SUBCARRIER ALLOCATION SCHEMES FOR MULTIUSER OFDM SYSTEMS

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ABSTRACT

We consider the problem of subcarrier allocation to multiple users in a downlink OFDM access system. We study allocation schemes that provide varying degrees of trade-off between (i) achieving throughput fairness among users, (ii) meeting tolerable latency requirements specified by the user applications, and (iii) achieving higher system capacity. The performance of the subcarrier allocation schemes is evaluated through simulation results.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) based systems have been proposed as a solution for broadband wireless access [5]. Simultaneous downlink transmission of data to different user terminals can be provided using wireless OFDM by assigning different subcarriers to different users. A good subcarrier allocation scheme should account for channel fading characteristics and balance user quality of service (QoS) requirements. In this work we study a subcarrier allocation scheme with these features in the context of a multiuser OFDM access (OFDMA) system.

Consider a single cell with a base station (BS) employing an OFDM based multiple access scheme to transmit data to K users. In a wireless environment, the channels between the OFDM system at the BS and the users vary both with time and across frequency. Further, due to geographical location differences, users experience different channel conditions even though on the same subcarrier frequency. In a naive fixed equal subcarrier allocation scheme, a fixed equisized set of subcarriers is allocated to each of the users. The time-frequency channel variations can however be exploited for multiuser diversity gains by allocating the OFDM subcarriers to the users as follows. Assuming that the BS has knowledge of each of the user's channel condition, a subcarrier (or a group of subcarriers) is allocated to the user that has the best instantaneous channel condition. Diversity gain arises from the fact that in an OFDMA system with multiple users, whose channels vary independently, there is a high probability of having a user whose channel quality is above a certain threshold. Such a subchannel assignment clearly leads to a larger system capacity as compared to the fixed allocation scheme. However this scheme tends to excessively favor users that have good channel conditions and neglects users that may be farther from the BS and hence have poorer channel quality. Moreover users supporting real-time traffic have delay constraints that need to be respected; such QoS considerations are not accounted for in the best user-channel selection scheme. Thus there is a need for a subcarrier allocation scheme with the following characteristics.

(I) Exploit multiuser diversity offered by variations across the time, frequency and spatial domains of the user channels.

(II) Apportion transmissions to users in a fair manner.

(III) Handle delay requirements of real-time traffic.

In this paper, we consider subcarrier allocation schemes that attempt to embody the above mentioned features and study their performance in a multiuser OFDMA environment.

The capacity of a multiuser OFDMA system can be increased by (i) intelligent subcarrier allocation, and (ii) power and bit loading. Our focus in this work is only on the former aspect. Algorithms for power and bit loading in a multiuser OFDM system can be found in [4], [7] and [14]. Joint optimization of subcarrier assignment and power/bit allocation is however impractical due to prohibitive computational complexity. A suboptimal approach would be decouple the two problems. In particular by doing so, the multiuser power and bit loading problem reduces to a set of single user power and bit loading problems, for a given subcarrier allocation. Efficient algorithms for power and bit loading in the single user case are available for example in [6]. We hence restrict our attention to the subcarrier allocation problem. Some previous approaches to this problem can be found in [4], [8], [10], [12], [14]. While the approach in [10]

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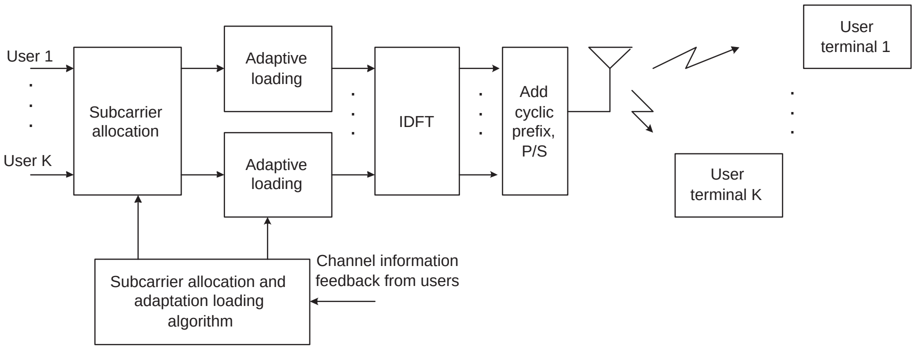

Fig. 1. Single antenna OFDMA system supporting multiple users

is based on exploiting multiuser diversity to increase capacity, the subcarrier allocation schemes in [4], [8], [14] aim at minimizing transmitted power for given bit rate and error rate constraints. A more general approach to allocating subcarriers is adopted in [12] by considering user-utility functions. Scheduling algorithms for CDMA and multiple antenna systems have been treated in [1], [11], [13]. Multiuser diversity with proportional fairness was treated in [13]. In [1], [11] different criteria for scheduling in high data rate (HDR) systems have been proposed.

In the next section, we describe the multiuser OFDMA system under consideration. We then describe in section 3 a subcarrier allocation scheme specifically targeted at real-time user applications. Simulation results comparing the performance of the subcarrier allocation are shown in section 4. Conclusions are drawn in section 5.

Notation: The union of P sets $\mathcal{A}_1, \ldots, \mathcal{A}_P$ is denoted by $\cup_{j=1}^{P} \mathcal{A}_j$. The number of elements in a set $\mathcal{A}$ is denoted by $|\mathcal{A}|$. $Pr(A)$ denotes the probability of occurrence of event A.

2. PRELIMINARIES

We consider the downlink service in a single cell with K users occupying an OFDMA single antenna system as shown in Fig. 1. The BS uses an OFDM system with M subcarriers to provide access to the K users. This is done through frequency division multiple access (FDMA) by allocating a portion of the subcarriers to each user. More precisely, let $\mathcal{I}_j$ be the set containing the indices of the subcarriers allotted to user j; $\cup_{j=1}^{K} \mathcal{I}_j = \{1, 2, \ldots, M\}$. Let N_j be the number of subcarriers assigned to user j, i.e. $|\mathcal{I}_j| = N_j$. We assume that users are assigned non-overlapping sets of subcarriers, i.e. $\mathcal{I}_j$, $j = 1, \ldots, K$ are disjoint sets.

As discussed in the Introduction, adaptive power/bit loading is not treated in this paper; we consider equi-powered transmissions over each subcarrier. Normalizing the total input power to unity, the power on each subcarrier is $1/M$.

Let $\rho_i(k, n)$ denote the channel gain to noise ratio for the channel between the BS OFDM's k-th subcarrier and the i-th user at time instant n. The capacity on this link is given by the well-known expression

$$C_i(k, n) = \log_2 \left(1 + \frac{1}{M} . \rho_i(k, n) \right). \tag{1}$$

Denote the average throughput of user i corresponding to the k-th subcarrier by $C_{i,av}(k, n)$. We assume that the BS keeps track of $C_i(k, n)$ and $C_{i,av}(k, n)$ for each user i on each subcarrier k.

We model QoS delay requirements for real-time users along the lines of [1], [11]. For user i, denote $W_i(k)$ to be the symbol delay on subcarrier k, T_i the threshold delay and δ_i the maximum probability of exceeding it. Note that by enforcing a worst-case delay requirement on each subcarrier, a delay of $\max_k W_i(k)$ is enforced on each user. For each user i, we define its probabilistic delay constraint in the form

$$Pr\{\max_k W_i(k) > T_i\} \leq \delta_i. \tag{2}$$

On each subcarrier at the BS, a scheduler needs to decide as to which user to allocate transmission on that frequency. This has to be done so that system capacity is increased by exploiting channel variations while maintaining fairness in subcarrier allocation among users and meeting the QoS delay constraints expressed in (2).

3. SUBCARRIER ALLOCATION SCHEMES

We now describe different schemes for subcarrier assignment to users. On each subcarrier k and time instant n, the packet scheduler has knowledge of $C_i(k, n)$, $C_{i,av}(k, n)$, δ_i, T_i, and $W_i(k)$. Different priorities are assigned to users, with the priority functions reflecting the characteristics (I-III) specified in the Introduction. Thus, consider for each user i, and subcarrier k at time slot n, the following priority functions. These functions embody attributes (I-III) with varying degrees.

(A) Instantaneous capacity based priority:

$$f_i(k) = C_i(k,n). \tag{3}$$

(B) Proportional fair based priority:

$$f_i(k) = \frac{C_i(k,n)}{C_{i,av}(k,n)}, \tag{4}$$

with $C_{i,av}(k,n)$ updated according to the rule

$$C_{i,av}(k,n{+}1) = \begin{cases} (1-\frac{1}{L})C_{i,av}(k,n) + \frac{1}{L}C_i(k,n) \\ \text{if user } i \text{ is chosen by subcarrier } k, \\ (1-\frac{1}{L})C_{i,av}(k,n) \qquad \text{else.} \end{cases} \tag{5}$$

L is a time window over which fairness is reflected.

(C) Exponential delay based proportional fair priority:

$$f_i(k) = \left[\frac{C_i(k,n)}{C_{i,av}(k,n)}\right] \cdot \left[\alpha_i \exp\left\{\frac{W_i(k) - W_{av}(k)}{1+\sqrt{W_{av}(k)}}\right\}\right] \tag{6}$$

where $\alpha_i = -\frac{\log \delta_i}{T_i}$ and $W_{av}(k)$ is the average of the delay, $W_i(k)$, values.

(D) Modified largest weighted delay first (M-LWDF) based priority:

$$f_i(k) = \alpha_i.\frac{C_i(k,n)}{C_{i,av}(k,n)}.W_i(k). \tag{7}$$

(E) Delay based priority:

$$f_i(k) = W_i(k). \tag{8}$$

The subcarrier allocation scheme then works as follows. Allocate subcarrier number k at time instant n to the $i^{\star}$-user where

$$i^{\star} = \arg \max_{1 \le i \le K} f_i(k) \tag{9}$$

The subcarrier allocation scheme based on priority criterion (A) exploits multiuser diversity by serving users that have the best channel conditions. Though this scheme enhances system capacity considerably, it fails to meet requirements (II-III). Similarly, subcarrier schemes based on priority functions (B) and (E) only have features (II) and (III) respectively. On the other hand, priority functions (C) and (D) exhibit QoS-aware characteristics and tend to balance features (I-III). In particular, the priority function (6) in (C) balances long-term fairness in terms of throughput (expressed by the first term) and QoS delay requirements (second term) with nominal degradation of system capacity. These observations are validated in the next section through simulations. Further as shown in [1], [11] scheduling based on priority functions (6) and (7) are attractive practically as well due to their throughput optimality, meaning that the queues at the BS do not blow-up if at all stable queues are feasible using any other algorithm.

4. SIMULATION RESULTS

In this section, we evaluate the performance of the subcarrier allocation schemes with respect to (i) average throughput, (ii) QoS delay requirements, and (iii) fairness in number of subcarriers allocated to users.

We consider an OFDM system with 64 usable subcarriers supporting 8 users in an isolated cell. To minimize signalling overhead, the subcarrier allocation is done by dividing the 64 subcarriers into four groups of 16 subcarriers each, under the following pattern: the ith group contains the set of subcarriers with index set $I_i = \{1+i, 5+i, 9+i, \ldots, 57+i, 61+i\}, i = 0, \ldots, 3$. As channel model for simulation evaluation, the exponential decay Rayleigh fading channel proposed in [2] is used. For this statistical model, we used a root-mean-square (RMS) delay spread, T_{RMS} of 30 ns. The channel is assumed static throughout the packet and generated independently for each packet. The impulse response of the channel is composed of complex samples with random uniformly distributed phase and Rayleigh distributed magnitude with average power decaying exponentially

$$h_i \sim \mathcal{N}(0, \sigma_k^2) + j\mathcal{N}(0, \sigma_k^2)$$

with $\sigma_k^2 = \sigma_0^2 e^{kT_s/T_{RMS}}$ and σ_0^2 chosen so that $\sum_k \sigma_k^2 = 1$. It is assumed that the sampling time T_s in the simulation is shorter than a symbol time by at least a factor of four. The number of samples that was taken in the impulse response $(10T_{RMS}/T_s)$ ensures sufficient decay of the impulse response tail. Moreover, the signals undergo an independent path loss PL_k that includes large scale fading and shadowing as expressed by the relation [9]

$$PL_k = PL(d_0) + 10\alpha \log_{10}(d_k/d_0) + X_\sigma$$

where d_0 is a reference distance, d_k is the distance between the transmitter and the receiver (user k), α is the path loss exponent and X_σ is a zero-mean Gaussian variable in the logarithmic scale responsible for shadowing with a standard deviation σ. The actual rate is chosen to be the largest available rate such that SNR is satisfied for a certain target BER. The rate distribution of the users are such that user 1 has most of the time the worst channel conditions, whereas user 8 has the best channel conditions. The allocation decisions are made once every slot, and simulations were performed by averaging over 100,000 independent channel realizations. For the proportional fairness scheme the value of the averaging window size L is chosen equal to 1000 [13]. The total transmit power is equally distributed to all subcarriers, and the service rates change in time randomly for different users. All active users have infinite backlog. The delay requirements are chosen as follows: users 1 and 3 have delay threshold 3 symbol duration with a violation

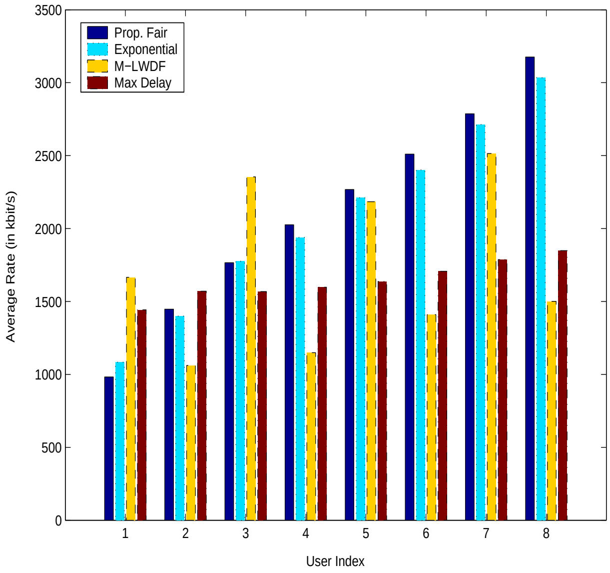

Fig. 2. Average rate

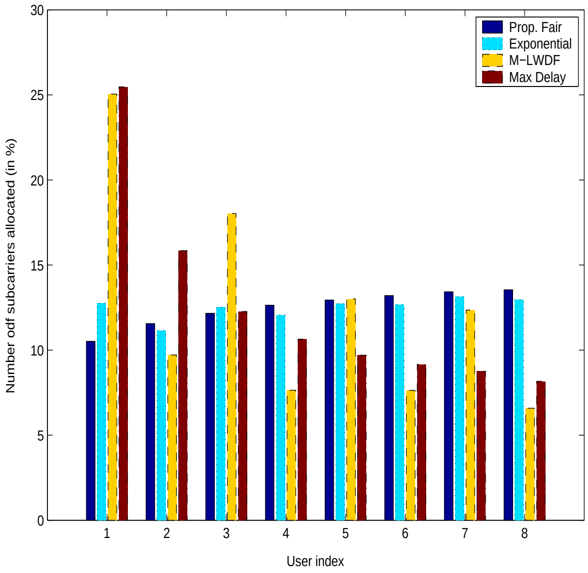

Fig. 3. Average number of allocated subcarriers

probability of 0.01, and users 5 and 7 have delay threshold 5 symbol duration with the same probability of exceeding it. The value of the probability is a reasonable value for Internet streaming audio [13]. Users 2 and 6 have delay threshold 10 symbol duration whereas users 4 and 8 have 12 symbol duration, all four with dropping probability equals to 0.05. We enforce this delay requirement on each subcarrier in the allocation scheme; by doing so, the user delay requirements are automatically met.

We compare the subcarrier allocation schemes (A-E) detailed in Section 3. In Figures 2 and 3 the performance of the five subcarrier allocation schemes with respect to the average capacity and the average number of allocated subcarriers is shown. The best user-channel selection scheme is not shown in these figures; it selects user 8 over most transmission slots as it has substantially higher SNRs. As such the rest of users are not allocated too many subcarriers due to which their average capacity is close to zero. On the other hand, the subcarrier allocation schemes (B)-(D) tend to allocate a fair portion of subcarriers among all eight users, refraining low SNR users from starvation. As a result, the capacity of users 1 to 7 is substantially higher under these allocation schemes. In Figure 3, we see that under subcarrier allocation schemes (D) and (E), user 1 is allocated high number of subcarriers as its delay requirements are the tightest. These two schemes are particularly delay-sensitive, sacrificing a large portion of the capacity to users with very demanding delay requirements. The same phenomenon is also present for users 2 and 3 but in smaller extent. Note also from the same figure that user 8 receives more average number of subcarriers under the proportional fairness scheme than under the exponential rule based scheme. This is a result of the looser delay requirements of user 8 as compared to the other users due to which its priority gets lowered under the exponential rule. The opposite behavior stands for user 1 whose delay requirements are the tightest. In Figure 2 we see that the schemes (B) and (C) deliver capacity to users with similar way. The only difference is that the proportional fairness allocation scheme provides larger portion of the overall capacity than the exponential rule based scheme as seen in Figure 5, its performance however deteriorating in terms of latency.

Figure 4 shows the average delay in symbol duration for each user. Under scheme (A) the queues for users 1 to 6 blow up and hence this scheme is not shown in the figure. Under the proportional fairness scheme, the delay requirements of user 1 are not met. Though this scheme meets the delay requirements of the other seven users, we note that the performance of this scheme tends to get worse as the number of users is increased and delay requirements are more stringent. In comparison, the exponential delay based rule meets delay requirements for all users. The subcarrier allocation scheme (E) allocates subcarriers depending only on the latency of users on each slot, resulting in an excellent delay performance but providing very low system capacity. The subcarrier allocation scheme (D) results in delay performance close to that of scheme (E) at the expense of low throughput performance. The exponential rule based scheme satisfies the delay constraints with slight concession in the system sum rate. This observation is of significance

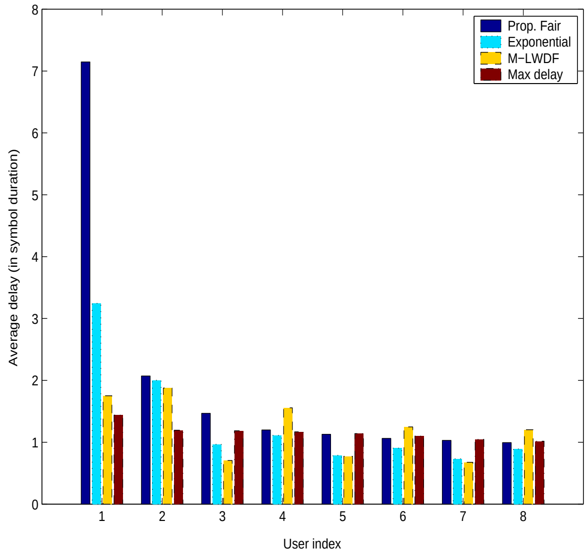

Fig. 4. Average delay

for designing practical subcarrier allocation schemes.

5. CONCLUSIONS

We considered different priority functions for allocation of subcarriers among multiple users in a single cell OFDMA system. We showed through supporting simulations that the exponential delay based proportional fairness scheme was a good choice for providing QoS delay guarantees with nominal system rate degradation. Future work will involve a more in-depth study of QoS-endowed priority functions and will consider extending results to multiple antenna systems.

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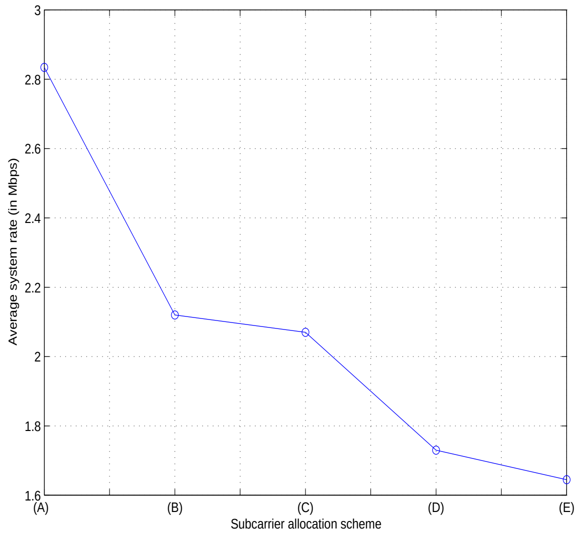

Fig. 5. Average system rate

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