An Interference Cancelling Multichannel Matched Filter

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ABSTRACT We consider mobile radio communications with one user of interest and possibly interfering users and noise, over several discrete-time channels obtained either by oversampling or from multiple antennas. The optimal receiver structure for one signal of interest plus spatially and temporally correlated noise is MLSE equalization with an appropriately weighted metric for vector signals. We show however that we can alternatively pass the vector received signal through both a MISO (multi-input single output) matched filter and a MIMO blocking equalizer. The blocking equalizer output is independent of the signal of interest and is used as the input to a MISO Wiener filter that reduces the noise in the matched filter ouput. The resulting structure is called the Interference Cancelling Matched Filter (ICMF). The training sequence of the signal of interest can be used to estimate the corresponding channel, from which matched filter and blocking equalizer can be determined. The remaining quantities can be adapted from the available signals. The performance of the ICMF is analyzed in a number of scenarios. The extension to the case of noncircular noise and interference is developed and is shown to be particularly of interest in the case of 1D constellations.

1 Multiple Channels

The multiple FIR channels we consider here are due to oversampling of a single received signal and/or the availability of multiple received signals from an array of antennas (in the context of mobile digital communications). To further develop the case of oversampling, consider linear digital modulation over a linear channel with additive noise so that the cyclostationary received signal can be written as

$$y(t) \;=\; \sum_k h(t-kT)a_k \;+\; v(t) \tag{1}$$

where the a_k are the transmitted symbols, T is the symbol period and $h(t)$ is the channel impulse response. The channel is assumed to be FIR with duration NT (approximately). If the received signal is oversampled at the rate $\frac{m}{T}$ (or if m different samples of the received signal are captured by m sensors every T seconds, or a combination of both), the discrete input-output relationship can be written as:

$$\begin{gathered}\mathbf{y}_k \;=\; \sum_{i=0}^{N-1}\mathbf{h}_i a_{k-i} + \mathbf{v}_k \;=\; \mathbf{H}_N A_N(k) + \mathbf{v}_k\;,\\ \mathbf{y}_k = \begin{bmatrix} y_{1,k}\\ \vdots \\ y_{m,k}\end{bmatrix}, \mathbf{v}_k = \begin{bmatrix} v_{1,k}\\ \vdots \\ v_{m,k}\end{bmatrix}, \mathbf{h}_k = \begin{bmatrix} h_{1,k}\\ \vdots \\ h_{m,k}\end{bmatrix}\\ \mathbf{H}_N = [\mathbf{h}_0\cdots\mathbf{h}_{N-1}]\,, A_N(k) = \left[a_k^H\cdots a_{k-N+1}^H\right]^H\end{gathered} \tag{2}$$

where the first subscript i denotes the i^{th} channel and superscript ^{H} denotes Hermitian transpose. $y_{i,k}\,, i=1,\ldots,m$ represent the m phases of the polyphase representation of the oversampled signal: $y_{i,k} = y(t_0+(k+\frac{i}{m})T)$. In the polyphase representation of the oversampled signals, we get a discrete-time circuit in which the sampling rate is the symbol rate. Its output is a vector signal corresponding to a SIMO or vector channel consisting of m SISO discrete-time channels where m is the sum of the oversampling factors used for the possibly multiple antenna signals, see Fig. 1.

2 Previous Work

It is well-known that the thus available frequential or spatial diversity can be exploited to cancel or diminish multi-user interference. A decision feedback equalizer (DFE) consisting of m feedforward filters and a feedback filter can be used to achieve this. In TDMA mobile communications, the channel can vary fairly rapidly. Therefore the data is sent in fairly short time-slots over which the channel can be considered time-invariant. A midamble of training sequence symbols is provided in the slot to allow for receiver adaptation. From a design point of view, the number of parameters in the multichannel DFE to be estimated though increases with m since there are m feedforward filters. Hence, a training sequence that is designed for $m=1$ will not allow a reliable design of the spatio-temporal feedforward filter.

We shall consider optimal and suboptimal receiver structures for the case when the additive zero mean noise is both spatially and temporally correlated. Strict optimality will only hold when the noise is considered Gaussian. When the noise actually consists of multiuser interference plus Gaussian noise, the optimal receiver performs joint detection of all

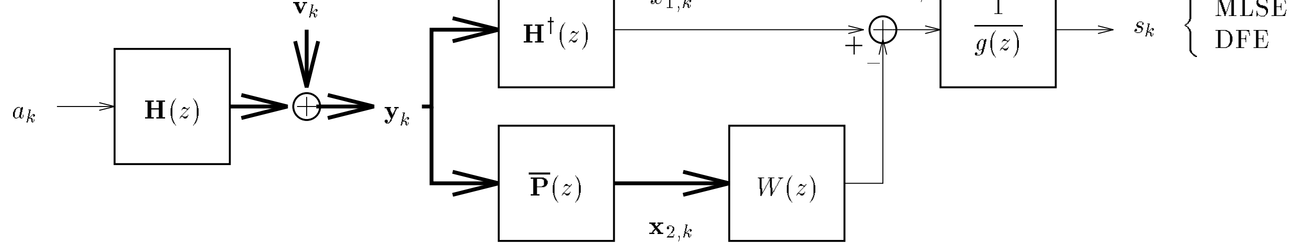

Figure 1: ICMF optimal receiver structure for one user received through multiple channels in colored additive Gaussian noise.

users. However, the estimation of the matrix transfer function from all users to all antennas (and/or sampling phases) is a formidable and often prohibitive task. Furthermore, the complexity of MLSE can be enormous in this case. We will accept the suboptimality induced by the Gaussian assumption. We will find that the suboptimality disappears in certain cases. We shall assume (short-term) stationarity of the vector received process. For that, we assume the transmitted symbol sequence to be stationary, the channel to be time-invariant and the additive noise to be a combination of stationary and cyclostationary components with period T (co-channel interference). In a first instance, we shall assume that the additive noise is circular (for the noise component corresponding to interfering users, most 2D constellations are circular). Extensions to the non-circular case will be discussed in the last section. References to more previous work can be found in [1], in which the circular case was developed.

3 ICMF Derivation

Assume we receive M samples:

$$\mathbf{Y}_M \;=\; \mathcal{T}_M(\mathbf{H}_N)\, A_{M+N-1}(M) + \mathbf{V}_M \tag{3}$$

where $\mathbf{Y}_M = [\mathbf{y}_M^H \cdots \mathbf{y}_1^H]^H$ and similarly for $\mathbf{V}_M$, and $\mathcal{T}_M(\mathbf{H}_N)$ is a block Toepliz matrix with M block rows and $[\mathbf{H}_N \;\; 0_{m\times(M-1)}]$ as first block row. With $\mathrm{R}_M^{\mathbf{V}} = \mathrm{E}\mathbf{V}_M\mathbf{V}_M^H$, the proper distance function to be used with the Viterbi algorithm is

$$(\mathbf{Y}_M - \mathcal{T}_M(\mathbf{H}_N)\, A_{M+N-1})^H \mathrm{R}_M^{-\mathbf{V}} (\mathbf{Y}_M - \mathcal{T}_M(\mathbf{H}_N)\, A_{M+N-1}) \tag{4}$$

where $A_{M+N-1} = A_{M+N-1}(M)$. When $\mathbf{v}_k$ is (modeled as) a multivariate AR process, $\mathrm{R}_M^{-\mathbf{V}}$ is banded and can be easily expressed recursively. Making abstraction of finite length effects, we can say that we need to pass the received signal $\mathbf{y}_k$ through a noise whitening filter $\mathrm{S}_{\mathbf{v}}^{-\frac{1}{2}}(z)$ where $\mathrm{S}_{\mathbf{v}}(z)$ is the power spectral density matrix of the noise $\mathbf{v}_k$ and $\mathrm{S}_{\mathbf{v}}^{\frac{1}{2}}(z)$ is a (minimum-phase) spectral factor. Alternatively, consider the transformation

$$\mathbf{x}_k \;=\; \mathbf{A}(z)\,\mathbf{y}_k \;=\; \begin{bmatrix} x_{1,k} \\ \mathbf{x}_{2,k} \end{bmatrix} \;=\; \begin{bmatrix} \mathbf{H}^\dagger(z)\mathbf{H}(z)\, a_k + w_{1,k} \\ \mathbf{w}_{2,k} \end{bmatrix} \tag{5}$$

where

$$\mathbf{A}(z) \;=\; \begin{bmatrix} \mathbf{H}^\dagger(z) \\ \overline{\mathbf{P}}(z) \end{bmatrix},\;\; \mathbf{w}_k \;=\; \begin{bmatrix} w_{1,k} \\ \mathbf{w}_{2,k} \end{bmatrix} \;=\; \mathbf{A}(z)\,\mathbf{v}_k\,. \tag{6}$$

$\mathbf{H}(z) = \sum_{i=0}^{N-1} \mathbf{h}_i z^{-i} = [H_1^H(z)\cdots H_m^H(z)]^H$ is the SIMO channel transfer function and $\mathbf{H}^\dagger(z) = \mathbf{H}^H(1/z^*)$ is its matched filter. $\overline{\mathbf{P}}(z)$ is an $(m{-}1)\times m$ transfer function such that $\overline{\mathbf{P}}(z)\mathbf{H}(z) = 0$ and is therefore also called a set of $m{-}1$ blocking equalizers. One possible choice is

$$\overline{\mathbf{P}}(z) \;=\; \begin{bmatrix} -H_2(z) & H_1(z) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -H_m(z) & 0 & \cdots & H_1(z) \end{bmatrix} \tag{7}$$

which is FIR. A better (lower order) choice is based on the multivariate forward prediction error filter $\mathbf{P}(z)$ for the noiseless received signal. One can show ([2] and references therein) that $\mathbf{P}(z)\,\mathbf{H}(z) = \mathbf{h}_0$. Hence, if $\mathbf{h}_0^\perp$ is a $m\times(m{-}1)$ matrix such that $\mathbf{h}_0^{\perp\,H}\,\mathbf{h}_0 = 0$, then we can take $\overline{\mathbf{P}}(z) = \mathbf{h}_0^{\perp\,H}\mathbf{P}(z)$. $\mathbf{A}(z)$ is an invertible transformation in general. Note that $\mathbf{w}_{2,k}$ contains no signal of interest but only filtered noise. However, since $\mathbf{w}_{2,k}$ is correlated with $w_{1,k}$, we can use $\mathbf{w}_{2,k}$ to lower the noise level on $x_{1,k}$. Hence consider the transformation

$$\mathbf{u}_k \;=\; \mathbf{B}(z)\,\mathbf{x}_k \;=\; \begin{bmatrix} u_{1,k} \\ \mathbf{u}_{2,k} \end{bmatrix} \;=\; \begin{bmatrix} \mathbf{H}^\dagger(z)\mathbf{H}(z)\, a_k + \widetilde{w}_{1,k} \\ \mathbf{w}_{2,k} \end{bmatrix} \tag{8}$$

where

$$\mathbf{B}(z) \;=\; \begin{bmatrix} 1 & -W(z) \\ 0 & I_{m-1} \end{bmatrix},\;\; \widetilde{\mathbf{w}}_k \;=\; \begin{bmatrix} \widetilde{w}_{1,k} \\ \mathbf{w}_{2,k} \end{bmatrix} \;=\; \mathbf{B}(z)\,\mathbf{w}_k \tag{9}$$

and $W(z) = \mathrm{S}_{w_1\mathbf{w}_2}\mathrm{S}_{\mathbf{w}_2\mathbf{w}_2}^{-1}$ is the Wiener filter for estimating $w_{1,k}$ from $\mathbf{w}_{2,k}$. Note that $W(z) = \mathrm{S}_{x_1\mathbf{x}_2}\mathrm{S}_{\mathbf{x}_2\mathbf{x}_2}^{-1}$ also. For Gaussian noise, $\mathbf{w}_{2,k}$ is independent of $u_{1,k}$ and a_k. Hence the $u_{1,k}$ constitute a set of sufficient statistics for the detection of the a_k. The cascade $\mathbf{B}(z)\mathbf{A}(z)$ leads to the Interference Cancelling Matched Filter (ICMF) structure depicted in Fig. 1. It will be convenient to process $u_{1,k}$ further by a whitening filter $1/g(z)$ (see Fig. 1) (this filter can be combined with any other filter that may follow): $g(z) = (\mathbf{H}^\dagger(z)\mathbf{H}(z))^{\frac{1}{2}}$. We get for the resulting signal s_k:

$$s_k \;=\; g^\dagger(z)\, a_k + n_k \tag{10}$$

N. The power spectral density of the additive noise can be shown to be

$$\mathrm{S}_{nn}(z) = \frac{\mathbf{H}^\dagger \mathrm{S}_{\mathbf{vv}}\mathbf{H} - \mathbf{H}^\dagger \mathrm{S}_{\mathbf{vv}}\overline{\mathbf{P}}^\dagger \left(\overline{\mathbf{P}}\mathrm{S}_{\mathbf{vv}}\overline{\mathbf{P}}^\dagger\right)^{-1} \overline{\mathbf{P}}\mathrm{S}_{\mathbf{vv}}\mathbf{H}}{\mathbf{H}^\dagger\mathbf{H}} \tag{11}$$

If the colored noise $\mathbf{v}_k$ consists of $d \leq m-1$ interfering users (that also have symbol period T) plus temporally and spatially white noise then

$$\mathrm{S}_{\mathbf{vv}}(z) \ = \ \mathbf{G}(z)\mathbf{G}^\dagger(z) + \sigma_v^2 I_m \tag{12}$$

where $\mathbf{G}(z)$ $(m \times d)$ regroups the channel transfer functions of the d interferers. In this case we have

$$\mathrm{S}_{nn}(z) = \sigma_v^2 \left(1 + \mathrm{tr}\left\{\mathbf{G}^\dagger P_{\mathbf{H}}\mathbf{G}\left(\mathbf{G}^\dagger P_{\overline{\mathbf{P}}^\dagger}\mathbf{G} + \sigma_v^2 I_d\right)^{-1}\right\}\right) \tag{13}$$

where $P_{\mathbf{H}(z)} \ = \ \mathbf{H}(z)\left(\mathbf{H}^\dagger(z)\mathbf{H}(z)\right)^{-1}\mathbf{H}^\dagger(z)$. Note that when $\mathbf{G}(z) = 0$, $\mathrm{S}_{nn}(z) = \sigma_v^2$ and $W(z) = 0$. The resulting structure with MLSE from s_k is optimal and consists simply of the multichannel whitened matched filter. When $\mathbf{G}(z) \neq 0$ but $\sigma_v^2 = 0$ ($\mathrm{S}_{\mathbf{vv}}(z)$ singular), then $\mathrm{S}_{nn}(z) = 0$: the resulting structure is again optimal even though the additive noise is not Gaussian because up to $m-1$ interfering users can be eliminated in the noise-free case!

4 Conservation of MFB

Considering the interferers as colored noise and the transmitted symbols to be uncorrelated ($\mathrm{S}_{aa}(z) = \sigma_a^2$), the Matched Filter Bound (MFB) using the received signal $\mathbf{y}_k$ is

$$\mathrm{MFB} = \frac{\sigma_a^2}{2\pi j}\oint \frac{dz}{z}\mathbf{H}^\dagger(z)\mathrm{S}_{\mathbf{vv}}^{-1}(z)\mathbf{H}(z) \ . \tag{14}$$

At the output of the ICMF, the MFB is (see (10))

$$\mathrm{MFB} = \frac{\sigma_a^2}{2\pi j}\oint \frac{dz}{z}\frac{\mathbf{H}^\dagger\mathbf{H}}{\mathrm{S}_{nn}} \ . \tag{15}$$

For the ICMF to be optimal, these two expressions should be identical which we now show. We introduce a lossless transfer function $\Theta(z)$ ($\Theta^\dagger\Theta = I_m$)

$$\Theta(z) = \mathbf{D}^{-1}\mathbf{A} = \left[\mathbf{H}\, g^{-\dagger} \quad \overline{\mathbf{P}}^\dagger\,(\overline{\mathbf{P}}\overline{\mathbf{P}}^\dagger)^{-\dagger/2}\right]^\dagger \tag{16}$$

with $\mathbf{D}(z)$ some obvious block diagonal transfer matrix. Then we get

$$\begin{aligned}&\mathbf{H}^\dagger\mathrm{S}_{\mathbf{vv}}^{-1}\mathbf{H} = \mathbf{H}^\dagger\Theta^\dagger\Theta\mathrm{S}_{\mathbf{vv}}^{-1}\Theta^\dagger\Theta\mathbf{H}\\ &= [g\ 0]\left(\Theta\mathrm{S}_{\mathbf{vv}}\Theta^\dagger\right)^{-1}[g\ 0]^\dagger = [g\ 0]\,\mathbf{D}^\dagger\mathrm{S}_{\mathbf{ww}}^{-1}\mathbf{D}\,[g\ 0]^\dagger\\ &= (\mathbf{H}^\dagger\mathbf{H})^2[1\ 0]\,\mathrm{S}_{\mathbf{ww}}^{-1}[1\ 0]^H = (\mathbf{H}^\dagger\mathbf{H})^2[1\ 0]\mathbf{B}^\dagger\mathrm{S}_{\widetilde{\mathbf{w}}\widetilde{\mathbf{w}}}^{-1}\mathbf{B}[1\ 0]^H\\ &= (\mathbf{H}^\dagger\mathbf{H})^2[1\ 0]\,\mathrm{S}_{\widetilde{\mathbf{w}}\widetilde{\mathbf{w}}}^{-1}[1\ 0]^H = (\mathbf{H}^\dagger\mathbf{H})^2\mathrm{S}_{\widetilde{w}_1\widetilde{w}_1}^{-1} = \mathbf{H}^\dagger\mathbf{H}\ \mathrm{S}_{nn}^{-1} \ .\end{aligned} \tag{17}$$

When the channels for all users are memoryless, then the ICMF corresponds to the (narrowband) generalized sidelobe canceller (GSW) [3]. The ICMF can therefore be considered as a particular instance of the broadband GSC beamformer. The GSW is itself a particular implementation of the linearly-constrained minimum variance (LCMV) beamformer. We shall now elucidate which constrained optimization problem the ICMF is the solution of. Consider obtaining s_k as the output of a MISO filter $\mathbf{F}(z)$: $s_k = \mathbf{F}(z)\mathbf{y}_k$. The unit-energy filter $\mathbf{F}$ ($\oint \frac{dz}{z}\mathbf{F}\,\mathbf{F}^\dagger = 1$) that maximizes the variance of the signal part of s_k (σ_s^2 if $\mathbf{v}_k \equiv 0$) is $\mathbf{F}_o = \frac{f}{g}\mathbf{H}^\dagger$ where f is any unit-energy scalar transfer function (in the previous development we considered the specific choice $f(z) = 1$) . All filters $\mathbf{F}$ that have the same influence on the signal part of s_k as $\mathbf{F}_o$ satisfy the constraint

$$\mathbf{F}\,\mathbf{H} = \frac{f}{g}\,\mathbf{H}^\dagger\mathbf{H} \ . \tag{18}$$

An arbitrary parameterization of $\mathbf{F}(z)$ is

$$\begin{aligned}&\mathbf{F} = \mathbf{F}\,\Theta^\dagger\Theta = [f_1\ \ \mathbf{F}_2]\Theta\\ &= \tfrac{f_1}{g}\mathbf{H}^\dagger + \mathbf{F}_2(\overline{\mathbf{P}}\overline{\mathbf{P}}^\dagger)^{-1/2}\overline{\mathbf{P}} = \tfrac{f_1}{g}\left(\mathbf{H}^\dagger - W\,\overline{\mathbf{P}}\right)\end{aligned} \tag{19}$$

where we can alternatively take f_1 and $\mathbf{F}_2$ or f_1 and W as free parameters (1×1 and $1 \times (m-1)$ transfer functions resp.). We shall consider the second parameterization. In order to satisfy the constraint (18), we require $f_1(z) = f(z)$. Hence $W(z)$ represents the free parameters. We shall choose these parameters to minimize the variance $\sigma_s^2 = \oint \frac{dz}{z}\mathrm{S}_{ss}(z)$. We find

$$\mathrm{S}_{ss} = f^\dagger f\left(g^\dagger g\,\sigma_a^2 + \mathrm{S}_{nn}\right),\ \mathrm{S}_{nn} = \frac{1}{g^\dagger g}[1\ -W]\mathrm{S}_{\mathbf{ww}}[1\ -W]^\dagger. \tag{20}$$

Minimization of S_{ss} at every frequency leads to minimization of σ_s^2 and hence the optimal solution is obtained for $W(z) = \mathrm{S}_{w_1\mathbf{w}_2}\mathrm{S}_{\mathbf{w}_2\mathbf{w}_2}^{-1}$ as before.

We can give one more interpretation of the ICMF in terms of SNR maximization. We can write as before any F as $F = \frac{f_1}{g}(\mathbf{H}^\dagger - W\,\overline{\mathbf{P}})$. We have for S_{ss}

$$\mathrm{S}_{ss} = f_1^\dagger f_1\, g^\dagger g\,\sigma_a^2 + f_1^\dagger f_1\,\mathrm{S}_{nn} = \mathrm{S}_{ss}^a + \mathrm{S}_{ss}^v \tag{21}$$

which we have decomposed into signal and noise contributions. The SNR in S_{ss} is

$$\frac{\mathrm{S}_{ss}^a}{\mathrm{S}_{ss}^v} = \frac{g^\dagger g\,\sigma_a^2}{\mathrm{S}_{nn}} \tag{22}$$

which is again maximized for the same $W(z)$ and $f_1(z)$ is arbitrary (as before). Remark that we consider the SNR in S_{ss} and not in σ_s^2 because the further processing of s_k is not limited to instantaneous detection, arbitrary filtering (by $f(z)$) in s_k is possible.

As far as the design of the various filters is concerned, the channel transfer function $\mathbf{H}(z)$ can be estimated with the training sequence for the user of interest. From $\mathbf{H}(z)$, one can determine the whitened matched filter and the blocking equalizers. The theoretical expression for $W(z) = \mathrm{S}_{w_1\mathbf{w}_2}\mathrm{S}^{-1}_{\mathbf{w}_2\mathbf{w}_2}$ using (12) is

$$W(z) = \mathbf{H}^\dagger\mathbf{G}\left(\mathbf{G}^\dagger P_{\overline{\mathbf{P}}^\dagger}\mathbf{G} + \sigma_v^2 I_d\right)^{-1}\mathbf{G}^\dagger\overline{\mathbf{P}}^\dagger\left(\overline{\mathbf{P}}\,\overline{\mathbf{P}}^\dagger\right)^{-1} . \quad (23)$$

If $\sigma_v^2 = 0$, then $W(z)$ satisfies

$$W(z)\overline{\mathbf{P}}(z)\mathbf{G}(z) = \mathbf{H}^\dagger\mathbf{G}(z) . \quad (24)$$

This system of equations allows an FIR solution for $W(z)$ if the number of interferers is limited to $d \leq m-2$. In general, $W(z)$ is IIR and will be approximated by an FIR filter. The $1 \times (m-1)$ Wiener filter $W(z)$ can be estimated from the signals $\mathbf{x}_k$. Even though $W(z)$ can contain quite a few coefficients, the samples of $\mathbf{x}_k$ over the whole time slot can be used for the estimation of $W(z)$. Alternatively, $W(z)$ can be adapted to track changes in the interference scenario during the time slot.

For implementing an actual receiver, we need to estimate $\mathrm{S}_{ss}(z)$ which can be done from the signal s_k observed over the time slot. For a MLSE receiver, we can estimate the psd of the colored noise as $\mathrm{S}_{nn}(z) = \mathrm{S}_{ss}(z) - \sigma_a^2\mathbf{H}^\dagger(z)\mathbf{H}(z)$. For MMSE equalizers, we consider the transfer function (Wiener filter)

$$\mathrm{S}_{as}(z)\mathrm{S}^{-1}_{ss}(z) \; = \; \sigma_a^2\, g(z)\, \mathrm{S}_{ss}^{-\frac{\dagger}{2}}(z)\mathrm{S}_{ss}^{-\frac{1}{2}}(z) . \quad (25)$$

This is the transfer function of the MMSE linear equalizer (LE). For the MMSE DFE, we consider the last expression in which the first two factors correspond to the feedforward filter while the last factor, the feedback filter, gets implemented in decision feedback form. Note that $\mathrm{S}_{ss}^{-\frac{1}{2}}(z)$ is proportional to the prediction filter for the psd $\mathrm{S}_{ss}(z)$.

The proposed receiver structure is appropriate for the downlink at the mobile unit (where only the training sequence for the user is assumed known). For instance in the GSM system, using multiple antennas at the mobile unit may not be realistic, but oversampling with a factor of $m = 2$ can be applied in a meaningful fashion. This would imply that if only one (dominant) interferer is present, it could be perfectly canceled with the ICMF, whose implementation requires no changes to the GSM standard. The ICMF could also be used as a suboptimal receiver structure for treating the users separately in the uplink at the basestation.

7 IC Performance Investigation

If the noise consists of d interferers plus white noise as in (12), then the MFB is obtained by combining the expressions

$$\mathrm{MFB} = \frac{\sigma_a^2}{2\pi j\sigma_v^2}\oint\frac{dz}{z}\frac{\mathbf{H}^\dagger\mathbf{H}}{1+\mathrm{tr}\left\{\mathbf{G}^\dagger P_{\mathbf{H}}\mathbf{G}\left(\mathbf{G}^\dagger P_{\overline{\mathbf{P}}^\dagger}\mathbf{G}+\sigma_v^2 I_d\right)^{-1}\right\}} \quad (26)$$

Considering the orientation of $\mathbf{G}$ w.r.t. $\mathbf{H}$, we can distinguish two extreme cases:

(i) $\mathbf{G} \perp \mathbf{H}$: $\mathbf{G}^\dagger P_{\mathbf{H}}\mathbf{G} = 0$, $\mathbf{G}^\dagger P_{\overline{\mathbf{P}}^\dagger}\mathbf{G} = \mathbf{G}^\dagger\mathbf{G}$

$$\mathrm{MFB} = \mathrm{MFB}_{JD} = \frac{\sigma_a^2}{2\pi j\sigma_v^2}\oint\frac{dz}{z}\mathbf{H}^\dagger\mathbf{H} \quad (27)$$

the Joint Detection MFB: assuming the interferers get detected correctly, then their contribution can be subtracted exactly.

(ii) $\mathbf{G} \parallel \mathbf{H}$: $\mathbf{G}^\dagger P_{\mathbf{H}}\mathbf{G} = \mathbf{G}^\dagger\mathbf{G}$, $\mathbf{G}^\dagger P_{\overline{\mathbf{P}}^\dagger}\mathbf{G} = 0$

$$\mathrm{MFB} = \mathrm{MFB}_{\parallel} = \frac{\sigma_a^2}{2\pi j\sigma_v^2}\oint\frac{dz}{z}\frac{\mathbf{H}^\dagger\mathbf{H}}{1+\frac{1}{\sigma_v^2}\mathrm{tr}\left\{\mathbf{G}^\dagger\mathbf{G}\right\}} \quad (28)$$

which is the integrated frequency-dependent SINR.

For a high INR, $\mathrm{MFB}_{\parallel}$ can be much smaller than MFB_{JD}. In order to get a feeling for the MFB in an average situation, we shall compute the expected MFB for the following scenario. Let

$$\mathbf{G} = \overline{\mathbf{G}}(\mathbf{G}^\dagger\mathbf{G})^{\dagger/2} , \; \mathbf{H} = \overline{\mathbf{H}}(\mathbf{H}^\dagger\mathbf{H})^{\dagger/2} \quad (29)$$

where the normalized versions $\overline{\mathbf{G}}$ and $\overline{\mathbf{H}}$ constitute together $d{+}1$ vectors spanning a $(d{+}1)$-dimensional subspace. We shall take these $d{+}1$ vectors to be uniformly distributed and i.i.d. at any frequency. The normalizing factors $(\mathbf{G}^\dagger\mathbf{G})^{\dagger/2}$ and $(\mathbf{H}^\dagger\mathbf{H})^{\dagger/2}$ are still deterministic. It can be shown that (26) can also be written as

$$\mathrm{MFB} = \oint\frac{dz}{z}\frac{\sigma_a^2\mathbf{H}^\dagger\mathbf{H}}{2\pi j\sigma_v^2}\left(1-\mathrm{tr}\left\{\left(\mathbf{G}^\dagger\mathbf{G}+\sigma_v^2 I_d\right)^{-1}\mathbf{G}^\dagger P_{\mathbf{H}}\mathbf{G}\right\}\right) \quad (30)$$

and as a result of our random model that

$$\mathrm{E}\,\overline{\mathbf{G}}^\dagger\overline{\mathbf{H}}\,\overline{\mathbf{H}}^\dagger\overline{\mathbf{G}} = \frac{1}{d+1}I_d . \quad (31)$$

Using some work, (30) and(31) lead to

$$\mathrm{E\,MFB} = \frac{d}{d+1}\mathrm{MFB}_{JD} + \frac{1}{d+1}\mathrm{MFB}_{\parallel} . \quad (32)$$

This shows that, depending on the number of interferers, the average MFB can be close to the one for the case of no interferers.

8 Non-circular Noise

When the additive noise consists of interfering users, the noise may be non-circular. This occurs if the symbol constellations for the interferers are not circular. Indeed, it can

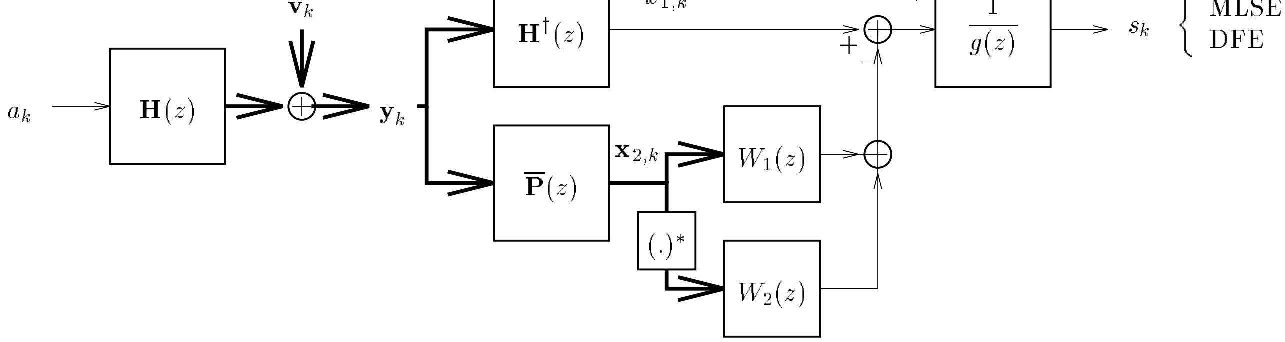

Figure 2: ICMF optimal receiver structure when the colored additive Gaussian noise is non-circular.

be shown that the output of the LTI channel is circular iff its input is circular. A major case of non-circular constellations are 1D constellations (e.g. BPSK). Paralleling the previous developments, one can show the optimality of the receiver structure shown in Fig. 2. We shall use the following notation:

$$\underline{\mathbf{x}}_k = \begin{bmatrix} \mathbf{x}_k \\ \mathbf{x}_k^* \end{bmatrix} \tag{33}$$

and similarly for other signals. Then

$$\underline{W}(z) = [W_1(z)\; W_2(z)] = S_{w_1\underline{\mathbf{w}}_2} S^{-1}_{\underline{\mathbf{w}}_2\underline{\mathbf{w}}_2} = S_{x_1\underline{\mathbf{x}}_2} S^{-1}_{\underline{\mathbf{x}}_2\underline{\mathbf{x}}_2} \tag{34}$$

is the Wiener filter for estimating $w_{1,k}$ from $\underline{\mathbf{w}}_{2,k}$. Again, because of Gaussianity, $\underline{\mathbf{w}}_{2,k}$ is independent of $u_{1,k}$ and a_k. Hence the $u_{1,k}$ constitute a set of sufficient statistics for the detection of the a_k. After noise whitening, we get again the signal s_k as in (10) but this time the non-circular noise n_k is characterized by

$$S_{\underline{nn}}(z) = \frac{\underline{\mathbf{H}}^\dagger S_{\underline{\mathbf{v}}\underline{\mathbf{v}}}\underline{\mathbf{H}} - \underline{\mathbf{H}}^\dagger S_{\underline{\mathbf{v}}\underline{\mathbf{v}}}\underline{\overline{\mathbf{P}}}^\dagger \left(\underline{\overline{\mathbf{P}}} S_{\underline{\mathbf{v}}\underline{\mathbf{v}}}\underline{\overline{\mathbf{P}}}^\dagger\right)^{-1} \underline{\overline{\mathbf{P}}} S_{\underline{\mathbf{v}}\underline{\mathbf{v}}}\underline{\mathbf{H}}}{\mathbf{H}^\dagger\mathbf{H}} \tag{35}$$

where

$$\underline{\mathbf{H}} = \begin{bmatrix} \mathbf{H} & 0 \\ 0 & \mathbf{H}^\star \end{bmatrix} \tag{36}$$

with $\mathbf{H}^\star(z) = \mathbf{H}^*(z^*)$ and $\underline{\overline{\mathbf{P}}}$ is similarly defined. Consider now the case in which $\mathbf{v}_k$ consists of d_1 interferers with 1D constellations and d_2 interferers with 2D constellations such that $d_1+2d_2 \leq 2m-2$, plus temporally and spatially white circular noise:

$$S_{\underline{\mathbf{v}}\underline{\mathbf{v}}} = \underline{\mathbf{G}}_1 S_{\underline{\mathbf{b}}_1\underline{\mathbf{b}}_1}\underline{\mathbf{G}}_1^\dagger + \underline{\mathbf{G}}_2 S_{\underline{\mathbf{b}}_2\underline{\mathbf{b}}_2}\underline{\mathbf{G}}_2^\dagger + \sigma_v^2 I_{2m} \tag{37}$$

where $\mathbf{b}^*_{1,k} = \mathbf{b}_{1,k}$ and $\underline{\mathbf{G}}_i$ are defined like $\underline{\mathbf{H}}$. As before, we have that $S_{\underline{nn}} = 0$ when $\sigma_v^2 = 0$ and $d_1+2d_2 \leq 2m-2$ (to show this, it is advantageous to work with real and imaginary parts of signals instead of with the signal and its complex conjugate). This means that for $1D$ constellations the number of interferers that can be cancelled doubles compared to 2D constellations, at least if the "widely linear" estimation filter $\underline{W}(z) = [W_1(z)\; W_2(z)]$ of Fig. 2 is used.

To find the MFB for the non-circular case, consider the derivation of the matched filter $\underline{F}(z) = [F_1\; F_2]$ that filters $\underline{\mathbf{y}}_k$. The matched filter is found as the solution to the following problem:

$$\min_{\frac{1}{2\pi j}\oint \frac{dz}{z}\underline{F}\,\underline{\mathbf{H}} = [1\,0]} \oint \frac{dz}{z}\underline{F} S_{\underline{\mathbf{v}}\underline{\mathbf{v}}}\underline{F}^\dagger \tag{38}$$

the solution of which is

$$\underline{F} = [1\,0]\left(\underline{\mathbf{H}}^\dagger S^{-1}_{\underline{\mathbf{v}}\underline{\mathbf{v}}}\underline{\mathbf{H}}\right)^{-1}\underline{\mathbf{H}}^\dagger S^{-1}_{\underline{\mathbf{v}}\underline{\mathbf{v}}}\,. \tag{39}$$

We find for the MFB:

$$\mathrm{MFB} = \sigma_a^2 \left(\left[\left(\frac{1}{2\pi j}\oint \frac{dz}{z}\underline{\mathbf{H}}^\dagger S^{-1}_{\underline{\mathbf{v}}\underline{\mathbf{v}}}\underline{\mathbf{H}}\right)^{-1}\right]_{11}\right)^{-1}\,. \tag{40}$$

This expression reduces to (14) in the circular noise case. Due to the correlation of $\mathbf{v}_k$ and $\mathbf{v}_k^*$, the MFB increases in the non-circular case. As in the circular noise case, the conservation of MFB after the ICMF can be shown, namely

$$\mathrm{MFB} = \sigma_a^2 \left(\left[\left(\frac{1}{2\pi j}\oint \frac{dz}{z}\underline{g}^\dagger S^{-1}_{\underline{nn}}\underline{g}\right)^{-1}\right]_{11}\right)^{-1}\,. \tag{41}$$

The investigation of the relation to beamformers leads to conclusions that are similar to the circular case.

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