

Spatial Multiplexing over Correlated MIMO Channels with a Closed Form Precoder

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Abstract

This paper addresses the problem of MIMO spatial-multiplexing (SM) systems in the presence of antenna fading correlation. Existing SM (V-BLAST and related) schemes rely on the linear independence of transmit antenna channel responses for stream separation and suffer considerably from high levels of fading correlation. As a result such algorithms simply fail to extract the non-zero capacity that is present even in highly correlated spatial channels. We make the simple but key point that just one transmit antenna is needed to send several independent streams if those streams are appropriately superposed to form a high-order modulation (e.g. two 4-QAM signals form a 16-QAM)! The concept builds upon constellation multiplexing (CM) [1] whereby distinct QAM streams are superposed to form a higher-order constellation with rate equivalent to the sum of rates of all original streams. In contrast to SM transmission, the substreams in CM schemes are differentiated through power scaling rather than through spatial signatures.

We build on this idea to present a new transmission scheme based on a precoder adjusting the phase and power of the input constellations in closed-form as a function of the antenna correlation. This yields a rate-preserving MIMO multiplexing scheme that can operate smoothly at any degree of correlation. At the extreme correlation case (identical channels), the scheme behaves equivalent to sending a single higher-order modulation whose independent components are mapped to the different antennas.

I. INTRODUCTION

Multiple input and multiple output (MIMO) systems, employing several transmit and receive antennas at both ends, are capable of providing a large increase in capacity compared to traditional single antenna systems [2], [3], [4]. This increase in capacity is however dependent upon the fact that the channels from a transmitter to a receiver follow independent paths. The capacity of MIMO systems can be shown to degrade if there are for example severe correlations present at the transmitter and/or receiver side [5], [6]. At worst, the capacity falls back to that of a SIMO/MISO with, potentially, additional array gain. However the impact on actual *transmission algorithms* such as spatial multiplexing [3], [7], [8] can be dramatic. Indeed any correlation present at the transmitter effectively increases the linear dependence of the input streams' response and makes stream separation and decoding a difficult task. For example current schemes like V-BLAST literally break down in the presence of correlation levels close to one. Designing appropriate transmission techniques that can adjust smoothly to any level of correlation is therefore an important and practical issue. Although correlated scenarios have previously been considered [9], [10] the focus has mainly been on capacity issues rather than on robust practical algorithms. In order to take advantage of correlation knowledge, [10] and [11] discuss using the eigen-decomposition of the average MIMO channel and thereupon implementing a waterfilling approach across the eigenmodes of the correlation matrix. This results in widely unbalanced error-rates across streams unless some form of adaptive coding/modulation is implemented as well. Ideally, waterfilling requires a continuous bit allocation to substreams, with more bits transmitted on the dominant eigenmodes. In practice though, modulation techniques are discrete which

makes it challenging to realize optimal bit assignments.

To minimize the BER in the presence of transmit correlation, a transmit precoding scheme based on power allocation and per-antenna phase shifting was introduced in [12] for a 2×2 MIMO system, while [13] investigates a phase-shifting only strategy. However interesting, both of these concepts rely upon the use of numerical optimization in order to find appropriate solutions and require exhaustive search-based maximum likelihood (ML) decoding techniques. Note that other recent contributions (e.g. [14]) have addressed the transmit precoding problem assuming full instantaneous channel feedback, a situation we consider impractical in our scenario. In this article we revisit the issue of reliable transmission over correlated MIMO channels when only (long term) correlation properties are known to the transmitter while the receiver has full channel knowledge. We take on a new perspective to solve the problem in a simple and insightful manner.

MIMO channel input signal vectors can be viewed as multidimensional constellations. For instance, having N transmitters and M ($\geq N$) receivers allows transmission of signal constellations with dimension up to $2N$ (N real and N imaginary). Clearly, fading correlation acts as *continuous* dimension reduction factor, as seen by the receiver. In the extreme correlation one situation, the dimension offered by the channel for transmission simply falls down to 2, i.e. one complex scalar per channel use. This is also the dimension offered for transmission by a SISO or SIMO/MISO channel, which as we know does not offer spatial multiplexing capability. Yet, we wish to pose the problem of how to maintain data rate of a multiplexing system over a channel with possibly fluctuating correlation levels.

A simple but interesting point can therefore be made: When the instantaneous channels of the various transmit antennas become quasi-identical due to high correlation, the MIMO system becomes equivalent to a SIMO system. However, just one transmit antenna is needed to send several independent streams drawn from a given modulation constellation, if those streams are appropriately superposed to form together a regular higher order constellation. For example, two 4-QAM signals can be sent over a single transmitter, if these 4-QAM signals add up to form a proper 16-QAM signal. Although transmission with 16-QAM results in a loss in BER performance (about 4dB in fading channels for zero-forcing receiver), it has the considerable merit of requiring only one transmit antenna to be sent and detected.

This article describes a transmission scheme which, exploiting the idea above, bridges across spatial multiplexing on one hand and high-order constellation transmission (constellation multiplexing) on the other hand. This allows to maintain the same data rate and provide robustness against arbitrary levels of correlation. The solution to this problem is expressed in the form of a linear MIMO precoder. Each transmit antenna carries an independent signal drawn from a single fixed modulation, (i.e. a spatial multiplexing

BLAST¹ like scenario). Each constellation is adjusted in power and phase according to the transmit correlation knowledge. The unique features of this approach include:

- 1) The optimized transmitter is determined in *closed-form* from the correlation coefficients by solving a simple constrained linear equation. This is in contrast with other previous approaches which involved iterative/exhaustive search techniques for the precoder.
- 2) The transmission rate Nb (where b is the modulation's efficiency in Bits/Symb) of the spatial multiplexing system is preserved regardless of correlation level. Clearly, the error rate performance is adversely affected by the correlation, but to a much lesser extent than in the absence of the precoding.
- 3) When the correlation approaches 1, the signal is seen at a certain stage of the receiver gradually coincides with that of a regular (2D) constellation with an alphabet size of 2^{Nb} symbols.
- 4) The transmitter is optimized based on a simple criterion coined BER balancing criterion (BBC). This states that all components of the SM system should be detected with similar target error rate. Interestingly, most high-order constellations based on regular complex grids implicitly also follow this criterion.
- 5) Finally, a proof is given that the criterion above always admits one solution.

Note that, for the sake of deriving the precoding parameters, a specific hybrid Zero-Forcing/interference canceller receiver is employed. The results are shown however to be general in nature and applicable to a wider class of receivers, such as the maximum likelihood (ML) receiver.

Notations: The following notations are adopted for the remaining paper: All boldface capital letters represent a matrix while a lowercase boldface letter denote a vector. $*$ stands for the transpose conjugate operation while $\mathbf{A}_{:,l}$ expresses the l 'th column of the matrix \mathbf{A} . Additionally, \angle is used to denote the phase of the given expression, \dagger refers to the Moore-Penrose matrix pseudoinverse while E is the expectation operator. The minimum distance between two symbols for a given modulation is denoted by d_{min} while $d_{max}(\geq d_{min})$ points to the minimum distance between two constellation points with highest amplitude.

II. SIGNAL AND CHANNEL MODELS

We consider a MIMO system consisting of N transmit antennas and $M (\geq N)$ receive antennas with correlations present at the transmitter only, as in the case in other related works. The non-trivial extension to the case of receive correlation will be addressed elsewhere. This models an environment where elements placed at an elevated high-point basestation for example exhibit correlations while the receiver is located in a rich-scattering surrounding.

¹Bell Labs Layered Space Time

In this situation the channel can be described by

$$\mathbf{H} = \mathbf{H}_0 \mathbf{R}_t^{\frac{1}{2}}. \quad (1)$$

The channel matrix \mathbf{H}_0 of size $M \times N$ consists of complex Gaussian zero mean unit-variance independent and identically distributed (iid) elements while \mathbf{R}_t is the $N \times N$ transmitter correlation matrix. We assume that the transmitter is aware of the correlation matrix \mathbf{R}_t , while \mathbf{H}_0 is only known at the receiver. This is a practical situation for many wireless systems where only long term statistics such as the correlation matrix may change slowly enough to be fed back regularly from receiver to the transmitter. Estimation methods for correlated MIMO channels can be found in [15]. In certain circumstances the channel may be dominated by strong line-of-sight components i.e. the Ricean channel model, which is dealt with in [1].

The baseband equivalent of the transmitted N -dimensional signal vector, once observed at the receiver, can be expressed as:

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n} = \mathbf{H}_0 \mathbf{R}_t^{\frac{1}{2}} \mathbf{s} + \mathbf{n}, \quad (2)$$

where \mathbf{n} is the M -dimensional noise vector whose entries are assumed iid complex Gaussian with zero mean and a variance of σ_n^2 . Additionally we set

$$\mathbf{s} = [\sqrt{P_1} s_1 \sqrt{P_2} e^{j\phi_2} s_2 \dots \sqrt{P_N} e^{j\phi_N} s_N]^T. \quad (3)$$

P_1, \dots, P_N represent positive power levels allocated respectively to input symbols s_1, \dots, s_N , and are selected to satisfy $\sum_{i=1}^N P_i = 1$. The SNR per receiver antenna is thus $\frac{1}{\sigma_n^2}$. ϕ_2, \dots, ϕ_N correspond to phase shifts possibly applied by the transmit precoding module on each antenna. Notice that we assume the first symbol does not undergo a phase change and can be regarded as a reference point for all other phase components. We therefore define $\phi_1 = 0$. In conventional spatial multiplexing schemes, one typically assigns equal weights $P_i = \frac{1}{N}$ and $\phi_i = 0$ for $1 \leq i \leq N$. The symbols are selected from the same modulation alphabet (say r-QAM), with an average energy of one, $E\{|s_i|^2\} = 1$.

III. TRANSMITTER OPTIMIZATION

Since the instantaneous channel properties are assumed unknown to the transmitter, the objective becomes to design a precoder, in the form of a set of power coefficients P_1, \dots, P_N and phases, only function of the correlation matrix \mathbf{R}_t and independent of \mathbf{H}_0 . One approach to do so consists in optimizing a bit error rate related criterion (e.g. pairwise error probability) averaged over all realization of \mathbf{H}_0 . However this type of approach, although optimal from a BER point of view, results in cost optimization problems that are hardly tractable. Instead, in an attempt to derive a low complexity algorithm, we will here rely on

a particular decoding structure which allows to extract the unknown part of the channel out of the precoder optimization. It will become apparent that this results in near optimal power weights. To better understand the approach below, the following key point can be made: On a long term basis, \mathbf{H}_0 will generally be well-conditioned (and thus 'easy' to invert) while ill-conditioning related limitations of this system will more likely come from \mathbf{R}_t . Indeed, in the fully correlated case, \mathbf{R}_t is rank one and non-invertible. At high correlation levels the ZF receiver would otherwise result in substantial noise-amplification and coloring.

A. Hybrid Zero-Forcing/MRC SIC

Following the remarks above and in the interest of deriving our closed-form precoding algorithm, a particular receiver structure is next assumed denoted hybrid zero-forcing maximum-ratio-combiner successive-interference-canceler (HZM-SIC). The HZM-SIC decoder simply cascades a zero-forcing filter with a V-BLAST type receiver. The idea behind the HZM-SIC structure is that the well conditioned and ill-conditioned components of the channel ought to be treated differently: \mathbf{H}_0 , being typically well-conditioned, is inverted out through a zero-forcing filter while $\mathbf{R}_t^{\frac{1}{2}}$, being possibly very ill-conditioned, is dealt with in a MRC manner rather than matrix inversion.

It is fundamental to emphasize at this point that the main goal for such a receiver structure is to lead to an insightful and closed-form deriving of the solution to the transmitter optimization problem that is fully independent of the instantaneous channel fading. Thus we do not claim optimality in any sense for this particular linear receiver, although the differentiation of well-conditioned from ill-conditioned channel components is a promising approach. Finally, given the general and intuitive nature of the obtained solutions (described in section IV) one may claim that the resulting precoding coefficients can be used for a much wider range of receiver algorithms, beyond the one presented here, such as the ML receiver for instance. This fact is corroborated by simulations.

For exposition purposes the article next starts with a two by two antenna case and later generalizes the results.

B. HZM-SIC receiver for 2×2 case

The hermitian square-root correlation matrix for a 2×2 setup may be expressed as:

$$\mathbf{R}_t^{\frac{1}{2}} = \begin{bmatrix} \alpha & \beta e^{j\psi} \\ \beta e^{-j\psi} & \alpha \end{bmatrix} \quad (4)$$

where α and β are both real and satisfy by construction $\alpha^2 + \beta^2 = 1$. $\rho = 2\alpha\beta$ is the modulus of the antenna correlation coefficient ($\rho \leq 1$).

1) *Zero-forcing stage*: Applying a linear zero-forcing filter on (2) in order to neutralize \mathbf{H}_0 , one obtains:

$$\mathbf{z} = \mathbf{H}_0^\dagger \mathbf{y} = \mathbf{R}_t^{\frac{1}{2}} \mathbf{s} + \mathbf{H}_0^\dagger \mathbf{n}. \quad (5)$$

Equation (5) can be written out in full as:

$$z_1 = \alpha \sqrt{P_1} s_1 + \beta \sqrt{P_2} e^{j(\phi_2 + \psi)} s_2 + n_1 \quad (6)$$

$$z_2 = \beta \sqrt{P_1} e^{-\psi} s_1 + \alpha \sqrt{P_2} e^{j\phi_2} s_2 + n_2 \quad (7)$$

2) *MRC stage with correlation coefficients*: We next start with estimating s_1 by applying MRC on z_1 , z_2 from (6), (7), with conjugate coefficients from the first column of $\mathbf{R}_t^{\frac{1}{2}}$:

$$\eta = (\mathbf{R}_t^{\frac{1}{2}})_{:,1}^* \mathbf{z} = \alpha z_1 + \beta e^{j\psi} z_2 \quad (8)$$

$$= \sqrt{P_1} s_1 + 2\alpha\beta e^{j(\psi + \phi_2)} \sqrt{P_2} s_2 \quad (9)$$

$$+ \alpha n_1 + \beta e^{j\psi} n_2.$$

Assuming the energy in the s_1 term is larger than that in the s_2 term above, an estimate for the first symbol \hat{s}_1 can be obtained, from (9) for instance with a slicer (or alphabet search) over $\frac{1}{\sqrt{P_1}}\eta$.

3) *Successive interference canceler*: After obtaining \hat{s}_1 , the symbol can be subtracted from the correlated signal observation \mathbf{z} . For the sake of the derivation we assume no propagation of error ($\hat{s}_1 = s_1$), such that we can form

$$\hat{\mathbf{z}} = \mathbf{z} - [\alpha \beta e^{-\psi}]^T \sqrt{P_1} s_1. \quad (10)$$

Finally a second MRC, is performed on $\hat{\mathbf{z}}$ to estimate \hat{s}_2 :

$$\hat{\eta} = (\mathbf{R}_t^{\frac{1}{2}})_{:,2}^* \hat{\mathbf{z}} = e^{j\phi_2} \sqrt{P_2} s_2 + \beta e^{-j\psi} n_1 + \alpha n_2. \quad (11)$$

Observe that the decoding structure becomes identical to the decoding of a traditional r -QAM modulated symbol where successive decision are made over each quadrant. With no transmitter correlation, $\mathbf{R}_t = \mathbf{I}$, and the performance of the above algorithm is identical to standard MIMO ZF receiver. Note also that this algorithm, if used as a practical receiver, only requires a single matrix inversion of \mathbf{H}_0 common for all stages, in contrast to V-BLAST which requires an inversion at each stage. The complexity increase against ZF is therefore only minor.

C. BER Balancing Criterion (BBC)

The symbol error probability for s_1 is governed by the variance σ_η^2 of the additive noise term $\alpha n_1 + \beta e^{j\psi} n_2$ in 9 and the minimum possible distance from \hat{s}_1 to its corresponding symbol region boundary

(here for short referred to as 'minimum distance'). Assuming the symbols follow a rigid regular format (e.g. r-QAM), the phase of the factor $2\alpha\beta e^{j(\psi+\phi_2)}\sqrt{P_2}s_2$ must be selected to maximize the distance from the decision boundaries of s_1 . For an arbitrary QAM modulation, this is simply done by setting ϕ_2 at the emitter such that

$$\phi_2 = -\psi. \quad (12)$$

For a further justification of this choice note that this also corresponds to a transmit MRC with respect to the phase of the correlation matrix, a procedure known to be optimal capacity-wise as well [9], [10] at high correlation. This solution is though not unique and ϕ_2 may additionally be rotated $\frac{\pi}{4}$ an arbitrary number of times. The minimum distance for a decision from η in (9) becomes:

$$\delta_1 = \sqrt{P_1}d_{min} - 2\alpha\beta\sqrt{P_2}d_{max}. \quad (13)$$

$\sqrt{P_1}d_{min}$ refers to the smallest possible minimum distance for s_1 while the second factor $\sqrt{P_2}d_{max}$ scaled with the correlation coefficient is the maximum possible distance for the interfering symbol s_2 .

Interestingly, under standard V-BLAST with no power weighting, $P_1 = P_2 = \frac{1}{2}$, and the minimum distance, as a function of the correlation, is given as

$$\delta_1 = \frac{1}{\sqrt{2}}d_{min} - \rho\frac{1}{\sqrt{2}}d_{max}. \quad (14)$$

In the 4-QAM case, $d_{min} = d_{max}$, and as the correlation increases we have $\rho \rightarrow 1$ and therefore

$$\lim_{\rho \rightarrow 1} \delta_1 = 0, \quad (15)$$

showing how, at high correlation levels, the system performance degrades quickly in the presence of noise as the symbols totally overlap each other. For a general M-QAM modulation δ_1 may gradually start becoming negative, implying that the transmitted symbol will cross the axis.

The minimum distance for s_2 is from (11) given simply as

$$\delta_2 = \sqrt{P_2}d_{min}. \quad (16)$$

Noise variance: The noise elements of vector \mathbf{n} follow the same distribution. Similarly all components in \mathbf{H}_0^\dagger also have an identical statistical distribution by construction. Thus, the noise factors $\beta e^{-j\psi}n_1 + \alpha n_2$ and $\alpha n_1 + \beta e^{j\psi}n_2$ have identical variance when averaged over \mathbf{H}_0 .

We can therefore equate the average probability of error for s_1 and s_2 simply by equating the minimum distances previously obtained in (13) and (16), for any value of the correlation:

$$\delta_1 = \delta_2, \quad (17)$$

which leads to:

$$\sqrt{P_1}d_{min} - \rho\sqrt{P_2}d_{max} = \sqrt{P_2}d_{min} \quad (18)$$

under constraint

$$P_1 + P_2 = 1. \quad (19)$$

The weights for this 2×2 system can easily be computed as function of the correlation amplitude ρ for a given modulation:

$$P_1 = \frac{(1 + \frac{d_{max}}{d_{min}}\rho)^2}{1 + (1 + \frac{d_{max}}{d_{min}}\rho)^2}, \quad P_2 = \frac{1}{1 + (1 + \frac{d_{max}}{d_{min}}\rho)^2}. \quad (20)$$

Special cases: Certain special cases for this precoder turn out to be of immediate interest in their interpretation.

- **Uncorrelated:** With no correlation $\rho = 0$ which yields equal power transmission, justifying the standard equal power design.
- **Fully correlated:** With full correlation and $d_{min} = d_{max}$ (4-QAM) $\rho = 1$ we find $P_1 = 0.8$ and $P_2 = 0.2$. Interestingly, this corresponds to the power allocation for a regular 2D constellation. For instance a 16-QAM constellation can be seen as the superposition of two 4-QAM constellations with respective powers 0.8 and 0.2, the second 4-QAM points being centered about the constellation point of the first 4-QAM points (see figure 1). Hence spatial multiplexing is here replaced by constellation multiplexing.

The latter case indicates that if antennas are fully correlated (as in a SIMO case), one may still preserve the pre-designed spatial multiplexing data rate by simply sending the equivalent of a higher order (e.g. QAM) constellation with separate binary information components over each antenna. This of course makes good intuitive sense. In more realistic in-between scenarios the precoder adjusts the transmit constellation smoothly between those two cases, performing a mix of spatial and constellation-multiplexing, capable of extracting a non-zero capacity for any level of correlation between the antennas.

In contrast to (14) the minimum distance, with precoding, can be specified as

$$\delta_1 = \sqrt{P_1}d_{min} - \rho\sqrt{P_2}d_{max} = \frac{1}{\sqrt{1 + (1 + \frac{d_{max}}{d_{min}}\rho)^2}} \quad (21)$$

which converges to a non-zero value, as the correlation increases.

IV. TRANSMIT OPTIMIZATION FOR ARBITRARY NUMBER OF ANTENNAS

This section describes the procedure for finding precoding weights in a general setting with more than two antennas. In order to find a closed-form solution the general derivation relies upon the fact that the

correlations between transmitters either follow a Jakes model, with real correlations, or the exponential correlation structure; both models are widely used in the literature [5], [16], [17], [8]. Appendix B discusses the situation when the correlation matrices may not follow any of the assumed structures.

The assumed decoder starts by selecting the symbol corresponding to largest power P_i . Without loss of generality we rely on the fact that the power weights are set to satisfy

$$P_1 \geq P_2 \geq \dots \geq P_N. \quad (22)$$

Thus s_1 is the first symbol to be decoded, followed by s_2 etc. in a chronological order.

A. HZM-SIC algorithm:

Let us define \mathbf{z} by the same decoding procedure as the one described in section III, obtained from inverting out the i.i.d fading component part of the channel:

$$\mathbf{z} = \mathbf{H}_0^\dagger \mathbf{y} = \mathbf{R}_t^{\frac{1}{2}} \mathbf{s} + \mathbf{H}_0^\dagger \mathbf{n}. \quad (23)$$

To determine the precoding weights according to the BBC criterion we introduce η_1 , obtained through a MRC with coefficients taken from the first column of $\mathbf{R}_t^{\frac{1}{2}}$ applied on \mathbf{z} :

$$\eta_1 = (\mathbf{R}_t^{\frac{1}{2}})_{:,1}^* \mathbf{z} = \sum_{l=1}^N r_{l,1}^* z_l \quad (24)$$

$$= \sum_{l=1}^N r_{l,1}^* \left(\sum_{k=1}^N r_{l,k} \sqrt{P_k} e^{j\phi_k} s_k \right) + (\mathbf{R}_t^{\frac{1}{2}})_{:,1}^* \mathbf{H}_0^\dagger \mathbf{n} \quad (25)$$

$$= \sqrt{P_1} s_1 + \left(\sum_{l=1}^N r_{l,1}^* r_{l,2} \right) \sqrt{P_2} e^{j\phi_2} s_2 + \dots \\ + \left(\sum_{l=1}^N r_{l,1}^* r_{l,N} \right) \sqrt{P_N} e^{j\phi_N} s_N + (\mathbf{R}_t^{\frac{1}{2}})_{:,1}^* \mathbf{H}_0^\dagger \mathbf{n}, \quad (26)$$

where the short hand notation $r_{i,j}$ is used to represent element $(\mathbf{R}_t^{\frac{1}{2}})_{i,j}$ in the square-root correlation matrix. We further define $\tau_{i,j} = |r_{i,j}|$.

To minimize interference caused by other symbols, the phase-wise transmit MRC shown by equation (12) can be extended:

$$\phi_k = -\angle \left(\sum_{l=1}^N r_{l,1}^* r_{l,k} \right). \quad (27)$$

Equation (27) makes certain that when there are phase correlations then the superposed QAM symbols maximize their distance from the decision boundary. The error probability for s_1 is then, as previously, governed by the additive noise variance and the minimum distance. The minimum distance for s_1 is reached for instance by $s_2 = s_3 = \dots = s_N = -s_1$. Written out with weights and correlation coefficients

the minimum distance for s_1 is:

$$\delta_1 = \sqrt{P_1}d_{min} - \left(\sum_{l=1}^N \tau_{1,l}\tau_{2,l}\right)\sqrt{P_2}d_{max} - \dots - \left(\sum_{l=1}^N \tau_{1,l}\tau_{N,l}\right)\sqrt{P_N}d_{max} \quad (28)$$

where we have used the fact that $\mathbf{R}_t^{\frac{1}{2}}$ is hermitian.

Assuming no error propagation, s_1 is detected and subtracted from equation (23):

$$\hat{\mathbf{z}} = \mathbf{z} - \sqrt{P_1}\mathbf{R}_{t:,1}^{\frac{1}{2}}s_1. \quad (29)$$

Correlation phase At this point we make the following assumption, allowing to generalize (27) for use in the next and all other stages:

$$\phi_k = -\angle\left(\sum_{l=1}^N r_{l,i}^*r_{l,k}\right), \quad \forall i = 1, \dots, N \quad (30)$$

The equation above implies that a single set of phase coefficients can be used in the precoding (one phase per signal) and still lead to an optimal precoding. As one can show, this assumption is exactly valid, among others, in the general class of complex exponential correlation models, used by a majority of authors e.g. [16], [17]. A proof of this result can be found in Appendix A.

When the assumption above is not met, then the phase coefficients in the precoder may no longer be optimized in closed-form. However we make the following two points. First the power weights, which, we should note, carry the essential of the information destined to differentiating the signals sent over the correlated antennas, may still be optimized as is shown later. Second, a suboptimal choice of phase coefficients is still obtainable in closed form as shown in Appendix B. However, for the rest of the main body we expect (30) to hold.

In the next stage of SIC decoding an additional MRC with weights from $(\mathbf{R}_{t:,2}^{\frac{1}{2}})^*$ can then be used to obtain an estimate for s_2 ,

$$\eta_2 = (\mathbf{R}_{t:,2}^{\frac{1}{2}})^*\hat{\mathbf{z}} = \sum_{l=1}^N r_{l,2}^* \hat{z}_l \quad (31)$$

$$= \sum_{l=1}^N r_{l,2}^* \left(\sum_{k=2}^N r_{l,k} \sqrt{P_k} e^{j\phi_k} s_k\right) + (\mathbf{R}_{t:,2}^{\frac{1}{2}})^* \mathbf{H}_0^\dagger \mathbf{n} \quad (32)$$

$$= \sqrt{P_2} e^{j\phi_2} s_2 + \left(\sum_{l=1}^N r_{l,2}^* r_{l,3}\right) \sqrt{P_3} e^{j\phi_3} s_3 + \dots \\ + \left(\sum_{l=1}^N r_{l,2}^* r_{l,N}\right) \sqrt{P_N} e^{j\phi_N} s_N + (\mathbf{R}_{t:,2}^{\frac{1}{2}})^* \mathbf{H}_0^\dagger \mathbf{n}. \quad (33)$$

Based on the phase relation (30), and as with (28), we can find the minimum distance for s_2 from (33)

as follows:

$$\delta_2 = \sqrt{P_2}d_{min} - \left(\sum_{l=1}^N \tau_{2,l}\tau_{3,l}\right)\sqrt{P_3}d_{max} - \dots - \left(\sum_{l=1}^N \tau_{2,l}\tau_{N,l}\right)\sqrt{P_N}d_{max}. \quad (34)$$

By repeating this N times, one obtains expressions for N minimum distances, on a form analogous to (28) and (34).

B. BBC-based transmit optimization

To express the BBC criterion by simply equating minimum distance terms (28), (34), etc. one must assure that the additive noise terms affecting each symbol, namely $\mathbf{R}_t^{\frac{1}{2}} \mathbf{H}_0^\dagger \mathbf{n}$, $i = 1, \dots, N$, have identical variance, as done in the first section. Notice that $\mathbf{R}_t^{\frac{1}{2}*} \mathbf{R}_t^{\frac{1}{2}} = \mathbf{R}_t$ is an hermitian matrix with unit diagonal entries, and the norm of all rows/columns in $\mathbf{R}_t^{\frac{1}{2}}$ is therefore identical to unity. When averaged over \mathbf{H}_0 , all symbols are conclusively affected by the same noise variance.

In order to guarantee all symbols an equal error rate, it is therefore sufficient that values for $\sqrt{P_1}, \sqrt{P_2}, \dots, \sqrt{P_N}$ be selected so that the minimum symbol distance observed for each symbol is identical:

$$\delta_1 = \delta_2, \delta_2 = \delta_3, \dots, \delta_{N-1} = \delta_N, \quad (35)$$

or equivalently

$$\delta_1 = \delta_N, \delta_2 = \delta_N, \dots, \delta_{N-1} = \delta_N. \quad (36)$$

Based on (36) the following linear system can then be set up as part of the problem to find the appropriate power levels:

$$\Delta \mathbf{p} = \mathbf{0} \quad (37)$$

where

$$\Delta = \begin{bmatrix} d_{min} & -\sum \tau_{1,l}\tau_{2,l}d_{max} & -\sum \tau_{1,l}\tau_{3,l}d_{max} & \dots & -d_{min} - \sum \tau_{1,l}\tau_{N,l}d_{max} \\ 0 & d_{min} & -\sum \tau_{2,l}\tau_{3,l}d_{max} & \dots & -d_{min} - \sum \tau_{2,l}\tau_{N,l}d_{max} \\ & & \dots & & \\ 0 & 0 & 0 & d_{min} & -d_{min} - \sum \tau_{N-1,l}\tau_{N,l}d_{max} \end{bmatrix}, \quad (38)$$

$$\mathbf{p} = [\sqrt{P_1} \ \sqrt{P_2} \ \dots \ \sqrt{P_N}]^T \quad (39)$$

and $\mathbf{0}$ is a vector with N zero elements. All sums in Δ run from $l = 1$ to $l = N$. Notice that $\sum \tau_{1,i}\tau_{2,j}$ simply corresponds to the magnitude of the antenna correlation coefficient between antenna i and j . The

system (38) only contains $N - 1$ equations for N unknowns. Additionally the rank of Δ is obviously $N - 1$. A valid solution however must also satisfy $\sum_{i=1}^N P_i = 1$ and $\sqrt{P_i} \geq 0$. To show that there always exists a solution we resort to the lemma below.

Lemma : *There exists a solution to (37) where \mathbf{p} is an unit-norm all-positive vector in the null space of Δ .*

The proof of the lemma above is given in Appendix C which also describes an efficient numerical method on how to compute the precoding weights. When combined with (27), a full solution to the precoding problem is obtained.

In the remaining of the article we assume $d_{min} = d_{max}$ (4-QAM) which allows us to eliminate d_{min} and d_{max} from the equations to allow for a simplified derivation. Notice that for example with 16-QAM transmission $\frac{d_{max}}{d_{min}} = 3$ and this factor may easily be taken into account in the expressions.

Special cases For extreme cases one obtains:

- With no correlation, $\sum_{l=1}^N \tau_{m,l} \tau_{n,l} = 0$, ($1 \leq m, n \leq N, m \neq n$) thus

$$\Delta = \begin{bmatrix} 1 & 0 & 0 & \dots & -1 \\ 0 & 1 & 0 & \dots & -1 \\ & & \dots & & \\ 0 & 0 & & 1 & -1 \end{bmatrix} \quad (40)$$

which can be shown to give $P_i = \frac{1}{N}$, distributing the energy equally across all substreams. Again, this validates the standard V-BLAST approach over uncorrelated channels.

- At the other end, with full transmitter correlation, $\sum_{l=1}^N \tau_{m,l} \tau_{n,l} = 1$ and a closed form solution can easily be found by writing out Δ :

$$\Delta = \begin{bmatrix} 1 & -1 & -1 & \dots & -2 \\ 0 & 1 & -1 & \dots & -2 \\ & & \dots & & \\ 0 & 0 & & 1 & -2 \end{bmatrix}. \quad (41)$$

This system can directly be simplified by a simple recursion starting from the last equation (row) into

$$\sqrt{P_i} = 2^{N-i} \sqrt{P_N} \quad (42)$$

for $1 \leq i \leq N$. As the solution must satisfy the energy requirement we arrive to $\sum_{i=1}^N P_i = \sum_{i=1}^N 2^{2(N-i)} P_N = 1$. Solving with respect to P_N gives $P_N = \frac{1}{\sum_{i=1}^N 2^{2(N-i)}} = \frac{3}{4^{N-1}}$. Finally we

obtain:

$$P_i = \frac{3 \cdot 4^N}{4^i(4^N - 1)}. \quad (43)$$

The energy for this setup decreases by one quarter from symbol s_i to s_{i+1} .

Intriguingly, if each symbol follows a 4-QAM modulation, the final form of received signal η (post inversion of \mathbf{H}_0), for full correlation, simply correspond to respectively a standard 4^N -QAM modulation.

Therefore, this precoding assures that at high correlation the data rate is preserved by transmitting the equivalent of a higher order QAM modulation with its various binary components being transmitted on each of antennas.

C. ML decoding

Although the precoding approach is mainly designed with emphasis on a hybrid zero-forcing and SIC detection, the precoding weights may also be used with other receiver/decoding algorithms. For a 2×2 system, the weights are for example very close to that shown in [12] and the minimum error rate receiver of [18] also recommends unequal power allocation to independent streams. The ML detection scheme is likewise known to be stable against moderate levels of correlation [19], however, at high antenna correlation it too experiences problems in distinguishing the transmitted symbols. The proposed precoding makes certain that correlated scenarios where symbols who would otherwise mix up together at the receiver, do not occur and an ML decoding would therefore clearly benefit from unequal weighting of symbols. This fact is confirmed through simulations in the next section.

V. SIMULATIONS

In this section we demonstrate the effectiveness of the new weighting approach proposed in the article. We look at simulation results under quasi-static flat Rayleigh fading with 4-QAM symbol constellation and variable correlation at the transmitter. The transmitter is only assumed to be aware of the correlations, while the receiver has perfect channel knowledge.

Figure 2 and 3 display simulation results for a 2×2 system with correlation level of $\rho = 0.9$ and full correlation at the emitter respectively. In the first plots we compare the following approaches:

- Standard ZF: a straight inversion of \mathbf{H} is used as receiver, no precoding.
- HZM-SIC without precoding (equal symbol weights)
- Precoded HZM-SIC

The results show the increased robustness due to the proposed precoding.

In the presence of full correlation, the proposed precoding and decoding method (HZM-SIC) only performs 4 dB worse off than standard ZF with no transmitter correlation (not shown here) which is comparable to the loss experienced by going from a 4-QAM transmission to a 16-QAM transmission. At full transmitter correlation, the non-precoded schemes simply break down as expected, while the precoded versions continue to function.

In figure 4 we demonstrate the use of ML decoding at SNR of 15 dB for a 2×2 setup with transmitter correlation ranging from $\rho = 0$ to $\rho = 1$. The difference between ML with or without precoding is relatively small at low correlation levels ($\rho < 0.8$) but becomes very substantial with higher degrees of correlation. Finally, we compare with the exhaustive search approach presented in [12] which gives optimal power weights for $\rho = 0.95$ as $P_1 = 0.78$ and $P_2 = 0.22$. The deviation from expressions of (20), $P_1 = 0.791$, $P_2 = 0.208$, is thus small and any loss incurred by the closed form algorithm is marginal, resulting in virtually equal performance under decoding.

Figure 5 demonstrates the use of the proposed algorithms with/without precoding for a 4×4 MIMO system under the Jakes correlation model. For this simulation we assume full correlation between a group of first and second element and similarly full correlation between the third and the fourth emitters. The first group of antennas is assumed to be uncorrelated with the second group, which gives the optimal weights $P_1 = P_3 = 0.4$, $P_2 = P_4 = 0.1$. The last figure 6 demonstrates the use of ML decoding on the same setup. Overall, the precoding variants manage to withstand the effects of transmit correlation to a large degree.

VI. CONCLUSIONS

In this article we proposed a closed-form power/phase weighting approach making use of the correlation channel knowledge to adapt the transmitted constellation. The algorithm assumes a particular SIC decoding, similar to decoding of r -QAM modulated symbols, though the resulting precoding weights may be applied on a wider range of receivers such as ML. The obtained multiplexing scheme offers a method to preserve data rate, with smoothly degrading performance, for any correlation level. It draws the bridge between pure spatial multiplexing scenario (for uncorrelated arrays) and constellation multiplexing, i.e. single stream transmission using a higher-order modulation (for fully correlated arrays). Simulation results were shown that validate the algorithm performance.

APPENDIX

A - Exponential correlation model:

With the exponential correlation model the matrix $\mathbf{R}_t^{\frac{1}{2}}$ is known to follow the structure:

$$r_{m,n} = \begin{cases} \gamma^{n-m} & m \leq n \\ r_{n,m}^* & m > n \end{cases} \quad (44)$$

$\gamma = \gamma_0 e^{j\theta}$ and $|\gamma| \leq 1$.

We need to show that the phases (27) derived through η_1 also hold for subsequent iterations, e.g. (33).

If we replace the elements of (27) with (44) the optimal phases are given as function of γ :

$$\phi_k = -\angle \left(\sum_{l=1}^N r_{l,1}^* r_{l,k} \right) = -\angle \left(\sum_{l=1}^N (\gamma^*)^{(1-l)} \gamma^{k-l} \right) \quad (45)$$

$$= -\angle \sum_{l=1}^N \frac{\gamma^* \gamma^k}{(\gamma^*)^l \gamma^l} = -\angle \sum_{l=1}^N \frac{\gamma^* \gamma^k}{|\gamma|^{2l}}. \quad (46)$$

The phases are thus independent of l and described by

$$\phi_k = -\theta(k-1). \quad (47)$$

A straightforward extension of (27), through for example (33), prescribes that the optimal phases for $e^{-j\phi_n} \eta_n$, corresponding to the n 'th iteration, should be selected as

$$\phi_k = -\angle \left(\sum_{l=1}^N r_{l,n}^* r_{l,k} \right) + \phi_n, \quad k > n. \quad (48)$$

(48) written out in full gives,

$$\phi_k = -\angle \sum_{l=1}^N \frac{(\gamma^*)^n \gamma^k}{(\gamma^*)^l \gamma^l} + \phi_n = -\angle \sum_{l=1}^N \frac{(\gamma^*)^n \gamma^k}{|\gamma|^{2l}} + \phi_n. \quad (49)$$

As stated previously, the phases are independent of l and assuming ϕ_n is selected through (47), we have $\phi_n = -\theta(n-1)$. Inserting this into (49) returns the optimal phases for the n 'th iteration as:

$$\phi_k = -\theta(k-n) - \theta(n-1) = -\theta(k-1). \quad (50)$$

(50) coincides with (47), showing that the optimal phases which maximize their distance from the decision boundary derived through η_1 also hold for succeeding iterations.

APPENDIX

B - Closed form solution with arbitrary correlation structure:

If the correlation matrix does not follow any particular structure then the derived phases (27) are only optimal for the first iteration of the maximal ratio combining. For the subsequent iterations, e.g. η_2 (33), an aligned constellation can not be guaranteed. A solution to this problem is obtained if equation (27) is

applied for the first iteration. For the remaining iterations one can assume the "worst-case" scenario and take account of possible misplacement by multiplying factors in $\delta_2, \dots, \delta_N$ by $\sqrt{2}$. The matrix (38) then takes the following format (for simplicity we assume $d_{min} = d_{max}$):

$$\Delta = \begin{bmatrix} 1 & -\sum \tau_{1,l}\tau_{2,l} & -\sum \tau_{1,l}\tau_{3,l} & \dots & -1 - \sum \tau_{1,l}\tau_{N,l} \\ 0 & 1 & -\sqrt{2}\sum \tau_{2,l}\tau_{3,l} & \dots & -1 - \sqrt{2}\sum \tau_{2,l}\tau_{N,l} \\ & & \dots & & \\ 0 & 0 & 0 & 1 & -1 - \sqrt{2}\sum \tau_{N-1,l}\tau_{N,l} \end{bmatrix}. \quad (51)$$

In contrast to (38), this results in slightly stretched versions of higher order modulations. Explicit simulations however indicate that the performance, with arbitrary phase correlations, is virtually on the same level as standard algorithms under (38) with the Jakes or exponential correlation model. The simulations included in this paper therefore do not take account of that and (51) is primary included for the sake of completeness.

APPENDIX

C - Existence of an all-positive solution:

Due to the specific structure of (38) there exists a solution to (37) where all entries of \mathbf{p} are non-negative. Observe that the left $N-1 \times N-1$ submatrix of Δ is upper-triangular and contains unit entries on the diagonal while all other elements are non-positive. The last column of Δ however consists of all strictly negative entries.

From the format of (38) it is clear that $\sqrt{P_N} \neq 0$ otherwise all elements in \mathbf{p} would become zero. Without loss of generality, we can therefore set $\sqrt{P_N} = 1$. Moving the last column to the right hand side gives a strictly positive vector and the upper unit triangular system can be solved by backsubstitution to find $\sqrt{P_{N-1}}, \dots, \sqrt{P_1}$. A suitable scaling can then assure proper normalization and the result follows. The identical argument is also applicable for (51).

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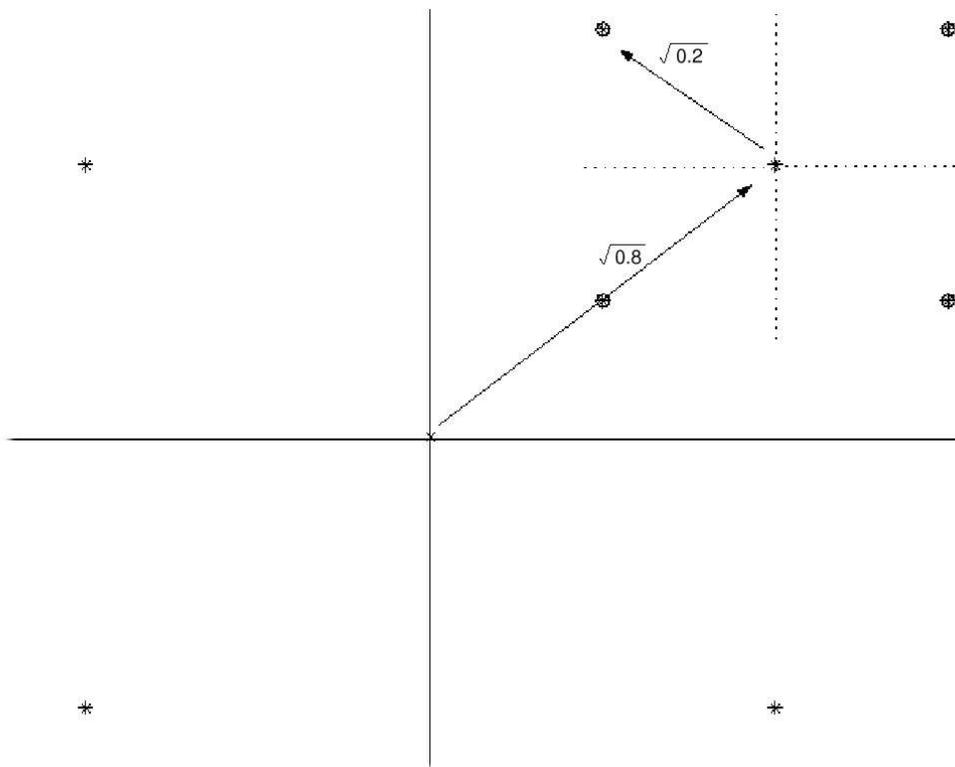


Fig. 1. Illustration of superposed 4-QAM constellations, $\rho = 1$. This yields a constellation equivalent to 16-QAM.

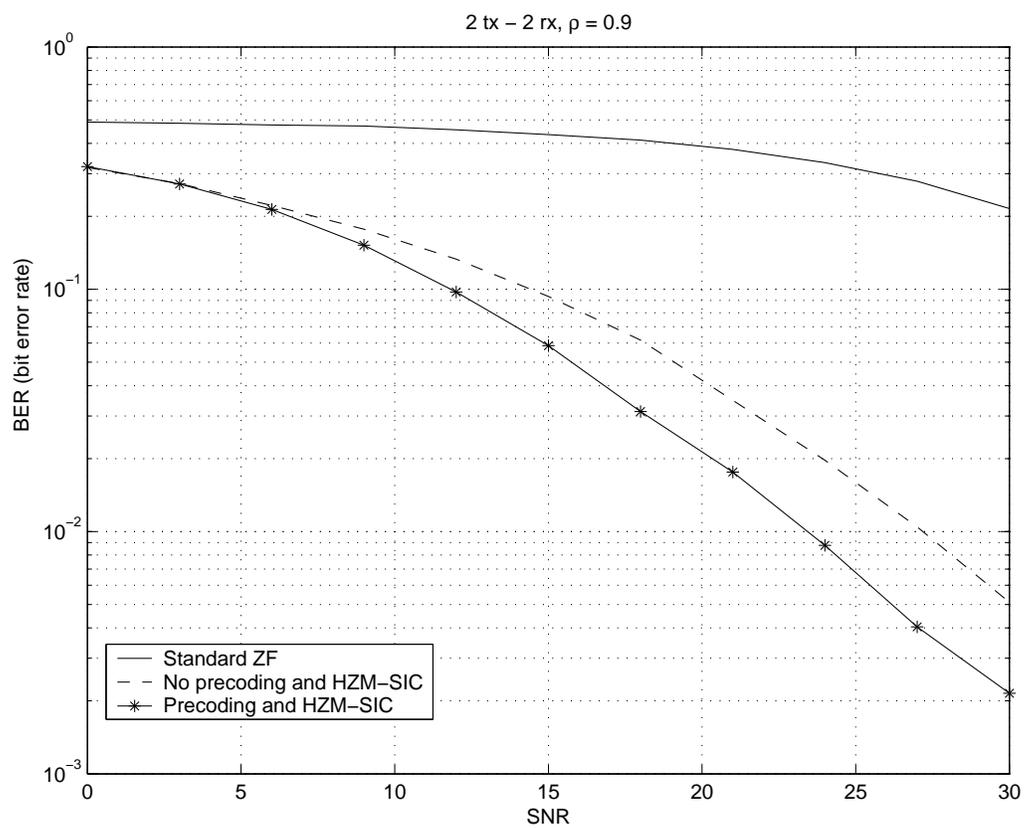


Fig. 2. 2 by 2 case, 4-QAM modulation. Improvement due to the proposed precoder with the HZM-SIC receiver and comparison with standard ZF receiver, for 0.9 correlation level.

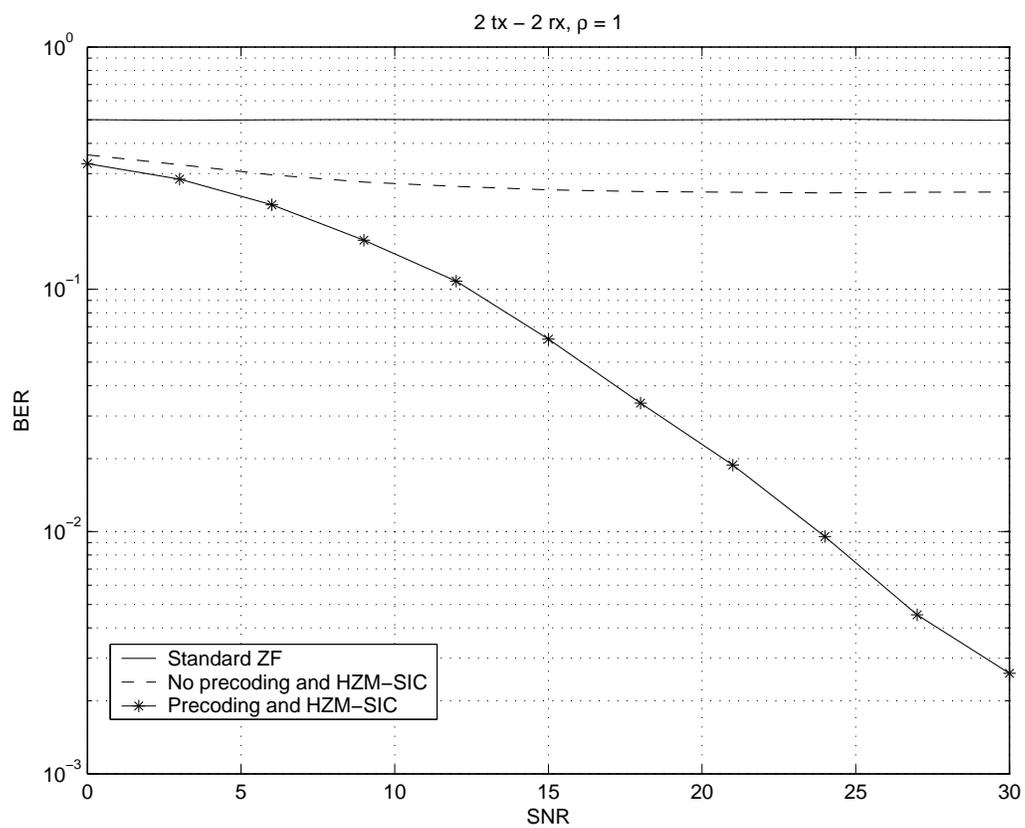


Fig. 3. 2 by 2 case, 4-QAM modulation. Improvement due to the proposed precoder with the HZM-SIC receiver and comparison with standard ZF receiver, for full correlation level.

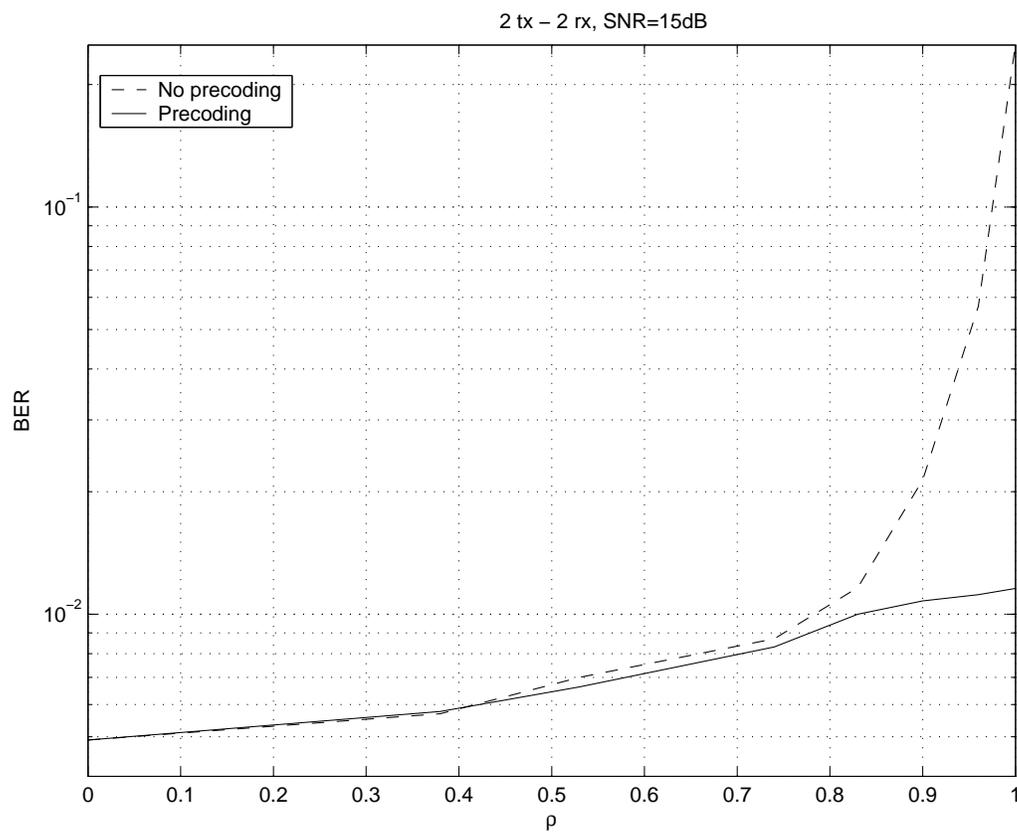


Fig. 4. 2 by 2 case, 4-QAM modulation. Improvement due to the proposed precoder with the ML receiver as function of correlation.

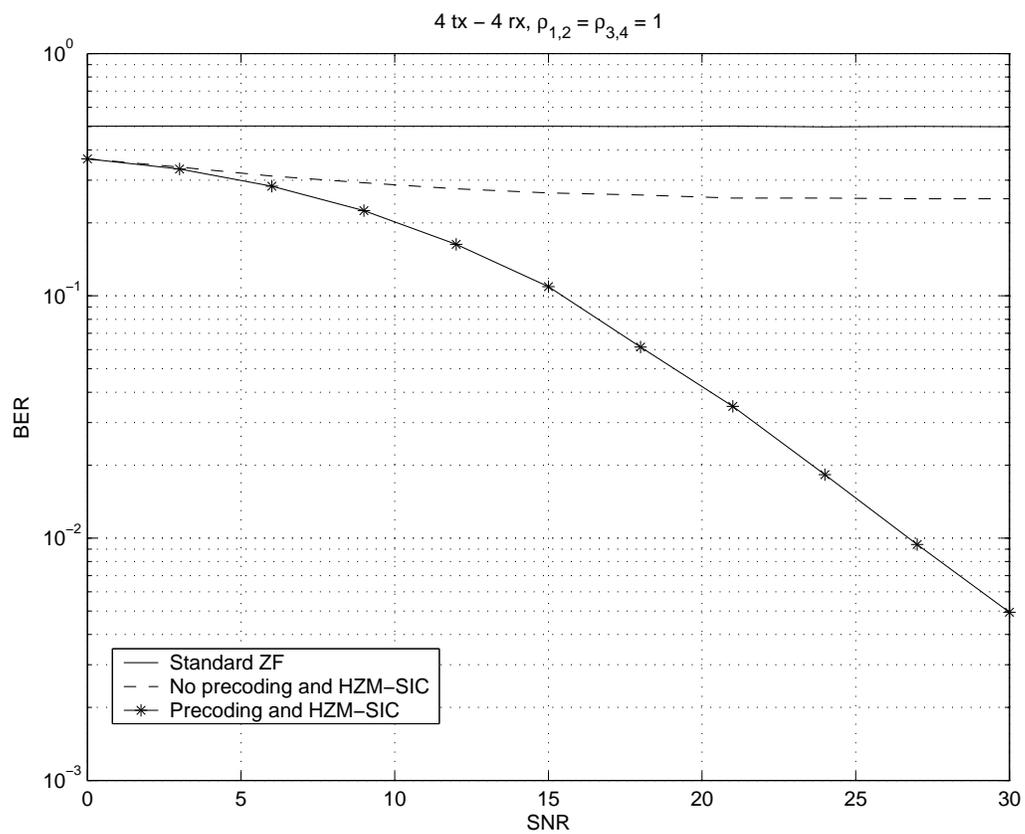


Fig. 5. 4 by 4 case, 4-QAM modulation. Improvement due to the proposed precoder with the HZM-SIC receiver and comparison with standard ZF receiver. Full group correlation: $\rho_{1,2} = \rho_{3,4} = 1, \rho_{1,3} = \rho_{1,4} = \rho_{2,3} = \rho_{2,4} = 0$.

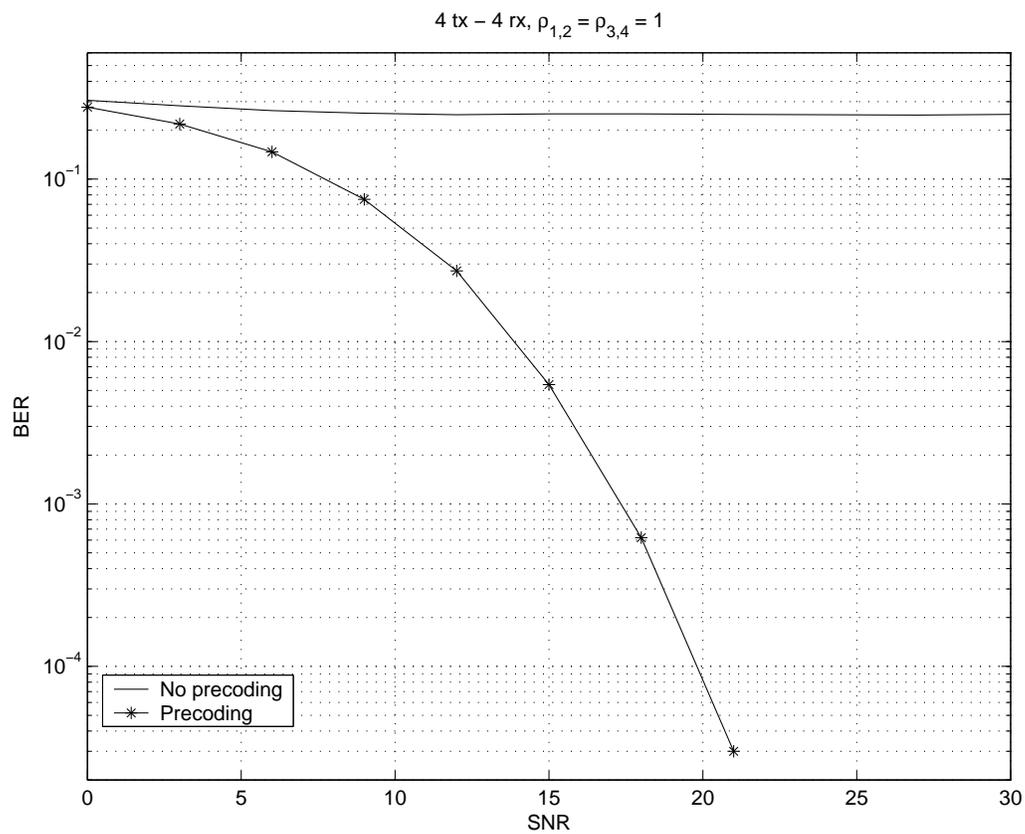


Fig. 6. 4 by 4 case, 4-QAM modulation. Improvement due to the proposed precoder with the ML receiver. Full group correlation: $\rho_{1,2} = \rho_{3,4} = 1, \rho_{1,3} = \rho_{1,4} = \rho_{2,3} = \rho_{2,4} = 0$, ML decoding.