DOWNLINK CDMA: TO CELL OR NOT TO CELL.

Mérouane Debbah

Mobile Communications Group, Institut Eurecom, 2229 Route des Cretes, B.P. 193, 06904 SOPHIA ANTIPOLIS CEDEX, France
E-mail: debbah@eurecom.fr

ABSTRACT

A common misconception asserts that to increase capacity in a CDMA network, one has to increase the number of cells. In this contribution, we show that, depending on the path loss, the size of the network and the fading channel statistics, the code orthogonal gain (due to the synchronization of all the users at the base station) can compensate and even compete in many cases with the drawbacks due to inter-cell interference. The results are especially realistic and useful for the design of dense small scale networks (stadium, mall...)

1. INTRODUCTION

An important problem that arises in the design wireless networks concerns the deployment of an efficient architecture to cover the users. Increasing the number of cells in a given area yields indeed a better coverage but increases at the same time inter-cell interference. The gain provided by a cellular approach is not at all straightforward and depends on many parameters: path loss, receiving filter, SNR, channel characteristics. Although very complex, this contribution is a first step into analyzing the problem. A useful framework is provided in order to determine the optimal base station coverage for a downlink CDMA network considering wireless frequency selective channels where each user is equipped with a matched filter. This framework illustrates the case where a service provider must optimize the deployment of base stations in an instantaneous overcrowded area to ease the traffic (for example, in a stadium during a football game). In order to obtain interpretable expression, the problem is analyzed in the asymptotic regime for very dense networks (the total number of users N tends to infinity, the number of users K per cell tends to infinity but the ratio $\frac{K}{N} \rightarrow \alpha$ is constant) by using tools of free probability theory [1]. Previous studies have already studied the capacity of a CDMA multi-cell network in the uplink scenario [2] with Wyner's model or with simple interference models [3]. However, none has taken explicitly into account the impact of the code structure (orthogonality or not) and the multi-path channel characteristics. The remainder of the paper is organized as follows: in section 2, the CDMA cellular model is described. In section 3, the capacity of a cellular CDMA scheme is derived. Finally, in section 4, some discussions are provided on the the behavior of the capacity with respect to the path loss and channel statistics

2. CDMA CELLULAR MODEL

2.1. Cellular Model

Without loss of generality and in order to ease the understanding, we focus our analysis on a one dimensional (1D) network. This scenario represents for example the case of the deployment of base stations along a motorway of length L (users i.e cars are supposed to move along the motorway). Some discussions on the 2D case are provided in section 4.4. Each base station is supposed to cover a region of length αL (see figure 1) ($0 \leq \alpha \leq 1$). The goal of this contribution is to determine the optimal number α. The total number of users[1], the number of users per base station[2], the density of the network (number of users per meter) are respectively denoted by N, K, d. The number of base stations in the network is given by: $\frac{L}{\alpha L} = \frac{1}{\alpha} = \frac{N}{K}$ (inverse of the load). Note that N, d and L are fixed whereas $K = d\alpha L$ varies with α. The coordinates of a base station $m_p, 0 \leq p \leq \frac{1}{\alpha} - 1$ are given by: $m_p = \alpha(\frac{L}{2} + pL)$.

2.2. Downlink CDMA Model

The receiving signal of user j in cell p of a downlink CDMA system can be written in the following form:

$$\mathbf{y}(x_j) = \sum_i \sqrt{P_i(x_j)} H_{ij} W_i \mathbf{s_i} + \mathbf{b} \quad (1)$$

x_j are the coordinates of user j in cell p. $\mathbf{y}(x_j)$ is the $N \times 1$ received vector, $\mathbf{s_i} = [s_i(1), ..., s_i(K)]^T$ is the $K \times 1$ transmit vector of cell i, $\mathbf{b_j} = [b_j(1), ..., b_j(N)]^T$ is an $N \times 1$ noise vector with zero mean variance σ^2 Gaussian independent entries. $P_i(x_j)$ represents the path loss between

[1] N is also the sprading length. Note that we are not neglecting border effects as all the users are within the region L.

[2] We assume that the users are uniformly distributed so that each base station has K users.

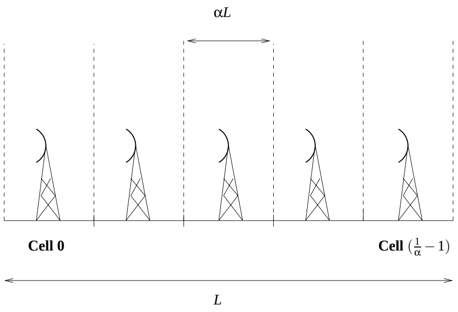

Fig. 1. Representation of the Cellular Model

base station m_i and the user j whereas matrix H_{ij} represents the frequency selective channel between user j and base station m_i. Each base station has an $N \times K$ code matrix $W_i = [\mathbf{w_i^1}, ..., \mathbf{w_i^K}]$. The user j is subject to intra-cell interference from cell p as well as inter-cell interference from cell $i = 0, ..., p-1, p+1, ..(\frac{1}{\alpha}-1)$.

2.3. Assumptions

The following assumptions are rather technical in order to simplify the analysis.

Channel model: Let $[\mid h_{ij}(1) \mid^2, ..., \mid h_{ij}(N) \mid^2]$ be the eigenvalues of $H_{ij}H_{ij}^{H}$[3]. The following ergodic assumption on the channel will be considered[4]. For each continuous bounded function $f : \mathbb{R} \to \mathbb{R}$, $\lim_{N\to+\infty} \frac{1}{N} \sum_{k=1}^{N} f(|h(k)|^2) = \int f(t)p(t)\, dt$ almost surely. In other words, we assume that the empirical channel distribution converges weakly to the limiting distribution given by the probability density function $p(t)$.

Code structure model: In the downlink scenario, Walsh-Hadamard codes are usually used. However, in order to get interpretable expressions of the SINR, isometric matrices[5] obtained by extracting $K < N$ columns from a Haar unitary matrix[6] will be considered.

Path loss: The model under consideration for the path loss will be of the exponential form $P_i(x_j) = Pe^{-\gamma|x_j - m_i|}$. The factor $\gamma \geq 0$ characterizes the type of attenuation[7].

[3] Note that since H_{ij} is Toeplitz, $h_{ij}(k)$ represents the frequency response of the channel when $N \to \infty$.

[4] The tools introduced can take into account the non ergodic case. However, the formulas have no simple interpretation.

[5] In [4], simulations show that these matrices provide similar performance as Walsh-Hadamard codes.

[6] A $N \times N$ random unitary matrix is said to be Haar distributed if its probability distribution is invariant by right (or equivalently left) multiplication by deterministic unitary matrices.

[7] The attenuation is usually of the polynomial form: $P_i(x_j) = \frac{P}{|x_j - m_i|}$. We use the exponential form for simplicity calculation sake and therefore put the framework in the most severe path loss scenario in favor of the multi-cell approach.

3. PERFORMANCE ANALYSIS

In all the following, without loss of generality, we will focus on user j of cell p. We assume that the user does not know the codes of the other cells as well as the codes of other users within the same cell. Moreover, the user is supposed to be equipped with the matched filter receiver: $\mathbf{g} = H_{pj}\mathbf{w_p^j}$.

3.1. General Capacity Formula

The output of the matched filter is given by:

$$\begin{aligned}
\mathbf{g}^H \mathbf{y}(x_j) &= \sqrt{P_p(x_j)}\mathbf{g}^H H_{pj}\mathbf{w_p^j} s_p(j) \\
&+ \sqrt{P_p(x_j)}\mathbf{g}^H H_{pj} U_p \begin{bmatrix} s_p(1) \\ \vdots \\ s_p(K) \end{bmatrix}_{(K-1)\times 1} \\
&+ \sum_{i \neq p} \sqrt{P_i(x_j)}\mathbf{g}^H H_{ij} W_i \mathbf{s}_i + \mathbf{g}^H \mathbf{b_j}
\end{aligned}$$

where $U_p = [\mathbf{w_p^1}, ..., \mathbf{w_p^{j-1}}, \mathbf{w_p^{j+1}}, ..., \mathbf{w_p^K}]$. The output $\text{SINR}(x_j, p)$ of user j with coordinates x_j in cell p has the following expression:

$$\text{SINR}(x_j, p) = \frac{\text{S}}{I_1 + I_2 + \sigma^2 \mathbf{g}^H \mathbf{g}} = \frac{\text{S}}{I + \sigma^2 \mathbf{g}^H \mathbf{g}}$$

$$\begin{aligned}
S &= P_p(x_j) \mid \mathbf{w_p^j}^H H_{pj}^H H_{pj} \mathbf{w_p^j} \mid^2 \\
I_1 &= \sum_{i \neq p} P_i(x_j) \mathbf{w_p^j}^H H_{pj}^H H_{ij} W_i W_i^H H_{ij}^H H_{pj} \mathbf{w_p^j} \\
I_2 &= P_p(x_j) \mathbf{w_p^j}^H H_{pj}^H H_{pj} U_p U_p^H H_{pj}^H H_{pj} \mathbf{w_p^j}
\end{aligned}$$

We would like to quantify the number of bits/s/Hz the system is able to provide to all the users. It has been shown [5] that the interference plus noise can be considered as Gaussian when K and N are large enough. In this case, the capacity of the network is given by:

$$C = \sum_{p=0}^{\frac{1}{\alpha}-1} \frac{1}{N} \mathbb{E}\left(\sum_{j=1}^{K} \log_2(1 + \text{SINR}(x_j, p)) \right) \quad (2)$$

For a fixed K and N, it is extremely difficult to get some insight on expression (2). In order to provide a tractable expression, we will analyze (2) in the asymptotic regime ($N \to \infty$, $K \to \infty$ but $\frac{K}{N} \to \alpha$) and show in particular that $\text{SINR}(x_j, p)$ converges almost surely to a deterministic value independent of the code $\mathbf{w_p^j}$.

3.2. Asymptotic Capacity Formula

Proposition 1 *When N grows towards infinity and $K/N \to \alpha$, the asymptotic capacity of downlink CDMA with random orthogonal spreading codes, exponential path loss and* *matched filter is given by:*

$$C(\alpha) = \frac{1}{L}\sum_{p=0}^{\frac{1}{\alpha}-1}\int_{-\frac{\alpha L}{2}}^{\frac{\alpha L}{2}} \log_2\left(1+\frac{Pe^{-\gamma|u|}(\mathbb{E}(|h|^2))^2}{I+\sigma^2\mathbb{E}(|h|^2)}\right)du \quad (3)$$

$$I = \alpha P(\mathbb{E}(|h|^2))^2\sum_{i=0,i\neq p}^{\frac{1}{\alpha}-1} e^{-\gamma|u+(p-i)L\alpha|} + \alpha Pe^{-\gamma|u|}\left(\mathbb{E}(|h|^4)-(\mathbb{E}(|h|^2))^2\right)$$

Proof 1 *Due to lack of space, we only give here the main steps of the proof. Four terms have to be derived separately in the SINR formula.*

$$S \xrightarrow{a.s} P_p(x_j)(\mathbb{E}(|h|^2))^2$$

$$\sigma^2\mathbf{g}^H\mathbf{g} \xrightarrow{a.s} \sigma^2\mathbb{E}(|h|^2)$$

$$I_1 \xrightarrow{a.s} \sum_{i\neq p} P_i(x_j)\frac{1}{N}\mathrm{Trace}(W_i^H H_{ij}^H H_{pj} H_{pj}^H H_{ij} W_i) \xrightarrow{a.s} \alpha(\mathbb{E}(|h|^2))^2\sum_{i\neq p} P_i(x_j)$$

$$I_2 \xrightarrow{a.s} \frac{P_p(x_j)}{N-K}\mathrm{Trace}\left(H_{pj}^H H_{pj} U_p U_p^H H_{pj}^H H_{pj}(\mathbf{I}-U_pU_p^H)\right) \xrightarrow{a.s} P_p(x_j)\left(\frac{\alpha}{1-\alpha}\mathbb{E}(|h|^4)-\frac{1}{1-\alpha}\int t^2 d\mu(t)\right)$$

μ is the limiting eigenvalue distribution of matrix $H_{pj}^* H_{pj} U_p U_p^H$. The proof of term I_2 follows a specific procedure as w_p^j is not independent of U_p. Using results from free probability, one can show that: $\int t^2 d\mu(t) = \alpha^2\mathbb{E}(|h|^4) + (1-\alpha)\alpha\left(\mathbb{E}(|h|^2)\right)^2$.

In the case of an infinite number of cells ($\alpha \to 0$) where each cell accommodates only one user (and therefore, no intra-cell interference occurs), the capacity simplifies to:

$$C(0) = \frac{1}{L}\int_{-\frac{L}{2}}^{\frac{L}{2}}\log_2\left(1+\frac{P\mathbb{E}(|h|^2)}{\frac{2P\mathbb{E}(|h|^2)}{\gamma L}(1-e^{\frac{-\gamma L}{2}}\cosh(\gamma u))+\sigma^2}\right)du$$

4. DISCUSSION

In all the following discussion, $P = 1$ and $\sigma^2 = 10^{-5}$.

4.1. Multipath versus Orthogonality

We would like to quantify in this section the cellular orthogonal gain with respect to the channel statistics and determine the optimal number of cells. Hence, in the case of no path loss ($\gamma = 0$), the capacity formula (3) is equal to:

$$C_{\text{multipath}}(\alpha) = \log_2\left(1+\frac{P\left(\mathbb{E}(|h|^2)\right)^2}{I+\sigma^2\mathbb{E}(|h|^2)}\right)$$

$$I = P\left(\mathbb{E}(|h|^2)\right)^2 + \alpha P\left(\mathbb{E}(|h|^4) - 2\left(\mathbb{E}(|h|^2)\right)^2\right)$$

Remarkably, the optimum number of cells depends only on how "peaky" the channel is through the kurtosis $T = \frac{\mathbb{E}(|h|^4)}{\left(\mathbb{E}(|h|^2)\right)^2}$. If $T > 2$, orthogonality is severally destroyed by the channel and one must have an infinite ($\alpha = 0$) number of cells (in fact, N cells) whereas if $T \leq 2$, one can use only one cell ($\alpha = 1$) to accommodate all the users[8].

4.2. Path Loss versus Orthogonality

In this section, we determine the minimum path loss that preserves the orthogonality gain. In the case of no multipath (Orthogonal CDMA in a AWGN channel), the capacity is given by:

$$C_{\text{path loss}}(\alpha) = \frac{1}{L}\sum_{p=0}^{\frac{1}{\alpha}-1}\int_{-\frac{\alpha L}{2}}^{\frac{\alpha L}{2}}\log_2\left(1+\frac{Pe^{-\gamma|u|}}{I+\sigma^2}\right)du$$

$$I = \frac{\alpha P\left(e^{-\gamma u}(e^{-\gamma pL\alpha}-1)+e^{\gamma u}(e^{-\gamma(\frac{1}{\alpha}-p-1)L\alpha}-1)\right)}{1-e^{\gamma L\alpha}}$$

The capacity of one cell and infinite number of cells is respectively given by:

$$C_{\text{path loss}}(1) = \frac{1}{\gamma L}\int_{-\frac{\gamma L}{2}}^{\frac{\gamma L}{2}}\log_2\left(1+\frac{Pe^{-|v|}}{\sigma^2}\right)dv$$

$$C_{\text{path loss}}(0) = \frac{1}{\gamma L}\int_{-\frac{\gamma L}{2}}^{\frac{\gamma L}{2}}\log_2\left(1+\frac{P}{\frac{2P}{\gamma L}(1-e^{-\frac{\gamma L}{2}}\cosh(v))+\sigma^2}\right)dv$$

In figure 2, we have plotted $C_{\text{path loss}}(1)$ and $C_{\text{path loss}}(0)$ versus γL. As one can observe, there is critical point $(\gamma L)_c$ below which multi-cell is not advantageous. Note that $(\gamma L)_c$ is an increasing function of $\frac{P}{\sigma^2}$. As a matter of fact, orthogonality pays off, even with path loss especially when the ratio $\frac{P}{\sigma^2}$ is high.

4.3. General Case

In the following case, we consider a realistic case with Rayleigh fading and $\gamma = 2$, $L = 500m$. In figure 3, the capacity $C(\alpha)$ is plotted versus $\frac{1}{\alpha}$: the network provider should theoretically consider an infinite number of base stations for achieving the optimal value $C(0)$. However, even in this case, there is an optimum number of base stations. Indeed, network providers have economic constraints expressed by a predetermined cost $\frac{dC(\alpha)}{d(\frac{1}{\alpha})}$ (i.e each base station added must provide at least L bits/s/Hz). The cost (see figure 3) determines the optimum number of cells to be deployed. The inter-cell distance is then given by: $\alpha_{\text{opt}}L$.

[8]Note that the breaking point $T = 2$ corresponds to the complex Rayleigh fading channel.

4.4. Extension to the 2D Case

The extension of the analysis to the 2D case is straightforward and similar curves can be obtained. However, in this case, the capacity depends crucially on the positions of the base stations (base station configuration) in the area considered. The formulas provided in this contribution and their extension to the 2D case give means to determine the optimal configuration. For example, in figure 4, the optimal position of the base stations for $2, 3, 4$ and 5 base stations is provided for a square grid of surface A. The positions have been determined using a numerical optimization in the case of an exponential path loss and Rayleigh fading.

5. CONCLUSION

Using asymptotic arguments, an explicit expression of the capacity was derived and was shown to depend only on a few meaningful parameters. Contrarily to past belief, the question "To cell or not to cell?" is meaningful. Indeed, for "smooth channels", we have shown that orthogonality pays off and one should on the contrary accommodate all the users within one cell. However, if the channel is "peaky", then a multi-cell approach should be sought. But even in this case, the capacity gain with respect to the number of base stations is not linear and therefore, based on economic constraints, the optimal inter-base station distance can be determined. Note finally that similar conlusions are obtained in the case of an infinite network [6].

6. ACKNOWLEDGMENT

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7. REFERENCES

[1] D.V. Voiculescu, K.J. Dykema, and A. Nica, *Free Random Variables*, American Mathematical Society, CRM Monograph Series, Volume 1, Providence, Rhode Island, USA, 1992.

[2] S. Shamai, S.Verdu, and B.M Zaidel, "Multicell uplink spectral efficiency of coded DS-CDMA with random signatures," *Selected Areas in Communications, IEEE Journal on*, vol. 19, pp. 1556–1569, 2001.

[3] A. Sendonaris and V. Veeravalli, "The Capacity-Coverage Trade-off in CDMA Systems with Soft Handoff," in *Proc. of the 31st Asilomar Conference*, Pacific Grove, USA, 1997, pp. 625–629.

[4] M. Debbah, W. Hachem, P. Loubaton, and M. de Courville, "MMSE Analysis of Certain Large Isometric Random Precoded Systems," *IEEE Transactions on Information Theory, Volume: 49 Issue: 5, Page(s): 1293 -1311*, May 2003.

[5] J. Zhang, E. Chong, and D. Tse, "Output MAI Distributions of Linear MMSE Multiuser Receivers in CDMA Systems," *IEEE Trans. on Information Theory*, pp. 1128–1144, Mar. 2001.

[6] T. B. Abdallah and M. Debbah, "Downlink CDMA: to cell or not to cell," *submitted to IEEE Transactions on Communications, download at http://www.eurecom.fr/∼debbah*, January 2004.

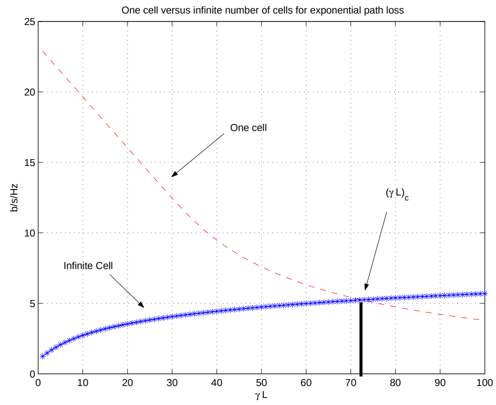

Fig. 2. Capacity versus γL in the case of one and infinite number of cells with no fading.

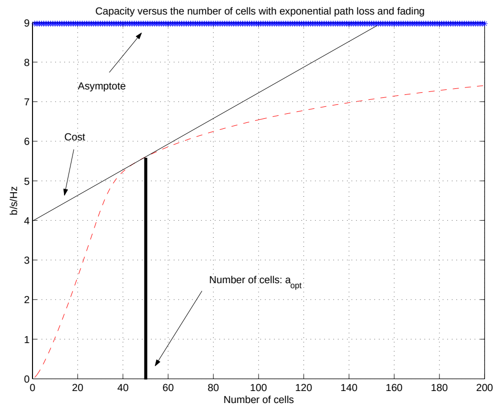

Fig. 3. Capacity behavior in the case of Rayleigh fading and $\gamma = 2$, $L = 500m$.

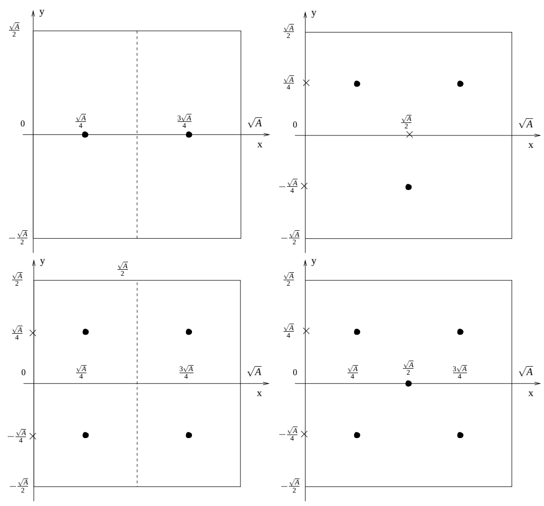

Fig. 4. Optimal Position of the base stations in the case of a square grid.