Further Results on Selective Multiuser Diversity*

Lin Yang
Depart. of ECE
University of Minnesota
200 Union Street SE
Minneapolis, MN 55455, USA.
lyang@ece.umn.edu

Mohamed-Slim Alouini
Depart. of ECE
University of Minnesota
200 Union Street SE
Minneapolis, MN 55455, USA.
alouini@ece.umn.edu

David Gesbert
Depart. of Mobile Commun.
Eurecom Institute
2229 R. des Cretes, B.P. 193,
F-06904 Sophia Antipolis, Fr.
gesbert@eurecom.fr

ABSTRACT

In this paper we study the performance of a recently proposed scheduling technique, known as selective multiuser diversity scheme, when the users in the system have unequal average signal-to-noise ratios (SNRs). Numerical examples confirm that selective multiuser diversity reduces dramatically the feedback load but maintains a good portion of multiuser diversity gain.

Categories and Subject Descriptors

H.1.1 [**Information Systems**]: MODELS AND PRINCIPLES—*Systems and Information Theory*

General Terms

Performance

Keywords

(1) Fading Channels, (2) Multiuser Diversity, (3) Proportional Fair Scheduling, (4) Scheduling Outage, (5) Feedback Load, and (6) System Capacity.

1. INTRODUCTION

Wireless channels are time-varying due to the multipath fading phenomenon. Traditionally, channel fading is viewed as a destructive factor that reduces the communication reliability. An effective way to combat fading is to obtain multiple independent replicas of the transmitted signal at the receiver by means of diversity. In a wireless network, independent paths between a base station and individual users form a new type of diversity. Recent studies on this multiuser diversity were motivated by Knopp and Humblet who showed in [1] that the total uplink (mobile to base) capacity can be maximized by picking the user with the best channel to transmit. Essentially, independent variations of channels for many users make it very likely that the communication always occurs over a strong channel. As such, the system throughput can benefit from the randomness due to the fading effect. The study of [1] was extended in [2] which showed that the same access scheme is valid also for the downlink case.

In a more recent paper [3], this type of multiuser selection diversity was studied from a different perspective, namely, the trade-off between the system throughput and the fairness among different users. It was argued that although the selection diversity based on the "best channel" criterion maximizes the system throughput, it can cause unfair scheduling of the system resources across users. Indeed with this system, users with the strongest channels in average will end up monopolizing the resources most of time. For this reason, *proportional fair scheduling* that uses a modified selection criterion based on the "relative channel strength" was then proposed to exploit the multiuser diversity while maintaining the fairness among users. The basic idea is to pick the user with the best channel compared to its own average.

One of the basic limitation facing multiuser diversity is the required feedback that carries the instantaneous channel quality estimates from all active users to the access point. In order to manage the priorities among the users, the scheduler requires theoretically the channel quality information of all users at all time. This makes traditional multiuser diversity system hardly practical when the number of simultaneously active users becomes large. As such, a recent paper [4] presented a new scheduling technique, referred to as *selective multiuser diversity scheduling* in which each user compares its own channel quality to a predetermined threshold and requests to be considered for channel access only if its channel quality is above this threshold. These qualified users then feedback the transmission rate that they can achieve to the access point. Relying on this self-qualification by the users, it was shown in [4] that the total amount of feedback is significantly reduced in comparison with the traditional multiuser diversity while most of the multiuser diversity gain is still achieved. The study in [4] was limited to the scenario in which all the users have identical average channel quality in terms of average signal-to-noise ratio

*This work was supported in part by the National Science Foundation Grant No. CCR-9983462 and in part by the Center of Transportation Studies (CTS) through the Intelligent Transportation Systems (ITS) Institute, Minneapolis, Minnesota.

MSWiM'04, October 4-6, 2004, Venezia, Italy.

(SNR). However users in practical systems are likely to be distributed randomly over the coverage area and therefore their average SNRs can be quite disparate. Moreover, the trade-off between the multiuser selection diversity gain and the scheduling fairness among all users is a more prominent issue when channels of different users are statistically non-identical.

In this paper, we investigate the performance of this new scheduling technique when the users in the system have unequal average SNRs. In the proportional fair scheduling setup, the relative channel strength in [3] is defined as the ratio of the data rate each individual's channel can afford to its average throughput, which is tracked by the base station over a certain time window. In this paper, we take an alternative point of view and assume that the scheduling is based on the relative SNR, that is, each mobile user can measure its received SNR and feeds it back to the base station. The base station tracks the average SNR of each user over a time window. In each time slot, the base station chooses the user with the largest ratio of SNR to its own average SNR. Further, as argued in [5], the power constraint at the base station is usually based on the maximum power rather than the long term average power which is typical in battery-limited applications. Therefore, we assume that the transmitting power is constant over all time slots.

The remainder of this paper is organized as follows. Section II describes the system model and the problem under consideration. Section III studies the performance of selective multiuser diversity in presence of unequal average SNRs among users when an absolute thresholding scheme is employed. Section IV studies on the other hand the performance of a normalized thresholding scheme. Finally, section V ends the paper with some preliminary numerical examples and concluding remarks.

2. SYSTEM MODEL AND PROBLEM STATEMENT

2.1 System and Channel Models

We consider a single-cell multiuser diversity system in which K simultaneously active users are communicating with an access point/base station. The baseband channel model can be written as

$$r_i(t) = h_i(t)x(t) + n_i(t), \qquad i = 1, 2, \cdots, K, \qquad (1)$$

where $x(t) \in \mathcal{C}$ is the transmitted signal in time slot t and $r_i(t) \in \mathcal{C}$ is the received signal of user i in time slot t.

We assume that $x(t)$ has the same constant normalized transmit power over time, i.e., $E\left(|x(t)|^2\right) = 1$. This is a valid power assumption for the base station where the power constraint is regulated by its peak power rather than the long-term average power constraint (eg. battery-limited) as for mobile units. $\{n_i(t)\}$ is an independent and identically distributed (i.i.d.) sequence of zero mean complex Gaussian noise with variance σ_n^2. $h_i(t)$ is the fading channel gain from the base station to the ith user in time slot t. We adopt the quasi-static fading channel model where $h_i(t)$ is independent identically distributed (i.i.d.) from burst to burst but remains roughly constant over each burst. We consider a flat fading model and assume that the fading coefficients of all users are independent but allow these coefficients not to be necessarily identically distributed. Let $\gamma_i(t) \triangleq \frac{|h_i(t)|^2}{\sigma_n^2}$ denote the instantaneous (in time slot t) received SNR and $\bar{\gamma}_i \triangleq \frac{\Omega_i}{\sigma_n^2}$ be the short-term average received SNR for the user i, respectively, where Ω_i is the short-term average fading power of the ith user which we assume in this paper to be not the same from user to user.

2.2 Problem Statement

If the sequences $\{h_i(t)\}$, $i = 1, \cdots, K$, can be tracked perfectly at the transmitter and the receiver, then the downlink channel for each channel realization can be viewed as a set of parallel Gaussian channels. The sum capacity (the maximum achievable sum of data rates transmitted to all users) can be achieved by transmitting to the user with the strongest channel (or equivalently, the user with the largest $\gamma_i(t)$) [2]. The main problem with this scheduling scheme is the lack of fairness among users if some of them experience the best fading conditions most of time [3]. As a remedy, a proportional fair scheduling scheme was proposed in [3] to transmit to the user k^* with the largest

$$\frac{R_k(t)}{T_k(t)},$$

where $R_k(t)$ and $T_k(t)$ are the requested data rate and the average throughput of users k, respectively. Alternatively, in this paper we assume that the base station transmits to user k^* with the largest

$$\frac{\gamma_k(t)}{\bar{\gamma}_k}. \qquad (2)$$

This scheduler uses the same idea as the original proportional fair scheduling algorithm in [3] except that it operates based on the SNR criterion rather than the data rate. In what follows, we refer to the proportional fair scheduling based on $\frac{\gamma_k(t)}{\bar{\gamma}_k}$ as the "normalized SNR-based scheduling".

In [4], selective multiuser diversity scheduling was proposed to reduce the amount of total feedback while maintaining a good portion of multiuser diversity gain. Specifically, all active users first compare their instantaneous channel condition to a predetermined threshold to decide whether or not they would request channel access from the access point. Only those users whose channel condition qualifies for an access request will then feedback information to the access point. This subset of qualified users are referred to as "feedback users". The access point schedules the transmission among the feedback users. Let $P(t)$ be the number of feedback users at slot t. When $P(t) > 0$, the scheduler conducts its selection on the set of feedback users. On the other hand, when $P(t) = 0$, no user feeds back any information to the base station, and in this case the scheduler randomly picks a user for access and communication. The feedback threshold can be optimized by a predetermined scheduling outage probability, by a feedback load specification, or by a quality of service requirement for reliable transmission. The former case corresponds to the feedback condition $\frac{\gamma_k(t)}{\bar{\gamma}_k} \geq A$ with a fixed threshold A for normalized SNR, and we refer to this scheme as "normalized thresholding scheme". In the latter case, the user k feeds back its channel quality if and only if $\gamma_k(t) \geq \gamma_{\text{th}}$, and this is referred to as "absolute thresholding". In the next two sections, we will investigate the performance of selective multiuser diversity in presence of unequal SNRs among users for both the absolute and the normalized thresholding schemes.

3. ABSOLUTE THRESHOLDING SCHEME

3.1 System Capacity

In the selective multiuser diversity system with absolute thresholding scheme, the number of feedback users, $P(t)$, is defined as,

$$P(t) = \text{card}\{k, \text{such that } \gamma_k(t) \geq \gamma_{\text{th}}\}, \tag{3}$$

where card is the cardinal operator. The selection process can be mathematically described as

$$\frac{\gamma_{k^*}(t)}{\bar{\gamma}_{k^*}} = \begin{cases} \max_i \left\{\frac{\gamma_i(t)}{\bar{\gamma}_i}\right\}, & \text{if } P(t) > 0 \\ \text{rand}_i \left\{\frac{\gamma_i(t)}{\bar{\gamma}_i}\right\}, & \text{if } P(t) = 0 \end{cases}, \tag{4}$$

where rand is the random pick operator. Therefore the cumulative distributive function (CDF) of the SNR post scheduling $\gamma*$ under absolute thresholding scheme, $F_{\gamma*}(\gamma)$, can be shown to be given by

$$\begin{aligned} F_{\gamma*}(\gamma) &= \frac{1}{K}\sum_{k=1}^{K}\prod_{j=1;j\neq k}^{K} F_{\gamma_j}(\gamma_{\text{th}})F_{\gamma_k}(\gamma);\ \text{if } \gamma \leq \gamma_{\text{th}} \\ F_{\gamma*}(\gamma) &= \prod_{j=1}^{K} F_{\gamma_j}(\gamma_{\text{th}}) + \frac{1}{K} \\ &\times \sum_{k=1}^{K}\left\{\left[F_j\left(\frac{\gamma}{\bar{\gamma}_k}\right)\right]^K - \left[F_j\left(\frac{\gamma_{\text{th}}}{\bar{\gamma}_k}\right)\right]\right\};\ \text{if } \gamma > \gamma_{\text{th}}, \end{aligned} \tag{5}$$

where $F_{\gamma_k}(\cdot)$ is the CDF of the k-th user SNR and $F_k(\cdot)$ is the CDF of the normalized SNR $\frac{\gamma}{\bar{\gamma}}$ of each individual user, which is identical for all users. For example, for the Rayleigh fading case, $F_{\gamma_k}(\gamma) = 1 - \exp\left(-\frac{\gamma}{\bar{\gamma}_k}\right)$ and $F_k(x) = 1 - \exp(-x)$. Then we can obtain the probability density function (PDF) of $\gamma*$, $f_{\gamma*}(\gamma)$, by taking the derivative of (5) with respect to γ yielding

$$f_{\gamma*}(\gamma) = \begin{cases} \frac{1}{K}\sum_{k=1}^{K}\prod_{j=1;j\neq K}^{K} F_{\gamma_j}(\gamma_{\text{th}})\, f_{\gamma_k}(\gamma), & \gamma \leq \gamma_{\text{th}} \\ \sum_{k=1}^{K}\frac{1}{\bar{\gamma}_k}\left[F_j\left(\frac{\gamma}{\bar{\gamma}_k}\right)\right]^{K-1} f_j\left(\frac{\gamma}{\bar{\gamma}_k}\right), & \gamma > \gamma_{\text{th}} \end{cases}, \tag{6}$$

where $f_j(\cdot)$ is the PDF of the normalized SNR $\frac{\gamma}{\bar{\gamma}}$ and which is for example equal to $f_j(x) = \exp(-x)$ for the Rayleigh fading case. In (6), $f_{\gamma_k}(\cdot)$ is the PDF of the k-th user SNR and which is for example equal to $f_{\gamma_k}(\gamma) = \frac{1}{\bar{\gamma}_k}\exp(-\gamma/\bar{\gamma}_k)$ in the Rayleigh fading case.

With the PDF of (6) in hand, one can show that the system average capacity for the Rayleigh fading case is given by taking the expectation with respect to $\log_2(1+\gamma*)$ yielding

$$\begin{aligned} E(C^*) &= \frac{\log_2 e}{K}\sum_{k=1}^{K}\prod_{j=1;j\neq k}^{K}\left[1 - \exp\left(-\frac{\gamma_{\text{th}}}{\bar{\gamma}_j}\right)\right] \\ &\times \left\{\exp\left(\frac{1}{\bar{\gamma}_k}\right)\left[E_1\left(\frac{1}{\bar{\gamma}_k}\right) - E_1\left(\frac{\gamma_{\text{th}}}{\bar{\gamma}_k} + \frac{1}{\bar{\gamma}_k}\right)\right]\right. \\ &- \left.\ln(1+\gamma_{\text{th}})\exp\left(-\frac{\gamma_{\text{th}}}{\bar{\gamma}_k}\right)\right\} \\ &+ \sum_{k=1}^{K}\sum_{i=0}^{K-1}(-1)^i\binom{K-1}{i}\frac{\log_2 e}{1+i} \\ &\times \left\{\ln(1+\gamma_{\text{th}})\exp\left(-(1+i)\frac{\gamma_{\text{th}}}{\bar{\gamma}_k}\right)\right. \\ &+ \left.\exp\left(\frac{1+i}{\bar{\gamma}_k}\right)E_1\left(\frac{\gamma_{\text{th}}(1+i)}{\bar{\gamma}_k} + \frac{1+i}{\bar{\gamma}_k}\right)\right\}, \end{aligned} \tag{7}$$

where $E_1(\cdot)$ is the exponential-integral function of the first order defined by

$$E_1(x) = \int_1^\infty \frac{e^{-xt}}{t}dt, \quad x \geq 0,$$

which is related to the exponential-integral function $E_i(x)$ [6, Eqn. (8.21)] by $E_1(x) = -E_i(-x)$.

3.2 Scheduling Outage Probability

When $P(t) = 0$ and no user feeds back any information, we declare a scheduling outage. The probability of this event is given by

$$P_o = \Pr(P(t) = 0) = \prod_{k=1}^{K} F_{\gamma_k}(\gamma_{\text{th}}). \tag{8}$$

3.3 Feedback Load

According to [4], the normalized feedback load is defined as

$$\bar{F} = \frac{E(P(t))}{K}, \tag{9}$$

where $E(P(t))$ for selective multiuser diversity system can be shown to be given under the absolute thresholding scheme by

$$\begin{aligned} E(P(t)) &= \sum_{k=0}^{K} k\ \Pr[P(t) = k] \\ &= \sum_{k=0}^{K} k \sum_{i_1=1}^{K}\sum_{i_2=1;i_2\neq i_1}^{K}\cdots\sum_{i_k=1;i_k\neq i_1,\cdots,i_{k-1}}^{K} \\ &\times \prod_{j=1}^{k}\left[1 - F_{\gamma_{i_j}}(\gamma_{\text{th}})\right] \\ &\times \prod_{i_{k+1}=1;i_{k+1}\neq i_1,\cdots,i_k}^{K} F_{\gamma_{i_{k+1}}}(\gamma_{\text{th}}). \end{aligned} \tag{10}$$

Another statistics of the feedback load is the feedback load variation, which is defined as [4]

$$V_F = \frac{\text{Var}(P(t))}{[E(P(t))]^2} \tag{11}$$

Contrary to V_F in [4] for the i.i.d. case, no closed-form can be obtained for V_F in this case.

3.4 Threshold Choice

The threshold γ_{th} can be optimized by reaching a predetermined scheduling outage probability P_0 or by meeting certain normalized average feedback load specification $\bar{F}$, which correspond to the inverse functions of (8) and (9). Note that this inversion can not be done in closed-form in this case and as such it has to be done numerically.

4. NORMALIZED THRESHOLDING SCHEME

4.1 System Capacity

Under the normalized thresholding scheme, the number of feedback users, $P(t)$, is defined as,

$$P(t) = \text{card}\{k, \text{such that } \frac{\gamma_k(t)}{\bar{\gamma}_k} \geq A\}. \tag{12}$$

The selection process is the same as that described in (4). Without loss of generality, let us assume $\bar{\gamma}_1 \leq \bar{\gamma}_2 \leq \cdots \leq \bar{\gamma}_K$. The CDF of the SNR post scheduling $\gamma*$ under the normalized thresholding scheme, $F_{\gamma*}(\gamma)$, can be shown to be given piece by piece as

$$
\begin{aligned}
F_{\gamma*}(\gamma) &= F_{0,\gamma*}(\gamma) = \sum_{k=1}^{K} \frac{1}{K} \prod_{j=1;j\neq k}^{K} F_j(A) F_{\gamma_k}(\gamma);\ \text{if } \gamma < A\bar{\gamma}_1 \\
F_{\gamma*}(\gamma) &= F_{1,\gamma*}(\gamma) = F_{0,\gamma*}(A\bar{\gamma}_1) + \prod_{j=2}^{K} F_j(A)\left[F_{\gamma_1}(\gamma) - F_{\gamma_1}(A\bar{\gamma}_1)\right] \\
&+ \sum_{k=2}^{K} \frac{1}{K} \prod_{j=1;j\neq k}^{K} F_j(A)\left[F_{\gamma_k}(\gamma) - F_{\gamma_k}(A\bar{\gamma}_1)\right];\ \text{if } A\bar{\gamma}_1 \leq \gamma < A\bar{\gamma}_2 \\
F_{\gamma*}(\gamma) &= F_{2,\gamma*}(\gamma) = F_{1,\gamma*}(A\bar{\gamma}_2) + \frac{1}{2} \prod_{j=3}^{K} F_j(A) \\
&\times \sum_{k=1}^{2} \left[\left(F_k\left(\frac{\gamma}{\bar{\gamma}_k}\right)\right)^2 - \left(F_k\left(\frac{A\bar{\gamma}_2}{\bar{\gamma}_k}\right)\right)^2\right] \\
&+ \sum_{k=3}^{K} \frac{1}{K} \prod_{j=1;j\neq k}^{K} F_j(A)\left[F_{\gamma_k}(\gamma) - F_{\gamma_k}(A\bar{\gamma}_2)\right];\ \text{if } A\bar{\gamma}_2 \leq \gamma < A\bar{\gamma}_3 \\
F_{\gamma*}(\gamma) &= \cdots\cdots \\
F_{\gamma*}(\gamma) &= F_{K-1,\gamma*}(\gamma) = F_{K-2,\gamma*}(A\bar{\gamma}_{K-1}) + \frac{1}{K-1} F_K(A) \\
&\times \sum_{k=1}^{K-1} \left[\left(F_k\left(\frac{\gamma}{\bar{\gamma}_k}\right)\right)^{K-1} - \left(F_k\left(\frac{A\bar{\gamma}_{K-1}}{\bar{\gamma}_k}\right)\right)^{K-1}\right] \\
&+ \frac{1}{K} \prod_{j=1}^{K-1} F_j(A)\left[F_{\gamma_k}(\gamma) - F_{\gamma_k}(A\bar{\gamma}_{K-1})\right];\ \text{if } A\bar{\gamma}_{K-1} \leq \gamma < A\bar{\gamma}_K \\
F_{\gamma*}(\gamma) &= F_{K,\gamma*}(\gamma) = F_{K-1,\gamma*}(A\bar{\gamma}_K) \\
&+ \frac{1}{K} \sum_{k=1}^{K} \left[\left(F_k\left(\frac{\gamma}{\bar{\gamma}_k}\right)\right)^K - \left(F_k\left(\frac{A\bar{\gamma}_K}{\bar{\gamma}_k}\right)\right)^K\right];\ \text{if } \gamma \geq A\bar{\gamma}_K.
\end{aligned} \tag{13}
$$

The corresponding PDF can be deduced by taking the derivative of (13) with respect to γ yielding

$$
\begin{aligned}
f_{\gamma*}(\gamma) &= f_{0,\gamma*}(\gamma) = \sum_{k=1}^{K} \frac{1}{K} \prod_{j=1;j\neq k}^{K} F_j(A) f_{\gamma_k}(\gamma);\ \text{if } \gamma < A\bar{\gamma}_1 \\
f_{\gamma*}(\gamma) &= f_{1,\gamma*}(\gamma) = \prod_{j=2}^{K} F_j(A) f_{\gamma_1}(\gamma) \\
&+ \sum_{k=2}^{K} \frac{1}{K} \prod_{j=1;j\neq k}^{K} F_j(A) f_{\gamma_k}(\gamma);\ \text{if } A\bar{\gamma}_1 \leq \gamma < A\bar{\gamma}_2 \\
f_{\gamma*}(\gamma) &= \cdots\cdots \\
f_{\gamma*}(\gamma) &= f_{K-1,\gamma*}(\gamma) = F_K(A) \sum_{k=1}^{K-1} \frac{1}{\bar{\gamma}_k} f_k\left(\frac{\gamma}{\bar{\gamma}_k}\right)\left[F_k\left(\frac{\gamma}{\bar{\gamma}_k}\right)\right]^{K-2} \\
&+ \frac{1}{K} \prod_{j=1}^{K-1} F_j(A) f_{\gamma_k}(\gamma);\ \text{if } A\bar{\gamma}_{K-1} \leq \gamma < A\bar{\gamma}_K \\
f_{\gamma*}(\gamma) &= f_{K,\gamma*}(\gamma) = \sum_{k=1}^{K} \frac{1}{\bar{\gamma}_K} f_k\left(\frac{\gamma}{\bar{\gamma}_k}\right)\left[F_k\left(\frac{\gamma}{\bar{\gamma}_k}\right)\right]^{K-1};\ \text{if } \gamma \geq A\bar{\gamma}_K.
\end{aligned} \tag{14}
$$

The system average capacity for selective multiuser diversity system under the normalized thresholding scheme over Rayleigh fading channels can thereby be shown to be given by

$$
\begin{aligned}
E(C*) &= \frac{\log_2 e}{K}\left[1 - \exp(-A)\right]^{K-1} \\
&\times \sum_{k=1}^{K} \left\{\exp\left(\frac{1}{\bar{\gamma}_k}\right)\left[E_1\left(\frac{1}{\bar{\gamma}_k}\right) - E_1\left(\frac{A\bar{\gamma}_1}{\bar{\gamma}_k} + \frac{1}{\bar{\gamma}_k}\right)\right]\right. \\
&- \left.\ln(1 + A\bar{\gamma}_1)\exp\left(-\frac{A\bar{\gamma}_1}{\bar{\gamma}_k}\right)\right\} \\
&+ \log_2 e\left[1 - \exp(-A)\right]^{K-1} \\
&\times \left\{\exp\left(\frac{1}{\bar{\gamma}_1}\right)\left[E_1\left(\frac{1 + A\bar{\gamma}_1}{\bar{\gamma}_1}\right) - E_1\left(\frac{1 + A\bar{\gamma}_2}{\bar{\gamma}_1}\right)\right]\right. \\
&- \left.\ln(1 + A\bar{\gamma}_2)\exp\left(-\frac{A\bar{\gamma}_2}{\bar{\gamma}_1}\right) + \ln(1 + A\bar{\gamma}_1)\exp(-A)\right\} \\
&+ \frac{\log_2 e}{K}\left[1 - \exp(-A)\right]^{K-1} \\
&\times \sum_{k=2}^{K} \left\{\exp\left(\frac{1}{\bar{\gamma}_k}\right)\left[E_1\left(\frac{1 + A\bar{\gamma}_1}{\bar{\gamma}_k}\right) - E_1\left(\frac{1 + A\bar{\gamma}_2}{\bar{\gamma}_k}\right)\right]\right. \\
&- \left.\ln(1 + A\bar{\gamma}_2)\exp\left(-\frac{A\bar{\gamma}_2}{\bar{\gamma}_k}\right) + \ln(1 + A\bar{\gamma}_1)\exp\left(-\frac{A\bar{\gamma}_1}{\bar{\gamma}_k}\right)\right\} \\
&+ \cdots\cdots \\
&+ \left[1 - \exp(-A)\right] \sum_{k=1}^{K-1} \sum_{i=0}^{K-2} (-1)^i \binom{K-2}{i} \frac{\log_2 e}{1+i} \\
&\times \left\{\exp\left(\frac{1+i}{\bar{\gamma}_k}\right)\left[E_1\left(\frac{[1 + A\bar{\gamma}_{K-1}](1+i)}{\bar{\gamma}_k}\right)\right.\right. \\
&- \left.E_1\left(\frac{[1 + A\bar{\gamma}_K](1+i)}{\bar{\gamma}_k}\right)\right] \\
&- \ln(1 + A\bar{\gamma}_K)\exp\left(-\frac{A\bar{\gamma}_K(1+i)}{\bar{\gamma}_k}\right) \\
&+ \left.\ln(1 + A\bar{\gamma}_{K-1})\exp\left(-\frac{A\bar{\gamma}_{K-1}(1+i)}{\bar{\gamma}_k}\right)\right\} \\
&+ \frac{\log_2 e}{K}\left[1 - \exp(-A)\right]^{K-1} \\
&\times \left\{\exp\left(\frac{1}{\bar{\gamma}_k}\right)\left[E_1\left(\frac{1 + A\bar{\gamma}_{K-1}}{\bar{\gamma}_k}\right) - E_1\left(\frac{1 + A\bar{\gamma}_K}{\bar{\gamma}_k}\right)\right]\right. \\
&- \ln(1 + A\bar{\gamma}_K)\exp\left(-\frac{A\bar{\gamma}_K}{\bar{\gamma}_k}\right) \\
&+ \left.\ln(1 + A\bar{\gamma}_{K-1})\exp\left(-\frac{A\bar{\gamma}_{K-1}}{\bar{\gamma}_k}\right)\right\} \\
&+ \sum_{k=1}^{K} \sum_{i=0}^{K-1} (-1)^i \binom{K-1}{i} \frac{\log_2 e}{1+i} \\
&\times \left\{\ln(1 + A\bar{\gamma}_K)\exp\left(-(1+i)\frac{A\bar{\gamma}_K}{\bar{\gamma}_k}\right)\right. \\
&+ \left.\exp\left(\frac{1+i}{\bar{\gamma}_k}\right) E_1\left(\frac{A\bar{\gamma}_K(1+i)}{\bar{\gamma}_k} + \frac{1+i}{\bar{\gamma}_k}\right)\right\}.
\end{aligned} \tag{15}
$$

4.2 Scheduling Outage Probability

The probability of scheduling outage under normalized thresholding scheme can be shown to be given by

$$P_o = \Pr(P(t) = 0) = \prod_{k=1}^{K} F_k(A) = \left[F_k(A)\right]^K. \tag{16}$$

Note that in last equation, $F_k(\cdot)$ is identical for all users.

4.3 Feedback Load

When employing normalized thresholding scheme, the normalized feedback load in (9) can be calculated with $E(P(t))$ given by

$$\begin{aligned} E(P(t)) &= \sum_{k=0}^{K} k \Pr\left[P(t)=k\right] \\ &= \sum_{k=0}^{K} k \binom{K}{k} [1-F_i(A)]^k [F_i(A)]^{K-k} \\ &= K\ (1-F_i(A)) \qquad (17) \end{aligned}$$

Finally, the normalized feedback load in this case can be shown to be given by

$$\bar{F} \;=\; 1-F_i(A), \qquad (18)$$

and the feedback load variation can be obtained in closed-form in this case as

$$V_F = \frac{F_i(A)}{K[1-F_i(A)]}. \qquad (19)$$

4.4 Threshold Choice

The threshold A can be chosen to meet a certain required outage probability or feedback load by solving equations (16) and (18), respectively. For instance, in the Rayleigh fading case, inverting (16) and (18) lead to

$$A = -\ln(\bar{F}) \qquad (20)$$

and

$$A = -\ln\left(1-P_o^{1/K}\right). \qquad (21)$$

5. NUMERICAL EXAMPLES

We present in this section some initial/preliminary results. A more detailed set of numerical results and their corresponding discussion will be presented in the journal version of this paper [7]. In Fig. 1-4, the short-term average SNR $\bar{\gamma}_k$, $k=1,\cdots,K$ are non-identical. The setup is as follows. Each time we increase the number of users by 2. For the non-iid case, the average SNR values of these two new users are generated from uniform$(0,1)$ and then normalized so that they add up to 2 SNR (due to this randomness, the curves in Fig. 1, 2 and 4 are not smooth), where SNR equals 1 at this time. Therefore the total SNR of K users is $K\ SNR$. The same set of values of $\bar{\gamma}_k$ at each K are used for all curves in Fig. 1 and 2 to ensure a fair comparison.

Specifically, Fig. 1 and 2 plot the average system capacity and corresponding feedback load versus number of users for selective multiuser diversity scheme with absolute thresholding for various values of the threshold $\gamma_{\rm th}$. The capacity performance of the traditional full feedback proportional fair scheduling scheme is also provided for comparison. Note that the feedback load for the traditional full feedback proportional fair scheduling scheme is 1. One can see that for $\gamma_{\rm th}=1$, a negligible loss in capacity performance is observed while the feedback load is reduced as low as 35%.

Fig. 3 shows the scheduling outage probability as a function of the absolute threshold $\gamma_{\rm th}$. Note that when $\gamma_{\rm th}=1$ and $K=10$, the scheduling outage probability is very small and is approximately equal to 10^{-2}.

Finally, Fig. 4 plots the average system capacity versus the required feedback load under the normalized thresholding scheme for various values of K. These figures confirm that selective multiuser diversity scheme reduces dramatically the feedback load but maintains most of the performance. For instance note that for K above 20 or so, a feedback load greater than 10% results in a very little additional capacity gain and is therefore not necessary.

6. REFERENCES

[1] R. Knopp and P. Humblet, "Information capacity and power control in single-cell multiuser communications," in *Proc. IEEE Int. Conf. Commun. (ICC'95), Seattle, WA*, vol. 1, pp. 331–335, Jun. 1995.

[2] D. N. C. Tse, "Optimal power allocation over parallel Gaussian channels," in *Proc. Int. Symp. Inform. Theory (ISIT'97), Ulm, Germany*, p. 27, Jun. 1997.

[3] P. Viswanath, D. Tse, and R. Laroia, "Opportunistic beamforming using dumb antennas," *IEEE Trans. Inform. Theory*, vol. 48, pp. 1277–1294, Jun. 2002.

[4] D. Gesbert and M. -S. Alouini, "How much feedback is Multi-User Diversity really worth?," in *Proc. IEEE Int. Conf. Commun. (ICC'04), Paris, France*, June 2004.

[5] D. Piazza and L. Milstein, "Multiuser diversity-mobility tradeoff: Modeling and performance analysis of a proportional fair scheduling," in *Proc. IEEE Global Telecommun. (Globecom'02), Taipei, Taiwan*, vol. 1, pp. 906–910, Nov. 2002.

[6] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*. Orlando, FL: Academic Press, corr. and enlarg. ed., 1980.

[7] D. Gesbert, L. Yang, and M. -S. Alouini, "How much feedback is multi-user diversity really worth?." Submitted to *IEEE Trans. Wireless Comm.*, 2004.

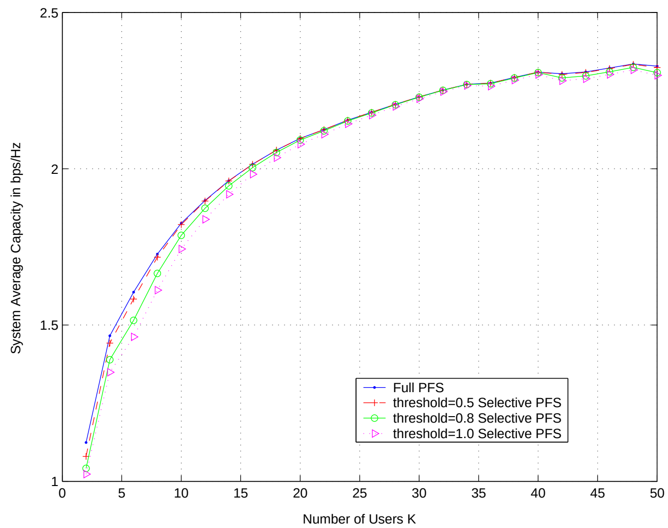

Figure 1: Average capacity comparison of traditional proportional fair scheduling and selective multiuser diversity scheme with absolute thresholding.

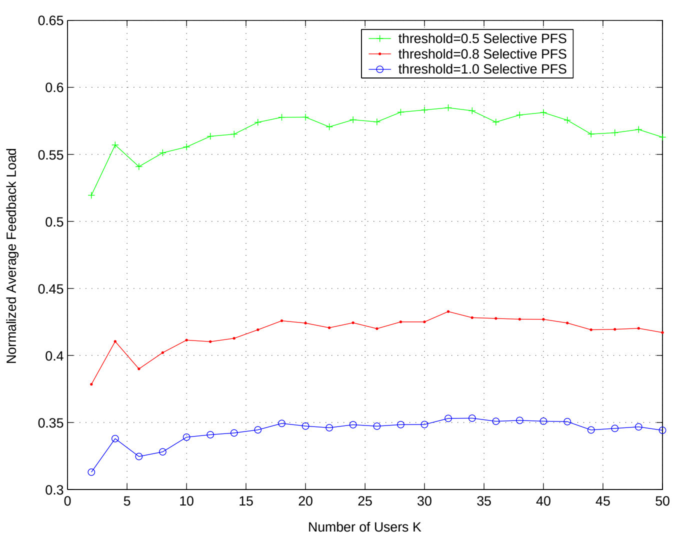

Figure 2: Normalized average feedback load of selective multiuser diversity scheme with absolute thresholding.

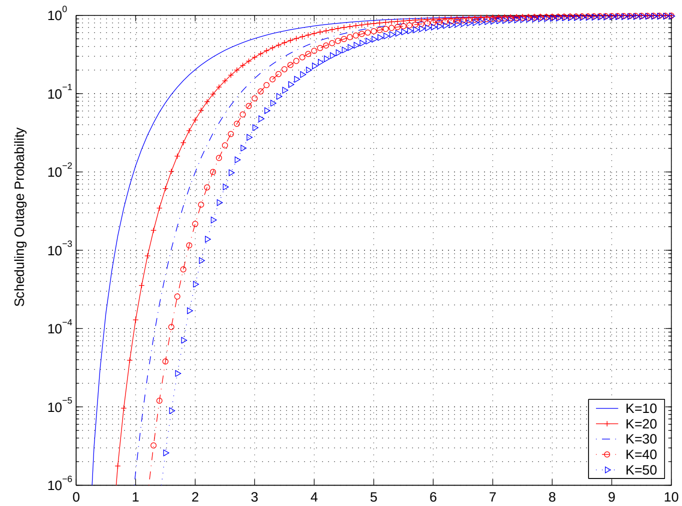

Figure 3: Scheduling outage vs. threshold for the selective multiuser diversity scheme with absolute thresholding.

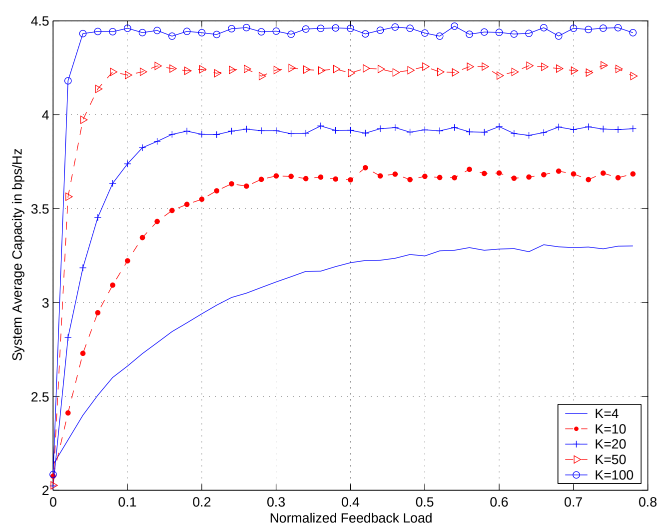

Figure 4: Average system capacity vs. required feedback load for selective multiuser diversity scheme with normalized thresholding.