

# Multiuser Channel Allocation Algorithms Achieving Hard Fairness

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**Abstract**—Multiple transmit antennas and dynamic channel resource allocation are targets of current research. A system that combines these two techniques can provide data transmission with very high spectral efficiency data transmission, and thereby meet the high-speed requirements of future generations of wireless networks. This paper investigates the performance of combined orthogonal channel and antenna allocation algorithms in multiple-antenna multi-channel systems. In [1] a Max-Min allocation algorithm is proposed for a  $N$ -user system with  $N$  parallel sub-channels. Here we extend this algorithm to the multiple-antenna systems and compare its performance in two different transmission scenarios (Spatial multiplexing and space time coding). The techniques are applicable, for instance, in MIMO systems using Orthogonal Frequency Division Multiple-Access (OFDMA) systems with dynamic sub-carrier allocation. We show that multiuser diversity, and thus an increase of aggregate data rates with the size of the user population, can still be successfully achieved even under a hard fairness constraint. Moreover multiple-antennas permit spatial multiplexing. The techniques considered here do not require phase information in the channel allocation process, which, from a practical point-of-view is particularly important for time-division duplex systems exploiting channel reciprocity.

**Index Terms**—Multiuser diversity, fairness, channel allocation, multiple antennas.

## I. INTRODUCTION

Wireless transmission is impaired by signal fading and interference. The increasing requirements on data and quality of service for wireless communication systems call for new techniques to improve link reliability and increase spectrum efficiency. The use of dynamic resource allocation (DRA) promises significant improvements in terms of spectral efficiency by taking advantage of channel variations and user diversity. DRA uses channel state information (CSI) to schedule users. The method for making this CSI available at the transmitter depends strongly on the considered system architecture. In systems such as HDR (also known as IS-865) the receiver estimates the CSI based on a common pilot and feeds the information back to the transmitter [3]. In systems employing time-division duplexing (TDD), channel reciprocity allows the transmitter to use the CSI estimated during reception for

transmission, which is the case for instance in the DECT cordless telephone system and for power-control in UMTS-TDD. In practical TDD systems, amplitude information is reasonably simple to estimate from the opposite link, while for accurate phase information this is not the case, mainly due to the difficulty in calibrating the difference in phase response between the transmitter and receiver chains. The techniques considered in our study do not require phase information in the channel allocation process. The authors in [2] show that in multiple-antenna systems, even with this partial knowledge of the channel, a gain proportional to the number of transmit antennas can still be achieved.

Many studies deal with dynamic allocation strategies. For instance, [4]- [7] present the concept of Multiuser Diversity and give power allocations strategy for maximizing the total sum-rate of multiuser systems which consists of scheduling at any one time the user which would make the best use of the channel (i.e the user with the best channel response). It has also been shown that multiuser diversity yields an increase of the total throughput as a function of the number of users. The most remarkable result from these studies is that for multiuser systems significantly more information can be transmitted across a fading AWGN channel than a non-fading AWGN channel for the same average signal power at the receiver. Spectral efficiency can be increased by more than a factor of two for small signal-to-noise ratios (around 0dB). This is due to the fact that at a given time and frequency, the channel gain is random and can be significantly higher than its average level. One can take advantage of this by using a proper dynamic time-frequency allocation based on the time/frequency varying characteristics of the channels. The main practical issue arising from channel-dependent resource allocation schemes is fairness. Users (or the base station) must wait until their channel conditions are favorable to transmit. In [8], the authors treat the fairness problem between users in the slow fading environment and discussed the implementation in the IS-865 system and propose methods to enhance fairness. Their approach consists in using multiple antennas to induce fast channel fluctuations combined with the proportionally fair allocation policy used in IS-865. In a similar vein for multi-cell systems, [9]- [11] study combined power control and base station assignment in multi-cell systems with fixed vector rate. This is also a form of fairness, since these algorithms allow users to transmit with their desired rates.

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Similar opportunistic techniques for multi-cell systems are briefly alluded to in [8]. In [12], the authors consider the sub-carrier assignment problem in OFDMA systems and compare the simplicity and fairness properties of different allocation algorithms. In [1], an algorithm that performs fair allocation of users across sub-channels according to a Max-Min criterion for OFDM-like systems on frequency selective channels is described. It was shown that this algorithm approaches, even under a hard fairness constraint, the performance of the optimal unfair algorithm maximizing the total throughput. Multiple-Input Multiple-output (MIMO) systems have recently emerged as one of the most promising technical breakthroughs in current wireless communications. The spectral efficiency of future wireless communications can be significantly improved by using MIMO by exploiting random fading and multipath delay spread [13]- [15]. There have been many studies of MIMO systems in a multi-user network environment including proposals for scheduling algorithms [16]. In [17] authors study optimal strategy of multiple access with multiple antennas at the base station. The proposed scheduling algorithm incorporates both the physical and low level protocol layer. In [18] the authors study greedy scheduling in multi-user MIMO systems and show that it leads to lower average user experienced delays compared to single-user greedy scheduling. The motivation of our study comes from the system capacity improvement that can be gained from the combination of MIMO and adaptive channel allocation. The use of multiple antennas the base station side provide a spatial dimension that can be seen as an additional system resource (as bandwidth and power). At the mobile side, the use of multiple antennas permits interference mitigation and provides an increase of the data rate. The adaptive channel allocation offers an increase of the system spectral efficiency [1].

The organization of this paper is as follows: Section II presents the system model. In Section III, we propose an overview of the single transmitting antenna case using the Max-Min algorithm as in [1]. We extend this study to the case of multiple transmitting antennas for the down-link in Section IV. We also compare different spatial combining methods when using multiple antennas at reception. Finally, in Section V we present our conclusions and outline ongoing extensions and future perspectives.

## II. SYSTEM MODELS

We consider a system with  $L$  antennas transmitting over  $M$  parallel channels with  $N$  users accommodated, where  $N$ , depending on the considered system and allocation, is a function of  $M$  and  $L$ . This could represent the case of any wideband OFDM system, such as Mobile Broadband Wireless Access (MBWA) systems, for instance the evolving IEEE 802.16 standard where an Orthogonal Frequency Division Multiple Access (OFDMA) technique is used. An other example of such system model could be the UTRAN HSDPA (high-speed data packet-access) 3GPP proposal using an OFDM(A) physical layer instead of WCDMA, proposed in [19] for the downlink channel. HSDPA also envisages multiple-antenna terminals. In

the context of these systems, the algorithms proposed in this paper would be used to allocate the different frequency sub-bands and transmit antennas to users. We can also imagine the use of these techniques in extensions of IEEE802.11a/g, Hiperlan2 or multiband-OFDM for UWB systems.

We consider that each sub-channel is a fading AWGN channel with noise variance  $N_0$ . As has been mentioned previously, we assume that the amplitude response for all users over all sub-carriers are known at the transmitter. For uplink transmissions, the base station estimates the CSI for each user from received pilots which are known sequences transmitted by the users on each antenna and are *spread over the entire available bandwidth*. The estimated CSI is used to carry out the sub-channel allocation algorithm and a message is fed back to inform each user of its assigned sub-channel/transmit antenna (Note that for slowly-varying channels this is reasonably simple to accomplish and consumes little signaling bandwidth since the allocation remains invariant for long periods). The received signal in a given antenna  $l$  over a given sub-channel  $m$  is given by

$$r_{l,m}^{UL} = \sum_{n=0}^{N-1} \sqrt{P_{l,m,n}^{UL}} H_{l,m,n}^{UL} x_{l,m,n}^{UL} + z_{m,l}^{UL} \quad (1)$$

where  $x_{l,m,n}^{UL}$ ,  $P_{l,m,n}^{UL}$  and  $H_{l,m,n}^{UL}$  are respectively the signal, the transmit power and the channel gain from user  $n$  for antenna  $l$  on sub-channel  $m$  and  $z_{m,l}^{UL}$  is the noise in sub-channel  $m$ . For downlink transmissions and reciprocal channels (for instance in TDD systems), the channel estimation is performed in the same manner as for uplink transmissions. In the case of non-reciprocal channels, each user has to estimate its CSI over all available sub-channels and from all transmitting antennas based on known pilots and feeds this information back to the base station which carries out the antenna and sub-channel allocation algorithm. The received signal for a user  $n$  over a given sub-channel  $m$  is given by

$$r_{n,m}^{DL} = \sum_{l=0}^{L-1} \sqrt{P_{l,m,n}^{DL}} H_{l,m,n}^{DL} x_{l,m,n}^{DL} + z_{m,n}^{DL} \quad (2)$$

where  $x_{l,m,n}^{DL}$ ,  $P_{l,m,n}^{DL}$  and  $H_{l,m,n}^{DL}$  are respectively the signal, the transmit power and the channel gain for user  $n$  from antenna  $l$  on sub-channel  $m$  and  $z_{m,n}^{DL}$  is the noise in sub-channel  $m$ . For symmetric channels, we have that  $H_{l,m,n}^{UL} = H_{l,m,n}^{DL}$ . In the spirit of OFDM-based systems, we model each channel gain  $H_{m,n}^{DL}$  (or  $H_{m,n}^{UL}$ ) as a frequency sample of a discrete multipath channel having  $\Gamma$  significant uncorrelated paths with delays:  $\tau_1, \tau_1, \dots, \tau_\Gamma$ , that is  $h_{l,m,n}^{DL}(t) = \sum_{i=0}^{\Gamma-1} \alpha_i \delta(t - \tau_i)$ , where the path gains  $\alpha_i$  are zero mean Gaussian random variables with variance  $\sigma_i^2$ . The channel is assumed stationary for the duration of coded transmission blocks, but may vary from block to block. The samples of the frequency response are given by  $H_{l,m,n}^{DL} = H_{l,n}^{DL}(f_m) = \sum_{i=0}^{\Gamma-1} \alpha_i e^{-j \frac{2\pi \tau_i f_m}{M}}$ , and have covariance

$$E \left\{ H_{l,m,n}^{DL} H_{l,m',n}^{DL*} \right\} = \sum_{i=0}^{\Gamma-1} E \left\{ |\alpha_i|^2 \right\} e^{-j \frac{2\pi \tau_i (f_m - f_{m'})}{M}}$$

where  $f_m$  is the frequency corresponding to sub-carrier  $m$ . Channel gains for different antennas over the same sub-channel and for the same user are assumed to be uncorrelated. The goal of the following sections will be to study allocation algorithms of users to sub-carriers according to optimization criteria based on mutual information.

### III. OVERVIEW: ORTHOGONAL ALLOCATION ALGORITHMS WITH HARD FAIRNESS FOR SINGLE ANTENNA SYSTEM

Let us first consider a  $N$  user with  $N$  parallel sub-channels system and impose a hard fairness constraint for that system, namely that each user is granted one sub-channel and only one user is scheduled in each sub-channel at any given time instant (i.e. orthogonal multi-access). There exists  $N!$  possible allocations of sub-channels to users each one represented by a vector  $\mathbf{c}_p = \{c_{p,0}, c_{p,1}, \dots, c_{p,N}\}$ , where  $c_{p,n}$  is the sub-channel assigned to user  $n$  and thus  $H_{c_{p,n},n}$  is the channel gain from the base station to user  $n$  over its assigned sub-channel when allocation  $\mathbf{c}_p$  is applied, for  $p = 0, \dots, N! - 1$ . The Max-Min allocation strategy consist of choosing the permutation  $\mathbf{c}_{p^*}$  where

$$p^* = \arg \max_{p=0, \dots, N!-1} \min_{n=0, \dots, N-1} H_{c_{p,n},n}$$

which achieves the ergodic sum rate

$$\sum_{n=0}^{N-1} R_n = E \left\{ \sum_{n=0}^{N-1} \log_2 \left( 1 + \frac{P}{N_0} |H_{c_{p^*,n},n}|^2 \right) \right\} \quad (\text{bits/dim}) \quad (3)$$

This policy guarantees that at any given time instant the minimum channel gain allocated is the best possible among all allocations. It was shown in [20] that this criterion achieves multiuser diversity and provides a non-negligible gain with respect even to a non-fading channel. The details of the Max-Min allocation algorithm are given in [1] where it was compared with other allocation policies.

### IV. MULTIPLE-ANTENNA MULTI-USER SYSTEM

Let us now consider the case with  $M$  parallel sub-channels and  $L$  transmitting antennas at the base station side for down-link transmissions. MIMO techniques can be divided into two groups: Space Time Coding which increases the performances of the communication system by coding the data over different transmitter branches. The second group is Space division multiplexing systems, which achieves a higher throughput by transmitting independent data over different transmit branches simultaneously. For a detailed study of MIMO systems, one can refer to [21] where different classes of techniques and algorithms which attempt to realize the various benefits of MIMO including spatial-multiplexing and space-time coding. In the following we compare the performance of these two transmission techniques using the Max-Min allocation algorithm, by first considering the use of a single antenna at reception. We then consider a system with  $K$  receiving antennas and compare different spatial combining techniques of

(antenna, sub-channel)

Users

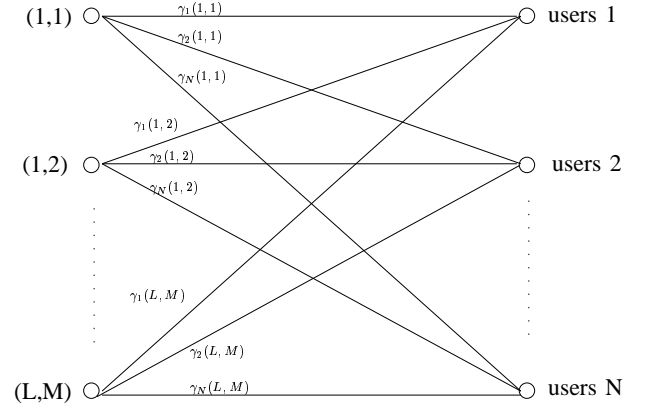


Fig. 1. System1 representation by a graph

the received signals. The Max-Min allocation in the multiple-antenna transmission system is the allocation that guarantees that the minimum SINR allocated is the best possible among all allocations.

#### A. Single antenna receivers

We consider two systems, the first attempts to achieve spatial-multiplexing whereas the second is a space-time coding approach.

1) **system1**: In the first system (*System1*) we assume that each user  $n$  is assigned one sub-channel and one antenna from which it receives its signal. In this system we accommodate up to  $N = L.M$  users and the allocation consists of the assignment of both sub-channel and antennas to users. We use the Min-Min allocation algorithm described in [1] to schedule users on antennas and sub-channels. The unique difference with single antenna case in the algorithm application is in the construction of the graph corresponding to the system (Figure 1). Here the right hand side set of vertices represents the couples (antenna, sub-channel) to assign to users instead of only sub-channels in the single antenna case. The weight of the edge between each tuple (antenna, sub-channel) =  $(l, m)$  and user  $n$  is the SINR:

$$\gamma_n(l, m) = \frac{|H_{l,m,n}|^2 \frac{P}{L}}{N_0 + \sum_{l' \neq l} |H_{l',m,n}|^2 \frac{P}{L}} \quad (4)$$

We assume that each antenna transmits with power  $P/L$  of over each sub-channel, thus the total transmitted power over each sub-channel is  $P$ . Under the assumption of Gaussian signals, the achievable sum rate can be written as

$$\sum_{n=0}^{N-1} R_n = E \left\{ \sum_{n=0}^{N-1} \log_2 (1 + \gamma_n(l_n^*, m_n^*)) \right\} \quad (\text{bits/dim}) \quad (5)$$

where  $l_n^*$  and  $m_n^*$  are respectively the antenna and sub-channel assigned to user  $n$  according to the Max-Min allocation policy.

2) **system2**: In the second system (*System2*), we assume that each user is assigned a single sub-channel and receives its signal from all antennas. *system2* can contain up to  $N = M$  users. As for *System1* we assume that each antenna transmits

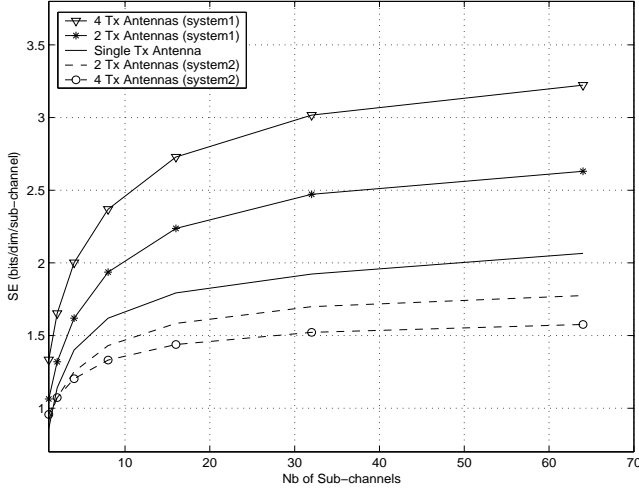


Fig. 2. The rate per sub-channel for both systems as a function of the number of sub-channels with 1, 2 and 4 Tx antennas, 1 Rx antenna, SNR=0dB

with power  $P/L$ . This system can be seen as a  $N$  user system with  $N$  parallel sub-channels where the channel gain of user  $n$  over sub-channel  $m$  is  $\frac{\sum_{l=1}^L |H_{l,m,n}|^2}{L}$ . This channel gains are the weights of the edges in the graph corresponding to the system. The sum rate of this system is

$$\sum_{n=0}^{N-1} R_n = E \left\{ \sum_{n=0}^{N-1} \log_2 \left( 1 + \frac{\sum_{l=1}^L |H_{l,m_n^*,n}|^2 \frac{P}{L}}{N_0} \right) \right\} \quad (\text{bits/dim}) \quad (6)$$

where  $m_n^*$  is the sub-channel assigned to user  $n$  using Max-Min allocation policy. In the following section, we compare the two transmission techniques of the multiple-antenna system in terms of the spectral efficiency using the Max-Min Fair Allocation.

3) **System Comparison:** Figure 2 shows the spectral efficiency SE (averaged per sub-channel sum rate) as a function of the number of sub-channels for both system1 and system2 with 1, 2 and 4 antennas using the Max-Min allocation algorithm. We assume a frequency selective channel with correlated frequency channel gains with a maximum path delay  $\tau_{max} = 2\mu s$  and an exponentially-decaying multipath intensity profile. The system bandwidth is assumed equal to  $B = 20\text{MHz}$ . We first note that SE increases with the number of sub-channels, in all cases, which is due to multi-user diversity. We can also note that *system1* permits transmission at a higher rate than *system2*. This is due to the ‘‘opportunistic’’ spatial-multiplexing offered by multiuser-diversity as described in [2], [8]. An other interesting remark is that the throughput increases with number of antennas in *system1* but decreases in *system2* which is due to the fact that channel variation is reduced by antenna diversity (i.e. the benefits of multiuser diversity are reduced when less channel variation is present).

The remainder of the paper will focus on system1 since we are interested in increasing the system throughput. Figure 3 shows the spectral efficiency of this system on a frequency selective channel with correlated frequency channel gains for different values of the system bandwidth and with 1 and 2 antennas

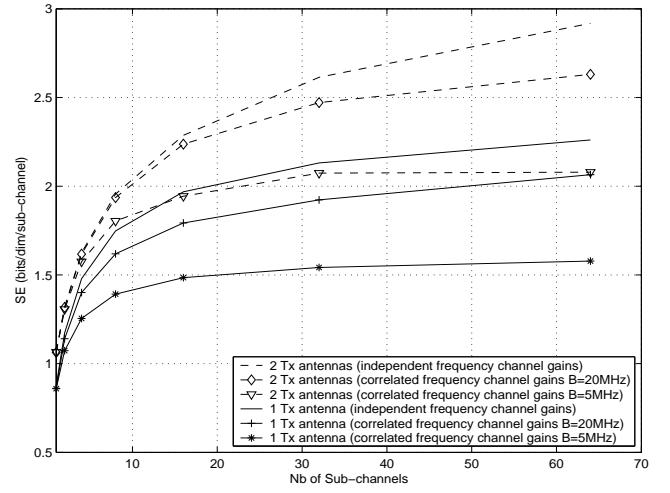


Fig. 3. System1 spectral efficiency for different bandwidth values with 1 and 2 Tx antennas, 1 Rx antenna, SNR=0dB

at the transmission side. The SE with independent frequency channel gains is given for comparison. Although unrealistic, this gives us an idea of the achievable rates as a function of the number of uncorrelated channels or the approximate number of degrees of freedom of the propagation environment in the available system bandwidth. This figure confirms the results highlighted in [1] for single antenna transmission. Bandwidth plays an important role on how much scheduling users on sub-channels can increase spectral efficiency. We note that the benefit from using multiple antennas at transmission can be limited by the amount of bandwidth. For example, for a large number of sub-carriers, a single antenna 20MHz system can outperform a double antenna 5MHz system.

### B. Multiple-Antenna receivers

In this section we consider the use of multiple antennas at the receiver (Mobile) side and we limit our study to *System1* (spatial multiplexing). We assume that each user has  $K$  receiving antennas. The signal to interference plus noise ratio (SINR) corresponding to the signal received by user  $n$  from antenna  $l$  on sub-channel  $m$  is [22]

$$\gamma_n(l, m) = \frac{\frac{P}{L} \cdot |\mathbf{w}_{l,m,n} \mathbf{H}_{l,m,n}|^2}{\mathbf{w}_{l,m,n} \left( \sum_{l' \neq l} \frac{P}{L} \mathbf{H}_{l',m,n} \mathbf{H}_{l',m,n}^H + N_0 \mathbf{I} \right) \mathbf{w}_{l,m,n}^H}$$

where  $\mathbf{H}_{l,m,n}$  is the  $K \times 1$  channel gain vector from transmitting antenna  $l$ , to all receiving antennas of user  $n$  over sub-channel  $m$  and  $\mathbf{w}_{l,m,n}$  is a weight vector performing spatial combining. In the case we are considering (simple detection), the expression of  $\mathbf{w}_{l,m,n}$  for Maximum Ration Combining is given by  $\mathbf{w}_{l,m,n} = \mathbf{H}_{l,m,n}^H$ .

For MMSE, the filter for the detection of  $x_i$  is given by

$$\mathbf{w}_{l,m,n} = \left( \frac{\sqrt{P/L}}{\mu_n} \Sigma_n^{-1} \mathbf{H}_{l,m,n} \right)^H$$

where:  $\Sigma_n^{-1} = N_0 \mathbf{I} + \frac{P}{L} \sum_{l' \neq l} \mathbf{H}_{l',m,n} \mathbf{H}_{l',m,n}^H$  and where  $\mu_n = \frac{P}{L} \mathbf{H}_{l,m,n}^H \Sigma_n^{-1} \mathbf{H}_{l,m,n}$

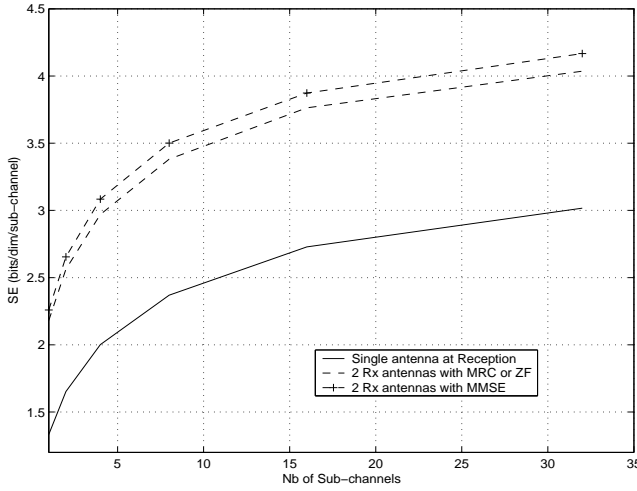


Fig. 4. Reception techniques comparison with Max-Min allocation algorithm (4 Tx antennas, SNR=0dB, Correlated frequency channel gains).

The filter in the MMSE receiver requires the estimation of the channel gains from interfering antennas and takes advantage of this to mitigate interference.

#### • Numerical results

Figure 4 shows the system spectral efficiency for a MIMO system using spatial multiplexing (system1) with different receivers (MMSE and MRC). We consider a system with 4 transmitting antennas at the base station and each receiver has  $K = 2$  antennas. The allocation of sub-channels and transmitting antennas is made according to the Max-Min allocation policy. We assume again a frequency-selective channel with correlated frequency channel gains resulting from a maximum path delay  $\tau_{max} = 2\mu s$  and an exponentially-decaying multipath intensity profile. The system bandwidth is assumed equal to  $B = 20\text{MHz}$ . This figure highlights the spectral efficiency gain that can be reached by using multiple antennas at the reception. Concerning the reception techniques comparison, as expected, MMSE receiver takes advantage from the knowledge of the interferer channel gains and yields slightly better performance than MRC.

#### V. CONCLUSION

In this paper we treated multiuser allocation algorithms for multi-carrier multiple-antenna systems. We proposed an extension of the Max-Min allocation algorithm described in [1]. We showed that spatial multiplexing and interference mitigation in addition to multiuser-diversity can also be achieved through similar allocation algorithms. We showed also the gain of using multiple antennas at the receiver. These results are pertinent for any type of system for which bandwidth can be allocated to a large population of users in a centralized fashion and supports multiple-antennas transceivers. This could be, for instance for wideband OFDMA systems or potentially future systems allocating users with multiple radio-interfaces across large portions of radio spectrum using potentially different radio-access technologies. In this paper, we assumed that

all users have the same traffic load. An extension of this work would be to investigate inter-layer scheduling techniques taking into account the traffic load of different users [17], [23]

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