L = T/T_c$.

Multi-modulation scheme. Two different coded-modulation schemes are assigned to low-rate and high-rate users with the same symbol interval T. Denoting γ' and γ'' the spectral efficiencies (bit/s/Hz), the *capacity region* of the two-rate system is defined by

$$\mathcal{R} = \left\{(\alpha', \alpha'') \in \mathbb{R}_+^2 : \text{SIR}' \geq \text{SIR}_0', \text{SIR}'' \geq \text{SIR}_0''\right\} \quad (3)$$

where SIR$'$ and SIR$''$ are the receiver output SIRs for low-rate and high-rate users, while SIR_0' and SIR_0'' are the required output SIRs for the low-rate and high-rate users, respectively.

Since the two subsets of users transmit with different coded-modulation schemes, their SIR requirements differ. In particular, since the required SIR is an increasing function of the coded-modulation spectral efficiency, we have $\text{SIR}_0' \leq \text{SIR}_0''$. In order to obtain results independent of the specific coded-modulation schemes used, we assume that optimal Gaussian codes (i.e., approaching Shannon's capacity limit[2]) are used. Then, the SIR requirements are

$$\text{SIR}' = 2^{\gamma'} - 1 \quad , \quad \text{SIR}'' = 2^{\gamma''} - 1 \quad (4)$$

We now apply the asymptotic-analysis of [8] for $L \to \infty$ and fixed α' and α'' with random spreading sequences. With SUMF receiver, the asymptotic capacity region is defined by

$$\alpha'\text{SIR}' + \alpha''\text{SIR}'' \leq \min\left\{1 - \frac{\text{SIR}'}{\text{SNR}'}, 1 - \frac{\text{SIR}''}{\text{SNR}''}\right\}$$

[2]More precisely, we consider a single-user Gaussian channel with equivalent SNR equal to the output SIR of the CDMA receiver, so that $\gamma' = \log_2(1 + \text{SIR}')$ and $\gamma'' = \log_2(1 + \text{SIR}'')$

where SNR$'$ and SNR$''$ are the maximum allowed SNRs for low-rate and high-rate users, respectively.

With LMMSER, the asymptotic capacity region is defined by the inequality

$$\alpha'\frac{\text{SIR}'}{1+\text{SIR}'} + \alpha''\frac{\text{SIR}''}{1+\text{SIR}''} \leq \min\left\{1 - \frac{\text{SIR}'}{\text{SNR}'}, 1 - \frac{\text{SIR}''}{\text{SNR}''}\right\}$$

(observe that this includes the SUMF asymptotic capacity region).

The MM approach may not be practical for satellite applications, especially if the rate ratio r is large. In fact, in mobile-satellite systems the only practical modulation formats are BPSK and QPSK, whose spectral efficiencies are limited to 1 and 2 bits per symbol, respectively. Nevertheless, the analysis of MM may pave the ground for the study of more practical MC and VS schemes.

Multi-code scheme. Here every high-rate user divides its data stream into r substreams ("virtual low-rate users") Each substream is individually spread and transmitted, and each virtual user is detected by an independent receiver. The *capacity region* $\mathcal{R}$ of the MC system is defined by

$$\mathcal{R} = \left\{(\alpha', \alpha'') \in \mathbb{R}_+^2 \ : \ \text{SIR} \geq \text{SIR}'\right\} \quad (5)$$

where SIR denotes the receiver output signal-to-interference ratio for the equivalent single-rate system with loading $\alpha_{\text{eq}} = \alpha' + r\alpha''$ with low-rate users only, and SIR$'$ is the required output SIR for any user of the equivalent system. In analogy with what done in the MM case, the SIR requirement is given by

$$\text{SIR}' = 2^{\gamma'} - 1 \quad (6)$$

where γ' is the spectral efficiency of low-rate users. We can now exploit the results of the asymptotic analysis of [8], for $L \to \infty$ and fixed α_{eq}, with random spreading sequences.

With SUMF receiver, the asymptotic capacity region is defined by the inequality

$$\alpha' + r\alpha'' \leq \frac{1}{\text{SIR}'} - \frac{1}{\text{SNR}'} \quad (7)$$

where SNR$'$ is the maximum allowed SNR for a low-rate user of the equivalent single-rate system.

With LMMSER, the asymptotic capacity region is defined by the inequality

$$\alpha' + r\alpha'' \leq (1 + \text{SIR}')\left(\frac{1}{\text{SIR}'} - \frac{1}{\text{SNR}'}\right) \quad (8)$$

(observe that this includes the SUMF asymptotic capacity region).

Notice that in our model we implicitly assume that a high-rate user transmits with a total power equal to the total power of r low-rate users. Thus, only the constraint on the maximum allowed SNR for low-rate users is relevant to determine the above capacity regions. Also, we should not forget that the capacity regions derived by the asymptotic analysis of [8] are valid for random spreading sequences. With MC, all the r virtual users corresponding to the same high-rate user could be transmitted with perfect coordination (i.e., perfect time and phase synchronous) and hence made mutually orthogonal. The random-signature sequence approach followed in this paper cannot take this orthogonality constraint into consideration. Simulation results show that orthogonality has little impact for the uplink, where non-orthogonal MAI dominates, while it has a major impact on the downlink, where all signals transmitted by the same satellite (cell) can be made orthogonal.

Variable-spreading scheme. With this scheme, high-rate users transmit with a symbol rate r times larger than low-rate users. Thus, the effective spreading sequence length for a high-rate user is L/r.

This is conceptually similar to the multi-code scheme: in fact, a high-rate user can be decomposed into r virtual low-rate users whose sequences are zero in a part of the interval.

From the above, it should be clear that the capacity region of VS is defined exactly as that of MC. However, here the asymptotic analysis is not applicable exactly, because the spreading sequences, being constrained to be zero on certain symbols, are not random.

Comparisons

We consider a system with two classes of users transmitting 8 kbit/s and 64 kbit/s, respectively, so that $r = 8$. The SNR is assumed to be $\text{SNR}' = 6$ dB. The basic low-rate scheme consists of a binary code with rate 1/4 and QPSK, so that the low-rate-user spectral efficiency turns out to be $\gamma' = 0.5$ bits per complex symbol. The Shannon-limit SIR for such a spectral efficiency is $\text{SIR}'_0 = \sqrt{2} - 1$, corresponding to -3.82 dB. With MM, high-rate users have spectral efficiency $\gamma'' = 4$ bit/complex symbol, achieved for example through a rate-4/5 trellis-coded modulation scheme based on 32QAM as in the V.32 modem standard [6]. The corresponding Shannon-limit SIR is $\text{SIR}'' = 15$, corresponding to 11.76 dB. For a fair comparison, the SNR constraint for high-rate users in MM is $\text{SNR}'' = r\,\text{SNR}'$, as in MC and VS schemes. Fig. 3 shows the asymptotic capacity regions of MM and MC with SUMF and LMMSER. MM with SUMF has a poor capacity. With LMMSER, MM outperform MC and VS only for large α'' and small α', an unlikely situation because a real system is expected to operate with a large number of low-rate users and a small number of high-rate users. In the latter situation, MC and VS are distincly better than MM (this result is in agreement with the experimental results of [5]).

For a system expected to operate with a large number of low-rate users and a small number of high-rate users, MC (and VS) outperform MM.

Observation. The results described above prompt the observation of a rather counter-intuitive fact. We have shown that with our system model, which involves a receiver formed by a linear detector and a single-user decoder, the MM scheme is far from optimum. Now, the MM scheme is indeed optimum if joint decoding of all users is performed: in fact, under this assumption the best transmission strategy consists of having each transmitter allocate his rate/power resources so as to remain within the capacity region of the Gaussian multiple-access channel, with no need of splitting users' rates. Yet, rate-splitting (which corresponds to MC or VS) performs better than MM under our model and we know [7] that it achieves all the points of the capacity region above, while allowing decoding techniques (stripping, or onion-peeling) that yield a complexity lower than with full joint decoding. Our results show that the combination of rate-splitting and single-user decoding with a linear front-end performs better than MM, and has an even lower complexity. In other words, MC and VS are better matched to the receiver of our model than MM, in spite of the latter being optimum when a different receiver is used.

Conclusions

We have described a technique that is based on some recent theoretical results of Tse and Hanly [8, 9] and allows analysis of CDMA system, thus avoiding time consuming simulations when it comes to camparing different modulation schemes, multicode techniques, and the like. In spite of the simplicity of the equations employed, resulting from the main assumptions made (random spreading sequences and large number of users) the results show that the technique we advocate here can be a useful tool for system design.

Further investigations that the authors have undertaken on this subject include the extension of the results in [8, 9] to a flat-fading channel. The basic assumption is that the fading is not slow enough to be compensated for by the power-control mechanism, thus leaving a residual shadowing, but two cases warrant separate treatments. In fact, the fading may or may not be fast enough to make the channel ergodic, and the presence or absence of ergodicity force conceptually different developments (see [2] for a discussion on this point).

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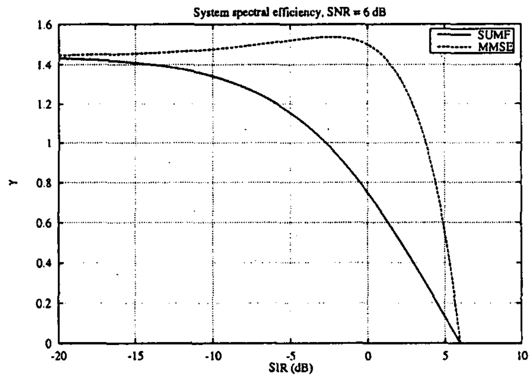

Figure 1: Asymptotic spectral efficiency γ of DS-CDMA vs. required SIR β_0 (SNR = 6 dB).

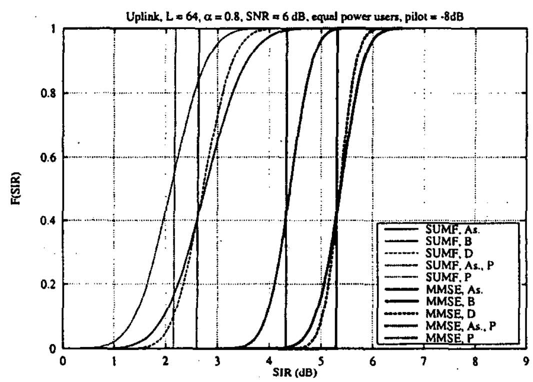

Figure 2: Cumulative distribution function of the SIR. signal-to-interference ratio. Here $L = 64$, $\alpha = 0.8$, SNR= 6 dB, and the pilot power to data power ratio equals −8 dB.

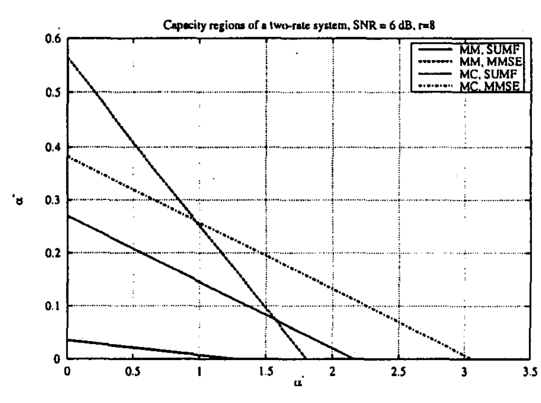

Figure 3: Asymptotic capacity regions for MM, MC (and approximately for VS) for random spreading sequences, $SNR' = 6$ dB, and rate ratio $r = 8$.