A Lower Bound for Vehicles' Trajectory Duration

Jerome Haerri and Christian Bonnet

Eurecom Institute, Mobile Communication Laboratory

06904 Sophia-Antipolis, France

{Jerome.Haerri,Christian.Bonnet}@eurecom.fr

Abstract— In Palm Calculus, the Palm intensity of a particular transition is the model's expected number of transitions per time unit when considered at stationary regime. Considering transitions as the event of vehicles reaching a waypoint in MANETs' Mobility Models, the Inverse Palm Intensity (IPI) is defined as the mean interval between two waypoints, or the expected time spent by vehicles to reach a predefined target. We propose in this paper to study this Palm intensity because such information might be very useful in MANET for example to efficiently adjust the refreshing intervals of ad hoc routing and topology control protocols. We obtain in this paper a lower bound for the IPI situated at $\approx$ 7s averaged on the Random-Waypoint mobility model (RWM) and the City Section mobility model (CityM), both considered at steady state. Therefore, by setting a refreshing period to this time interval, it is possible to improve the global performance of topology control and routing protocols, since control overhead is reduced.

I. INTRODUCTION

Palm calculus[5] is a set of formula that relate time averages versus event averages. Time averages are obtained by sampling the system at arbitrary time instants. The event average viewpoint is obtained by sampling the system when selected state transitions occur. In MANETs, Palm calculus is applied to mobility models in order to avoid subtle problems, such as speed decay of average speed as simulation progresses, or such as getting rid of differences between the long term distribution of nodes and the initial one. One important concept in Palm Calculus is the Palm intensity. It is defined as the expected number of state transitions per time unit. When Palm calculus is applied to mobility models, a state transition is defined as the time instant when new parameters are set (direction, speed,..). The *Inverse Palm Intensity (IPI)* is therefore defined as the mean interval between two successive state transitions. Although the Palm distribution of speeds and positions of mobile nodes have already been asserted in [3], to our knowledge, the Palm intensity has never been analyzed in MANET's mobility models.

In this paper, we make use of Palm Calculus to provide a theoretical lower bound on the mean interval between two successive waypoints, also called **trajectory duration**[1], for vehicular motion. We show that this value never falls below $7s$ on average. This result is validated through simulations using the Random Waypoint Mobility model and the City Section Mobility model belonging to the Random Trip Framework [3]. It therefore motivates the use of *aperiodic*[2] topology maintenance strategies, since setting a lower bound on topology updates to 7 seconds makes the number of maintenance messages drop dramatically. Accordingly, it becomes conceivable to consider prediction-based models [1] to reach optimal aperiodic maintenances.

II. PALM INTENSITY

A. Random Waypoint

Palm calculus is now well established, but not widely used or even known in applied areas[3]. We do not use all the Palm calculus

---

[1]Therefore, Inverse Palm Intensity, Mean Interval between two successive waypoints, and Trajectory Duration will be later interchangeably used

[2]Aperiodic strategies adapt their refreshing intervals to the motion of nodes

[3]For a quick overview of Palm calculus, refer to [2]; for a full fledged theory, see [5]

---

framework here but only concentrate on the Palm Intensity λ. We apply Palm calculus to the random waypoint model. We assume that this model has a stationary regime for a minimum velocity strictly greater than zero (see [2] for a complete proof) and consider as selected transitions instant T_n the time at which waypoints are reached. Since the simulation is in stationary regime, we imagine that, at time 0, the simulation has been running for some time. We take as convention $T_0 \leq 0 < T_1$. In other words, T_0 is the last time a transition occurred before time 0 and T_1 is the next one starting from 0. Considering $T_0 = 0$, the Palm intensity formula is given by

$$\frac{1}{\lambda} = \mathbb{E}^0(T_1 - T_0) = \mathbb{E}^0(T_1)$$

The inverse Palm intensity (IPI), or the mean interval between two successive waypoints, is therefore given by

$$\lambda^{-1} = \mathbb{E}^0(D_1)\mathbb{E}^0(\frac{1}{V_0}) = \overline{\Delta} \int_0^\infty \frac{1}{v} f^0(v) dv \quad (1)$$

where $\overline{\Delta}$ is the average distance between two points in the simulation area, and $f^0(v)$ is the Palm density distribution of speeds. The intensity is finite if and only if $\mathbb{E}^0(\frac{1}{V_0})$ is finite, which, for the uniform speed case, means $v_{min} > 0$. There exists a closed form for $\overline{\Delta}$ when the simulation area is a rectangle [6]. We consider here the simulation area as a $a \times a$ square and use the closed form $\overline{\Delta} \approx 0.5214a$.

B. Uniform Time Stationary Distribution of Speeds

When speeds are chosen from the uniform distribution with a low minimum speed, then at any given time, a large proportion of nodes will be moving very slowly. Since the average distance between nodes is fixed, this can create a nearly stable backbone that could make the Palm intensity seem unrealistically good. Therefore, the worst case for the intensity would be to have a uniform time stationary distribution of speeds which keeps vehicles velocities uniformly distributed through the simulation. Accordingly, the Palm intensity is reduced to the ratio between $\overline{\Delta}$ and the mean time stationary distribution of speeds. We also consider this case in our theoretical values and consider an appropriate choice of $f^0(v)$ proposed in [4]:

$$f^0(v) = \frac{2s}{v_{max}^2 - v_{min}^2} \text{ for } v_{min} < s < v_{max} \quad (2)$$

Therefore, the inverse intensity is reduced to

$$\lambda^{-1} = \overline{\Delta} \frac{2}{v_{max} + v_{min}}$$

C. Random Waypoint with Pausing

A final consideration would be the pause times included in the Random mobility model. When a mobile reaches a waypoint, it picks a pausing duration according to the density f_{pa}^0, stays immobile for this duration, and then moves again. To analyze this model, we consider as selected transition times the time instants at which either a waypoint is reached or a pausing time is expired. From [2], the intensity formula gives

$$\frac{1}{\lambda} = 0.5 \frac{1}{\lambda_{pa}} + 0.5 \frac{1}{\lambda_{mo}} \quad (3)$$
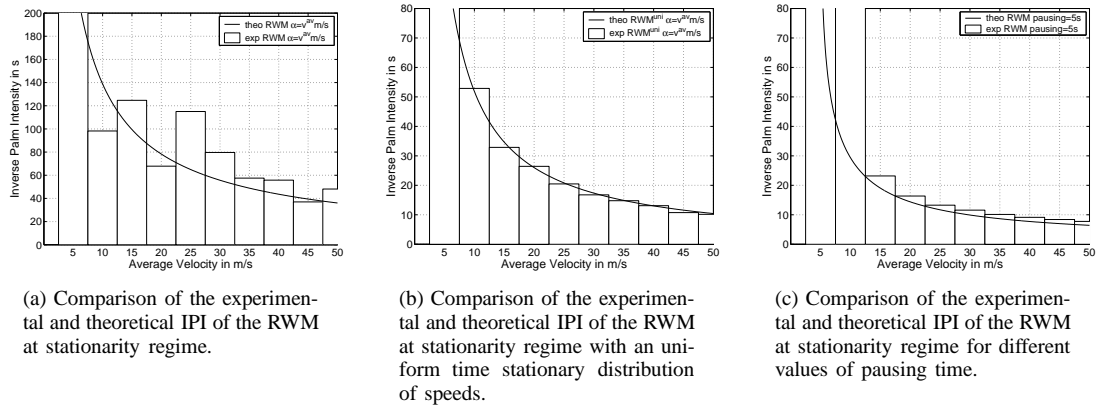
(a) Comparison of the experimental and theoretical IPI of the RWM at stationarity regime.

(b) Comparison of the experimental and theoretical IPI of the RWM at stationarity regime with an uniform time stationary distribution of speeds.

(c) Comparison of the experimental and theoretical IPI of the RWM at stationarity regime for different values of pausing time.

Fig. 1. Comparison of the experimental and theoretical IPI in a Random Waypoint mobility model.

III. PRELIMINARY EXPERIMENTAL RESULTS

We assessed the Inverse Palm intensity through simulations and compared it with the theoretical values obtained at the previous section (See solid lines in Figure 1). We have evaluated the IPI for both the RWM and the City Section in which we have used two different samples of Houston, TX. Since both mobility models are extracted from the Random Trip Framework [3], the obtained IPI is when both models are at steady state. We have simulated $900s$ of both models for different pausing times, but due to the space limitation, we only include here one value. Nodes were assumed to be moving in a flat squared area of $1000m \times 1000m$. Finally, we also have simulated the RWM using the uniform time stationary distribution of speeds given in (2).

Figure 1 shows the characteristics of the inverse Palm intensity given the mean speed on the RWM and City Section evaluated at regular $5m/s$ intervals. The solid line represents the theoretical value for the IPI, while the boxes are the experimental ones. Each box represents 10 runs. We can see that, compared to Figures 1(a), Figure 1(b) is, as expected, the worst case configuration for the RWM given its uniform time-stationary distribution of speeds. The IPI values are on average 75% smaller than IPI obtained with a uniform non time-stationary distribution. Figure 1(c) gives smaller values for the IPI than all other configurations for the RWM due to the IPI of the pause transitions (Eq. 3). As we could expect, the smallest IPIs are obtained for large velocities. When considering the RWM with small pausing times, the IPI is $\approx 7s$. Yet, it gets improved when the average pausing time increases.
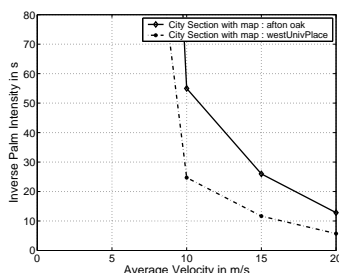

Fig. 2. Inverse Palm intensity in a City Section Model for different city configurations

We also have simulated the City section in Figure 2 with two different road topologies. Vehicles are moving at constant speed on a road and each intersection represents a waypoint. Since the mean distance between intersections is far lower in our maps than in our RWM simulations, we obtained smaller values for the IPI than those in the RWM. We only considered here a range of average velocity varying up to $20m/s$, since larger velocities would not be acceptable for realistic situations. Averaged on both maps, when the mean speed is $20s$, the IPI is $\approx 9s$. However, in a configuration where mean speeds could vary depending on the street category (similar to a speed limit for example), the IPI would be bigger than $9s$.

IV. COMMENTS AND CURRENT WORK

We have provided a lower bound on the average trajectory duration, or inverse Palm intensity, that is $\approx 7s$ using extreme values for the configuration parameters of the mobility models. In more realistic situations, this value is rather $\approx 30s$. Therefore, topology control and routing protocols' refreshing process may be optimized. For example, considering recommendations of the RFC3626, if we set OLSR[7]'s topology update intervals to $10s$, the corresponding overhead would be reduced up to 85% .

Although very interesting, these values depend on nodes average velocity and on the distance between two successive waypoints. Even though it is not an easy task to obtain a good estimate of their values in real situations, we can find a dual behavior for pedestrian and vehicular motions. When nodes move fast, they usually follow predefined routes and their trajectories may be easily predicted. But when nodes experience random walks, they usually move at a lower speed and results obtained in this paper give estimates on their average trajectory duration. Therefore, nodes mobility assessment depends on the application for the deployment of mobile ad-hoc networks.

In the final version of the paper, we shall include a larger diversity of maps used to simulate the City Section Mobility model and provide extensive results notably showing the effects of velocity and pause time on the average trajectory duration. We shall also provide some simulation results of the well-known MPR (Multi-Points Relay) protocol used in OLSR [7] adapted to handle mobility predictions, quantifying the performance increase of protocols using the knowledge of vehicles' average trajectory duration.

REFERENCES

[1] J. Haerri, Navid Nikaein, and Christian Bonnet, "Trajectory knowledge for improving topology management in mobile ad-hoc networks", in *Research Report RR_4_112*, Eurecom Institue, France, 2004.

[2] Jean-Yves Le Boudec, "Understand the simulation of mobility models with palm calculus", in *Technical Report EPFL/IC/2004/53*, EPFL, 06 2004.

[3] Jean-Yves Le Boudec and Milan Vojnovic, "Perfect Simulation and Stationarity of a Class of Mobility Models", to appear in *Proc. of the Infocom'05*, USA, 2005.

[4] William Navidi and Tracy Camp, "Stationarity Distributions for the Random Waypoint Model", in *IEEE Transactions on Mobile Computing*, 3(1), 2004.

[5] F. Baccelli and Pierre Bremaud, "Palm Probabilities and Stationary Queues", in *Springer LNS*, 1987.

[6] B. Ghosh, "Random distances within a rectangle and between two rectangles", *Bulletin Calcutta Mathematical Society*, 43:17-24, 1951.

[7] T. Clausen *et al*, "Optimized Link State Routing Protocol", *IEEE INMIC*, Pakistan, 2001.