Blind Optimal MMSE Receiver for Asynchronous CDMA in the Presence of Multipath *

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Abstract

We consider multiple users in a DS-CDMA system operating in a multipath environment. The received cyclostionary spread signal sampled at the chip rate is converted to a stationary vector signal, leading to a linear multichannel model. Prior knowledge of the transmission pulse (spreading sequence) is exploited to obtain the desired user channel estimate blindly. It is shown that the multiple user problem can be decoupled and a single user scenario can be obtained to apply the blind estimation and interference cancelation algorithms banking on limited a priori knowledge. The FIR channel estimation problem is investigated and the optimal MMSE receiver for multipath channels is presented along with a computationally more efficient Interference Canceling Rake Receiver (ICRR). Both the MMSE receiver and the ICRR are parameterized in terms of the desired user's channel and quantities that can be estimated from second-order statistics.

1. Multiple User Data Model

The u users are assumed to transmit linearly modulated signals over a linear channel with additive Gaussian noise. It is assumed that the receiver employs a single antenna to receive the mixture of signals from all users. Oversampling is inherent to CDMA systems due to the large (extra) bandwidth and the need to resolve chip pulses. Therefore, the use of multiple antennas, while certainly bringing extra diversity beyond the spread bandwidth [5], might not be as fetching as in other multiple access methods. The received signal can be written in baseband notation as

$$y(t) = \sum_{j=1}^{u} \sum_{k} a_j(k) g_j(t - kT_s) + v(t), \qquad (1)$$

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where $a_j(k)$ are the transmitted symbols from the user j, T_s is the common symbol period, g_j is the overall channel impulse response for the jth user. Assuming the $\{a_j(k)\}$ and $\{v(t)\}$ to be jointly wide-sense stationary, the process $\{y(t)\}$ is wide-sense cyclostationary with period T_s. Oversampling the received signal at L times the symbol rate (LT_c here is the processing gain), we obtain the wide-sense stationary $L \times 1$ vector signal $\mathbf{y}(k)$ at the symbol rate. The overall channel impulse response for jth user's signal, g_j, is the convolution of the transmitter pulse (spreading code) and $h_j(t)$, the convolution of the pulse shape and the actual channel (assumed to be FIR) representing the multipath fading environment. This can be expressed as

$$g_j(t) = \sum_{l=0}^{L-1} c_j(l) h_j(t - lT_c), \qquad (2)$$

where T_c is the chip duration. We consider that the longest FIR channel length among all users is L_c (in T_c's). For channels that are shorter, we can still consider them to be of length L_c with zeros concatenated at the end. Let k_j be the chip-delay index for the jth user: $h_j(k_jT_c)$ is the first non-zero chip-rate sample of $h_j(t)$. The parameter N_j is the duration of $g_j(t)$ in symbol periods. It is a function of L_c and k_j. If $L_c < L$, then in asynchronous conditions ($k_j \neq 0$), $2 \leq N_j \leq 3$: the overall channel spans at most three symbol periods. We consider user 1 as the user of interest and assume that $k_1 = 0$ and $N_1 = 2$ (synchronization to user 1). Let $N = \sum_{j=1}^{u} N_j$. The vectorized chip-rate samples lead to a discrete-time $L \times 1$ vector signal at the symbol rate that can be expressed as

$$\begin{aligned} \mathbf{y}(k) &= \sum_{j=1}^{u} \sum_{i=0}^{N_j-1} \mathbf{g}_j(i) a_j(k-i) + \mathbf{v}(k) \\ &= \sum_{j=1}^{u} \mathbf{G}_{j,N_j} A_{j,N_j}(k) + \mathbf{v}(k) \\ &= \mathbf{G}_N \mathbf{A}_N(k) + \mathbf{v}(k) \end{aligned} \qquad (3)$$

where

$$\mathbf{y}(k) = \begin{bmatrix} y_1(k) \\ \vdots \\ y_L(k) \end{bmatrix}, \ \mathbf{g}_j(k) = \begin{bmatrix} g_{1j}(k) \\ \vdots \\ g_{Lj}(k) \end{bmatrix}$$

$$\mathbf{v}(k) = \begin{bmatrix} v_1(k) \\ \vdots \\ v_L(k) \end{bmatrix} \quad \begin{matrix} \mathbf{G}_{j,N_j} = \left[\ \mathbf{g}_j(N_j-1)\dots\mathbf{g}_j(0) \ \right] \\ \mathbf{G}_N = \left[\ \mathbf{G}_{1,N_1}\dots\mathbf{G}_{u,N_u} \ \right] \end{matrix}$$

$$A_{j,N_j}(k) = \left[\ a_j^H(k-N_j+1) \quad \dots \quad a_j^H(k) \ \right]$$

$$\mathbf{A}_N(k) = \left[\ A^H_{1,N_1}(k) \quad \dots \quad A^H_{u,N_u}(k) \ \right],$$

and the superscript ^{H} denotes Hermitian transpose. The matrix $\mathbf{G}_{1,N_1}$ (for user 1) can be written in terms of the spreading code and the channel vector $\mathbf{h}_1$ as $\mathbf{G}_{1,N_1} = [\mathbf{g}_1(1) \ \ \mathbf{g}_1(0)]$ with $\mathbf{g}_1(i) = \mathbf{C}_1(i)\mathbf{h}_1$, and where,

$$\mathbf{C}_1(0) = \begin{bmatrix} c_1(0) & & 0 \\ c_1(1) & \ddots & c_1(0) \\ \vdots & & \vdots \\ c_1(L-1) & & c_1(L-L_c-1) \end{bmatrix}, \text{ and,}$$

$$\mathbf{C}_1(1) = \begin{bmatrix} 0 & c_1(L-1) & \dots & c_1(L-L_c) \\ \vdots & \ddots & \ddots & \vdots \\ & & & c_1(L-1) \\ & & & \vdots \\ 0 & \dots & & 0 \end{bmatrix}$$

For the interfering users, we have a similar setup except that owing to asynchrony, a third coefficient might appear in $\mathbf{G}_{j,N_j}$.

It is clear that the signal model as described above addresses a multiuser setup with a possiblity of joint interference cancelation for all sources simultaneously [6] provided the timing information and spreading codes of all of them are available. As we shall see later, it is possible to decompose the problem into single user ones thus making the implementation suitable for applications such as at mobile terminals or as pre-processing stage at the base station.

2. Previous Work

Blind solutions for CDMA systems have received considerable attention since the pioneering work of [2]. The desirable feature of such a scheme is that its informational complexity is the same as that of a matched filter detector, i.e., only the desired user signature waveform and timing are required for its operation. Besides, it seems intuitively correct to employ at the mobile terminal an algorithm that banks simply on the information destined for itself.

The problem addressed in [2] is that of DS-CDMA communications over the a channel without multipath. Other recent work has been concentrated on investigating solutions for multipath channels. Constrained optimization schemes were proposed in [8] [3] where the receiver's output energy is minimized subject to appropriate constraints. Convergence to the optimal MMSE receiver is sought in these receivers. The above mentioned receivers have been shown to converge asymptotically (SNR$\rightarrow \infty$) to the decorrelating solution. However, the solution in [3] handles the synchronous case while the solution in [8] degrades as the number of asynchronous users increases.

In this paper, we present the optimal MMSE receiver for multipath channels and asynchronous conditions, obtained by applying multichannel linear prediction to the received cyclostationary signal. An alternative *rake*-like configuration is also presented and blind implementation of these structures is explored.

3. MMSE Interference Suppression by Linear Prediction

3.1. Multichannel Linear Prediction

Consider multichannel linear prediction with the predictor error transfer function $\mathbf{P}(z)$ yielding prediction error $\tilde{\mathbf{y}}_k = \mathbf{P}(z)\mathbf{y}_k$. We then have, $\mathbf{P}(z)S_{\mathbf{yy}}(z)\mathbf{P}^\dagger(z) = S_{\tilde{\mathbf{y}}\tilde{\mathbf{y}}}(z)$. The prediction errors are white for infinite prediction order [6].

3.2. The Optimal MMSE Equalizer

Let $\mathbf{G}(z) = \mathbf{G}_1(z)$ be the channel transfer function for the desired user The structure of the optimal MMSE linear equalizer is given in fig. 1. We find,

$$\mathbf{F}_{MMSE}(z) = S_a\mathbf{y}(z)S^{-1}_{\mathbf{yy}}(z) = \sigma_a^2\mathbf{G}^\dagger(z)S^{-1}_{\mathbf{yy}}(z), \quad (4)$$

where, $\mathbf{G}(z)$ can be estimated *blindly* using second-order statistics of the received signal as explained in section 5 and $\mathbf{G}^\dagger(z) = G^H(1/z^H)$. As for $S^{-1}_{\mathbf{yy}}(z)$, a finite number of correlation lags of $\mathbf{y}(k)$ are adequate to determine the prediction error filters $\mathbf{P}(z)$ leading to the FIR model $S^{-1}_{\mathbf{yy}}(z) \approx \mathbf{P}^\dagger(z)R^{-1}_{\tilde{y}\tilde{y}}\mathbf{P}(z)$ [7]. It is seen that $S^{-1}_{\mathbf{yy}}$ is Infinite Impulse Response (IIR) in general. In the noiseless (singular) case, however, the FIR assumption on $S^{-1}_{\mathbf{yy}}(z)$ turns out to be exact. In other words, a finite number of correlation lags of $\mathbf{y}(k)$ are adequate for its estimation. Let us denote

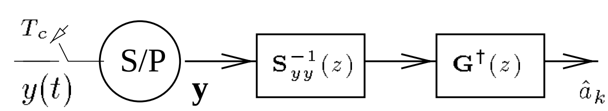

Figure 1. Optimal MMSE Receiver Structure

by u_1 and u_2, the number of users with channel lengths spanning two and three symbol periods respectively. Then, for the decorrelating condition ($\mathbf{P}(z)\mathbf{G}(z) = constant$) to hold, the shortest FIR predictor that suffices is of order

$$\underline{M} = \left\lceil \frac{2u_1 + 3u_2}{L - u_1 - u_2} \right\rceil \quad . \tag{5}$$

From the above discussion, it is clear that the optimal receiver length is not only a function of the number of symbols spanned by ISI but also of the number of active users. Furthermore, the effect of future symbols renders the structure non-causal.

3.3. The Noise Subspace

We stack M successive $\mathbf{y}(k)$ vectors in a super vector $\mathbf{Y}_M(k) = \mathbf{Y}_M(k) = \mathcal{T}_{M,u}(\mathbf{G}_N)A_{N+u(M-1)}(k) + \mathbf{V}_M(k)$, where, $\mathcal{T}_{M,u} = [\mathcal{T}_M(\mathbf{G}_{1,N_1}) \ \cdots \ \mathcal{T}_M(\mathbf{G}_{u,N_u})]$ and $\mathcal{T}_M(\mathbf{x})$ is a banded block Toeplitz matrix with M block rows and $\left[\mathbf{x} \quad \mathbf{0}_{n\times(M-1)}\right]$ as first block row (n is the number or rows in $\mathbf{x}$). Assuming the chip rate sampled noise to be white, the covariance matrix of the received signal can be written as

$$\mathbf{R}^Y_M = \mathcal{T}_{M,u}(\mathbf{G}_N)\mathbf{R}^a_{N+u(M-1)}\mathcal{T}^H_{M,u}(\mathbf{G}_N) + \sigma^2_v \mathbf{I}_{LM}. \tag{6}$$

Let us consider the noiseless covariance matrix ($v(t) \equiv 0$). We further suppose that the transmitted symbols are uncorrelated $\mathbf{R}^a = \sigma^2\mathbf{I}$. Upon applying Gram-Schmit orthogonalization scalar component by scalar component, on the elements of $\mathbf{Y}_M(k)$, we build the UDL factorization of $(\mathbf{R}^y_M)^{-1}$, and obtain the consecutive prediction error filters and variances. We can write $\mathbf{P}\mathbf{Y}_M(k) = \widetilde{\mathbf{Y}}_M(k) \Rightarrow \mathbf{P}\mathbf{R}^Y\mathbf{P}^H = \mathbf{R}^{\widetilde{\mathbf{Y}}}$, where, $\widetilde{\mathbf{Y}}(k)$ contains the prediction errors and the rows of $\mathbf{P}$ the prediction error filters of consecutive orders. $\mathbf{R}^{\tilde{Y}}$ is diagonal and contains the prediction error variances. Referring to section 3.2, we see that the first singularities will be encountered within the block row $\underline{M}$ of $\mathbf{R}^Y$ in which the elements of $\mathbf{y}(k + \underline{M} - 1)$ are processed. If $\mathcal{T}_{M,u}(\mathbf{G}_N)$ is of full column rank, then we shall come across $\overline{M} = M(L-u) - N + u$ singularities where $M \in \{0, 1, \cdots, L-u-1\}$. The corresponding elements in $\mathbf{R}^{\tilde{Y}}$ are zero. The rows in the lower triangular matrix $\mathbf{P}$ corresponding to these zero diagonal elements are termed *singular prediction error filters* [6]. For $M = \underline{M} + 1$, $\mathcal{T}_{M+1,u}(\mathbf{G}_N)$ will have L more columns while just u more rows; hence the rank of $\mathbf{R}_{YY}$ increases by u. We stack all singular prediction error filters in a $(M(L-u) - N + u) \times LM$ matrix, $\mathcal{G}_M$, the row space of which is the transpose of the noise subspace.

4. Interference Canceling Rake Receiver

Let us investigate the case where the interferers plus the channel noise are treated as Gaussian noise of unknown color. In such a scenario, the interference canceling scheme shown in fig. 2 is optimal. The transformation

$$\mathbf{x}_k = \mathbf{A}(z)\mathbf{y}_k = \begin{bmatrix} x_{1,k} \\ \mathbf{x}_{2,k} \end{bmatrix} = \begin{bmatrix} \mathbf{G}^\dagger(z)\mathbf{G}(z)a_k + w_{1,k} \\ \mathbf{w}_{2,k} \end{bmatrix} \tag{7}$$

where,

$$\mathbf{A}(z) = \begin{bmatrix} \mathbf{G}^\dagger(z) \\ \mathbf{G}^{\perp\dagger}(z) \end{bmatrix}, \quad \text{and,}$$

$$\mathbf{G}^{\perp\dagger}(z) = \begin{bmatrix} -G_2(z) & G_1(z) & 0 & \cdots & 0 \\ 0 & -G_3(z) & G_2(z) & \cdots & \vdots \\ \vdots & & \ddots & \ddots & \\ G_L(z) & 0 & \cdots & & -G_1(z) \end{bmatrix}$$

splits $\mathbf{y}_k$ into the desired signal and interference components. Many other configurations for the signal blocking matrix, $\mathbf{G}^{\perp\dagger}(z)$, are possible [7], the one above being convenient when the channel coefficients are known (estimated *a priori*). It is clear that $\mathbf{w}_{2,k}$ contains no signal of interest but some of its components are correlated with $w_{1,k}$ and can be used to lower the interference level in the latter. In order to accomplish this, consider the transformation

$$\mathbf{b}_k = \mathbf{B}(z)\mathbf{x}_k = \begin{bmatrix} 1 & -W(z) \\ 0 & \mathbf{I}_{L-1} \end{bmatrix} \begin{bmatrix} x_{1,k} \\ \mathbf{x}_{2,k} \end{bmatrix}, \tag{8}$$

where, $W(z) = S_{x_1\mathbf{x}_2}S^{-1}_{\mathbf{x}_2\mathbf{x}_2} = S_{w_1\mathbf{w}_2}S^{-1}_{\mathbf{w}_2\mathbf{w}_2}$ is the Wiener filter for interference cancelation, since $b_{1,k}$ are uncorrelated with and independent of $\mathbf{b}_{2,k} = \mathbf{x}_{2,k} = \mathbf{w}_{2,k}$ and since $b_{1,k}$ depends on a_k, then only $b_{1,k}$ is required for further processing. Drawing connections with the MMSE equalizer described in the previous section, it is seen that

$$F_{MMSE}(z) = S_{as}S^{-1}_{ss}(z)\left[\mathbf{G}^\dagger(z) \ \mathbf{G}^{\perp\dagger}(z)\right], \tag{9}$$

Hence, the interference canceler followed by a scalar MMSE linear equalizer corresponds to the optimal MMSE equalizer of (4). We refer to this structure as the interference canceling (IC) rake receiver for obvious reasons.

The ICRR maximizes the ratio of signal to noise power spectral densities. With negligible ISI in the desired signal and *i.i.d.* symbols, the signal power spectral density is constant and the ICRR maximizes the SNR, i.e., ICRR ≈ MMSE.

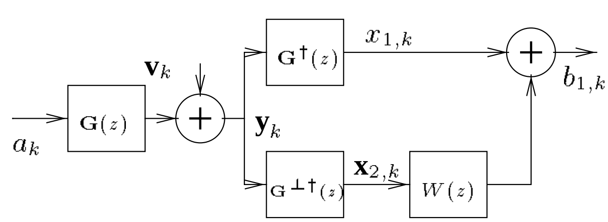

Figure 2. Inter. Canceling Rake Receiver

4.1. IC Filter Length

Once again consider the noiseless case. In order to decorrelate, we need

$$W(z)\mathbf{G}^{\perp\dagger}(z)\mathbf{G}_{int}(z) = z^{-1}\mathbf{G}^{\dagger}(z)\mathbf{G}_{int}(z), \qquad (10)$$

where, $\mathbf{G}_{int}(z)$ are the contributions (possibly rank deficient) from the interfering users' channels, and m is the order of the channel moving average (MA) process. Then the length of the interference canceling Wiener filter $W(z)$ is [6]

$$M \geqslant \left\lceil \frac{\lambda(u-1)}{L-u} \right\rceil \;, \qquad (11)$$

where, λ equals two or three depending upon the prevalent channel length N_j. It can be seen that for fully loaded systems ($u \approx L$), M can be fairly large. This effect can be alleviated by using multiple diversity (antenna) channels. The delay d for the FIR filter $W(z) = w_0 z^d \cdots w_{M-1} z^{-(M-1)+d}$ in order to decorrelate can be shown to be constrained within $0 \leqslant d \leqslant M-1$.

5. Blind Channel Identification

Exploitation of the prior knowledge available in terms of the spreading code (transmitter pulse) has been shown to improve the estimation [1].

5.1. Noise Subspace Method

In [6], it was shown that joint estimation, using subspace techniques, of u cochannel users transmitting is possible (blindly) upto a $u \times u$ non-singular matrix (an instantaneous mixture). In contrast to a TDMA system, the direct sequence spreading waveform is different for every user in DS-CDMA systems. There is, therefore, enough structure in the problem to identify the channel without ambiguity using subspace techniques [4].

Given knowledge of the noise subspace $\mathcal{G}_M$, the channel vector $\mathbf{h}_1$ can be identified as the solution of

$$\min_{\mathbf{h}_1} ||\mathcal{G}_M \mathcal{T}_M(G_{1,N_1})||. \qquad (12)$$

The noise subspace is determined from the singular prediction error filters as explained before.

5.2. Sub-Response Matching (SRM)

The SRM algorithm is elaborated upon in [1]. Here, it suffices to say that the sub-channels $G_i(z)$ obtained as a result of oversampling are stacked together in a signal blocking matrix $\mathbf{G}^{\perp\dagger}(z)$ (7) in order to obtain an expression of the form $\mathbf{G}^{\perp\dagger}(z)\mathbf{G}_(z) = 0$, which can be solved under an appropriate constraint. Let $\mathrm{g} = \left[\mathrm{g}_1^H(1) \;\; \mathrm{g}_1^H(0)\right]^H$ be the overall channel vector, and $\mathcal{T}(\mathrm{g}^\perp)$, the time domain correspondent of $\mathbf{G}^{\perp\dagger}(z)$. Then the SRM criterion $||\mathcal{T}(\mathrm{g}^\perp)\mathbf{Y}_M||_2^2$, can be written as the minimization with respect to g of

$$\begin{aligned} & \mathsf{tr}\,\{\mathcal{T}(\mathrm{g}^\perp)\mathbf{Y}_M\mathbf{Y}_M^H\mathcal{T}^H(\mathrm{g}^\perp)\} \\ = \; & \mathsf{tr}\,\{\mathrm{g}^\perp\left(\sum_{k=1}^{M}\mathbf{Y}_2(k)\mathbf{Y}_2^H(k)\right)\mathrm{g}^{\perp H}\}, \end{aligned} \qquad (13)$$

where, tr denotes trace.

This is a quadratic cost function in the actual channel coefficients, $\mathbf{h}_1$. Implementation details for SRM can be looked up in [1]. It is worth pointing out however, that this method banks on asymptotically removing the contribution of white noise. Hence for the multiuser problem, the SRM estimate could turn out to be biased.

5.3. Identifiability Conditions

Although the general identifiability conditions [6] i.e., constraining $\mathcal{T}_{M,u}(\mathbf{G}_N)$ to be full column rank largely suffice, they are by no means necessary for the channel estimation in this particular problem. To show this, let us split the matrix $\mathcal{T}_{M,u}(\mathbf{G}_N)$ as $[\mathcal{T}_{M,1}(\mathbf{G}_{N_1}) \quad \mathcal{T}_{M,u-1}(\mathbf{G}_{N_{2:u}})]$. Then we can state the identifiability conditions as follows:

- **(A.1)** $\mathcal{T}_{M,1}(\mathbf{G}_{N_1})$ must have full column rank
- **(A.2)** Let W_1 and W_2 be the respective range spaces of $\mathcal{T}_{M,1}(\mathbf{G}_{N_1})$ and $\mathcal{T}_{M,u-1}(\mathbf{G}_{N_{2:u}})$. Then, $dim(W_1 + W_2) = rank\{\mathcal{T}_{M,u}(\mathbf{G}_N)\}$, where, $\mathcal{T}_{M,u-1}(\mathbf{G}_{N_{2:u}})$ could be rank deficient. Then, the condition $dim(W_1 \cap W_2) = 0$ along with **(A.1)** is necessary in order to estimate the channel vector uniquely.

The first condition is evidently necessary and always holds for $L_c < L$, since $\mathbf{G}_{1,N_1} = [\mathrm{g}_1(1)\;\mathrm{g}_1(0)]$ with $\mathrm{g}_1(0) = \left[g_{1,1}^H(0)\ldots g_{1,L_c-1}^H(0)\;0\ldots0\right]^H$.

It is easy to see that the conditions above are rather relaxed for an environment with asynchronous interferers, i.e., they hold with probability 1!

5.4. Channel Order Over-estimation

In the channel estimation algorithm described above, it was assumed that the channel delay spread, L_c was known *a priori*. When this is not the case, we propose to overestimate the channel length for the desired user. The key observation is that as long as the overestimated L_c does not exceed $L+1$, we have $N_1 = 2$. Hence overestimating the channel does not lead to any uncertainty in the overall channel response which remains of length $N_1 = 2$. Hence, the solution for $\widehat{\mathbf{h}}_1$ obtained from one of the above methods coincides with the true channel vector $\mathbf{h}_1$ followed by a number of zeros accounting for the overestimated part of the channel impulse response.

6. Simulations

We tested the Interference Canceling Rake Receiver (ICRR) over a frequency selective channel modeled as an FIR filter with $L_c = 12$ taps. A spreading factor of 31 was used. Near-far conditions prevail in that the interfering users are randomly (ranging from 3 to 10 dB.) stronger than the user of interest.

Fig.3 compares the ICRR performance to the linear prediction based MMSE receiver with $S^{-1}_{\mathbf{yy}}(z)$ of three and five symbol lengths for 9 asynchronous interferers. For reference, a plot of the constrained beamformer-like receiver of [8] is also shown. Referring to (5) (with $u_1 = 7$ and $u_2 = 3$ in the simulations), we obtain an $\underline{M} = 1$ symbol periods. It is observed that all receivers give more or less a performance approaching the optimal MMSE (prediction based) receiver. However, it is seen in fig. 4 that as the number of users increases, so should the receiver length, conform with (5). For this plot the interference canceler is of substantial length (five symbol periods) while the beamformer is no longer capable of canceling out the interference.

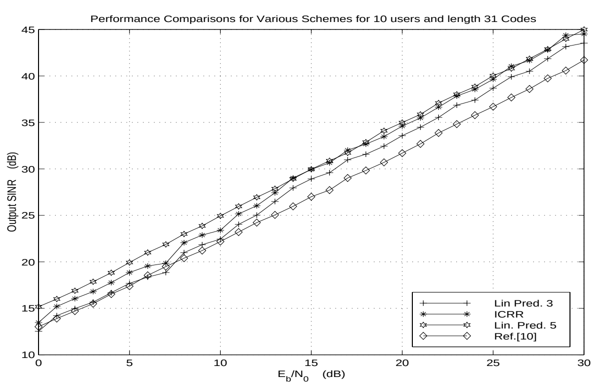

Figure 3. Performance of Various Receivers u =10, L =31

7. Concluding Remarks

We presented the linear prediction based MMSE receiver and the ICRR. Both receivers require blind channel estimate for the user of interest which can be obtained using one of the two methods presented. The MMSE requires the estimation of a linear predictor $\mathbf{P}(z)$ and the interference canceler $W(z)$ respectively. Both can be estimated from second order statistics. The former is an $L \times L$ matrix transfer function and its estimation requires more data and its implementation is more complex as compared to the $1 \times L$ interference canceler $W(z)$. For limited delay spread $L_c < L$, the ICMF behaves like the MMSE receiver.

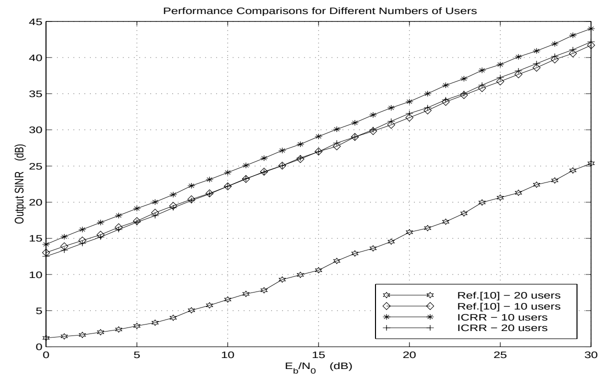

Figure 4. Receiver Performances for different u's, L =31

References

[1] J. Ayadi and D. T. M. Slock. Crámer-Rao Bounds for Knowledge Based Estimation of Multiple FIR Channels. In *Proc. IEEE Conf. on Signal Proceesing Advances in Wireless Communications*, Paris, France, April 1997.

[2] M. Honig, U. Madhow, and S. Verdú. Blind Adaptive Multiuser Detection. *IEEE Trans. on Info. Theory*, 41(4):944–960, July 1995.

[3] H. Liu and K. Li. Low Complexity Receivers for CDMA Communications in Frequency-selective Channels. In *Proc. IEEE Conf. on Signal Proceesing Advances in Wireless Communications*, Paris, France, April 1997.

[4] H. Liu and G. Xu. A Subspace Method for Signature Waveform Estimation in Synchronous CDMA Systems. *IEEE Trans. on Signal Communications*, 44(10):1346–1354, October 1996.

[5] H. Liu and M. D. Zoltowsky. Blind Equalization in Antenna Array CDMA Systems. *IEEE Trans. on Signal Processing*, 45(1):161–172, January 1997.

[6] D. T. M. Slock. Blind Joint Equalization of Multiple Synchronous Mobile Users using Oversampling and/or Multiple Antennas. In *Proc. 28th Asilomar Conf. on Signals, Systems & Computers*, Pacific Grove, CA, November 1991.

[7] D. T. M. Slock. Spatio-Temporal Training-Sequence-Based Channel Equalization and Adaptive Interference Cancellation. In *Proceedings of Int. Conf. Acoustics, Speech and Signal Processing*, Atlanta, GA, May 1996.

[8] M. K. Tsatsanis. Performance Analysis of Minimum Variance CDMA Receivers. In *Proc. of International Conference on Digital Signal Processing*, pages 102–112, Santorini, Greece, July 1997.