Memory-based Opportunistic Multi-user Beamforming

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***Abstract*— A scheme exploiting memory in opportunistic multi-user beamforming is proposed. The scheme builds on recent advances realized in [1] in the area of multi-user downlink precoding and scheduling based on partial transmitter channel state information (CSIT). Although the precoding and scheduling done in [1] is optimal within the set of unitary precoders, it is only so asymptotically for large number of users. Secondly, this scheme is unable to exploit potential time correlation of the channel. In this paper, we show 1- that exploiting memory in the transmitter allows to fill the gap to optimality for fixed (even low) number of users for time correlated channels, 2- how such schemes ([1] and ours) can be extended to take fairness into account in the proportional fair sense.**

I. Introduction

The design of MIMO transmit/receive physical layer schemes that lend themselves well to integration with efficient protocols for resource allocation at medium access control (MAC) layer represents a critical open area for current research. In this framework of cross layer design, two problems deserve particular attention: 1- the joint design of antenna combining schemes at the transmitter together with scheduling protocols, 2- resolution of the problem above under constraint of reasonably low feedback of CSIT.

In several recent papers, including [2], it was shown how the use of space-time codes for the downlink of multi-user MISO (multiple antenna at the base station, single antenna at the mobile) does not combine well with MAC layer multi-user diversity TDMA-like scheduling algorithms such as [3], unless the scheduler operates under severely corrupted CSIT information. In other words, for a system with reasonably accurate CSIT feedback at the base station it is essential to exploit the multiple transmit antennas for broadcasting information to several users at once rather than trying to maximize the reliability/diversity of a single-user link. To realize this, optimal schemes based on the dirty paper coding approach have been proposed [4], as well as suboptimal greedy techniques for solving the precoding and multi-user power allocation problem [5]. Unfortunately, the applicability of such schemes is limited due to the need for full CSIT which may lead to prohibitive feedback requirements in FDD systems and/or lack of robustness to CSIT errors in TDD setups. In [1] a scheme was proposed to obtain a unitary precoder (multi-user beamforming matrix) together with an optimally selected set of N_t spatially multiplexed users per slot, where N_t is the number of transmit antennas at the base station. The idea of [1] builds on the concept of opportunistic beamforming as initially shown in [6], to the difference that it is extended to the multi-user multi-beam situation. The scheme of [1] offers optimal scaling laws of capacity when the number of users is large. Additionally, it requires only little feedback from the users (in the form of individual SINRs), making it a simple yet powerful scheme for MIMO space-time scheduling. A limitation of [1] is that it is far from optimal for low to moderate number of users. If full CSI is available in the transmitter, the sum rate capacity scales linearly with N_t, even when N_t is of the order of the number of users K [7]. However, because only partial CSIT is used in [1], it was shown that linear scaling in N_t can only be guaranteed if N_t does not grow faster than $\log K$. Furthermore the scheme is unable to exploit temporal correlation that exists in most realistic channel settings.

In this paper we present a new space-time scheduling scheme, coined Memory-based Opportunistic Beamforming (MOB). This scheme exploits memory in the channel as a means to attempt filling the capacity gap to optimality in the fixed (arbitrary) number of user case. In a nutshell, MOB replaces the random selection of precoding matrices with a combination of random and past feedback-aided beamforming matrices. In summary, we make the following points:

- Given traffic is normally bursty with long silent periods in data-access networks, the scheduler may not count on a large number of simultaneously active users at all times.
- There is a capacity gap between a purely opportunistic multi-user beamforming scheme and a fully channel aware precoding and scheduling scheme for low to moderate number of users.
- In this paper, we propose a framework allowing to fill this capacity gap partially to fully, for any number of users, by exploiting the channel time correlation (i.e. limited Doppler spread).
- The simple concept is that of *feedback aggregation* which states that information derived from a low rate feedback channel can be cumulated over time to approach the performance of a full CSIT scenario.
- When the coherence time of the channel is large, MOB approaches the sum capacity of a system with optimal unitary precoding and full CSIT.

- For i.i.d. channels, the performance of the proposed scheme (MOB) remains superior to that of [1] at the expense of moderate additional feedback (two SINR values per user instead of one).

Interestingly, the scheme can be seen to relate also to recent useful results [8] presented to improve the delay performance of the single-beam opportunistic beamforming of [6]. In [8] the scheduling is limited to TDMA and time correlation of the channel is exploited through the use of a fixed data base of beams determined in advanced, and does not automatically reach the performance of a full CSIT scenario.

In the second part of this paper we address the practically relevant issue of user fairness. We point out that unlike [6], the multi-user scheme of [1] does not guarantee proportional fairness over users, and the fairness that offers arises when the system becomes interference dominated. Specifically, fairness is achieved if the number of antennas is large enough and/or the number of transmit antennas grows faster than log$K$. However, when the noise increases, fairness is undermined by noise, and therefore it is meaningful to extend the proportional fair scheduling to this scheme.

II. Network model

We consider a multiple antenna broadcast (downlink) channel with K users in which the transmitter (base station) is equipped with N_t antennas, and each user terminal with N_r antennas. The received signal $\mathbf{y}_k(t) \in \mathbb{C}^{N_r \times 1}$ at user k at time slot t is mathematically described as

$$\mathbf{y}_k(t) = \sqrt{\frac{\rho_k}{N_t}}\mathbf{H}_k\mathbf{x}(t) + \mathbf{n}_k \tag{1}$$

where $\mathbf{x}(t) \in \mathbb{C}^{N_t \times 1}$ is the transmitted vector signal at time slot t, $\mathbf{H}_k \in \mathbb{C}^{N_r \times N_t}$ is the complex channel matrix, and $\mathbf{n}_k \in \mathbb{C}^{N_r \times 1}$ is the circularly symmetric complex Gaussian noise at receiver k. We assume that the channel matrix $\mathbf{H}_k$ is perfectly known to the receiver, and that the elements of $\mathbf{H}_k$ and $\mathbf{n}_k$ have a zero mean and unit variance complex Gaussian distribution. We assume that the total power is equally distributed to the transmit antennas. The total transmit power is assumed to be $\mathbb{E}(x^* x) = N_t$, thus the transmit power per antenna is one. Due to the noise variance normalization, ρ_k takes the meaning of signal-to-noise ratio (SNR) of user k. In the following sections, for simplicity, we assume N_r=1.

III. Capacity of multi-user MIMO Broadcast Channels

If full channel knowledge is available at the transmitter for all K users, the sum rate is equal to [9], [10]

$$C_{DPC} = \mathbb{E}\left\{\max_{P_1,\ldots,P_K,\sum_k P_k = P} \log\det(\mathbf{I} + \sum_{\mathbf{k=1}}^{\mathbf{K}} \mathbf{H_k^* P_k H_k})\right\} \tag{2}$$

where $\mathbf{H}_k$ is $1 \times N_t$ channel matrix with i.i.d. $\mathbb{C}\mathcal{N}(0,1)$ distributions, P_k is the optimal power allocated to user k, and P is the total power. Furthermore, it is also known that when both the transmitter and the receiver have full CSI, the sum rate scales linearly with N_t.

A. Capacity-achieving schemes and Optimum Unitary Beamforming

The sum rate capacity of MIMO BC channel has been examined by several authors [4], [9], [10], and it was shown that the capacity-achieving strategy in multi-user MIMO downlink is dirty paper coding (DPC) [11]. A low complexity suboptimal approach to maximize the broadcast multi-antenna channel capacity is obtained by restricting the set of precoding matrices to unitary matrices and narrowing the power allocation strategy to a step function, i.e. only N_t users among the K users have a non-zero and equal power and all other users have zero power. The optimum sum rate under unitary precoding can be expressed by the following: Let $\mathbf{I}$ be a scheduling vector of size N_t, i.e a set of N_t user indexes among the K active users (a user cannot be scheduled twice in the vector). Then, the capacity is given as follows:

$$C_{UnitBF} = \mathbb{E}\left\{\max_{\mathbf{Q},\mathbf{I}} SR(\mathbf{Q},\mathbf{H_I})\right\} = \mathbb{E}\{SR_{unit}(\mathbf{H})\} \tag{3}$$

where the maximization is done over the set of all unitary complex matrices $\mathbf{Q}$ as well as the set of all scheduling vectors $\mathbf{I}$, and where SR is the rate summed over the users pointed by the scheduling vector, with combined channel matrix denoted $\mathbf{H_I}$; under precoding by unitary matrix $\mathbf{Q}$. For a given channel matrix $\mathbf{H}$ combined for all users, the maximum sum rate achievable with unitary precoding is denoted $SR_{unit}(\mathbf{H})$. When the number of users is asymptotically large, the random beamforming vectors are nearly matched to the users' channel vectors, and the capacity under optimum unitary precoding has the same scaling as the capacity defined earlier [12]. However this approach still requires full CSIT.

IV. Memoryless opportunistic multi-user beamforming

We now briefly review the concepts of memoryless opportunistic multi-user beamforming [1]. In this framework, the transmitter generates at each time slot N_t random beams and serves the users with the highest signal-to-interference-plus-noise ratios (SINRs). The random beams are generated independently from one time slot to the other. Assuming $N_r = 1$, a $N_t \times N_t$ unitary matrix $\mathbf{Q}$ is generated according to an isotropic distribution. At time slot t, the transmitted signal is

$$\mathbf{x}(t) = \sum_{m=1}^{N_t} \mathbf{q}_m(t) s_m(t) \tag{4}$$

where $s_m(t)$ is the m-th transmit symbol at time slot t and $\mathbf{q}_m \in \mathbb{C}^{N_t \times 1}$ are random orthonormal vectors (beams) for

$m = 1, \ldots, N_t$ (columns of $\mathbf{Q}$). Therefore, the k-th receiver's signal is

$$y_k = \sum_{m=1}^{N_t} \mathbf{H}_k \mathbf{q}_m s_m + n_k, \quad k = 1, \ldots, K \tag{5}$$

Each user calculates its SINRs on each one of the N_t random beams:

$$SINR_{k,m} = \frac{|\mathbf{H}_k \mathbf{q}_m|^2}{1/\rho_k + \sum_{j \neq m} |\mathbf{H}_k \mathbf{q}_j|^2} \tag{6}$$

and feeds back its maximum SINR and the index m of the beam for which its SINR is maximized. For each beam $\mathbf{q}_m$, the transmitter assigns the beam to the user with the highest corresponding SINR on that beam, i.e. $\max SINR_{k,m}$. The sum rate of the above scheme is given by [1],

$$C_{sum} \approx \mathbb{E} \left\{ \sum_{m=1}^{N_t} \log_2(1 + \max_{1 \leq k \leq K} SINR_{k,m}) \right\} \tag{7}$$

In [1] it was shown that the sum rate of memoryless opportunistic multi-user beamforming scales as $N_t \log \log K$, which is the same scaling having full channel knowledge on both sides and using DPC. In other words this technique solves the CSIT problem but only with a large number of users, which may not be realized in practice. Recently, we presented a power control approach to that problem [13]. The contribution of this paper is to recognize that exploiting channel memory can do the trick to solve the CSIT problem at fixed (even low) number of users.

V. Memory based opportunistic beamforming (MOB)

Our proposed scheme is built on the transmission scheme of [1], yet attempts to exploit memory in the channel by making at each time slot an improved selection of the unitary matrix based on past information.

We form a set of N 'preferred' unitary matrices $\mathcal{Q} = \{\mathbf{Q}_1, .., \mathbf{Q}_N\}$, satisfying $\mathcal{Q} \subseteq \mathcal{U}(N_t, N_t)$ (the set of unitary matrices defining the complex Stiefel manifold). At each time slot, the unitary matrix, denoted $\mathbf{Q}_i*$, of the preferred set, which has provided the highest sum rate at its time of use, is applied and its sum rate is updated under current channel conditions. We compare its sum rate with that of a new, randomly generated unitary matrix. The beamforming matrix that gives the higher sum rate is chosen for transmission. In the phase of updating the set, the value of the sum rate of $\mathbf{Q}_i*$ is updated, and the new random beamforming matrix is added into the set if its sum rate is higher than that of the matrix of the set that has offered the minimum sum rate. Let the notation $SR(\mathbf{Q}, \mathbf{H})$ imply the sum rate calculated using (7) for a unitary beamforming matrix $\mathbf{Q}$ conditioned on the channel matrix $\mathbf{H}$. Below, we give the steps of the proposed algorithm:

Memory based opportunistic beamforming (MOB) Algorithm

First phase ('best' unitary matrix selection)
Initialize $\mathcal{Q}$ with random unitary beamforming matrices $\mathbf{Q}_i$, each one with sum rate $SR(\mathbf{Q}_i)$.

At each time slot t,

- Generate a new random matrix, $\mathbf{Q}_{rand}$, with sum rate given by $SR(\mathbf{Q}_{rand})$
- Select from $\mathcal{Q}$ the matrix $\mathbf{Q}_i*$, such that $i^* = arg \max_{\mathbf{Q}_i} SR(\mathbf{Q}_i)$
- Apply $\mathbf{Q}_i*$, collect updated feedback and calculate $SR(\mathbf{Q}_{i*})$ given new channel (using (7))
- If $[SR(\mathbf{Q}_{i*}) > SR(\mathbf{Q}_{rand})]$ use $\mathbf{Q}_{i*}$, else use $\mathbf{Q}_{rand}$

Second phase (update of the set $\mathcal{Q}$)

- Update the value $SR(\mathbf{Q}_{i*})$ in the set $\mathcal{Q}$
- If $[SR(\mathbf{Q}_{rand}) > SR(\mathbf{Q}_{i_{min}})]$, replace $\mathbf{Q}_{i_{min}}$ with $\mathbf{Q}_{rand}$, where $\mathbf{Q}_{i_{min}}$ is the unitary matrix of $\mathcal{Q}$ giving the minimum sum rate ($i_{min} = arg \min_{\mathbf{Q}_i} SR(\mathbf{Q}_i)$)

The performance of our algorithm depends on the distribution of the sum rate conditioned to the channel realization $\mathbf{H}$. Let $X_i = SR(\mathbf{Q}_i, \mathbf{H})$ be the sum rate when random unitary matrix $\mathbf{Q}_i$ is used conditioned to the channel $\mathbf{H}$. The random variables $\{X_i\}$ are i.i.d. for i with associated probability density function (pdf) f_X and cumulative distribution function (cdf) F_X. For N i.i.d. random unitary matrices $\mathbf{Q}_i$, $i = 1, \ldots, N$, the performance is equivalent to

$$X^* = \max_{1 \leq i \leq N} X_i = N \int_0^\infty x \left[F_X(x)\right]^{N-1} f_X(x) dx \tag{8}$$

Proposition *1:* For a channel with memory $M = \frac{T_{coh}}{T_{slot}}$, where T_{coh} is the coherence time of the channel, and T_{slot} is the slot duration, for $M \to \infty$, the sum rate X^* converges to the optimal capacity of unitary beamforming:

$$SR_{unit} = \max_{\mathbf{Q}_i \in \mathcal{U}} SR(\mathbf{Q}_i, \mathbf{H}) \tag{9}$$

Proof To prove this proposition, we can equivalently show that for the set of unitary matrices $\{\mathbf{Q}_1, \ldots, \mathbf{Q}_N\} \subset \mathcal{U}$, $\max\limits_{1 \leq i \leq N} X_i$ converges to SR_{unit} for N sufficiently large and fixed number of users K. Thus, we want to show that $\forall \epsilon, \delta > 0$, $\exists N$ such that $\Pr(\max\limits_{1 \leq i \leq N} X_i \leq SR_{unit} - \epsilon) \leq \delta$.

As the sequence $\{\mathbf{Q}_i\}_1^N$ contains i.i.d. random variables, and $\{X_i\}_1^N$, are also i.i.d. for i, using order statistics we have that

$$\Pr(\max_{1 \leq i \leq N} X_i \leq SR_{unit} - \epsilon) = [F_X(SR_{unit} - \epsilon)]^N \tag{10}$$

For a channel with memory M, it is evidently meaningful to have $N = M$. As $0 \leq F_X(x) \leq 1$, asymptotically for $M \to \infty$, we have that:

$$\Pr(\max_{1 \leq i \leq N} X_i \leq SR_{unit} - \epsilon) \to 0 \tag{11}$$

In practice the channel coherence time is finite and thus our scheme will converge to a suboptimal solution. The convergence of our algorithm is evaluated through Monte Carlo simulations.

VI. Proportional Fair Multi-user Opportunistic Beamforming

To the best of our knowledge, proportional fair scheduling (PFS) has been derived only for single-user scheduling. In this section we propose an extension of PFS for multi-user scheduling, and combine PFS with our proposed scheme as well as [1].

A. Proportional Fairness in single-user transmission

We consider the single-user case (e.g., TDMA). Let $R_k(t)$ be the data rate and $T_k(t)$ be the average throughput of k-th user at time slot t, respectively. At each slot, user k^* is scheduled, i.e. such that

$$k^* = arg \max_{1 \leq k \leq K} \frac{R_k(t)}{T_k(t)} \tag{12}$$

The average throughputs are updated as follows:

$$T_k(t+1) = \begin{cases} (1-\frac{1}{t_c})T_k(t) + \frac{1}{t_c}R_k(t), & k = k^* \\ (1-\frac{1}{t_c})T_k(t), & k \neq k^* \end{cases} \tag{13}$$

The parameter t_c defines the time horizon in which we want to achieve fairness. Obviously, the larger t_c, the less stringent the fairness constraint, and thus longer delays start appearing between successive transmissions to the same user. In TDMA scheduling, it was shown that PFS maximizes the sum of the logarithms of the average throughputs, $T_k(t)$, over all users, thus $\sum_k \log(T_k)$, where $k = 1, \ldots, K$.

B. Proportional Fairness in multiple-user transmission

We define I to be the set of scheduled users over one slot, with cardinality $card(I) = N_t$. We make the following claim:

Proposition *2:* Under proportional fairness, the scheduling policy that maximizes the sum of the logarithms of the average throughputs is such that the users are scheduled as

$$I^* = \arg\max_{I} \prod_{k \in I} \left(1 + \frac{R_{k|I}(t)}{(t_c - 1)T_k(t)}\right) \tag{14}$$

where $R_{k|I}(t)$ is the data rate of user $k \in I$, when selecting I as scheduling vector, and $T_k(t)$ is its average throughput at time slot t.

Proof Let $J = \sum_{k \in S} \log(T_k(t+1))$ be the system objective function. Then, we have:

$J = \sum_k \log(T_k(t+1)) = \sum_{k \notin I} \log((1-\frac{1}{t_c})T_k(t)) +$

$+ \sum_{k \in I} \log((1-\frac{1}{t_c})T_k(t) + \frac{1}{t_c}R_{k|I}(t)) =$

$= \sum_{k \notin I} \log((1-\frac{1}{t_c})T_k(t)) + \sum_{k \in I} \log((1-\frac{1}{t_c})T_k(t)) +$

$+ \sum_{k \in I} \log(1 + \frac{R_{k|I}(t)}{(t_c-1)T_k(t)}) =$

$= \sum_k \log((1-\frac{1}{t_c})T_k(t)) + \sum_{k \in I} \log(1 + \frac{R_{k|I}(t)}{(t_c-1)T_k(t)})$

The first part of the expression does not depend on the choice of the scheduling vector *I*. Hence, to select the users that maximize the objective function, we choose I that maximizes $\sum_{k \in I} \log(1 + \frac{R_{k|I}(t)}{(t_c-1)T_k(t)}) = \log\left[\prod_{k \in I}\left(1 + \frac{R_{k|I}(t)}{(t_c-1)T_k(t)}\right)\right]$, which is equivalent to eq.(14) as the logarithmic function is monotonically increasing.

In a multi-beam scheme, such as [1], suppose that each user feeds back its SINR on each one of the beams. Then, we can calculate the rate that each user can support on each beam m, for $m = 1, \ldots, N_t$.

Proposition *3:* As the rate of user k does not depend on the rate of user $j \in I, j \neq k$, on the other beams, the set of users that maximizes the sum of the logarithms of the average throughputs over all users is such that:

$$I^* = \arg\max \sum_{k \in I} \left(\frac{R_{k|I}(t)}{T_k(t)}\right) \tag{15}$$

The average throughput of user k is updated as follows:

$$T_k(t+1) = \begin{cases} (1-\frac{1}{t_c})T_k(t) + \frac{1}{t_c}R_{k|I}(t), & k \in I \\ (1-\frac{1}{t_c})T_k(t), & k \notin I \end{cases} \tag{16}$$

VII. Simulation Results

We consider a time-varying Rayleigh fading channel where the fading $\mathbf{H}_k(t)$ are i.i.d. among users and for different antennas. The plots are obtained through Monte-Carlo simulations and ergodic capacity is considered for different values of Doppler spread. We use the Jakes Doppler model, with autocorrelation function $J_0(2\pi f_D T_{slot}\ell)$, and f_D and T_{slot} denoting the one-sided Doppler bandwidth (in Hz) and the slot duration (in seconds), respectively. We let T_{slot}=1ms and the average SNR is set to 0dB for all users.

In Fig. 1, the sum rate performance of the two schemes for different Doppler spreads and 20 users is plotted. As expected, the capacity of our scheme increases as the memory of channel is increasing. Furthermore, MOB has the same capacity scaling as that of random beamforming (RBF) [1]. The worst case scenario is to have a fast fading channel with M=1. In that case, the probability that the 'preferred' matrix will be valid if reapplied, falls to $1/2$, resulting only to selection diversity gain. This performance is also equivalent to that of RBF where two randomly generated sets of beams are generated and the one with the best sum rate is chosen. The sum rate of MOB is also plotted for a quasi-static channel (very large M). In that case, our scheme is able to 'learn' the channel directions of users, approaching thus to the case where full CSI is available. Note that out scheme achieves high sum rate even for fixed, but not necessarily large, number of users.

In Fig. 2, we show the sum rate of MOB as a function of number of users for $N_t = 8$. As expected, the gap between MOB and RBF is bigger for small number of users. As K becomes large, the sum rate of RBF increases as it is more

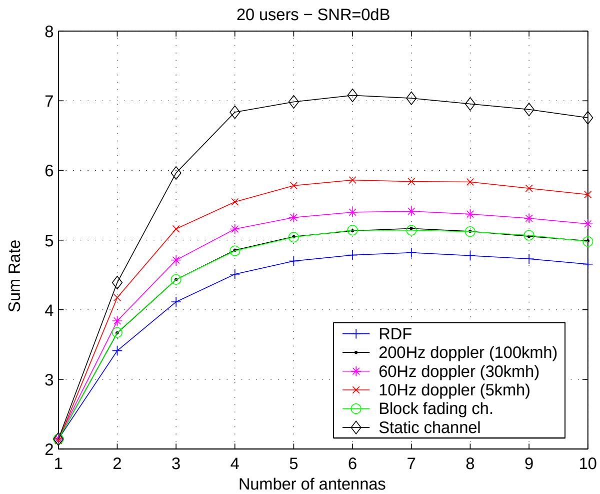

Fig. 1. Sum rate (bps/Hz) vs. number of transmit antennas for 20 users and various Doppler spreads

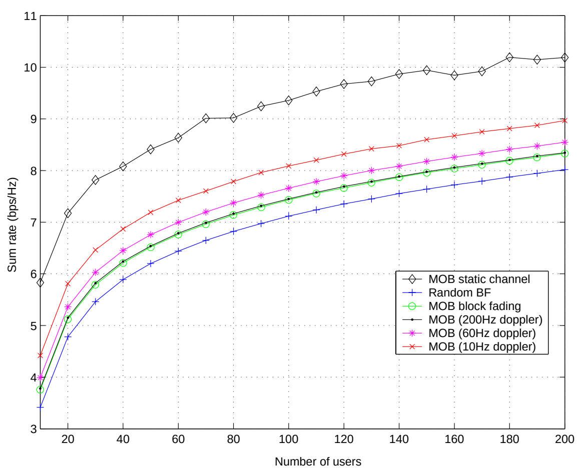

Fig. 2. Sum rate as a function of number of active users

likely that the random beams will find almost perfectly the users, achieving multiplexing gain.

Finally, we simulate the performance of our proposed scheme and RBF with the extended PFS algorithm. We set t_c=100 slots, which corresponds to a channel memory with approximately 10Hz Doppler spread. In Fig. 3, we plot the capacity of both schemes with PFS versus the number of transmit antennas. We assume $f_D = 10$ Hz and K=20 users. Note that MOB with PFS has the same scaling as RBF with PFS, and that even under PFS, our scheme can exploit the time correlation to provide significant gain in terms of sum rate.

VIII. Conclusion

The performance of opportunistic beamforming schemes degrades severely with low number of users. We show how exploiting channel time correlation we can alleviate this problem at minimal cost. The proposed scheme provides a way to close the gap to optimality for arbitrary number of users when the channel coherence time is large.

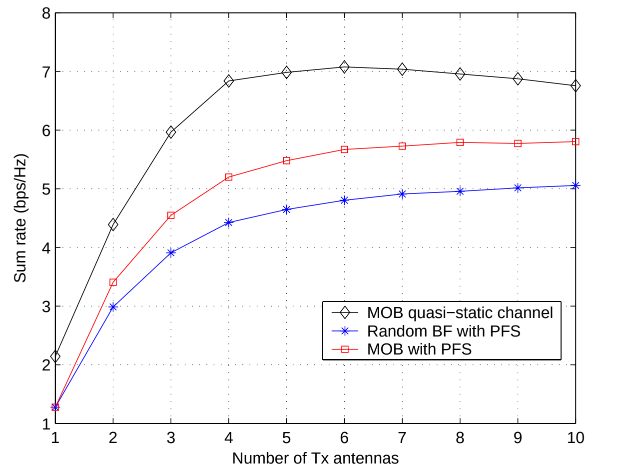

Fig. 3. Sum rate of both schemes with PFS vs. number of antennas for 20 users and 10Hz Doppler spread

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