Spectral Efficiency of CDMA Downlink Cellular Networks with Matched Filter

Nicolas Bonneau[1], Mérouane Debbah[2], Eitan Altman[1]

Abstract

In this contribution, the performance of a downlink Code Division Multiple Access (CDMA) system with orthogonal spreading and multi-cell interference is analyzed. A useful framework is provided in order to determine the optimal base station coverage for wireless frequency selective channels with dense networks where each user is equipped with a matched filter. Using asymptotic arguments, explicit expressions of the spectral efficiency are obtained and provide a simple expression of the network spectral efficiency based only on a few meaningful parameters. Contrarily to a common misconception which asserts that to increase spectral efficiency in a CDMA network, one has to increase the number of cells, we show that, depending on the path loss and the fading channel statistics, the code orthogonal gain (due to the synchronization of all the users at the base station) can compensate and even compete in some cases with the drawbacks due to inter-cell interference. The results are especially realistic and useful for the design of dense networks.

Keywords

Random Matrix Theory, CDMA, Multi-cell Network, Matched Filter, Spectral Efficiency

[1]MAESTRO, INRIA Sophia Antipolis, 2004 Route des Lucioles, B.P. 93, 06902 Sophia Antipolis, {nicolas.bonneau, eitan.altman}@sophia.inria.fr

[2]Mobile Communications Group, Institut Eurecom, 2229 Route des Cretes, B.P. 193, 06904 Sophia Antipolis Cedex, merouane.debbah@eurecom.fr

I. Introduction

An important problem that arises in the design of CDMA systems concerns the deployment of an efficient architecture to cover the users. Increasing the number of cells in a given area yields indeed a better coverage but increases at the same time inter-cell interference. The gain provided by a cellular network is not at all straightforward and depends on many parameters: path loss, type of codes used, receiving filter, channel characteristics. Previous studies have already studied the spectral efficiency of an uplink CDMA multi-cell network with Wyner's model [1] or with simple interference models [2], [3], [4], [5], [6], [7], [8], [9]. However, none has taken explicitly into account the impact of the code structure (orthogonality) and the multi-path channel characteristics.

This contribution is a first step into analyzing the complex problem of downlink CDMA multi-cell networks, using a new approach based on unitary random matrix theory. The purpose of this contribution is to determine, for a dense and infinite multi-cell network, the optimal distance between base stations. A downlink frequency selective fading CDMA scheme where each user is equipped with a linear matched filter is considered. The users are assumed to be uniformly distributed along the area. Only orthogonal access codes are considered as the users are synchronized within each cell. The problem is analyzed in the asymptotic regime: very dense networks are considered where the spreading length N tends to infinity, the number of users per meter d tends to infinity but the load per meter $\frac{d}{N} = \alpha$ is constant. The analysis is mainly based on asymptotic results of unitary random matrices [10]. One of the great features of this tool is that performance measures such as Signal to Interference plus Noise Ratio (SINR) [11] or spectral efficiency [12] have very simple forms in the large system limit, independent of the particular CDMA code structure. Moreover, the theoretical results were shown to be very accurate predictions of the system's behavior in the finite size case (spreading length N of 256 for the SINR [13] or number of antennas 8 for the mutual information [14]).

This paper is structured as follows: in section II, the cellular CDMA model is introduced. In section III, the SINR expression is derived and an asymptotic analysis of the spectral efficiency with matched filter in case of downlink orthogonal CDMA is provided. Finally in section IV, discussions as well as numerical simulations are provided in order to validate our analysis.

II. CDMA Cellular Model

A. Cellular Model

Without loss of generality and in order to ease the understanding, we focus our analysis on a one dimensional (1D) network. This scenario represents for example the case of the deployment of base stations along a motorway (users i.e. cars are supposed to move along the motorway). Discussions on the two dimensional (2D) case are given in section III-C. An infinite length base station deployment is considered (see figure 1). The base stations are supposed equidistant with inter-base station distance a. The spreading length N is fixed and is independent of the number of users. The number of users per cell is $K = da$ (d is the density of the network). Note that as the size of the cell increases, each cell accommodates more users (with the constraint $da \leq N$). However, the total power emitted by the network does not increase since the same number of users is served by the network.

B. Downlink CDMA Model

In the following, upper case and lower case boldface symbols will be used for matrices and column vectors, respectively. $(.)^T$ will denote the transpose operator, $(.)^\star$ conjugation and $(.)^H = \left((.)^T\right)^\star$ hermitian transpose. $\mathbb{E}$ denotes the expectation operator. The general case of downlink wide-band CDMA is considered where the signal transmitted by the base station in cell p to user j has complex envelope

$$x_{pj}(t) = \sum_n s_{pj}(n) v_{pj}(t - nT). \tag{1}$$

In (1), $v_{pj}(t)$ is a weighted sum of elementary modulation pulses $\psi(t)$ which satisfy the Nyquist criterion with respect to the chip interval T_c ($T = NT_c$)

$$v_{pj}(t) = \sum_{\ell=1}^{N} v_{p\ell j} \psi(t - (\ell - 1)T_c). \tag{2}$$

The signal is transmitted over a frequency selective channel with impulse response $c_{pj}(\tau)$. Under the assumption of slowly-varying fading, the continuous time signal $y_j^p(t)$ received by user j in cell p has the form:

$$y_j^p(t) = \sum_q \sum_n \sum_{k=1}^{K} s_{qk}(n) \int \sqrt{P_q(x_j)} c_{qk}(\tau) v_{qk}(t - nT - \tau) d\tau + n(t). \tag{3}$$

where $n(t)$ is the complex white Gaussian noise. In (3), the index q stands for the cells, the index n for the transmitted symbol and the index k for the users (in each cell there are K users). User j is determined by his position x_j. The signal (after pulse matched filtering by $\psi^*(-t)$) is sampled at the chip rate to get a discrete-time received signal of user j in cell p of a downlink CDMA system that has the form:

$$\mathbf{y}^p(x_j) = \sum_q \sqrt{P_q(x_j)} \mathbf{C}_{qj} \mathbf{W}_q \mathbf{s}_q + \mathbf{n}. \tag{4}$$

x_j are the coordinates of user j in cell p. $\mathbf{y}^p(x_j)$ is the $N \times 1$ received vector, $\mathbf{s}_q = [s_q(1), \ldots, s_q(K)]^T$ is the $K \times 1$ transmit vector of cell q, $\mathbf{n} = [n(1), \ldots, n(N)]^T$ is an $N \times 1$ noise vector with zero mean and variance σ^2 Gaussian independent entries. $P_q(x_j)$ represents the path loss between base station q and user j whereas matrix $\mathbf{C}_{qj}$ represents the $N \times N$ Toeplitz structured frequency selective channel between base station q and user j. Each base station has an $N \times K$ code matrix $\mathbf{W}_q = [\mathbf{w}_q^1, \ldots, \mathbf{w}_q^K]$. User j is subject to intra-cell interference from other users of cell p as well as inter-cell interference from all the other cells.

C. Assumptions

The following assumptions are rather technical in order to simplify the analysis.

C.1 Code structure model:

In the downlink scenario, Walsh-Hadamard codes are usually used. However, in order to get interpretable expressions of the SINR, isometric matrices obtained by extracting $K < N$ columns from a Haar unitary matrix will be considered. A $N \times N$ random unitary matrix is said to be Haar distributed if its probability distribution is invariant by right (or equivalently left) multiplication by deterministic unitary matrices. Although of limited practical use, these random matrices represent a very useful analytical tool as simulations [13] show that their use provides similar performances as Walsh-Hadamard codes. **Note that each cell uses a different isometric code matrix**.

C.2 Multipath channel

We consider the case of a multipath channel. Under the assumption that the number of paths from base station q to any user j is given by L_{qj}, the model of the channel is given by

$$c_{qj}(\tau) = \sum_{\ell=0}^{L_{qj}-1} \eta_{qj}(\ell)\psi(\tau - \tau_{qj}(\ell)). \tag{5}$$

where we assume that the channel is invariant during the time considered. In order to compare channels at the same signal to noise ratio, we constrain the distribution of the i.i.d. fading coefficients $\eta_{qj}(\ell)$ such as:

$$\mathbb{E}\left[\eta_{qj}(\ell)\right] = 0 \text{ and } \mathbb{E}\left[|\eta_{qj}(\ell)|^2\right] = \frac{\varrho}{L_{qj}}. \tag{6}$$

Usually, fading coefficients $\eta_{qj}(\ell)$ are supposed to be independent with decreasing variance as the delay increases. In all cases, ϱ is the average power of the channel, such as $\mathbb{E}\left[|c(\tau)|^2\right] = \sum_{\ell=0}^{L_{qj}-1} \mathbb{E}\left[|\eta_{qj}(\ell)|^2\right] = \varrho$. For each base station q, let $h_{qj}(i)$ be the Discrete Fourier Transform of the fading process $c_{qj}(\tau)$. The frequency response of the channel at the receiver is given by:

$$h_{qj}(f) = \sum_{\ell=0}^{L_{qj}-1} \eta_{qj}(\ell)e^{-j2\pi f\tau_{qj}(\ell)} \left|\Psi(f)\right|^2. \tag{7}$$

where we assume that the transmit filter $\Psi(f)$ and the receive filter $\Psi^*(-f)$ are such that, given the bandwidth W,

$$\Psi(f) = \begin{cases} 1 & \text{if } -\frac{W}{2} \leq f \leq \frac{W}{2} \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

Sampling at the various frequencies $f_1 = -\frac{W}{2}$, $f_2 = -\frac{W}{2} + \frac{1}{N}W$, $\ldots$, $f_N = -\frac{W}{2} + \frac{N-1}{N}W$, we obtain the coefficients $h_{qj}(i)$, $1 \leq i \leq N$, as

$$h_{qj}(i) = h_{qj}(f_i) = \sum_{\ell=0}^{L_{qj}-1} \eta_{qj}(\ell)e^{-j2\pi\frac{i}{N}W\tau_{qj}(\ell)}e^{j\pi W\tau_{qj}(\ell)}. \tag{9}$$

Note that $\mathbb{E}\left[|h_{qj}(i)|^2\right] = \varrho$.

Notice that the assumption on the code structure model enables us to simplify the model (4) in the following way. Let $\mathbf{C}_{qj} = \mathbf{U}_{qj}\mathbf{H}_{qj}\mathbf{V}_{qj}$ denote the singular value decomposition of $\mathbf{C}_{qj}$. $\mathbf{U}_{qj}$ and $\mathbf{V}_{qj}$ are unitary, while $\mathbf{H}_{qj}$ is a diagonal matrix with elements $\{\tilde{h}_{qj}(1), \ldots, \tilde{h}_{qj}(N)\}$.

Since $\mathbf{U}_{qj}$ and $\mathbf{V}_{qj}$ are unitary, $\mathbf{U}_{qj}^H\mathbf{n}$ and $\mathbf{V}_{qj}\mathbf{W}_q$ have respectively the same distribution as $\mathbf{n}$ and $\mathbf{W}_q$ (the distribution of a Haar distributed matrix is left unchanged by left multiplication by a constant unitary matrix). As a consequence, model (4) is equivalent to:

$$\mathbf{y}^p(x_j) = \sum_q \sqrt{P_q(x_j)}\mathbf{H}_{qj}\mathbf{W}_q\mathbf{s}_q + \mathbf{n}. \tag{10}$$

where $\mathbf{H}_{qj}$ is a diagonal matrix with diagonal elements $\{\tilde{h}_{qj}(i)\}_{i=1\ldots N}$. Note that for any user j in cell p, when $N \to \infty$, , the coefficients $\tilde{h}_{qj}(i)$ tend to the Discrete Fourier Transform of the channel impulse response $h_{qj}(i)$ given by (9) (see [15]).

C.3 Path loss:

The general path loss $P_q(x_j)$ depends on a path loss factor which characterizes the type of attenuation. The greater the factor, the more severe the attenuation. In section III, we will derive expressions for an exponential path loss $P_q(x_j) = Pe^{-\gamma|x_j - m_q|}$ [16], where m_q are the coordinates of base station q. Note that in the usual model, the attenuation is of the polynomial form: $P_q(x_j) = \frac{P}{(|x_j - m_q|)^\beta}$. The polynomial path loss will be considered through simulations in section IV to validate our results. We use the exponential form for the sake of calculation simplicity and therefore put the framework in the most severe path loss scenario in favor of the multi-cell approach.

III. Performance Analysis

A. General SINR formula

In all the following, without loss of generality, we will focus on user j of cell p. We assume that the user does not know the codes of the other cells as well as the codes of other users within the same cell. As a consequence, the user is equipped with the matched filter receiver: $\mathbf{g}_{pj} = \mathbf{H}_{pj}\mathbf{w}_p^j$.

The output of the matched filter is given by:

$$\begin{aligned}
\mathbf{g}_{pj}^H \mathbf{y}^p(x_j) &= \sqrt{P_p(x_j)}\mathbf{g}_{pj}^H \mathbf{H}_{pj} \mathbf{w}_p^j s_p(j) \\
&+ \sqrt{P_p(x_j)}\mathbf{g}_{pj}^H \mathbf{H}_{pj} \mathbf{W}_p^{(-j)} \begin{bmatrix} s_p(1) \\ \vdots \\ s_p(K) \end{bmatrix}_{(K-1)\times 1} \\
&+ \sum_{q\neq p} \sqrt{P_q(x_j)}\mathbf{g}_{pj}^H \mathbf{H}_{qj} \mathbf{W}_q \mathbf{s}_q + \mathbf{g}^H \mathbf{n}. \qquad (11)
\end{aligned}$$

where $\mathbf{W}_p^{(-j)} = [\mathbf{w}_p^1, \ldots, \mathbf{w}_p^{j-1}, \mathbf{w}_p^{j+1}, \ldots, \mathbf{w}_p^K]$. From (11), we obtain the expression for the output SINR of user j in cell p with coordinates x_j and code $\mathbf{w}_p^j$:

$$\mathrm{SINR}(x_j, \mathbf{w}_p^j) = \frac{S^*(x_j)}{I_1(x_j) + I_2(x_j) + \sigma^2 {\mathbf{w}_p^j}^H \mathbf{H}_{pj}^H \mathbf{H}_{pj} \mathbf{w}_p^j}. \qquad (12)$$

where

$$S^*(x_j) = P_p(x_j) \left| {\mathbf{w}_p^j}^H \mathbf{H}_{pj}^H \mathbf{H}_{pj} \mathbf{w}_p^j \right|^2. \qquad (13)$$

$$I_1(x_j) = \sum_{q\neq p} P_q(x_j) {\mathbf{w}_p^j}^H \mathbf{H}_{pj}^H \mathbf{H}_{qj} \mathbf{W}_q \mathbf{W}_q^H \mathbf{H}_{qj}^H \mathbf{H}_{pj} \mathbf{w}_p^j. \qquad (14)$$

$$I_2(x_j) = P_p(x_j) {\mathbf{w}_p^j}^H \mathbf{H}_{pj}^H \mathbf{H}_{pj} \mathbf{W}_p^{(-j)} {\mathbf{W}_p^{(-j)}}^H \mathbf{H}_{pj}^H \mathbf{H}_{pj} \mathbf{w}_p^j. \qquad (15)$$

Note that the SINR is a random variable with respect to the channel model. For a fixed d (or $K = da$) and N, it is extremely difficult to get some insight on expression (12). In order to provide a tractable expression, we will analyze (12) in the asymptotic regime ($N \to \infty$, $d \to \infty$ but $\frac{d}{N} \to \alpha$) and show in particular that $\mathrm{SINR}(x_j, \mathbf{w}_p^j)$ converges almost surely to a random value $\mathrm{SINR}_{\lim}(x_j, p)$ independent of the code $\mathbf{w}_p^j$. Usual analysis based on random matrices use the ratio $\frac{K}{N}$ [17] also known as the load of the system. In our case, the ratio $\frac{K}{N}$ is equal to αa.

Proposition 1: When N grows towards infinity and $\frac{d}{N} \to \alpha$, the SINR of user j in cell p in downlink CDMA with orthogonal spreading codes and matched filter is given by:

$$\mathrm{SINR}_{\mathrm{lim}}(x_j, p) = \frac{P_p(x_j)\left(\frac{1}{W}\int_{-\frac{W}{2}}^{\frac{W}{2}} |h_{pj}(f)|^2 df\right)^2}{I_1(x_j) + I_2(x_j) + \frac{\sigma^2}{W}\int_{-\frac{W}{2}}^{\frac{W}{2}} |h_{pj}(f)|^2 df}. \tag{16}$$

$$I_1(x_j) = \frac{\alpha a}{W}\sum_{q\neq p} P_q(x_j)\int_{-\frac{W}{2}}^{\frac{W}{2}} |h_{pj}(f)|^2 |h_{qj}(f)|^2 df.$$

$$I_2(x_j) = \frac{\alpha a}{W} P_p(x_j)\left(\int_{-\frac{W}{2}}^{\frac{W}{2}} |h_{pj}(f)|^4 df - \frac{1}{W}\left(\int_{-\frac{W}{2}}^{\frac{W}{2}} |h_{pj}(f)|^2 df\right)^2\right).$$

Proof: See Appendix. ■

B. Spectral Efficiency

We would like to quantify the number of bits/s/Hz the system is able to provide to all the users. It has been shown [18] that the interference plus noise at the output of the matched filter for randomly spread systems can be considered as Gaussian when K and N are large enough. In this case, the mean (with respect to the position of the users and the fading) spectral efficiency of cell p is given by:

$$C^p = \frac{1}{N}\mathbb{E}_{x,h}\left[\sum_{j=1}^{K}\log_2\left(1 + \mathrm{SINR}(x_j, \mathbf{w}_p^j)\right)\right]. \tag{17}$$

In the asymptotic case and due to invariance by translation, the spectral efficiency per cell is the same for all cells. As a consequence, the network spectral efficiency is infinite. Without loss of generality, we will consider a user in cell 0 ($x_j \in [-\frac{a}{2}, \frac{a}{2}]$) and the corresponding asymptotic SINR is denoted $\mathrm{SINR}_{\mathrm{lim}}(x_j)$. Assuming the same distribution for all the users in cell 0, we drop the index j. The measure of performance in this case is the number of bits per second per hertz per meter (b/s/Hz/meter) the system is able to deliver:

$$C = \frac{1}{a}\frac{K}{N}\mathbb{E}_{x,h}\left[\log_2\left(1 + \mathrm{SINR}_{\mathrm{lim}}(x)\right)\right] = \alpha\mathbb{E}_{x,h}\left[\log_2\left(1 + \mathrm{SINR}_{\mathrm{lim}}(x)\right)\right]. \tag{18}$$

According to the size of the network, the total spectral efficiency scales linearly with the factor C. If we suppose that in each cell, the statistics of the channels are the same, then denoting by $P_0(x) = P(x)$, $h_0(f) = h(f)$ and $L_0 = L$, we obtain the following proposition from (16):

Proposition 2: When the spreading length N grows towards infinity and $\frac{d}{N} \to \alpha$, the asymptotic spectral efficiency per meter of downlink CDMA with random orthogonal spreading codes, general path loss, and matched filter is given by:

$$C(a) = \frac{\alpha}{a}\mathbb{E}_h \left[\int_{-\frac{a}{2}}^{\frac{a}{2}} \log_2 \left(1 + \frac{P(x)\left(\frac{1}{W}\int_{-\frac{W}{2}}^{\frac{W}{2}} |h(f)|^2 df\right)^2}{I(x) + \frac{\sigma^2}{W}\int_{-\frac{W}{2}}^{\frac{W}{2}} |h(f)|^2 df} \right) dx \right]. \tag{19}$$

$$I(x) = \frac{\alpha a}{W}\sum_{q\neq 0} P_q(x) \int_{-\frac{W}{2}}^{\frac{W}{2}} |h(f)|^2 |h_q(f)|^2 df$$
$$+ \frac{\alpha a}{W} P(x) \left(\int_{-\frac{W}{2}}^{\frac{W}{2}} |h(f)|^4 df - \frac{1}{W}\left(\int_{-\frac{W}{2}}^{\frac{W}{2}} |h(f)|^2 df \right)^2 \right)$$

with $a \in [0, \frac{1}{\alpha}]$.

C. 2-D Network

In the case of a 2-D network, the expression for the general SINR (16) from Proposition 1 is still valid, if we admit that $x_j = (x_j^1, x_j^2)$ represents the coordinates of the user considered, d is the density of users *per square meter* and $a = |\mathcal{C}|$ is the *surface* of the cell $\mathcal{C}$. The expression (19) for the spectral efficiency from Proposition 2 can be immediately rewritten with a double integration over the surface of the cell:

$$C(a) = \frac{\alpha}{a}\mathbb{E}_h \left[\iint_{\mathcal{C}} \log_2 \left(1 + \frac{P(x^1,x^2)\left(\frac{1}{W}\int_{-\frac{W}{2}}^{\frac{W}{2}} |h(f)|^2 df\right)^2}{I(x^1,x^2) + \frac{\sigma^2}{W}\int_{-\frac{W}{2}}^{\frac{W}{2}} |h(f)|^2 df} \right) dx^1 dx^2 \right]. \tag{20}$$

$$I(x^1,x^2) = \frac{\alpha a}{W}\sum_{q\neq 0} P_q(x^1,x^2) \int_{-\frac{W}{2}}^{\frac{W}{2}} |h(f)|^2 |h_q(f)|^2 df$$
$$+ \frac{\alpha a}{W} P(x^1,x^2) \left(\int_{-\frac{W}{2}}^{\frac{W}{2}} |h(f)|^4 df - \frac{1}{W}\left(\int_{-\frac{W}{2}}^{\frac{W}{2}} |h(f)|^2 df \right)^2 \right)$$

with $a \in [0, \frac{1}{\alpha}]$.

D. Simplifying Assumptions

D.1 Equi-spaced Delays

For ease of understanding of the impact of the number of paths on the orthogonality gain, we suppose that in each cell q, all the users have the same number of paths L_q and the delays are uniformly distributed according to the bandwidth:

$$\tau_q(\ell) = \frac{\ell}{W}. \tag{21}$$

Hence, replacing $h(f)$ and $h_q(f)$ with their expression with respect to the temporal coefficients (7) and using (21), (19) from Proposition 2 reduces to

$$C(a) = \frac{\alpha}{ar}\mathbb{E}_\eta\left[\int_{-\frac{a}{2}}^{\frac{a}{2}} \log_2\left(1 + \frac{P(x)\left(\sum_{\ell=0}^{L-1}|\eta(\ell)|^2\right)^2}{I(x) + \sigma^2\sum_{\ell=0}^{L-1}|\eta(\ell)|^2}\right)dx\right]. \tag{22}$$

$$I(x) = \alpha a\sum_{q\neq 0} P_q(x)\left(\left(\sum_{\ell=0}^{L-1}|\eta(\ell)|^2\right)\left(\sum_{\ell=0}^{L_q-1}|\eta_q(\ell)|^2\right) + \sum_{\ell=0}^{\min(L,L_q)-1}\sum_{\ell'\neq\ell}\eta(\ell)\eta(\ell')^*\eta_q(\ell)^*\eta_q(\ell')\right)$$
$$+ \alpha a P(x)\left(\sum_{\ell=0}^{L-1}\sum_{\ell'\neq\ell}|\eta(\ell)|^2|\eta(\ell')|^2\right).$$

In the case of a single path (i.e. $L_q = 1$ for all q), the signal is only affected by flat-fading, therefore orthogonality is preserved and intra-cell interference vanishes.

$$C(a) = \frac{\alpha}{ar}\mathbb{E}_\eta\left[\int_{-\frac{a}{2}}^{\frac{a}{2}} \log_2\left(1 + \frac{P(x)|\eta|^2}{\alpha a\sum_{q\neq 0}P_q(x)|\eta_q|^2 + \sigma^2}\right)dx\right]. \tag{23}$$

D.2 Exponential path loss and ergodic case

In the case of an exponential path loss, explicit expressions of the spectral efficiency can be derived when $L_q \to \infty$ for all q, referred in the following as the ergodic case. Although L_q grows large, we suppose L_q to be negligible with respect to N (see Appendix). The impact of frequency reuse is also considered. In other words, r adjacent cells may use different frequencies to reduce the amount of interference. This point is a critical issue to determine the impact of frequency reuse on the spectral efficiency of downlink CDMA networks.

Proposition 3: When the spreading length N and the number of paths L_q (for all q) grow towards infinity with $\frac{d}{N} \to \alpha$ and $\frac{L_q}{N} \to 0$, the asymptotic spectral efficiency per meter of downlink CDMA with random orthogonal spreading codes, exponential path loss, frequency reuse r and matched filter is given by:

$$C(a) = \frac{\alpha}{ar}\int_{-\frac{a}{2}}^{\frac{a}{2}} \log_2\left(1 + \frac{Pe^{-\gamma|x|}\left(\mathbb{E}\left[|h|^2\right]\right)^2}{I(x) + \sigma^2\mathbb{E}\left[|h|^2\right]}\right)dx. \tag{24}$$

$$I(x) = \alpha a P\left(\mathbb{E}\left[|h|^2\right]\right)^2 \frac{2e^{-\gamma ar}}{1-e^{-\gamma ar}}\cosh(\gamma x) + \alpha a P e^{-\gamma|x|}\left(\mathbb{E}\left[|h|^4\right] - \left(\mathbb{E}\left[|h|^2\right]\right)^2\right)$$

with $a \in [0, \frac{1}{\alpha}]$.

Proof: See Appendix. ■

In the case where $a \to 0$, the number of b/s/Hz/meter depends only on the fading statistics, the path loss and the factor $\alpha = \frac{d}{N}$ through:

$$C(0) = \alpha \log_2\left(1 + \frac{P\mathbb{E}\left[|h|^2\right]}{\sigma^2 + \frac{2\alpha P\mathbb{E}\left[|h|^2\right]}{\gamma}}\right). \tag{25}$$

Proof: Let $a \to 0$ in (24). ■

IV. Discussion

In all the following discussion, $P = 1$, $\sigma^2 = 10^{-7}$, $\alpha = 10^{-2}$ and $r = 1$ (unless specified otherwise).

A. Path Loss versus Orthogonality

We would like to quantify the impact of path loss on the overall performance of the system when considering downlink unfaded CDMA. In this case,

$$C(a) = \frac{\alpha}{a}\int_{-\frac{a}{2}}^{\frac{a}{2}} \log_2\left(1 + \frac{P(x)}{I(x)+\sigma^2}\right)dx. \tag{26}$$

$$I(x) = \alpha a \sum_{q\neq 0} P_q(x)$$

with $a \in [0, \frac{1}{\alpha}]$.

In figure 2 and 3, we have plotted the spectral efficiency per meter with respect to the inter-cell distance for an exponential ($\gamma = 1, 2, 3$) and polynomial ($\beta = 4$) path loss, without frequency-selective fading. Remarkably, for each path loss factor, there is an optimum inter-cell distance which maximizes the users' spectral efficiency. This surprising result shows that there is no need into packing base stations without bound if one can remove completely the effect of frequency-selective fading. It can be shown that optimal spacing depends mainly on the path loss factor γ and increases with a decreasing path loss factor.

B. Ergodic fading versus Orthogonality

We would like to quantify the impact of the channel statistics on the inter-cell distance. In other words, in the case of limited path loss, should one increase or reduce the cell size? A neat framework can be formulated in the case of exponential path loss with vanishing values of the path loss factor γ and ergodic fading. Although the spectral efficiency tends to zero, one can infer on the behavior of the derivative of the spectral efficiency which is given by:

$$\frac{\partial C}{\partial a} \propto \left(\frac{3}{2} - \frac{\mathbb{E}\left[|h|^4\right]}{\left(\mathbb{E}\left[|h|^2\right]\right)^2} \right) \tag{27}$$

with $a \in [0, \frac{1}{\alpha}]$.

Proof: See Appendix. ■

This simplified case (exponential path loss with ergodic fading) is quite instructive on the impact of frequency-selective fading on orthogonal downlink CDMA. In the ergodic case and with limited path loss, the optimum number of cells depends only on how "peaky" the channel is through the kurtosis $T = \frac{\mathbb{E}\left[|h|^4\right]}{\left(\mathbb{E}\left[|h|^2\right]\right)^2}$. If $T > \frac{3}{2}$, orthogonality is severely destroyed by the channel and one has to decrease the cell size whereas if $T \leq \frac{3}{2}$, one can increase the cell size[1].

C. Number of paths versus Orthogonality

We would like to quantify the impact of the number of multi-paths on the overall performance of the system. In figure 4 and 5, we have plotted the spectral efficiency per meter with respect to the inter-cell distance for an exponential ($\gamma = 1$) and polynomial ($\beta = 4$) path loss, in each case for numbers of multi-paths $L = 1$, $L = 2$, $L = 10$ (supposing an equal number of paths is generated by each cell) and Rayleigh fading. For $L = 1$, fading does not destroy orthogonality and as a consequence, an optimum inter-cell distance is

[1]The value $\frac{3}{2}$ is mainly dependent on the type of path loss (exponential, polynomial...)

obtained as in the non-fading case. However, for any value of $L > 1$, the optimum inter-cell distance is equal to 0.

D. Impact of reuse factor

In figure 6, we consider a realistic case with ergodic Rayleigh frequency-selective fading and $\gamma = 2$ and reuse factor $r = 1, 2, 3$. The spectral efficiency has been plotted for various values of the inter-cell distance. The curve shows that the users' rate decreases with increasing inter-cell distance, which is mainly due to frequency-selective fading. Note that the best spectral efficiency is achieved for a reuse factor of 1, meaning that all base stations should use all the available bandwidth.

E. General discussion

We would like to show that, in a cellular system, multipath fading is in fact more dramatic than path loss and restoring orthogonality through diversity (multiple antennas at the base station) and equalization techniques (MMSE,...) pays off. To visually confirm this fact, figure 7 plots for a path loss factor $\gamma = 2$ the spectral efficiency per meter with respect to the inter-cell distance in the ergodic Rayleigh frequency-selective fading, non-fading and inter-cell interference free case (i.e. $\frac{1}{a}\int_{-\frac{a}{2}}^{\frac{a}{2}} \log_2(1 + \frac{P(x)}{\sigma^2})dx$). The figure shows that one can more than triple the spectral efficiency per meter by restoring orthogonal multiple access for any inter-cell distance. Moreover, for small values of the inter-cell distance, greater gains can be achieved if one removes inter-cell interference (by exploiting the statistics of the inter-cell interference for example). Note also that even with fading and inter-cell interference, the capacity gain with respect to the number of base stations is not linear and therefore, based on economic constraints, the optimal inter-base station distance can be determined. Hence, based on the quality of service targets for each user, the optimum inter-cell distance can be straightforwardly derived.

V. Conclusion

Using asymptotic arguments, an explicit expression of the spectral efficiency was derived and was shown to depend only on a few meaningful parameters. This contribution is also very instructive in terms of future research directions. In the "traditional point of view" of cellular systems, the general guidance to increase the cell size has always been related to an increase in the transmitted power to reduce path loss. However, these results show that path loss is only the second part of the story and the first obstacle is on the contrary frequency-selective fading since path loss does not destroy orthogonal multiple access whereas frequency-selective fading does. These considerations show therefore that all the effort must be focused on combating frequency-selective fading through diversity and equalization techniques in order to restore orthogonality. Finally, note that the results presented in this paper deal only with the downlink and any deployment strategy should take also into account the uplink traffic as in [19].

VI. Appendix

A. Preliminary Results:

Let $\mathbf{X}_p$ be a $N \times N$ random matrix with i.i.d. zero mean unit variance Gaussian entries. The matrix $\mathbf{X}_p \left(\mathbf{X}_p^H \mathbf{X}_p\right)^{-\frac{1}{2}}$ is unitary Haar distributed (see [13] for more details). The code matrix $\mathbf{W}_p = \left[\mathbf{w}_p^1, \ldots, \mathbf{w}_p^K\right]$ is obtained by extracting K orthogonal columns from $\mathbf{X}_p(\mathbf{X}_p^H \mathbf{X}_p)^{-\frac{1}{2}}$. In this case, the entries of matrix $\mathbf{W}_p$ verify [10]:

$$\mathbb{E}\left[\left|w_p^1(i)\right|^2\right] = \frac{1}{N},\ 1 \le i \le N, \tag{28}$$

$$\mathbb{E}\left[\left|w_p^1(i)\right|^2 \left|w_p^k(i)\right|^2\right] = \frac{1}{N(N+1)},\ k > 1, \tag{29}$$

$$\mathbb{E}\left[w_p^1(i)^* w_p^k(i) w_p^1(l) w_p^k(l)^*\right] = -\frac{1}{N(N^2-1)},\ k > 1,\ i \neq l. \tag{30}$$

B. Proof of Proposition 1

Term S^*: Let us focus on the term S^* of (13). As $N \to \infty$, (13) becomes:

$$S^* = \mathbf{w}_p^{j\,H} \mathbf{H}_{pj}^H \mathbf{H}_{pj} \mathbf{w}_p^j = \sum_{i=1}^{N} \left|h_{pj}(i)\right|^2 \left|w_p^j(i)\right|^2.$$

Using (28), it is rather straightforward to show that

$$S^* \to \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \left|h_{pj}(i)\right|^2$$

in the mean square sense. Therefore,

$$S^* \xrightarrow[N\to\infty]{} \frac{1}{W} \int_{-\frac{W}{2}}^{\frac{W}{2}} \left|h_{pj}(f)\right|^2 df. \tag{31}$$

(31) stems from the fact that as $N \to \infty$, the eigenvalues $|h_{pj}(i)|^2$ of $\mathbf{H}_{pj}^H \mathbf{H}_{pj}$ correspond to the squared frequency response of the channel in the case of a Toeplitz structure of $\mathbf{H}_{pj}$ (see [15]).

Term I_1: Let us now derive the term I_1 of (14). It can be shown that (since $\mathbf{w}_p^j$ is independent of $\mathbf{W}_q$, see proof in [13])

$$\mathbf{w}_p^{j\,H} \mathbf{H}_{pj}^H \mathbf{H}_{qj} \mathbf{W}_q \mathbf{W}_q^H \mathbf{H}_{qj}^H \mathbf{H}_{pj} \mathbf{w}_p^j - \frac{1}{N} \mathrm{trace}(\mathbf{W}_q \mathbf{W}_q^H \mathbf{H}_{qj}^H \mathbf{H}_{pj} \mathbf{H}_{pj}^H \mathbf{H}_{qj}) \to 0. \tag{32}$$

Therefore, each term I_q^* in the sum in (14) can be calculated as:

$$\begin{aligned} I_q^* &= \mathbf{w}_p^{j\,H} \mathbf{H}_{pj}^H \mathbf{H}_{qj} \mathbf{W}_q \mathbf{W}_q^H \mathbf{H}_{qj}^H \mathbf{H}_{pj} \mathbf{w}_p^j \\ &\to \frac{1}{N} \mathrm{trace}(\mathbf{W}_q \mathbf{W}_q^H \mathbf{H}_{qj}^H \mathbf{H}_{pj} \mathbf{H}_{pj}^H \mathbf{H}_{qj}) \\ &\to \frac{1}{N} \sum_{k=1}^{K} \sum_{i=1}^{N} |h_{pj}(i)|^2 \, |h_{qj}(i)|^2 \left| w_q^k(i) \right|^2. \end{aligned}$$

Using (28), it is rather straightforward to show that

$$I_q^* \to \lim_{N\to\infty} \frac{1}{N^2} \sum_{k=1}^{K} \sum_{i=1}^{N} |h_{pj}(i)|^2 \, |h_{qj}(i)|^2$$

in the mean square sense. Therefore,

$$I_q^* \xrightarrow[\substack{N\to\infty \\ d/N\to\alpha}]{} \frac{\alpha a}{W} \int_{-\frac{W}{2}}^{\frac{W}{2}} |h_{pj}(f)|^2 |h_{qj}(f)|^2 df.$$

Term I_2: Finally, let us derive the asymptotic expression of I_2 in (15). The proof follows here a different procedure as $\mathbf{w}_p^j$ is not independent of $\mathbf{W}_p^{(-j)}$. However, one can show that

$$\begin{aligned} I_2 &= P_p(x_j) \mathbf{w}_p^{j\,H} \mathbf{H}_{pj}^H \mathbf{H}_{pj} \mathbf{W}_p^{(-j)} \mathbf{W}_p^{(-j)\,H} \mathbf{H}_{pj}^H \mathbf{H}_{pj} \mathbf{w}_p^j \\ &= P_p(x_j) \sum_{\substack{k=1 \\ k\neq j}}^{K} \left(\mathbf{w}_p^{j\,H} \mathbf{H}_{pj}^H \mathbf{H}_{pj} \mathbf{w}_p^k \right)^2 \\ &= P_p(x_j) \sum_{\substack{k=1 \\ k\neq j}}^{K} \left(\sum_{i=1}^{N} |h_{pj}(i)|^2 \, w_p^j(i)^* w_p^k(i) \right)^2 \\ &= P_p(x_j) \sum_{\substack{k=1 \\ k\neq j}}^{K} \sum_{i=1}^{N} \sum_{l=1}^{N} |h_{pj}(i)|^2 \, |h_{pj}(l)|^2 \, w_p^j(i)^* w_p^j(l) w_p^k(i) w_p^k(l)^*. \end{aligned}$$

and using (29) and (30), it can be shown that I_2 converges in the mean square sense to

$$P_p(x_j) \lim_{N\to\infty} \frac{1}{N(N+1)} \sum_{\substack{k=1\\k\neq j}}^{K} \sum_{i=1}^{N} |h_{pj}(i)|^4 - \frac{1}{N(N^2-1)} \sum_{\substack{k=1\\k\neq j}}^{K} \sum_{i=1}^{N} \sum_{\substack{l=1\\l\neq i}}^{N} |h_{pj}(i)|^2 \, |h_{pj}(l)|^2 .$$

Therefore,

$$I_2 \xrightarrow[d/N\to\alpha]{N\to\infty} P_p(x_j) \frac{\alpha a}{W} \left(\int_{-\frac{W}{2}}^{\frac{W}{2}} |h_{pj}(f)|^4 - \frac{1}{W} \left(\int_{-\frac{W}{2}}^{\frac{W}{2}} |h_{pj}(f)|^2 df \right)^2 \right).$$

C. Proof of Proposition 3

Note that as $L_q \to \infty$,

$$\sum_{\ell=0}^{L_q-1} |\eta_q(\ell)|^2 \to \mathbb{E}\left[\sum_{\ell=0}^{L_q-1} |\eta_q(\ell)|^2 \right] = \mathbb{E}\left[|h|^2\right],$$

$$\sum_{\ell=0}^{L-1} \sum_{\ell'\neq \ell} |\eta(\ell)|^2 \, |\eta(\ell')|^2 = \left(\sum_{\ell=0}^{L-1} |\eta(\ell)|^2 \right)^2 + \sum_{\ell=0}^{L-1} \sum_{\ell'\neq\ell} |\eta(\ell)|^2 \, |\eta(\ell')|^2 - \left(\sum_{\ell=0}^{L-1} |\eta(\ell)|^2 \right)^2$$
$$\to \mathbb{E}\left[|h|^4\right] - \left(\mathbb{E}\left[|h|^2\right] \right)^2,$$

and $\sum_{\ell'\neq\ell} \eta(\ell)\eta(\ell')^* \eta_q(\ell)^* \eta_q(\ell') \to 0$.

The rest of the proof is mainly an application of Proposition 1 where we consider a path loss of the form $Pe^{-\gamma(|x-qar|)}$ (γ is a decaying factor) between the user x ($x \in \left[-\frac{a}{2}, \frac{a}{2}\right]$) and base station q ($q \in \mathbb{Z}$) with coordinates $m_q = qa$. In this case, the inter-cell interference has an explicit form:

$$\sum_{q\neq 0} P_q(x) = P \sum_{q=-\infty}^{+\infty} e^{-\gamma|x-qar|} = Pe^{-\gamma x} \sum_{q=1}^{+\infty} e^{-\gamma q a r} + Pe^{\gamma x} \sum_{q=1}^{+\infty} e^{-\gamma q a r} = \frac{2Pe^{-\gamma a r}}{1-e^{-\gamma a r}} \cosh(\gamma x).$$

D. Proof of Equation (27)

Let $\gamma \to 0$ in (24). In this case,

$$\lim_{\gamma\to 0} C(a) = \frac{\alpha}{a} \int_{-\frac{a}{2}}^{\frac{a}{2}} \log_2 \left(1 + \frac{P(1-\gamma\,|x| - \frac{\gamma^2}{2}\,|x|^2) \left(\mathbb{E}\left[|h|^2\right] \right)^2}{I + \sigma^2 \mathbb{E}\left[|h|^2\right]} \right) dx + O(\gamma^3)$$

where

$$I = \alpha a P \left(\mathbb{E}\left[|h|^2\right] \right)^2 \frac{2e^{-\gamma a}}{1-e^{-\gamma a}} + \alpha a P \left(\mathbb{E}\left[|h|^4\right] - \left(\mathbb{E}\left[|h|^2\right] \right)^2 \right)$$
$$= \alpha a P \left(\mathbb{E}\left[|h|^4\right] - 2\left(\mathbb{E}\left[|h|^2\right] \right)^2 \right) + \frac{2\alpha P}{\gamma} \left(\mathbb{E}\left[|h|^2\right] \right)^2$$

since $\lim_{\gamma\to 0}\frac{2}{e^{\gamma a}-1}=\frac{2}{\gamma a}-1$. We have therefore:

$$\lim_{\gamma\to 0} C(a)=\frac{\frac{\alpha}{\ln 2}P\left(\mathbb{E}\left[|h|^2\right]\right)^2\left(1-\frac{\gamma a}{4}-\frac{\gamma^2a^2}{24}\right)}{\frac{2\alpha P}{\gamma}\left(\mathbb{E}\left[|h|^2\right]\right)^2\left(1+\gamma\left(\frac{a}{2}\left(\frac{\mathbb{E}\left[|h|^4\right]}{\left(\mathbb{E}\left[|h|^2\right]\right)^2}-2\right)+\frac{\sigma^2}{2\alpha P\mathbb{E}\left[|h|^2\right]}\right)\right)}+O(\gamma^3)$$

$$=\frac{\gamma}{2\ln 2}\left(1-\frac{\gamma a}{4}-\frac{\gamma^2a^2}{24}\right)\left(1-\gamma\left(\frac{a}{2}\left(\frac{\mathbb{E}\left[|h|^4\right]}{\left(\mathbb{E}\left[|h|^2\right]\right)^2}-2\right)+\frac{\sigma^2}{2\alpha P\mathbb{E}\left[|h|^2\right]}\right)\right)+O(\gamma^3)$$

which gives

$$\lim_{\gamma\to 0} C(a)=\frac{\gamma}{2\ln 2}\left(1-\gamma\left(\frac{a}{2}\left(\frac{\mathbb{E}\left[|h|^4\right]}{\left(\mathbb{E}\left[|h|^2\right]\right)^2}-\frac{3}{2}\right)+\frac{\sigma^2}{2\alpha P\mathbb{E}\left[|h|^2\right]}\right)\right)+O(\gamma^3)$$

and

$$\frac{\partial C}{\partial a}=-\frac{\gamma^2}{4\ln 2}\left(\frac{\mathbb{E}\left[|h|^4\right]}{\left(\mathbb{E}\left[|h|^2\right]\right)^2}-\frac{3}{2}\right)+O(\gamma^3).$$

References

[1] B.M. Zaidel, S. Shamai, and S.Verdu, "Multicell Uplink Spectral Efficiency of Coded DS-CDMA with Random Signatures," *IEEE Journal on Selected Areas in Communications*, vol. 19, pp. 1556–1569, 2001.

[2] A. Sendonaris and V. Veeravalli, "The Capacity-Coverage Trade-off in CDMA Systems with Soft Handoff," in *Proc. of the 31st Asilomar Conference*, Pacific Grove, USA, 1997, pp. 625–629.

[3] N. Kong and L.B Milstein, "Error Probability of Multicell CDMA over Frequency Selective Fading Channels with power control Error," *IEEE Trans. on Communications*, vol. 47, no. 4, Apr. 1999.

[4] O. K Tonguz and M.M Wang, "Cellular CDMA Networks Impaired by Rayleigh Fading: System Performance with Power Control," *IEEE Trans. on Veh. Technol.*, vol. 43, 1994.

[5] K.S Gilhousen, I.M Jacobs, R. Padovani, A.J Viterbi, L.A Weaver Jr., and C.E Wheatley, "On the Capacity of a Cellular CDMA System," *IEEE Trans. on Veh. Technol.*, vol. 40, pp. 303–312, 1991.

[6] G.E Corazza, G. De Maio, and F. Vatalaro, "CDMA Cellular System Performance with Fading, Shadowing and Imperfect Power Control," *IEEE Trans. on Veh. Technol.*, vol. 47, pp. 450–459, 1998.

[7] D.K Kim and F. Adachi, "Theoretical Analysis of Reverse Link Capacity for an SIR-Based Power Controlled Cellular CDMA System in a Multipath Fading Environment," *IEEE Trans. on Veh. Technol.*, vol. 50, pp. 452–464, 2001.

[8] J. Zhang and V. Aalo, "Performance Analysis of a Multicell DS-CDMA System with Base Station Diversity," *IEE Proceedings on Communications*, vol. 148, pp. 112–118, Apr. 2001.

[9] Z. Li and M. Latva-Aho, "Performance of a Multicell MC-CDMA System with Power Control Errors in Nakagami Fading Channels," *IEICE Trans. Commun.*, vol. E86-B, pp. 2795–2798, Sept. 2003.

[10] F. Hiai and D. Petz, *The Semicircle Law, Free Random Variables and Entropy*, vol. 77 of *Mathematical Surveys and Monographs*, AMS, 2000.

[11] D.N.C Tse and S. Hanly, "Linear Multi-user Receiver: Effective Interference, Effective Bandwidth and User Capacity," *IEEE Trans. on Information Theory*, pp. 641–657, Mar. 1999.

[12] S. Shamai and S. Verdu, "The Impact of Frequency-Flat Fading on the Spectral Efficiency of CDMA," *IEEE Trans. on Information Theory*, pp. 1302–1326, May 2001.

[13] M. Debbah, W. Hachem, P. Loubaton, and M. de Courville, "MMSE Analysis of Certain Large Isometric Random Precoded Systems," *IEEE Trans. on Information Theory*, vol. 49, no. 5, pp. 1293–1311, May 2003.

[14] M. Debbah and R. Müller, "MIMO Channel Modeling and the Principle of Maximum Entropy," *IEEE Trans. on Information Theory*, vol. 51, no. 5, May 2005.

[15] R. Gray, *Toeplitz and Circulant Matrices*, Stanford University, Palo Alto, 1st edition, 1977.

[16] M. Franceschetti, J. Bruck, and L.J. Schulman, "A Random Walk Model Of Wave Propagation," *IEEE Trans. on Antennas and Propagation*, vol. 52, no. 5, pp. 1304–1317, May 2004.

[17] S. Verdu and S. Shamai, "Spectral Efficiency of CDMA with Random Spreading ," *IEEE Trans. on Information Theory*, pp. 622–640, Mar. 1999.

[18] D. Guo, S. Verdu, and L.K. Rasmussen, "Asymptotic Normality of Linear Multiuser Receiver Outputs," *IEEE Trans. on Information Theory*, vol. 48, no. 12, pp. 3080 – 3095, Dec. 2002.

[19] N. Bonneau, M. Debbah, E. Altman, and G. Caire, "Spectral Efficiency of Uplink CDMA Cellular Networks," in *IEEE International Conference on Acoustics, Speech, and Signal Processing*, Philadelphie, USA, Mar. 2005.

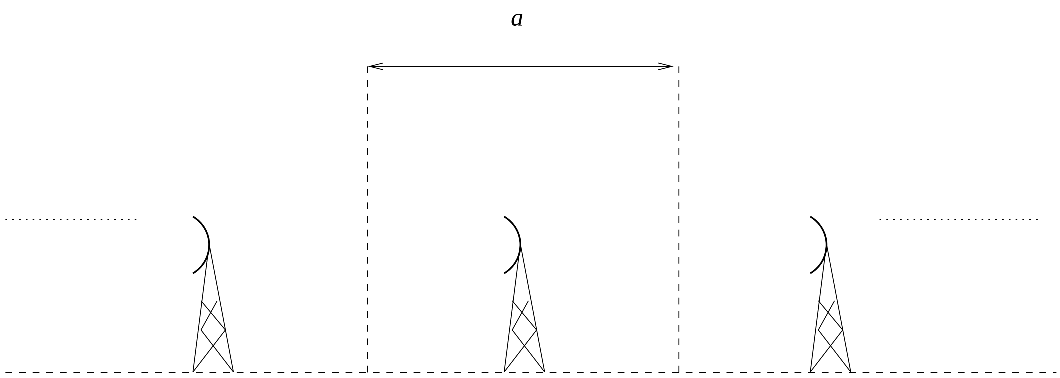

Fig. 1. Representation of a CDMA Cellular Network

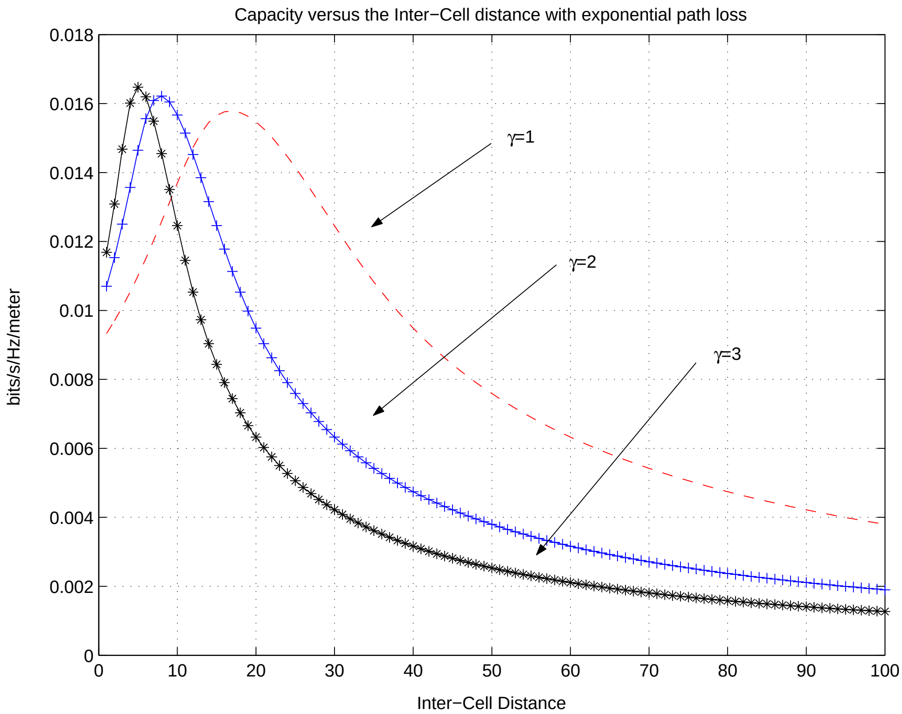

Fig. 2. Spectral efficiency versus inter-cell distance (in meters) in the case of exponential path loss and no fading: $\sigma^2 = 10^{-7}$, $P = 1$, $\gamma = 1, 2, 3$.

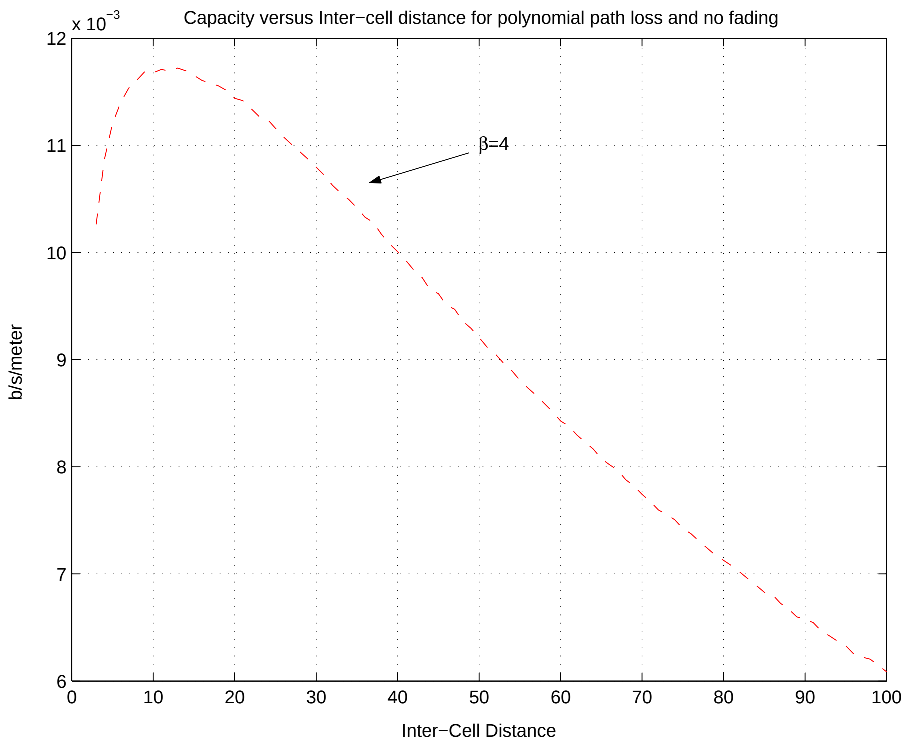

Fig. 3. Spectral efficiency versus inter-cell distance (in meters) in the case of polynomial path loss ($\beta = 4$) and no fading: $\sigma^2 = 10^{-7}$, $P = 1$.

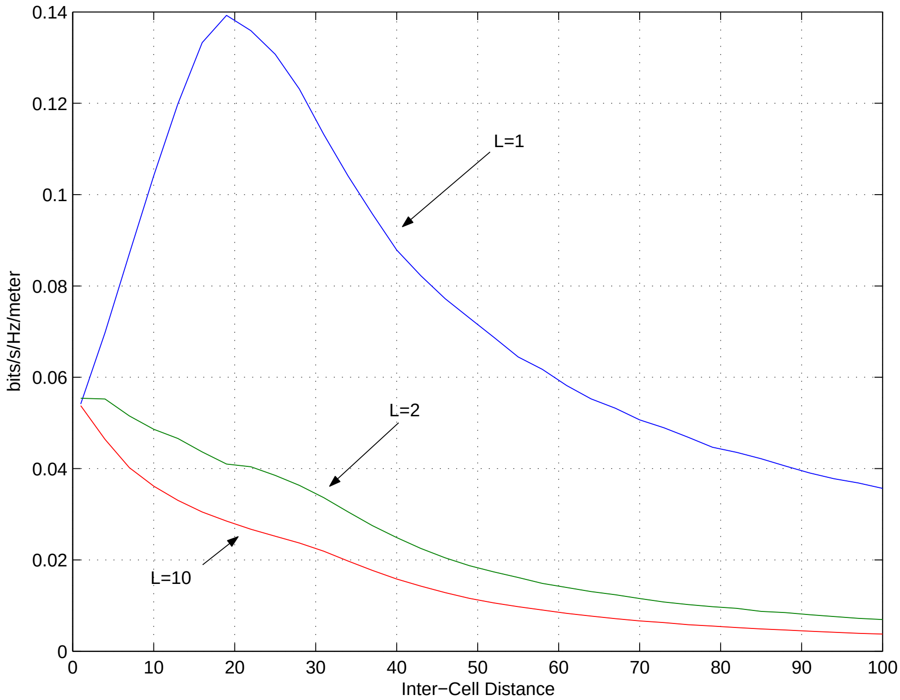

Fig. 4. Spectral efficiency versus inter-cell distance (in meters) in the case of exponential path loss and multipath fading: $\sigma^2 = 10^{-7}$, $P = 1$, $\gamma = 1$, $L = 1, 2, 10$.

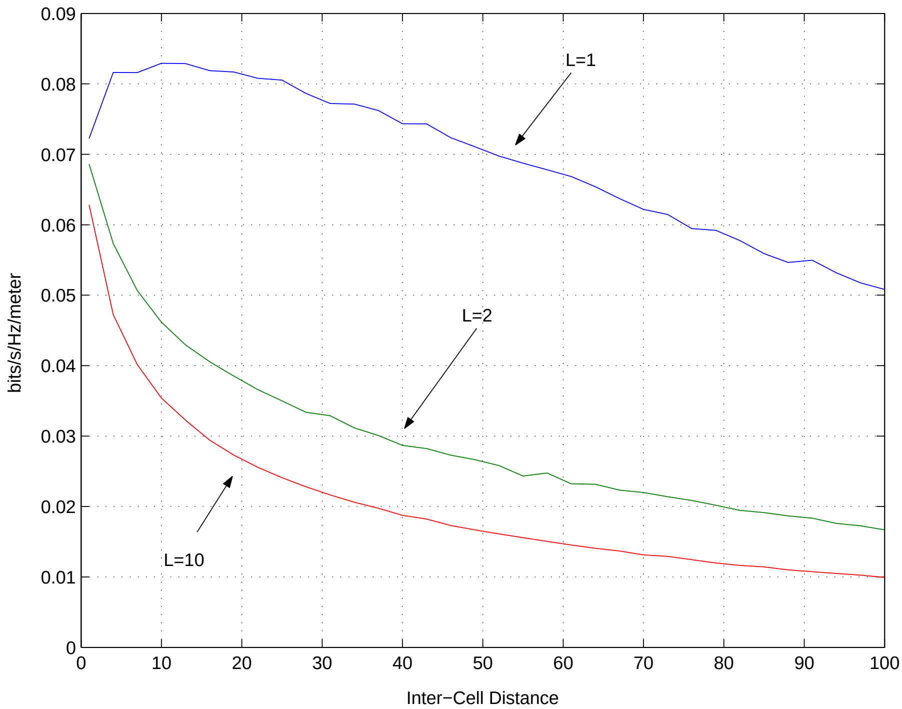

Fig. 5. Spectral efficiency versus inter-cell distance (in meters) in the case of polynomial path loss ($\beta = 4$) and multipath fading: $\sigma^2 = 10^{-7}$, $P = 1$, $L = 1, 2, 10$.

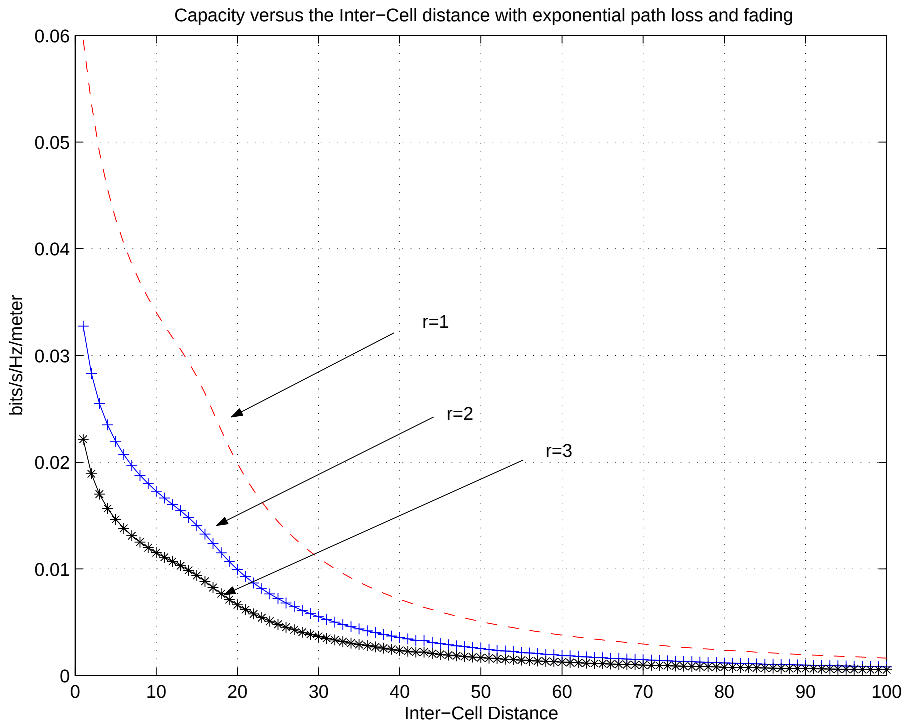

Fig. 6. Effect of the reuse factor: spectral efficiency versus inter-cell distance (in meters) in the case of exponential path loss and fading: $\sigma^2 = 10^{-7}$, $\gamma = 2$, $P = 1$, $r = 1, 2, 3$.

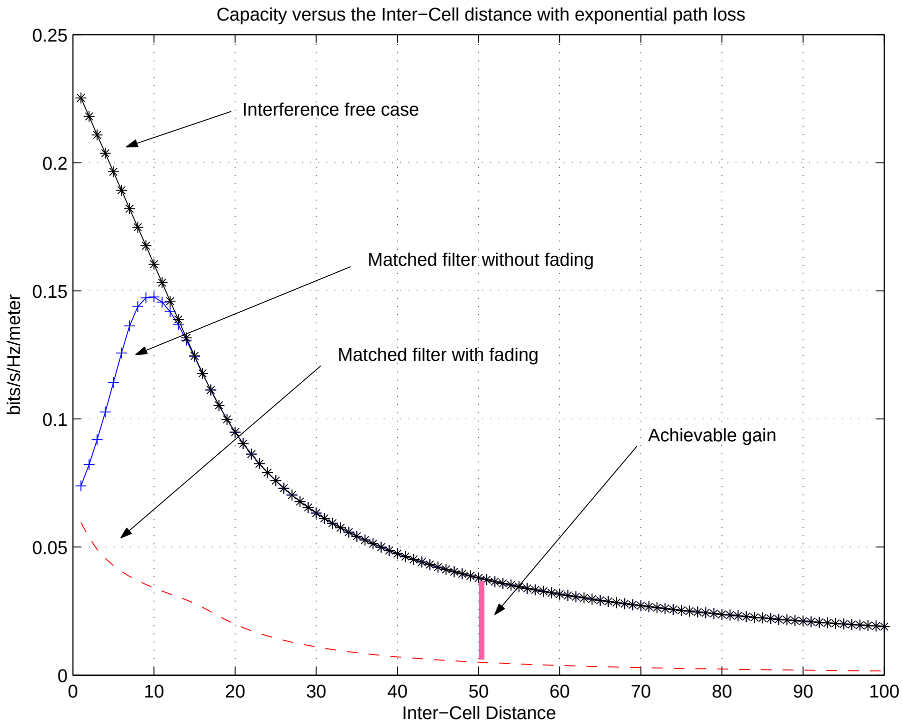

Fig. 7. Spectral efficiency versus inter-cell distance (in meters) in the case of exponential path loss with fading, without fading and interference free case (one cell): $\sigma^2 = 10^{-7}$, $P = 1$, $\gamma = 2$.