MMSE-GDFE Lattice Decoding for Solving Under-determined Linear Systems With Integer Unknowns

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Abstract — **Minimum mean square error generalized decision-feedback equalizer (MMSE-GDFE) lattice decoding is shown to be an efficient decoding strategy for under-determined linear channels. The proposed algorithm consists of an MMSE-GDFE front-end followed by a lattice reduction algorithm with a greedy ordering technique and, finally, a lattice search stage. By introducing flexibility in the termination strategy of the lattice search stage, we allow for trading performance for a reduction in the complexity. The proposed algorithm is shown, through experimental results in MIMO quasi-static channels, to offer significant gains over the state of the art decoding algorithms in terms of performance enhancement and complexity reduction. On the one hand, when the search is pursued until the best lattice point is found, the performance of the proposed algorithm is shown to be within a small fraction of a dB from the maximum likelihood (ML) decoder while offering a large reduction in complexity compared to the most efficient implementation of ML decoding proposed by Dayal and Varanasi (e.g., an order of magnitude in certain representative scenarios). On the other hand, when the search is terminated after the first point is found, the algorithm only requires linear complexity while offering significant performance gains (in the order of several dBs) over the linear complexity algorithm proposed recently by Yao and Wornell.**

I. The System Model and the Proposed Algorithm

The proposed algorithm can be used to solve the general problem of separating m sources from observations made by $n \leq m$ sensors through linear Gaussian channels. Such channels include under-determined multiple-input multiple-output (MIMO) systems with more transmitters than receivers, overloaded CDMA system or ISI channels, for example. The received signal can be written as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w} \tag{1}$$

where $\mathbf{H} \in \mathbb{R}^{n \times m}$ is the channel matrix, $\mathbf{x} = \mathbf{G}\mathbf{u}$ is an $m \times 1$ lattice point (generated by the lattice generator matrix $\mathbf{G} \in \mathbb{R}^{m \times m}$) with $\mathbf{u} \in \mathbb{Z}^m$ and $\mathbf{w}$ is an AWGN. Let $\mathbf{F}$ and $\mathbf{B}$ be the forward and backward filters of the MMSE-GDFE, respectively. Then, the algorithm is given by

1. Apply the MMSE-GDFE filtering on the received signal

$$\mathbf{y}' \quad \triangleq \quad \mathbf{F}\mathbf{y} = \mathbf{B}\mathbf{G}\mathbf{u} + \mathbf{n} \tag{2}$$

   where $\mathbf{n}$ is the noise which contains the self noise term $[\mathbf{B} - \mathbf{F}\mathbf{H}]\,\mathbf{G}\mathbf{u}$ resulting from MMSE-GDFE stage.

2. Apply the LLL reduction algorithm [1] on matrix $\mathbf{BG}$ to obtain $\mathbf{S} = \mathbf{BGU}$ with $\mathbf{U}$ a unimodular matrix and $\mathbf{S}$ has reduced columns.
3. V-BLAST greedy order the columns of $\mathbf{S}$ [2].
4. Apply lattice decoding on $\mathbf{S}$ and $\mathbf{y}'$ using the Schnorr-Euchner enumeration and a finite radius as in [2] to obtain $\hat{\mathbf{u}}$. In this stage, one can incorporate any predefined termination strategy or metric computation inside the Shcnorr-Euchner enumeration using the analogy with sequential decoding [2].
5. The estimated codeword is obtained as $\hat{\mathbf{c}} = \mathbf{GU}\hat{\mathbf{u}}$.

The power of the MMSE-GDFE front-end is manifested in the fact that (2) is a full rank linear system of m equations and m unknown since $\mathbf{B}$ is always invertible (and has all its eigenvalues larger than 1). Furthermore, it is straightforward to see that the first point found by the Schnorr-Euchner enumeration is the Babai point for the lattice in (2) after applying LLL algorithm. If the search is terminated at this point, the complexity is only linear in the number of variables (assuming the fading channel is very slow such that the overhead of MMSE-GDFE and LLL can be ignored). Recently, Yao and Wornell have proposed a decoding algorithm with a comparable complexity to this linear complexity variant of the proposed scheme. In this algorithm, the output codeword is the solution of zero-forcing decision-feedback equalizer, or the Babai point, of the LLL reduction of $\mathbf{HG}$ in (1) for systems with $m \leq n$ [3] (i.e., the difference between the decoder in [3] and the linear complexity variant of our scheme is the MMSE-GDFE front-end). Unlike our algorithm, one can easily see that the decoder [3] does not extend to under-determined systems. Moreover, simulation results show that our algorithm largely outperforms the decoder [3] while maintaining the linear complexity when $m \leq n$. For under-determined systems, the proposed algorithm offers a large reduction in complexity over the most efficient implementation of the ML decoder [4] with only a very samll loss in performance.

References

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