

Channel Allocation Algorithms For Multi-carrier Systems

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Abstract: This study focuses on orthogonal channel allocation strategies yielding multiuser diversity with a deterministic channel use in a N -user system with N parallel sub-channels. The techniques are applicable, for instance, in Orthogonal Frequency Division Multiple-Access (OFDMA) systems with dynamic sub-carrier allocation. We show that multiuser diversity, and thus an increase of aggregate data rates with the size of the user population, can still be successfully achieved even under a hard fairness constraint. Furthermore, we provide algorithms which perform channel allocation yielding variable-rate with constant power and fixed-rate with variable power. We show the effect of system bandwidth (and thus sub-channel correlation) on multiuser diversity. The techniques considered here do not require phase information in the channel allocation process, which, from a practical point-of-view is particularly important for time-division duplex systems exploiting channel reciprocity.

Keywords: Multiuser diversity, fairness, channel allocation, OFDM.

I. INTRODUCTION

In multiaccess wireless systems, fading is generally considered as a detrimental effect. An important means to deal with this effect is dynamic resource allocation (DRA), for example by optimizing transmit power, rate and bandwidth using channel state information (CSI). The method for making this CSI available at the transmitter depends strongly on the considered system architecture. In systems such as HDR (also known as IS-865) the receiver estimates the CSI based on a common pilot and feeds back the information to the transmitter [1]. In systems employing time-division duplexing (TDD), the channel reciprocity allows the transmitter to use the CSI estimated during reception for transmission, which is the case for instance in the DECT cordless telephone system and for power-control in UMTS-TDD. In practical TDD systems, amplitude information is reasonably simple to estimate from the opposite link, while for accurate phase information this is not the case, mainly due to the difficulty in calibrating

the difference in phase response between the transmitter and receiver chains.

Many studies deal with dynamic allocation strategies. For instance, [2]- [5] present the concept of Multiuser Diversity and give power allocations strategy for maximizing the total sum-rate of multiuser systems which consists of scheduling at any one time the user which would make the best use of the channel (i.e the user with the best channel response). It has also been shown that multiuser diversity yields an increase of the total throughput as a function of the number of users. The most remarkable result from these studies is that for multiuser systems significantly more information can be transmitted across a fading AWGN channel than a non-fading AWGN channel for the same average signal power at the receiver. Spectral efficiency can be increased by more than a factor of two for small signal-to-noise ratios (around 0dB). This is due to the fact that at a given time and frequency, the channel gain is random and can be significantly higher than its average level. One can take advantage of this by using a proper dynamic time-frequency allocation based on the time/frequency varying characteristics of the channels.

The main practical issue arising from channel-dependent resource allocation schemes is fairness. Users (or the base station) must wait until their channel conditions are favorable to transmit. In [6], the authors treat the fairness problem between users in the slow fading environment and discussed the implementation in the IS-865 system and propose methods to enhance fairness. Their approach consists in using multiple antennas to induce fast channel fluctuations combined with the proportionally fair allocation policy used in IS-865. In a similar vein for multi-cell systems, [7]- [9] study combined power control and base station assignment in multi-cell systems with fixed vector rate. This is also a form of fairness, since these algorithms allow users to transmit with their desired rates. Similar opportunistic techniques for multi-cell systems are briefly alluded to in [6]. In [10], the authors consider the subcarrier assignment problem in OFDMA systems and compare the simplicity and fairness properties of different allocation algorithms.

In [11], a fair allocation criterion yielding multiuser diversity with a deterministic channel use in a N -user system with N parallel sub-channels is proposed. We build our work on these results by addressing an algorithm that performs the allocation of users across sub-channels according to

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this criterion for OFDM-like systems on frequency selective channels. From a futuristic system point-of-view, another application of the ideas outlined in this work would be allocation of users equipped with various radio interfaces in different parts of the radio spectrum, potentially using different radio-access technologies, based on link quality and quality-of-service (QoS) constraints. We analyze both fixed-power/variable-rate and variable-power/fixed-rate cases. The addressed results are very pertinent for slowly-varying channels since frequency selectivity is exploited.

The organization of this paper is as follows: Section II presents the system model and formulates the fairness problem. In Section III, we provide an algorithm to achieve the criterion in [11] and compare this algorithm to the one achieving the maximum total throughput. We also outline an algorithm for power control under a fixed rate vector constraint. In Section VI we present numerical results and outline the effect of bandwidth on multiuser diversity. Finally, in Section V we present our conclusions and outline ongoing extensions and future perspectives.

II. SYSTEM MODELS

We consider a single-antenna system transmitting over M parallel channels and accommodating N users, where N , depending on the considered system and allocation, is a function of M . This could represent the case of any wideband OFDM system, such as Mobile Broadband Wireless Access (MBWA) systems, for instance the evolving IEEE 802.16 standard where an Orthogonal Frequency Division Multiple Access (OFDMA) technique is used. An other example of such system model could be the UTRAN HSDPA (high-speed data packet-access) 3GPP proposal using an OFDM(A) physical layer instead of WCDMA, proposed in [12] for the downlink channel. In the context of these systems, the algorithms proposed in this paper would be used to allocate the different frequency sub-bands to users. We can also imagine the use of these techniques in extensions of IEEE802.11a/g, Hiperlan2 or multiband-OFDM for UWB systems. In the futuristic scenario mentioned in the introduction, we could also envisage a centralized control of radio spectrum across very large bandwidths and co-localized radio-access technologies (e.g. GSM/UMTS-FDD/UMTS-TDD, 802.11, 802.16, DVB-T, etc.).

For clarity of presentation, we assume that each user is assigned only one subchannel ($k = 1$), but the proposed allocation strategies are still valid for the general case where $k > 1$. We have only to duplicate each user k times and run the algorithms, so that each user would be allocated k subcarriers. We consider that each sub-channel is a fading AWGN channel with noise variance N_0 . As has been mentioned previously, we assume that the amplitude response for all users over all subcarriers are known at the transmitter. For uplink transmissions, the base station estimates the CSI for each user from a received pilot which is a known sequence transmitted by the

users and is *spread over the entire available bandwidth*. The estimated CSI is used to carry out the subchannel allocation algorithm and a message is fed back to inform each user of its assigned subchannel (Note that for slowly-varying channels this is reasonably simple to accomplish and consumes little signaling bandwidth since the allocation remains invariant for long periods). The received signal on sub-channel m is given by:

$$r_m^{UL} = \sum_{n=0}^{N-1} \sqrt{P_{m,n}^{UL}} H_{m,n}^{UL} x_{m,n}^{UL} + z_m^{UL} \quad (1)$$

where $x_{m,n}^{UL}$, $P_{m,n}^{UL}$ and $H_{m,n}^{UL}$ are respectively the signal, the transmit power and the channel gain from user n on sub-channel m and z_m^{UL} is the noise in sub-channel m . For downlink transmissions and reciprocal channels (for instance in TDD systems), then the channel estimation is performed in the same manner as for uplink transmissions. In the case of non-reciprocal channels, each user has to estimate its CSI over all available subchannels based on a known pilot and feeds this information back to the base station which carries out the subchannel allocation algorithm. The received signal for a user n on sub-channel m is given by

$$r_{m,n}^{DL} = \sqrt{P_m^{DL}} H_{m,n}^{DL} x_m^{DL} + z_{m,n}^{DL} \quad (2)$$

where x_m^{DL} , P_m^{DL} and $H_{m,n}^{DL}$ are respectively the signal, the transmit power and the channel gain for user n on sub-channel m and $z_{m,n}^{DL}$ is the noise in sub-channel m for user n . In symmetric channels, we have that $H_{m,n}^{UL} = H_{m,n}^{DL}$. In the spirit of OFDM-based systems, we model each channel gain $H_{m,n}^{DL}$ (or $H_{m,n}^{UL}$) as a frequency sample of a discrete multipath channel having Γ significant uncorrelated paths with delays: $\tau_1, \tau_1, \dots, \tau_\Gamma$, that is

$$h_{m,n}^{DL}(t) = \sum_{i=0}^{\Gamma-1} \alpha_i \delta(t - \tau_i), \quad (3)$$

where the path gains α_i are zero mean Gaussian random variables with variance σ_i^2 . This channel is assumed stationary for the duration of coded transmission blocks, but may vary from block to block. The samples of the frequency response are given by

$$H_{m,n}^{DL} = H_n^{DL}(f_m) = \sum_{i=0}^{\Gamma-1} \alpha_i e^{-j \frac{2\pi \tau_i f_m}{M}} \quad (4)$$

and have covariance

$$E \{ H_{m,n}^{DL} H_{m',n}^{DL*} \} = \sum_{i=0}^{\Gamma-1} \sum_{j=0}^{\Gamma-1} E \left\{ \alpha_i \alpha_j e^{-j \frac{2\pi (\tau_i f_m - \tau_j f_{m'})}{M}} \right\} \quad (5)$$

$$= \sum_{i=0}^{\Gamma-1} E \{ |\alpha_i|^2 \} e^{-j \frac{2\pi \tau_i (f_m - f_{m'})}{M}} \quad (6)$$

where f_m is the frequency corresponding to subcarrier m . The goal of the following sections will be to study allocation algorithms of users to sub-carriers according to optimization criteria such as mutual information or transmitted power

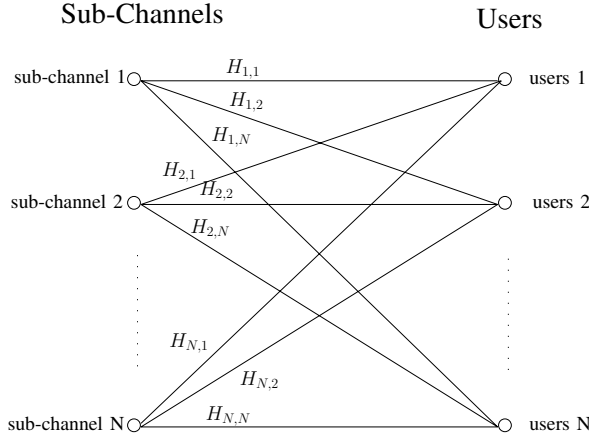


Fig. 1. System representation by a graph.

and to illustrate their performance for both uncorrelated and correlated channel gains.

III. ORTHOGONAL ALLOCATION ALGORITHMS WITH HARD FAIRNESS

We start by imposing a hard fairness constraint on the system, namely that each user is guaranteed one sub-channel at any given time instant and that there is only one user per sub-channel (i.e. orthogonal multiaccess). Thus, in this system we accommodate up to $N = M$ users. There exists $N!$ possible allocations of sub-channels to users each one represented by a vector $\mathbf{c}_l = \{c_{l,0}, c_{l,1}, \dots, c_{l,N}\}$, where $c_{l,n}$ is the sub-channel assigned to user n when allocation \mathbf{c}_l is applied, for $l = 0, \dots, N! - 1$.

An achievable ergodic sum rate is upper-bounded as

$$\sum_{n=0}^{N-1} R_n \leq E \left\{ \max_{l=0, \dots, N!-1} \sum_{n=0}^{N-1} \frac{1}{2} \log_2 \left(1 + \frac{P}{N_0} H_{c_{l,n},n} \right) \right\} \quad (7)$$

when the transmitters (guided by the receivers) jointly select the optimal allocation vector given the instantaneous channel gains. This rate can be achieved either by adapting the data rate with the variation of the channels or by coding over many independent channel realizations.

A. Max-Min Allocation (MMA) policy

1) *Allocation Criterion:* In [11] a simple allocation strategy was proposed where we choose the permutation \mathbf{c}_{l^*} where

$$l^* = \arg \max_{l=0, \dots, N!-1} \min_{n=0, \dots, N-1} H_{c_{l,n},n} \quad (8)$$

This policy guarantees that at any given time instant the minimum channel gain allocated is the best possible among all allocations. It was shown in [11] that this criterion achieves multiuser diversity and provides a non-negligible gain with respect even to a non-fading channel. In the following we give a description of one algorithm that permits us to achieve this allocation criterion in polynomial time. In practice this policy (and similarly the one which follows) allows the instantaneous information rate to vary but is strictly non-zero.

2) *Allocation Algorithm description:* Before giving the details of the considered allocation algorithm let first point out some general properties that will help in the algorithm's description.

A N -user system with N parallel channels can be represented by a weighted bipartite graph $G = (X, Y, E)$ where the left hand side (LHS) set of vertices X represents the sub-channels and the right hand side (RHS) set Y represents the users (Figure 1). E is the set of edges between X and Y . In our system all users can transmit on all sub-channels, thus the graph G is complete ($\text{card}(E) = N^2$). Each edge in the graph is weighted by the channel gain corresponding to the user and the sub-channel in the vertices of that edge i.e. $w(x, y) = H_{x,y}$. The max-min allocation algorithm that can be described as follows:

- 1) We first begin by constructing the graph $G = (X, Y, E)$ corresponding to our system as described previously.
- 2) The aim of our algorithm is to find the permutation \mathbf{c}_{l^*} such that $l^* = \arg \max_{l=0, \dots, N!-1} \min_{n=0, \dots, N-1} H_{c_{l,n},n}$. If we consider the N^2 order statistics of the channel gains, we have that:

$$\text{ord} \left(\min_{n=0, \dots, N-1} H_{c_{l^*,n},n} \right) \geq N - 1 \quad (9)$$

thus we can remove, from E , the $N - 1$ edges with the minimum weights

$$E = E - \{(x, y) / \text{ord}(w(x, y)) < N - 1\}$$

- 3) Find, in graph G , the edge $e = (x, y)$ with the minimum weight: $e = \arg \min_{(x,y) \in X \times Y} \{w(x, y)\}$. and remove this edge from G
- 4) In graph theory, a perfect matching $M \subseteq E$ in G is defined as a set of edges such that no two elements share a vertex and where all vertices are matched.

- If a perfect matching is possible in the new graph G , we go back to 3.
- If no perfect matching is possible in graph G , we allocate y to x , remove the vertices x and y and all their edges from G and go back to 3.

Complete details on how to verify the existence of a perfect matching in a graph G are given in section III-A-3.

- 5) we stop when all user are assigned one sub-channel.

3) Finding a Perfect Matching:

Theorem 1: (Hall's Theorem) Let $G(X; Y; E)$ be a bipartite graph with $\text{card}(X) \leq \text{card}(Y)$. Then G has a perfect matching saturating every vertex of X if and only if $\text{card}(S) \leq \text{card}(A(S))$ for every subset $S \subseteq X$, where $A(S)$ is the subset of vertices of Y that are adjacent to some vertex in S .

In practice, for $\text{card}(X)$ very large, verifying Hall's conditions for every subset in X is too complicated. Another method

to verify this condition is the computation of the permanent of the adjacency matrix of the graph, which is also very hard to compute. [13], [14] propose a method to construct a maximum cardinality matching for a bipartite graph: *the augmenting path algorithm*. Once, this matching is constructed, we have only to verify if its cardinality is equal or not to $\text{card}(X)$. Before describing this method let us give the following definitions.

Definition1: Given a matching \mathcal{M}_t , an alternating path comprises edges in \mathcal{M}_t and edges not in \mathcal{M}_t alternately.

Definition2: an augmenting path for \mathcal{M}_t is an alternating path which starts and ends at exposed vertices.

To find the maximum cardinality matching we start with a feasible matching \mathcal{M}_t (for example the empty matching), try to find repeatedly an augmenting path P , and replace \mathcal{M}_t by $\mathcal{M}_t \oplus P$. If there is no remaining augmenting paths, the maximum cardinality matching is found. Using this method, the complete matching can be found in $O(\text{card}(X)^3)$.

B. The maximum total rate Allocation (MTRA) Policy

The MTRA policy is the strategy that achieves this maximum sum-rate in (7). As in the previous section, we can model the considered system by a bipartite graph. The difference with the last representation is the edges weights. Here we set the weight of each edge to the $\log(1 + \gamma)$ of the corresponding user and sub-channel instead of the channel gains. Finding in the graph the matching that maximizes the total weight is equivalent to find the allocation the total sum rate. [14] describes an algorithm that permits to find a such matching in $O(N^3)$ time based in the well-known Hungarian method.

C. Fixed Rate Allocation (FRA) Policy

The objective here is to find the allocation of users to sub-channels minimizing the total transmit power while achieving some required rate-tuple $\mathbf{R} = (R_0, R_1, \dots, R_{N-1})$, (i.e the SINR tuple $(\gamma_0, \gamma_1, \dots, \gamma_{N-1})$) where R_i is the rate of user i . In this policy instantaneous power is allowed to vary in order to achieve a target per user information rate.

The Hungarian method, presented in the previous section, performs the desired assignment with a small change in edge weights of the corresponding graph. For instance, the weight of the edge between user i is and sub-channel j will be the negative of the power needed to achieve the desired SINR target γ_i if user i is assigned to sub-channel j , that is

$$w(i, j) = -P(R_i) = -\frac{\gamma_i \cdot N_0}{H_{i,j}} \quad (10)$$

Finding the matching that maximizes the sum of weights permits us to find the desired allocation of users to sub-channels.

IV. NUMERICAL RESULTS

Figure 2 shows the average per user throughput as a function of the number of users with the MMA and MTRA fair allocation algorithms in a Rayleigh fading environment, which we compare to the unfair allocation where for each given sub-channel we choose the user with the best channel

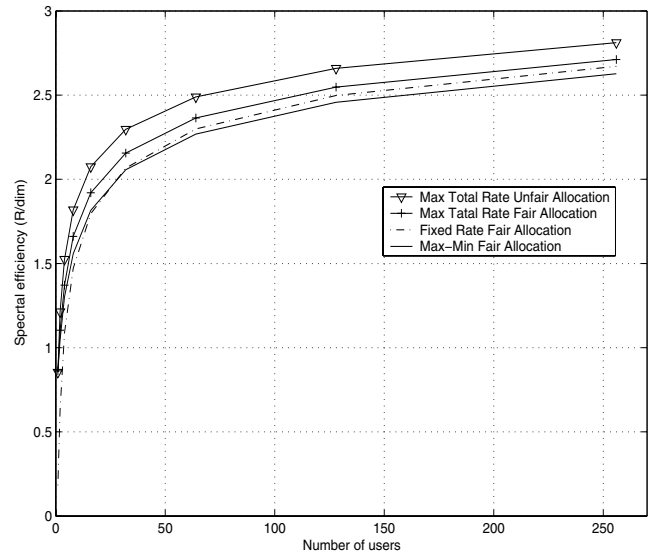


Fig. 2. Averaged throughput over Rayleigh fading at 0-dB with fair and unfair allocations

[2], [11]. For these results we have assumed that the correlations between frequency channel gains in (6) are zero. Although unrealistic, this gives us an idea of the achievable rates as a function of the number of uncorrelated channels (or the approximate number of degrees of freedom of the propagation environment in the available system bandwidth). We, first, note in Figure 2 that the per user average throughput increases with the number of users, in all cases, which is due to multi-user diversity. We can also see that even under a hard fairness constraint we can achieve performance which comes close to the optimal unfair policy. With a fixed rate (variable power) requirement we see that multiuser diversity can still be achieved and this additional constraint does not introduce any throughput degradation. This curve was computed for the same average SNR (0dB) as in the variable cases. Figure 3 shows the spectral efficiency (SE) as a function of the number of users accommodated, using the proposed max-min allocation algorithm on a frequency selective channel with correlated frequency channel gains, different values of the system bandwidth and with a fixed number of subcarriers equal to 64. Here the number of sub-carriers per user is M/N . For the correlated channel results, we assumed that the maximum path delay $\tau_{max} = 2\mu s$ and an exponentially-decaying multipath intensity profile. The performance of the algorithm with independent frequency channel gains is also given for comparison. As expected, bandwidth plays an important role in how much scheduling users on sub-channels can increase spectral efficiency. We see that when the system bandwidth is appropriately chosen spectral efficiency can be increased by more than a factor of 2 for moderate user populations even with hard fairness constraints.

We also note that for sufficiently large bandwidth (here $B = 50\text{MHz}$) we approach the system performance of

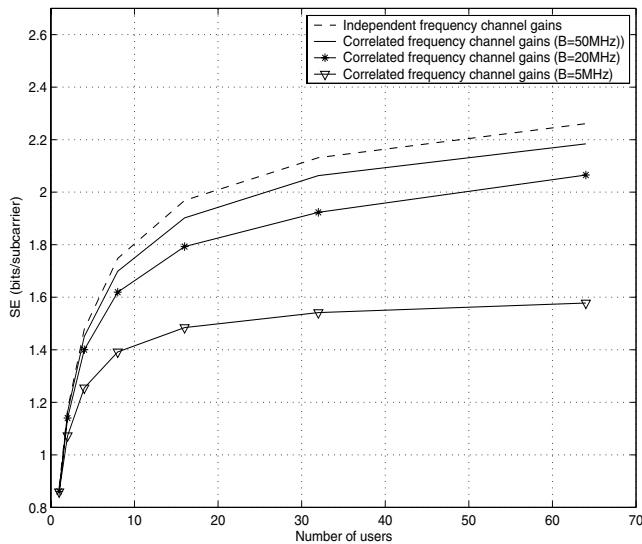


Fig. 3. Spectral Efficiency (SE) variation with user population for a fixed number of sub-carriers

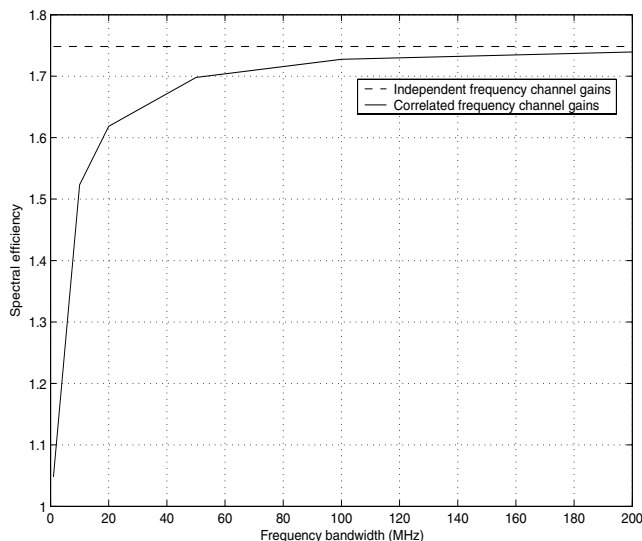


Fig. 4. Spectral Efficiency (SE) variation with system bandwidth for fixed number of users and sub-carriers with the max-min allocation policy

the independent frequency channel gains case. An other interesting result, given by the lower curve ($B = 5\text{MHz}$), is that for a number of users greater than the number of coherence bandwidths of the system, the increase of the number of user induces only a very slight improvement of the SE. This result is confirmed by figure IV which shows the SE as a function of the system bandwidth for a fixed number of users ($N = 8$) and a fixed number of subcarriers ($M = 64$). This curve shows that an increase in the system bandwidth yields a rapid improvement of the system SE and slowly approaches approaches the performances of the independent frequency channel gains case for system bandwidth greater than 50 MHz. This result is due to the

increase in subcarrier spacing which becomes greater than the coherence bandwidth for $B > 50\text{MHz}$ and thus the frequency correlation of the channel becomes small. The value of 50MHz is also the system bandwidth for which the number of coherence bandwidths is almost equal to the number of users accommodated in the system.

V. CONCLUSION

In this paper we treated multiuser allocation algorithms for multi-carrier systems. We proposed an algorithm that performs the Max-Min Fair allocation criterion described in [11] and have shown that even under hard fairness constraints, we can achieve performance close to that of optimal unfair allocation. These results are pertinent for any type of system for which bandwidth can be allocated to a large population of users in a centralized fashion. This could be, for instance for wideband OFDMA systems or potentially future systems allocating users with multiple radio-interfaces across large portions of radio spectrum using potentially different radio-access technologies. The results presented in this paper are generalized for multiple-input multiple-output (MIMO) transceivers in [15] where it is shown that spatial multiplexing and interference mitigation in addition to multiuser-diversity can also be achieved through similar allocation algorithms.

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