

A Specular Approach to MIMO Frequency-Selective Channel Tracking and Prediction

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Abstract — We propose a specular approach to MIMO channel tracking that yields a parsimonious channel representation where the time and space components of the channel variations are separated by singular value decomposition (SVD). We show that this separation enables simple short-term channel prediction. This is especially useful for systems using channel state information at the transmitter, since the latency necessary to the feedback process can be mitigated. We validate our approach through the use of computer simulations.

I. INTRODUCTION

The use of specular models for channel analysis and tracking has been proposed by various authors seeking to improve the ability to accurately estimate [1], represent and transmit [2], or predict [3, 4] Channel State Information (CSI). Specular methods constitute viable candidates for channel tracking and prediction, since the insight they provide into the actual channel structure – namely, separation of the channel variation into its space and time components – can improve the performance and decrease the complexity of channel tracking and prediction. Various methods have been proposed to estimate the underlying parameters, including MUSIC in [3], ESPRIT in [4] and SAGE in [1].

In this paper, we propose the use of the singular value decomposition (SVD) to separate time and frequency components, and use auto-regressive (AR) methods to predict the future evolution of the channel. We show that prediction can enhance the link quality in systems relying on the CSI available at the transmitter (CSIT), since duplex systems mainly rely on a feedback scheme to transmit the channel state information from the receiver, where the channel estimation is possible, to the transmitter, where the channel estimate is needed. Most of the literature on the topic of exploiting CSIT assumes that the channel state information can be fed back in a negligible amount of time. This is not necessarily the case, since sending frequent updates of the CSI increases the amount of uplink bandwidth consumed by this scheme. The ability for the transmitter to extrapolate CSI from past values can therefore be an important asset in the actual use of a CSIT-exploiting transmission scheme, since it actually allows to predict the current channel state. We explore these issues in the framework of Multiple-Input Multiple-Output (MIMO)

frequency-selective channels, and compare the performance of the proposed method to more rudimentary CSIT situations.

II. SPECULAR CHANNEL MODEL

Let us consider a Multiple-Input Multiple-Output (MIMO) frequency-selective channel, with N_t transmit (Tx) and N_r receive (Rx) antennas, as depicted in Fig. 1. In order to improve channel estimation and reduce MFB loss, it is advantageous to exploit correlations in the channel, if present. For time-varying channel, two channel models can be considered according to two transmission modes:

1. continuous transmission: in this case the vectorized channel impulse response can be modeled as a (locally) stationary vector signal; limited bandwidth usually allows downsampling w.r.t. symbol rate; stationarity can only be local due to slow fading
2. bursty transmission: in this case, the time axis is cut up in bursts, the channel (down)samples within each burst can be rerepresented in terms of Basis Expansion Models (BEMs); limited bandwidth leads to limited BEM terms.

Both models are equivalent as long as the temporal correlation structure in the continuous mode gets properly transformed to intra and inter burst correlation between BEM coefficients.

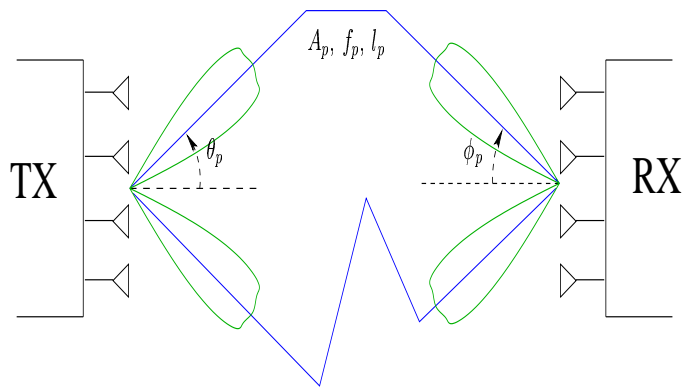


Figure 1: MIMO transmission with N_t transmit and N_r receive antennas.

The impulse response of the channel between Tx antenna i and Rx antenna j is denoted by $h_{i,j}(t, \tau)$, where t is the time and τ is the lag. Hence, $h_{i,j}(t, \cdot)$ is the channel impulse response as seen by the signal received at time t . Let us assume that the

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impulse response has finite support, and consider its discretized version

$$\underline{\mathbf{h}}_n^{(i,j)} = [h^{(i,j)}(nT_s, 0), h^{(i,j)}(nT_s, T_s), \dots, h^{(i,j)}(nT_s, (L-1)T_s)]^H \quad (1)$$

where T_s is the sampling interval at the receiver, and L is chosen such that all the channel coefficients outside the lag interval $[0 \dots (L-1)T_s]$ are zeros. Let us further stack these into a column vector with $N_t N_r L$ coefficients

$$\underline{\mathbf{h}}_n = [\underline{\mathbf{h}}_n^{(1,1)H} \dots \underline{\mathbf{h}}_n^{(1,N_r)H}, \underline{\mathbf{h}}_n^{(2,1)H} \dots \underline{\mathbf{h}}_n^{(N_t, N_r)H}]^H. \quad (2)$$

We emphasize the fact that $\underline{\mathbf{h}}_n$ constitutes a snapshot of all the channel coefficients at time nT_s .

We aim at decomposing the channel according to a specular model. According to such a model, each impulse response $h_{i,j}(t, \cdot)$ is the superposition of a finite number P of discrete paths at lag $\tau_p^{(i,j)} = l_p^{(i,j)} T_s$, $p = 1 \dots P$, resulting from either line-of-sight propagation, or one or several reflections. This model relies upon the fact that the paths between all the Tx-Rx antenna pairs have most of their characteristics in common, except for what happens near the antenna arrays. Hence, they share some properties, namely their speed w.r.t. the reflectors, and the reflection characteristics (hence their Doppler and gain are the same whatever antenna pair is considered). Note that we assume that the lag does not vary over time. Pursuant to this model, each path coefficient can be decomposed into a product of three components :

- a space component $\alpha_p^{(i,j)}$, which depends on the physical properties of path p between Tx antenna i and Rx antenna j , including antennas and reflectors position, and directions of departure (DoD) and arrival (DoA),
- a time component $\beta_p(t) = A_p(t) e^{j2\pi f_p t}$ reflecting the fast fading, which includes the Doppler shift f_p due to reflectors motion and the relative speed of the transmitter w.r.t. the receiver, as well as the short-term (but usually small-scale) amplitude variations $A_p(t)$. All the other parameters (including the Doppler frequency) vary on a slower time scale and correspond to slow fading.
- a gain component γ_p which remains constant over time.

Using the discretized version of the time component $\beta_{n,p} = \beta_p(nT_s)$, the proposed channel model yields

$$h^{(i,j)}(nT_s, lT_s) = \sum_{p=1}^{P(i,j)} \alpha_p^{(i,j)} \gamma_p \beta_{n,p} \delta_{l_p^{(i,j)}}(l). \quad (3)$$

Due to the Doppler shift, the phase of the path complex amplitude is varying rapidly. The actual path amplitude is not varying rapidly unless what we consider to be a specular path is already the superposition of multiple paths that are not resolvable in delay, Doppler and angles. With $f_p \in (-f_d, f_d)$, the Doppler shift for path p , the (fast fading) variation is bandlimited and hence the channel should be perfectly predictable! (not so due to the slow fading: the slow parameters such as delays and angles will vary eventually). When only the fast fading is taken into account as temporal variation, the matrix spectrum $S_{\mathbf{h}\mathbf{h}}(f)$ of the vectorized channel can be doubly singular:

1. if $A_p(t) \equiv A_p$ and P finite: spectral support singularity: sum of cisoids!

2. if $P < N_t N_r L$: matrix singularity, limited source of randomness (limited diversity)

When the channel spectral support becomes singular, the channel becomes perfectly predictable. Hence channel prediction should play an important role in channel estimation.

Since we consider the temporal evolution of the channel, let us gather N successive channel states in the matrix

$$\mathbf{H} = [\underline{\mathbf{h}}_1 \dots \underline{\mathbf{h}}_N]. \quad (4)$$

Using the following notations

$$\underline{\mathbf{a}}_p = (\alpha_p^{(1,1)} \dots \alpha_p^{(1,N_r)}, \alpha_p^{(2,1)} \dots \alpha_p^{(N_t, N_r)})^H, \quad (5)$$

$$\underline{\mathbf{b}}_p = (\beta_{1,p} \dots \beta_{N,p}), \quad (6)$$

and

$$\mathbf{A} = [\underline{\mathbf{a}}_1 \dots \underline{\mathbf{a}}_P], \quad (7)$$

$$\mathbf{B} = [\underline{\mathbf{b}}_1^H \dots \underline{\mathbf{b}}_P^H]^H, \quad (8)$$

$$\mathbf{G} = \text{diag}(\gamma_1 \dots \gamma_P), \quad (9)$$

$$(10)$$

\mathbf{H} can be rewritten, similarly to eq. (3), as

$$\mathbf{H} = \mathbf{A} \mathbf{G} \mathbf{B}, \quad (11)$$

where \mathbf{B} contains the fast fading part.

The important issue here is that the spectral modeling of the channel coefficient temporal variation should be done in a transform domain and not on the channel impulse response coefficients themselves. Since each such coefficient can be the result of the contributions of many paths, the dynamics of the temporal variation (with n) of the coefficients $\underline{\mathbf{d}}_n = (\beta_{n,1} \dots \beta_{n,P})^H$ are necessarily of higher order, compared to the variation of $A_p(nT_s) e^{j2\pi f_p nT_s}$ which can be of an order as low as one (when $A_p(nT_s)$ is constant; the cisoid $e^{j2\pi f_p nT_s}$ is perfectly predictable with first-order linear prediction). Also, if the impulse response coefficients are modeled directly, then their (spatial and delay-wise) correlation has to be taken into account: $S_{\mathbf{h}\mathbf{h}}(f)$ cannot be modeled accurately as diagonal, whereas $S_{\mathbf{d}\mathbf{d}}(f)$ can.

Hence, the elements of $\underline{\mathbf{d}}_n$ are modeled as decorrelated stationary scalar processes. The channel distribution is typically taken to be complex Gaussian. If the fast parameters $\underline{\mathbf{d}}_n$ are not too predictable, then the estimation errors of the slow parameters \mathbf{A} and \mathbf{G} should be negligible (change only with slow fading, hence their estimation error should be small). Since $\underline{\mathbf{h}}_n = \mathbf{A} \mathbf{G} \underline{\mathbf{d}}_n$, we obtain the spectrum

$$S_{\mathbf{h}\mathbf{h}}(f) = \mathbf{A} \mathbf{G} \underbrace{S_{\mathbf{d}\mathbf{d}}(f)}_{\text{diagonal}} (\mathbf{A} \mathbf{G})^H \quad (12)$$

The components of $\underline{\mathbf{d}}_n$ can conveniently be modeled as AR processes, each spanning only a fraction of the Doppler range $(-f_d, f_d)$. In fact, a subsampled version of the fast parameters $\underline{\mathbf{d}}_n$ could be introduced, with the subsampled rate corresponding to the (maximum) Doppler spread. A stationary (AR) model can be taken for the subsamples and the other samples can be obtained by linear interpolation from the subsamples. This is the case of a BEM with a single basis function: the interpolation filter response.

III. SPACE AND TIME CHANNEL DECOMPOSITION

Let $\hat{\mathbf{H}}$ denote the estimated channel realizations : $\hat{\mathbf{H}} = \mathbf{H} + \mathbf{E}$, where \mathbf{E} is the noise resulting from the estimation process. Let us recall that any matrix $\hat{\mathbf{H}}$ can be expressed as $\hat{\mathbf{H}} = \mathbf{U}\mathbf{S}\mathbf{V}^H$, where \mathbf{U} and \mathbf{V} are square, unitary matrices, and \mathbf{S} is a $N_t N_r L \times N$ real matrix with non-negative values s_i on its diagonal, and zeros elsewhere. This expression constitutes the SVD [5] of $\hat{\mathbf{H}}$. If the singular values in \mathbf{S} are in non-increasing order ($s_i \geq s_{i+1}$), the decomposition is unique in \mathbf{S} , but \mathbf{U} and \mathbf{V} are only unique up to an orthogonal transform inside each singular subspace (i.e. inside the singular subspaces spanned by the vectors associated to one singular value).

Notice at this point the similarity of the SVD of $\hat{\mathbf{H}}$ with eq. (11). We seek to use the SVD to extract the specular model parameters from the available channel estimates. Following the notations of eqs. (5) and (6), let us denote by

$$\underline{\mathbf{u}}_p = (u_p^{(1,1)} \dots u_p^{(1,N_r)}, u_p^{(2,1)} \dots u_p^{(N_t,N_r)})^H \quad (13)$$

$$\text{and } \underline{\mathbf{v}}_p = (v_{1,p} \dots v_{N,p}), \quad (14)$$

Note that the SVD can not always provide the exact form of eq. (11), even at high SNR, since it introduces extra constraints in the form of the unitary requirements for \mathbf{U} and \mathbf{V} . Obviously, the presence of the estimation noise \mathbf{E} will yield a noise subspace, and thus \mathbf{S} will be larger than \mathbf{G} . Actually, separability can be impaired by two issues :

- if the estimation noise \mathbf{E} is strong, the SVD can not separate the signal subspace from the noise subspace. In this case, all the values in \mathbf{U} , \mathbf{S} and \mathbf{V} are unreliable.
- if two paths have the same gain ($\gamma_p = \gamma_q$), the SVD can not discriminate them. In particular, time components $\underline{\mathbf{v}}_p$ and $\underline{\mathbf{v}}_q$ will contain a linear mixture of $\underline{\mathbf{b}}_p$ and $\underline{\mathbf{b}}_q$. This will impair the predictability of these time series.

Note that it is not necessary to estimate the dimension of the noise subspace for the prediction algorithm to work properly: if s_p is a singular value belonging to the noise subspace, any attempt to predict the time series \mathbf{b}_p will most definitely not yield any usable values, but will rather produce noise. Nevertheless, the small gain s_p will ensure that these values will not harm the predicted channel state. This is particularly relevant from a design point of view, since there is no need to dynamically adapt the number of tracked paths.

IV. PREDICTION

One of the interest of a specular channel model (aside from the low number of variables needed to accurately describe the channel), is its long-term validity. Since it closely follows the physical channel structure, and hence separates the spatial and temporal properties of the channel, prediction of the time coefficients only (the $v_p(t)$) is sufficient to extrapolate the channel state at a future time. Since the Doppler effect produces a phase rotation for each path, auto-regressive (AR) analysis seems appropriate as a prediction method. Since we expect only a few singular values to have an impact on the performance of the predictor, we assume that it is sufficient to predict the coefficients for a fixed number M of singular subspaces, associated with the strongest singular values. If $M < P$, the channel can not be accurately predicted, whereas $M > P$ will yield a waste of computing resources.

N successive known channel realizations are used to train the AR predictor. For each singular value $p = 1 \dots M$, a K-tap filter ($w_1^{(p)} \dots w_K^{(p)}$) is obtained as

$$\arg \min_{(w_1^{(p)} \dots w_K^{(p)})} \sum_{n=K+1}^N \left(v_{n,p} - \sum_{i=1}^K w_i^{(p)} v_{n-i,p} \right)^2. \quad (15)$$

The predicted values for “future” values of the $v_{N+l,p}$, $l > 0$, are recursively computed as

$$\tilde{v}_{N+l,p} = \sum_{i=1}^K w_i^{(p)} \tilde{v}_{N+l-i,p} \quad (16)$$

where

$$d_{N+l-i,p} = \begin{cases} v_{N+l-i,p} & \text{if } l-i \leq 0 \\ \tilde{v}_{N+l-i,p} & \text{if } l-i > 0 \end{cases} \quad (17)$$

Let \mathbf{U}' denote the truncated version of \mathbf{U} where only the first M columns are preserved. Let also \mathbf{S}' denote the $M \times M$ matrix obtained from \mathbf{S} by keeping only the first M rows and columns. Using these notations, and $\tilde{\mathbf{v}}_{N+l} = [\tilde{v}_{N+l,1} \dots \tilde{v}_{N+l,P}]$, the predicted channel is reconstructed according to

$$\tilde{\mathbf{h}}_{N+l} = \mathbf{U}' \mathbf{S}' \tilde{\mathbf{v}}_{N+l}^H \quad (18)$$

V. AR PREDICTOR SIMULATION RESULTS

We illustrate the behaviour of our prediction algorithm through computer simulations. We generated 120 consecutive samples of a 2×2 channel as described in Section II, with $P = 3$ paths, and a delay spread of $L = 8$ samples. The channel estimate $\hat{\mathbf{H}}$ was obtained through the addition of a -20dB random white Gaussian noise with i.i.d. coefficients. The training of the 20-taps AR predictor was done over the first $N = 100$ samples, and the prediction algorithm was used to generate 20 subsequent samples, that we compared to the ones generated using the channel model without noise.

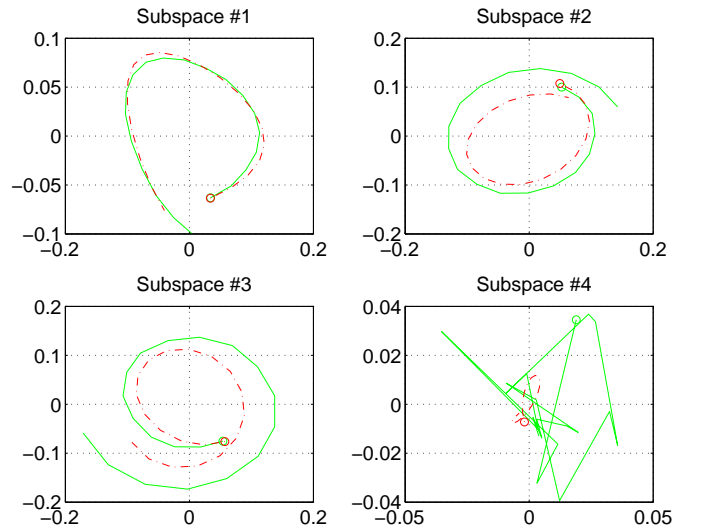


Figure 2: Time series predictors evolution for the first four singular subspaces

Figure 2 depicts (in solid lines) the temporal evolution in the complex plane of each of the predicted time-series $\hat{v}_{N+l,p}$ associated with the four strongest singular values ($p = 1 \dots 4$) for the first 20 steps ($l = 1 \dots 20$). The reference values, defined as the four first components of

$$\underline{v}_{N+l} = ((\mathbf{US})^\dagger \underline{h}_{N+l})^H, \quad (19)$$

where † denotes the pseudoinverse operator, are also plotted (in dashed lines) for comparison. The origin of each line (which corresponds to $l = 1$), is materialized by a circle. Note that the values corresponding to the training data do not appear on these plots. The first plot shows that the time values describing the evolution of the first (*i.e.* strongest) singular value of the channel is accurately predicted by the AR model. The time series from the second and third subspaces are accurately predicted near the beginning of the time series, and are shown to slowly diverge when the prediction time increases. The precision decreases when the subspace number grows since the associated singular values are in decreasing order, thus decreasing the effective SNR of the time series. The fourth subspace is a noise subspace, since the actual channel has 3 paths only. Hence, its time series lacks the predictability of the previous ones: the AR predictor generates random-looking values. Note that, as we already noticed, these values are associated with a relatively small singular value, and hence generate little perturbation on the predicted channel values.

VI. CAPACITY SIMULATION RESULTS

In order to evaluate the prediction performance of our method, we computed the ergodic mutual information of the channel with different levels of channel knowledge at the transmitter (but perfect channel knowledge at the receiver). Namely, we consider the following situations

- No channel information at the transmitter.
- Perfect instantaneous channel knowledge at the transmitter.
- Outdated channel knowledge at the transmitter. This corresponds to the case of a transmitter using CSI that was gathered through a feedback loop. This model takes into account the fact that the channel evolves during the time it takes for the receiver to estimate the channel and to transmit it back. Despite this temporal lag, the CSI is assumed to be known perfectly (*i.e.* we do not take into account estimation noise and bandwidth issues on the channel feedback link).
- Predicted CSIT with pure AR predictor. This setup assumes that the transmitter has knowledge of outdated and noisy channel information, and that AR prediction is applied independently to each channel coefficient.
- Predicted CSIT with specular method. This setup assumes that the transmitter has knowledge of outdated and noisy channel information, but uses the proposed specular decomposition algorithm to extrapolate the current channel state.

Since the channel has a non-zero delay spread, the ergodic mutual information is computed in the frequency domain. The transmitted signal covariance is adjusted according to the available channel state information. When none is available, the transmitted signal is assumed to be spatially white and

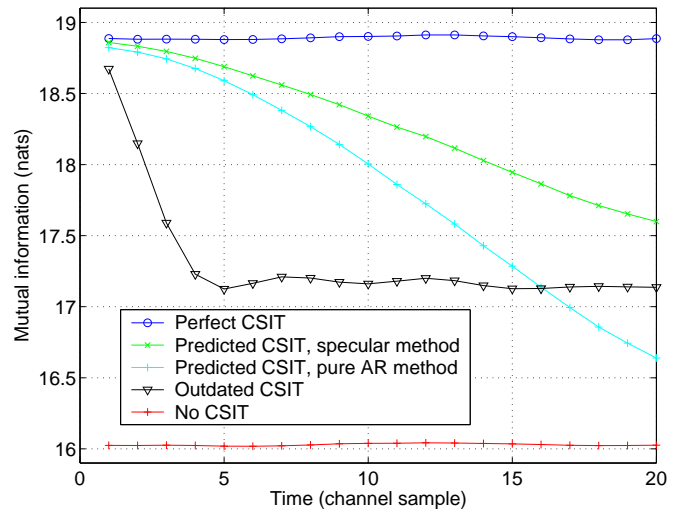


Figure 3: Ergodic mutual information vs. prediction time

with a flat power spectrum density, since this setting maximizes the expectation of the achieved mutual information in the case of a channel with i.i.d. random coefficients with equal variance [6]. When some CSI is available at the transmitter, the mutual information is maximized over all possible Tx covariance matrices using a waterfilling algorithm [7]. This mutual information is evaluated at a SNR of 0 dB. In these simulations, the spatial characteristics of the actual channel are randomly generated assuming a uniform law for directions of departure and arrival of the rays. The Doppler spectrum is a phase rotation of random frequency, randomly chosen within $\frac{1}{5}^{th}$ of the total transmission bandwidth. The other characteristics of the channel are as described in Section V.

Figure 3 depicts the evolution of the ergodic mutual information versus prediction time (*i.e.* the number of interpolated channel samples since the last known channel), for various levels of CSIT. It shows that the performance of using outdated CSIT drops sharply with time, whereas methods using predictors remain closer from the theoretical limit provided by the perfect instantaneous CSIT case, although the performance of the predictors decreases with the prediction length. The specular model outperform the pure AR predictor method, thanks to a better insight on the channel structure. The mutual information without CSIT is also plotted for reference.

VII. NOTE ON COMPLEXITY

The specular method proposed in this paper involves a lower number of predictors than the pure AR prediction method : while the former only has a limited number of coefficients to track (4 in the example simulation), the latter involves the use of one predictor per coefficient to track (32 in this case). The SVD operation, on the other hand, is rather complex. Nevertheless, the SVD needs not be completely recomputed if our method is applied to a sliding temporal window (where, at each time instant, a new channel measurement is available, whereas the oldest one is discarded). In this case, the SVD can be updated [8] with a much lower complexity than it would take to recompute it from scratch.

VIII. CONCLUSION

We showed that decoupling the spatial and temporal variations of the channel coefficients can lead to an efficient specular channel tracking and prediction method. We evaluated the performance of such a scheme using the SVD and AR prediction for MIMO frequency-selective channels. We believe that a whole class of algorithm relying on spatial and temporal decoupling is available, with variations including using another prediction method, or a decomposition other than the SVD.

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