Low Complexity ML Detection Based on the Search for the Closest Lattice Point

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Abstract — **The design of maximum likelihood detection algorithms for linear multi-dimensional constellations and lattice codes transmitted over Gaussian fading channels is considered. The linearity of the constellations over the field of complex numbers facilitates the design of maximum likelihood detectors using number theoretic tools for searching the closest lattice point. In particular, the Pohst enumeration method and the Schnorr-Euchner refinement of the Pohst strategy are used to develop two maximum likelihood detection algorithms. The first algorithm is shown to offer a significant reduction in complexity compared to the Viterbo-Boutros implementation of the Pohst strategy. The second algorithm is more suited to maximum likelihood detection than the Agrell *et al.* implementation of the Schnorr-Euchner strategy. Further, the two algorithms are compared to extract insights on the lowest complexity approach in different scenarios. The results obtained indicate that the distance between the nulling and cancellation solution and the maximum likelihood solution plays a major role in determining the lowest complexity algorithm.**

In this paper, we focus our attention on the maximum likelihood (ML) detection problem of lattice constellations. In particular, we propose two new ML detection algorithms with lower complexity compared to previously proposed techniques. Two versions of sphere decoding of Pohst [1] are of interest. First, a modified version of the Pohst strategy [1], or natural spanning, which reduces the sphere radius each time a point is found in side the sphere without re-examining previously tested nodes as done in [2]. Second, a modified version of the Schnorr-Euchner refinement of the Pohst strategy [3], which makes it more suitable for finding the ML solution of MIMO systems and lattice codes. The former version is referred to by Algorithm I, the latter by Algorithm II. For simplicity of presentation, we restrict ourselves to convex constellations S carved from $\mathbb{Z}^m$ and bounded in the m-dimensional cube $[(\mu_{1,\min},\ldots,\mu_{m,\min})^T : (\mu_{1,\max},\ldots,\mu_{m,\max})^T]$, $\mu_{i,\min} \le \mu_{i,\max} \in \mathbb{Z}, i = 1,\ldots,m$. Given the lattice generator matrix $\boldsymbol{B}$ [1], we first reorder its columns by a permutation Π such that $||\boldsymbol{b}_{\Pi(1)}||^2 \le \ldots \le ||\boldsymbol{b}_{\Pi(m)}||^2$, and we apply the QR decomposition on $\Pi(\boldsymbol{B})$. The algorithm then proceeds as follows:

Algorithm I (Input $C_0, \boldsymbol{\rho}, \boldsymbol{R}$. Output $\hat{\boldsymbol{x}}$ and $\hat{d}$):

1. Set $i := m$, $\text{dist}_m := 0$, $S_m := \rho_m$, $B = C_0$.
2. If $B < \text{dist}_i$ go to 4. Else, $LB(x_i) := \max\left(\left\lceil \frac{-\sqrt{B-\text{dist}_i}}{r_{ii}} + S_i \right\rceil, \mu_{i\min}\right)$, $UB(x_i) := \min\left(\left\lfloor \frac{\sqrt{B-\text{dist}_i}}{r_{ii}} + S_i \right\rfloor, \mu_{i,\max}\right)$, and set $x_i = LB(x_i) - 1$.
3. $x_i := x_i + 1$. If $x_i \le UB(x_i)$ go to 5, else go to 4.
4. If $i = m$ terminate, else set $i := i + 1$ and go to 3.
5. Set $\xi_i := \rho_i - x_i$, and $t := r_{ii}(S_i - x_i)$. If $i > 1$ $\Big\{$ $i := i - 1$, $S_i := \rho_i + \frac{1}{r_{ii}} \sum_{l=i+1}^{m} r_{il}\xi_l$, $\text{dist}_i := \text{dist}_{i+1} + t^2$, and go to 2 $\Big\}$. Else go to 6.
6. $\hat{d} := \text{dist}_1 + t^2$. If $\hat{d} < B$, set $B = \hat{d}$, save $\hat{\boldsymbol{x}} := \boldsymbol{x}$, $UB(x_k) := \min\left(\left\lfloor \frac{\sqrt{B-\text{dist}_k}}{r_{kk}} + S_k \right\rfloor, \mu_{i,\max}\right)$, $k = 1,\ldots,m$. Then go to 3.

Algorithm II is a modification of the Schnorr-Euchner refinement of the Pohst strategy [3] [2] that takes into account of the constellation boundaries and the lattice code constraints [5]. It does not compute upper and lower bounds on x_i, but only computes a step associated at the i-th level which takes account of the ordering of the nodes. Algorithm II zig-zags around the decision feedback equalizer (DFE) point, or the Babai point [3], until it finds the ML solution. During its zig-zags, one constrains the search with the lattice code boundaries once it "steps over" the ML solution when the closest lattice point to the received signal is not a valid code point. Algorithm II's complexity can be considerable improved when using minimum mean square error (MMSE) filtering with the MMSE-DFE matrix which makes the starting point of the Algorithm II the MMSE-DFE instead of zero forcinf (ZF)-DFE, which is much closer to the ML solution. Using the MMSE-DFE gives large improvement over ZF-DFE in terms of performances, and therefore a large reduction of the sphere decoder complexity. Our experiments shows that MMSE-DFE implies smaller sphere decoder complexity with almost no performance penality. Additional saving in complexity can be done by ordering the columns of the MMSE-DFE matrix in a "greedy" manner as in BLAST detection algorithms [6]. For example, in 4×4 BLAST system with a 16-QAM modulation, the complexity reduction factor is about 5 at small SNRs and 2 at large SNRs (at large SNR, the ZF-DFE tends to MMSE-DFE).

REFERENCES

[1] M. Pohst, "On the computation of lattice vectors of minimal length, successive minima and reduced basis with applications," in *ACM SIGSAM*, vol. 15, pp. 37–44, Bull, 1981.

[2] E. Viterbo and J. Boutros, "A universal lattice code decoder for fading channel," *IEEE Trans. Inform. Theory*, vol. 45, pp. 1639–1642, July 1999.

[3] E. Agrell, T. Eriksson, A. Vardy, and K. Zeger, "Closest point search in lattices," *IEEE Trans. Inform. Theory*, Aug. 2002.

[4] M. O. Damen, A. Chkeif, and J.-C. Belfiore, "Lattice codes decoder for space-time codes," *IEEE Commun. Letters*, vol. 4, pp. 161–163, May 2000.

[5] M. O. Damen, H. El Gamal, and G. Caire, "On ML detection and the search for closest points in lattices," accepted pending a revision in *IEEE Trans. Inform. Theory*, special issue on space-time transmission, reception, coding and signal design 2003.

[6] B. Hassibi, "An efficient square-root algorithm for BLAST," in *Proc.* ICASSP '00, Turkey, 2000.

[1] Matrix $\boldsymbol{B}$ contains the MIMO channel and the lattice code generator matrix [4].

[2] The main modifications over [3] are summarized here owing to space limitations. More details are available in the journal version [5].

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