Achievable Rates for UWB Peer-to-Peer Networks

Raymond KNOPP, Younes SOUILMI
Institut Eurécom
Mobile Communications Department
06904 Sophia Antipolis, France
Email: raymond.knopp@eurecom.fr,younes.souilmi@eurecom.fr

***Abstract*— Some achievable information rates for one-dimensional peer-to-peer networks using *Ultra-WideBand* signaling are presented. For a simple propagation model and coherent detection, achievable rates with and without interference mitigating techniques are provided showing the influence of node density and communication distance on achievable rates of peer-to-peer links. The model is made more precise by introducing multipath propagation and upper and lower bounds on achievable rates are provided for non-coherent receivers and impulsive *flash* signaling.**

I. Introduction

Consider the general wireless peer-to-peer (P2P) network shown in Figure 1. It is characterized by direct links between nodes in the network without the need for a central access-point. It can model *Wireless Personal Area Networks(WPAN)* or certain forms of *Sensor Networks*. The purpose of this study is to investigate the suitability of *Ultrawideband (UWB)* signaling techniques for such networks, and to determine the achievable rates as a function of the density of the network, channel bandwidth and propagation characteristics, and the distance between direct links.

Interest in UWB signaling techniques is motivated by recent regulatory events in the United States. The Federal Communications Commission(FCC) suggests the use of channels in excess of 500 MHz in the 3.1-10.6 GHz band with a maximal average power spectral density of -41 dBm/MHz. For a 500 MHz channel bandwidth this corresponds to an average transmit power of -14 dBm, thus limiting transmission distances to a few tens of meters depending on the target application. UWB signaling can also potentially be implemented with very low-cost and low power-consumption components, representing an interesting solution for remote control and sensor network applications.

The paper is organised as follows: Section II considers a simplistic model for a UWB network and gives an indication of the potential information rates of a UWB network with respect to network density and the use or not of interference mitigating detection techniques. A more precise model is given in Section III taking into account multipath propagation and focusing on non-coherent receivers. Upper and lower-bounds to the achievable rates are given. Finally in Section IV, some numerical results are presented and conclusions are drawn.

II. Decentralized 1-D UWB Networks

We restrict our treatment here to the idealized network configuration shown in Figure 2 which is a network con-

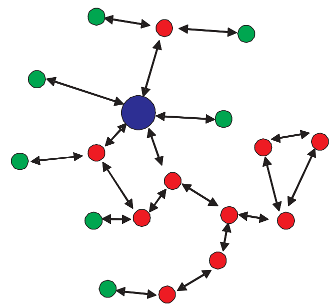

Fig. 1. Peer-to-Peer Network

sisting of $2N+1$ nodes equally spaced on the line with separation D_0. This type of network was studied in [1] in the context of cellular systems and in [2], a similar ad hoc setting was considered. The reason for considering regular linear networks is for analytical simplicity. Extensions to 2-D network topologies could be made without too much additional effort, but would not bring significantly more insight into the fundamental problem.

All nodes are assumed to transmit with the same constant power P_T and no centralized control is performed in the network (e.g. centralized power control, TDMA, etc.). Nodes communicate directly with peers within some distance $kD_0, k \in \mathbf{Z}$, and their information rate is chosen for this target distance. Nodes are assumed to transmit at the same information rate and co-operate fully. Communication at distances beyond kD_0 could be achieved by multihop routing which is not considered here, since it would be the task of higher layers to use nodes as relays. Nevertheless, we will compute achievable rates as a function of kD_0 which can be used in a subsequent study to determine the network throughput as a function of k. In such cases k would be thought of as the *relay distance* which would be a parameter to optimize to control the global throughput of the network. This has been done recently in the context of narrowband systems with retransmission protocols [3] for exponentially decaying traffic patterns.

The propagation model for path loss along a link of distance d is of the form $L(d) = Kd^{-\alpha}$ where K is a constant and α is the attenuation exponent, typically between 2 and 4 for indoor communications. In the following section we will enhance the propagation model to include multipath propagation.

Consider the following simplistic channel model for receiver

0-7803-8329-X/04/$20.00

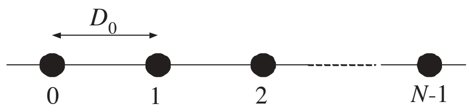

Fig. 2. 1-D Network

0

$$r(t)=\sqrt{P_T K k D_0{}^{-\alpha}}x_0(t)+\sum_{l\neq 0}\sqrt{P_T K(|l|D_0)^{-\alpha}}x_l(t)+z(t) \quad (1)$$

where $z(t)$ is additive white Gaussian noise with power spectral density $N_0/2$, $x_0(t)$ is the desired signal for user 0 (coming from a source at distance k) and the $x_l(t), l\neq 0$ are the interfering signals. Here we assume a worst-case scenario where all interferers are transmitting, where in reality a subset would be transmitting and another receiving at a particular instant. The information signals $x_l(t)$ are assumed to be bandlimited Gaussian processes with bandwidth W Hz and unit power-spectral density. Because of the Gaussianity of the transmitted signals, the following rate is achievable for user 0

$$R_0(k)=W\log_2\left(1+\frac{P_T K(kD_0)^{-\alpha}}{W(N_0+\frac{4P_T K D_0^{-\alpha}}{W}\sum_{l=1}^{N-1}l^{-\alpha}}\right)\ \text{bits/s} \quad (2)$$

By decoding the users whose received signal power is stronger than the desired signal (under the assumption that all users in the network use the same information rate), the receiver for user 0 can also strip out the strong interference and thus achieve the following rate

$$R_0(k)=W\log_2\left(1+\frac{P_T K(kD_0)^{-\alpha}}{W(N_0+\frac{4P_T K D_0^{-\alpha}}{W}\sum_{l=k}^{N-1}l^{-\alpha}}\right)\ \text{bits/s} \quad (3)$$

The rates with and without interference cancellation(IC) are shown in Figure 3 for a typical indoor channel with a path-loss exponent of $\alpha = 3.1$ and attenuation of 80dB at 10m. We note first of all that the rates are characterized by a noise-limited region (below -50 dBm without IC), where rate increases linearly with P_T. Secondly, there is an interference-limited region (above -50 dBm without IC) where increasing P_T yields a small improvement in rate. IC can yield significantly higher information rates and push the interference-limited region to beyond -30 dBm. Using high-power transmission (e.g. -15 dBm) close to the FCC limit is clearly not beneficial in dense networks, at least under this simplistic propagation model.

III. Achievable Rates for Flash-Signaling and Non-Coherent Detection

We now examine a more realistic propagation scenario with impulsive flash-signaling and non-coherent detection. We restrict our treatment to strictly time-limited memoryless complex baseband signals, both at the transmitter and receiver. The transmitted pulse for each user in the system, of duration T_p, is passed through a linear channel, $h(t)$ which is further assumed to be a zero-mean complex baseband circularly symmetric process(i.e. non line-of-sight communications).

The received signal bandwidth W is roughly $1/T_p$, in the sense that the majority of the signal energy is contained in this finite bandwidth. We consider a block fading channel model so that the received signal for a reference user, in each time-interval $[kT_c,(k+1)T_c)$,[1] is given by

$$r(t)=\int_0^{T_p}x_0(u)h_{k,0}(t-u)du+\sum_{l\neq 0}\int_0^{T_p}x_l(u)h_{k,l}(t-u)du+z(t) \quad (4)$$

where $z(t)$ is white circularly symmetric Gaussian noise with power spectral density N_0, T_d the channel delay spread and T_c its *coherence time*. It is assumed that the different users signals are loosely synchronized in the network, at least so that the duration of a typical channel impulse response (T_d) includes their asynchronism.

The transmitted signal for user l is written as

$$x_l(t)=\sum_{k=0}^{N}u_{k,l}\sqrt{E_s}p\left(t-kT_s\right) \quad (5)$$

where k is the symbol index, T_s the symbol duration, $E_s = P_T T_s$ the transmitted symbol energy, u_k is the unit-energy transmitted symbol at time k, and $p(t)$ is the associated pulse shape. For all k, $p(t)$ is a unit-energy pulse of duration T_p. A guard interval of length T_d is left at the end of each symbol (from our memoryless assumption) so that $T_s \geq T_p + T_d$.

Through a Karhunen-Loeve expansion we rewrite the channel model in (4), for each symbol k, as the equivalent set of parallel channels

$$r_{k,i} \;=\; h_{i,0}\sqrt{E_{s,0}\lambda_i}u_{k,0}+\sum_{l\neq 0}h_{i,l}\sqrt{E_{s,l}\lambda_i}u_{k,l}+z_i,\ i=1,...,\infty \quad (6)$$

where z_i is $\mathcal{N}(0,N_0)$ and $\{h_i\}$ are unit variance zero mean independent gaussian random variables. The $\{\lambda_i\}$ are the solution to $\lambda_i\phi_i(t)=\int_0^{T_d+T_p}R_h(t,u)\phi_i(u)du$,where $R_h(t,u)$ denotes the autocorrelation function of the channel, which is assumed to be the same for all users in the system.

Because of the bandlimiting nature of the channels in this study, the channel will be characterized by a finite number, D, of significant eigenvalues which for rich environments will be roughly equal to WT_d, in the sense that a certain proportion of the total channel energy will be contained in these D components. Based on measurement campaigns [4] it was shown that the number of significant eigenvalues can be large but significantly less than the approximate dimension of signal-space WT_d. This is due to insufficient scattering in short-range indoor environments. The number of significant eigenvalues can nonetheless be taken to be large.

In the case of vanishing signal to noise ratio, binary flash-signaling is a virtually optimal signaling format [5], [7], [6] for both coherent and non-coherent receivers. Using the notation from the previous section, we express the binary flash signaling scheme as

$$u_{k,l}=\begin{cases}1/\eta & \text{with probability} \quad \eta\\ 0 & \text{with probability} \quad (1-\eta)\end{cases} \quad (7)$$

and $T_s = T_d + T_p$. The probability of emission, η, is assumed to be the same for every user.

[1] the channel impulse response $h_k(t)$ is piecewise constant, with its values remaining fixed on time-intervals $[nT_c,(n+1)T_c)$, $n\in\mathbf{Z}$.

A. Genie-Aided Receiver - Upper bound

Under the constraint of the flash signaling transmission strategy the achievable rate without complete channel side information at the receiver of user 0 is given by $I(U_{k,0};\{R_{k,i}\})$. This is difficult to compute numerically but is upper-bounded as

$$\frac{1}{T_s}I(U_{k,0};\{R_{k,i}\}) \leq \frac{1}{T_s}I(U_{k,0};\{R_{k,i}\},\{U_{k,l\neq 0}\}) = \frac{1}{T_s}I(U_{k,0};\{R_{k,i}\}|\{U_{k,l\neq 0}\}) \quad (8)$$

This upper-bound is interpreted as a genie-aided receiver who has access to the symbols of the interference but not their channels. As a result it cannot strip out the interference (as in Section 2) but it knows where it occurs and can thus use this information in the decoding process. In relation to the receiver with IC in Section 2, this is also an upper-bound on the achievable rates with flash-signaling for the non-coherent receiver which can decode the interferers with received signal strengh stronger than the desired signal. The achievable rate for the genie-aided receiver can be shown to be given by (9) at the end of this document with Y a zero-mean Gaussian random vector with covariance matrix $\mathbf{I}$ and $E_I = \sum_{l\neq 0} E_{s,l}\mathbf{I}(u_{k,l} = 1)$ with $\mathbf{I}(\cdot)$ being the indicator function. This average mutual information can be efficiently computed numerically. In the noise-limited region this is achieved by numerical integration and in the interference-limited region by Monte-Carlo averaging.

B. Wideband Threshold Detection

In order to obtain a lower bound on $I(U_{k,0};\{R_{k,i}\})$ and at the same time evaluate achievable rates for simple receivers, we note that

$$I(U_{k,0};\{R_{k,i}\}) \geq I(U_{k,0};Y_k) \quad (10)$$

where $y_k = \mathbf{I}\left(\sum_{i=1}^{D} |r_{k,i}|^2 > \xi\right)$ and ξ is a threshold to be optimized. The lower-bound is guaranteed by the data-processing inequality [8]. This detector is a simple energy detector followed by hard-decision decoding and would model a minimalistic low-power UWB receiver which does not use an analog-to-digital converter. It would consist simply of filtering, amplification and a square-law device such as a *Schottkey diode*. In addition, the decoding algorithm would also operate on a binary alphabet which could potentially reduce implementation complexity.

The lower-bound is the capacity of a binary-input discrete-memoryless channel with transition probabilities depending on η and ξ. Since, conditioned on the interference level, the y_k are quadratic forms of complex Gaussian random variables, the transition probabilities can be efficiently computed numerically. As in the case of the upper-bound, in the noise-limited region, this can be done by numerical integration and the the interference-limited region by Monte-Carlo averaging. Optimization of eta and xi must also be done numerically.

IV. Numerical Results and Conclusions

Here we compute the upper and lower bounds on the achievable rates for the 1D-network model assuming an eigenvalue distribution with $D = WT_d = 20$ significant eigenvalues of equal amplitude which would correspond roughly to a system with bandwidth $W = 800$MHz and delay-spread $T_d = 25$ns. We consider the same configurations as in the simplified network analysis of Section 2. The upper and lower-bounds on the achievable rates are shown in Fig. 4 for a node separation of $D_0 = 1$m and relay distances of $k = 4, 8$. We see that similarly to the interference-cancelling receiver of Section 2, the interference-limited regions begins at transmission powers around -30 dBm. Achievable data rates with realistic channels and non-coherent detection are five and ten times smaller than the simplified channel model in the noise and interference-limited regions respectively. This loss is due to the spreading of signal energy across the unknown channel parameters and is similar to the loss in spectral-efficiency of DS-CDMA systems with increasing bandwidth (and channel uncertainty) as described in [7].

It is remarkable to note that interference-cancellation does not provide a significant increase in achievable information rates with respect to simple energy detection with hard-decision decoding. Moreover, the simple receiver can produce close to optimal information rates without the use of a analog-to-digital converter. Flash-signaling can also be implemented without the need for a digital-to-analog converter or power-amplifier in the transmitter, and as a result, very low-power and extremely simple UWB devices could be conceived for medium bit-rate densely populated P2P networks.

Data rates can be increased by coding across parallel flash-signaling channels, which would be possible, for instance, in a multi-band system. This is described for single-user systems in [9].

References

[1] S.Shamai and A.Wyner, "Information theoretic considerations for symmetric cellular, multiple-access fading channels - Part I," *IEEE Trans. on Inform. Theory,* Vol.43, No.6, pp. 1877-1894, Nov. 1997.

[2] Liang-Liang Xie and P.R. Kumar, "A network information theory for wireless communication: Scaling laws and optimal operation." *Submitted to IEEE Transactions on Information Theory*, April 12, 2002.

[3] Tabet T., Knopp R., "Throughput Analysis for Slotted Regular AdHoc Networks", in proc. Asilomar Conference on Signals, Systems and Communications, Nov. 2003.

[4] Menouni A., Knopp R. and Sadaane R., "Empirical Eigenanalysis of the Indoor UWB Propagation Channel", Institut Eurécom Internal Report, October 2003.

[5] Abou-Faycal,I. C. ; Trott, M. D. ; Shamai, S. , " The Capacity of Discrete-Time Memoryless Rayleigh-Fading Channels " *IEEE Transactions on Information Theory*, Vol. 47 Issue: 4 , May 2001 pp. 1290 -1301.

[6] Verdu, S. ," Spectral efficiency in the wideband regime " *IEEE Transactions on Information Theory*, Vol. 48 Issue: 6 , June 2002 pp. 1319 -1343.

[7] Telatar, I. E.; Tse, D. N. C. ,"Capacity and mutual information of wideband multipath fading channels " *IEEE Transactions on Information Theory*, Vol. 46, Issue:4, pp. 1384-1400, 2000.

[8] Gallager, R.G., "Information Theory and Reliable Communication", Wiley&Sons, 1968.

[9] Souilmi, Y., Knopp, R., "On the achievable rates of multiband UWB systems," to appear in the *Journal of Wireless Networks*, 2004.

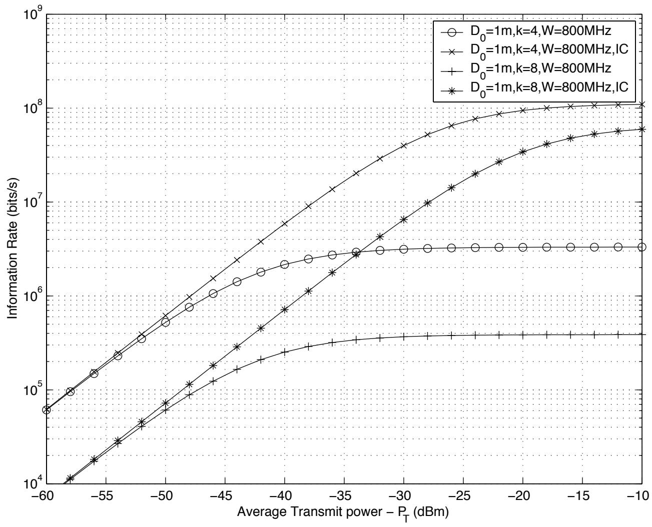

Fig. 3. Achievable Rates for Simplistic Decentralized 1-D Network with and without interference cancellation. In the case of interference cancellation, all interferers with a signal level stronger than the desired signal are stripped out. The propagation is characterized by $\alpha = 3.1$ with an attenuation at 10m set to -80dB.

$$\begin{aligned}\frac{1}{T_s} I(U_{k,0};\{R_{k,i}\}|\{U_{k,l\neq 0}\}) &= -\frac{1}{T_s} \underset{E_I}{E}\underset{Y}{E}\left[\eta \log\left(\eta + (1-\eta)\sqrt{\prod_{i=1}^{D} 1 + \frac{E_{s,0}\lambda_i}{\eta N_0 + E_I\lambda_i}}\, e^{\left(-Y^H (\mathbf{diag}(\frac{E_I\lambda_i + N_0\eta}{E_{s,0}\lambda_i}))^{-1} Y\right)}\right)\right. \\ + \quad & \left.(1-\eta)\log\left((1-\eta) + \frac{\eta}{\sqrt{\prod_{i=1}^{D} 1 + \frac{E_{s,0}\lambda_i}{\eta N_0 + E_I\lambda_i}}}\, e^{\left(Y^H (\mathbf{diag}(\frac{N_0\eta + E_I\lambda_i}{E_{s,0}\lambda_i}+1))^{-1} Y\right)}\right)\right] \ \text{bits/s} \qquad (9)\end{aligned}$$

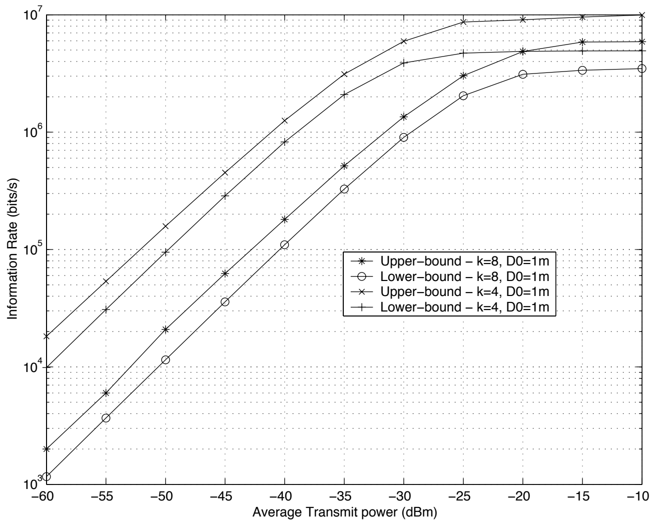

Fig. 4. Achievable Rates for Decentralized 1-D Network with unknown channels. The upper-bound represents the genie-aided receiver furnished with the information symbols of all interferers in the network (both strong and weak). The lower-bound is the achievable rate with threshold detection, where numerical optimization of eta and xi was performed. The propagation is characterized by $\alpha = 3.1$ and attenuation at 10m set to -80dB, and a delay-spread of $T_d = T_s = 25$ns.