Multipath: Curse or blessing?
A system performance analysis of MIMO wireless systems

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Abstract - In this paper, we look at the performance of noise-limited MIMO wireless systems and compare analytically line-of-sight (LOS) and non-LOS dominated environments on coverage and cell capacity. Through a few key examples, we show that neither a LOS scenario or a NLOS scenario always provides the best performance as it appears the most favorable propagation scenario heavily depends on the chosen metric and type of system.

1. INTRODUCTION

Over 7 years after the publication of the earlier results on multiple-input multiple-output (MIMO) antenna communication [1, 2], surprisingly heated discussions between researchers about the claimed benefits of MIMO systems can still be heard in many conferences and seminars. At heart of the debate often lies the concept of multipath propagation and the supposed impairments or benefits (depending on which side of the MIMO fence one stands) it carries for the sake of wireless networks performance. In basic terms, the question is as follows: If one could decide in favor of the presence of absence of line-of-sight in a wireless link otherwise equipped with a MIMO antennas, what is the strategy that would maximize system performance? Should one go for low rank MIMO channels combined with much greater signal-to-noise ratios brought by a favorable path loss or should one aim at high rank multipath channels in spite of a degraded SNR? In view of the lack of control one has in setting the degree of LOS in practice, the debate appears by much respect a philosophical, although an intriguing one. However, when faced to the problem of deciding whether MIMO systems should be made available in both LOS-dominant and NLOS areas and if yes with how many antennas, the question of evaluating quantitatively the impact of the degree of LOS on the system performance becomes a useful, and to the author's knowledge, non treated topic.

In this paper, we address these questions by evaluating and comparing analytically the system performance (coverage, capacity) of MIMO wireless systems in LOS and NLOS environments in a noise limited setting. We use recently developped Gaussian approximations of MIMO Shannon capacity. In order to frame our analysis, we consider two distinct service models. The first is a low-data rate maximum-coverage oriented model. The second is a high-data rate maximum-capacity+coverage model.

2. PERFORMANCE MODELS

In this paper, we consider a temporal snapshot of a point-to-multipoint data-oriented wireless network. Each MIMO link has N Txers and M rxers.

2.1. Link capacity model

We assume an ideal adaptive modulation and coding scheme such that the instantaneous spectral efficiency enjoyed by user u on his link to the AP is given by the Shannon capacity C^u of his $M \times N$ matrix channel $\mathbf{H}^u$ with no transmit CSI (this happens for a sufficiently fine sampling of the SNR scale by the operating regions of the various coding modes) [2]:

$$C^u = \log_2 \det \left[\mathbf{I}_M + \frac{\rho^u}{N} \mathbf{H}^u \mathbf{H}^{u*} \right] \quad (1)$$

where $\mathbf{I}_M$ is the identity matrix of size M and ρ^u is the SNR.

2.2. Cell capacity model

The cell capacity is defined here as the average of link capacities across users uniformly distributed over a cell disk: $\bar{C} = E^u\{C^u\}$. In what follows, we assume that the SINR is noise dominated (thus we consider a single cell scenario or we assume a large enough frequency reuse factor in a multi-cell scenario) and we ignore effects the macroscopic fading (shadowing) so that the SNR of user u is entirely characterized by the distance d from the user to the AP, and is denoted by $\rho(d)$. Thus the cell capacity is given for a uniformly distributed user population by:

$$\bar{C}(d_0) = \frac{1}{d_0^2} \int_0^{d_0} 2E(\log_2 \det \left[\mathbf{I}_M + \frac{\rho(d)}{N} \mathbf{H}\mathbf{H}^* \right]) d\delta d \quad (2)$$

where d_0 is the cell radius and where the expectation above is taken over the distribution of $\mathbf{H}$.

2.3. Coverage performance

While the cell capacity is a key performance metric for mature deployments with already a large subscriber population, coverage performance is the driving metric for initial roll-outs, governing the number of APs required per area (km2) to deliver a desired level of throughput C_{min} with a certain reliability p_o. A standard way of defining the achievable cell radius d_{max} is through enforcing that a large fraction p_o of the users on the *cell edge* reach a target link capacity level of C_{min}:

$$d_{max} \text{ chosen such that } \text{Prob}\{C(d_{max}) \geq C_{min}\} = p_o \quad (3)$$

where $C(d)$ is the random link capacity at distance d from the AP. Typically the area reliability level is selected close to one ($p_o = 0.9$) [3]. The reasoning above remains valid for mobile users, in which case the expectations are taken over both time and space.

3. IMPACT OF LOS ON LINK CAPACITY

The impact of LOS manifests itself in both the distributions of $\mathbf{H}$ and of the SNR. We investigate the latter in the next section and focus here on the MIMO channel properties. While real-life channels often follow a combination a LOS and NLOS components (i.e. Rice channels), we limit ourselves to the extreme cases of pure LOS and pure NLOS channels.

3.1. LOS case

The LOS channel is described by the angles of departure and arrival of the LOS path with respect to the array elements. It can be conveniently modeled through $\mathbf{H}_l = \mathbf{v}_{rx}\mathbf{v}_{tx}^*$ [4], where subscript l stands for 'LOS', $\mathbf{v}_{rx}$ (resp. $\mathbf{v}_{tx}$) is the vector of unit-modulus arrival (resp. departure) phase terms. In this case, the channel matrix has rank one and the deterministic capacity at distance d from the AP can be shown to be:

$$C_l(d) = log_2(1 + \rho(d)M) \quad (4)$$

which is the capacity of a AWGN channel and where the factor M above corresponds to the receive array gain.

3.2. NLOS

In this case, the channel is modeled via a matrix $\mathbf{H}$ of independent unit-variance Gaussian components. Recent results have shown that the random capacity in (1) can be approximated by a Gaussian variable when M,N are large enough [5, 6, 7]. Luckily, this results holds true even with very moderate values of M,N (2 or 3) [8]. To obtain tractable expressions for our analysis, we distinguish between a high SNR and a low SNR case and assume a symmetrical MIMO system ($M = N$). From [7] we obtain (subscript nl stands for 'NLOS'):

$$C_{nl}(d) \sim \mathcal{N}(m_C, \sigma_C^2) \quad (5)$$

Low SNR:

$$m_C = M\rho(d)\log_2(e),\ \sigma_C^2 = \rho(d)^2 \log_2(e)^2 \quad (6)$$

High SNR: (with $\gamma \approx 0.5772$)

$$m_C = M \log_2 \frac{\rho(d)}{e},\ \sigma_C^2 = log_2(e)^2(\log(M) + \gamma + 1) \quad (7)$$

3.3. Impact of LOS on the SNR

A standard way to model average SNR at any point in the cell is through the use of a so-called link budget. We define P_t, G_t, G_r, σ_{noise}^2, $L(d)$, to be the transmit power, transmit and receive antenna gains, thermal noise power, and path loss at distance d respectively, all in dB unit. The average SNR at distance d writes (in dB):

$$\rho(d)_{dB} = P_t + G_t + G_r - \sigma_{noise}^2 - L(d) \quad (8)$$

For both LOS and NLOS cases, the path loss is usually modeled through:

$$L(d) = A + 10\epsilon \log_{10}(d) \quad (9)$$

where constants A, ϵ depend on the LOS ratio. In particular ϵ is called the path loss exponent and varies from 2 in LOS (free space) to close to 4 in urban NLOS settings [9]. In linear scale, we write:

$$\rho(d)_l = \alpha_l d^{-\epsilon_l} \quad (10)$$

$$\rho(d)_{nl} = \alpha_{nl} d^{-\epsilon_{nl}} \quad (11)$$

where α_l, α_{nl} are the SNR for LOS and NLOS at 1km and depend on the link budget parameters (see Sec. 5).

4. SYSTEM PERFORMANCE

We now use the link performance models above to establish closed form expressions for system-level performance. We focus on two system scenarios:

- Low-rate system (low target capacity C_{min})
- High-rate system (high target capacity C_{min})

For the first system we analyze the impact of LOS on the achievable cell radius. For the second, we analyze the achievable capacity and cell radius.

4.1. Low rate system

From the coverage definition in (3) and from (4), (10), we derive easily the achievable cell radius for the LOS case:

$$d_{max,l} = \left(\frac{2^{C_{min}} - 1}{\alpha_l M}\right)^{-1/\epsilon_l} \quad (12)$$

which, for a small C_{min}, yields:

$$d_{max,l} \approx \left(\frac{C_{min}}{\alpha_l \log_2(e) M}\right)^{-1/\epsilon_l} \quad (13)$$

In the NLOS case, we use the Gaussian appxoximation above and from (3) we derive:

$$C_{min} - m_C = \sigma_C Q^{-1}(p_o) \quad (14)$$

where $Q^{-1}()$ is the inverse Q function. Because we consider users with low capacities, we can assume their SNR is also low. Thus we can use (6) and (11) to obtain:

$$d_{max,nl} = \left(\frac{C_{min}}{\alpha_{nl} \log_2(e)(M + Q^{-1}(p_o))}\right)^{-1/\epsilon_{nl}}, \quad (15)$$

which exhibits a strong analogy with the LOS case of (13), with the extra Q-function term dealing with the randomness nature of capacity in the NLOS case.

Interpretations: Interestingly, if $\alpha_{nl} < \alpha_l$, $\epsilon_{nl} > \epsilon_l$, then $Q^{-1}(p_o)$ being negative, it appears that the coverage performance in NLOS can never exceed that in LOS no matter how many MIMO antennas are used!

4.2. High rate system

We first assume a system that guarantees a high capacity level for most users and examine its maximum cell radius. Then we limit the cell radius to a constant common value (less than achievable radius) and determine the cell capacity for such a system.

4.2.1. Coverage performance

From (12) we find for the high rate ($C_{min} >> 1$) and LOS case:

$$d_{max,l} = 2^{-C_{min}/\epsilon_l}(\alpha_l M)^{1/\epsilon_l} \quad (16)$$

In the NLOS case, using the high SNR Gaussian approximation of (7) in (14)[1], we get

$$\begin{aligned} \rho(d_{max,nl}) &= 2^{\tilde{C}/M} \\ \tilde{C} = C_{min} &+ \log_2(e)(M - \sqrt{\log(M) + \gamma + 1}Q^{-1}(p_o)) \\ d_{max,nl} &= 2^{-\tilde{C}/\epsilon_{nl}} \alpha_{nl}^{1/\epsilon_{nl}} \end{aligned} \quad (17)$$

[1]large C_{min} means large SNR, provided M is less than C_{min}

Interpretations: Comparing (16) and (17) it can be shown that which of a LOS and NLOS system will provide the best coverage in a high-rate setting will depend on both the link budget parameters and the number of antennas, as is exemplified in the numerical results section.

4.2.2. Capacity performance

To obtain a fair capacity comparison between the LOS and NLOS cases, we must fix the cell radii to a common value d_0, compatible with the high-rate assumption. The cell capacity $\bar{C}(d_0)$ is then given by integrating over the cell area as in (2).

In the LOS case, the link capacity is deterministic and we find

$$\begin{aligned} \bar{C}(d_0)_l &= \frac{1}{d_0^2}\int_0^{d_0} 2\log_2(1 + M\rho(d))d\delta d \\ \bar{C}(d_0)_l &\approx \log_2(M\alpha_l d_0^{-\epsilon_l}) \ \text{ high SNR region} \end{aligned} \quad (18)$$

In the NLOS case, we use the mean of the Gaussian approximation. Note that, since we impose a high rate/SNR condition at the cell edge, the high SNR condition will be fulfilled over the entire cell. We combine (2) and (7), thus yielding after a few manipulations (not shown here):

$$\begin{aligned} \bar{C}(d_0)_{nl} &= \frac{1}{d_0^2}\int_0^{d_0} 2M\log_2(\rho(d)/e)d\delta d \\ \bar{C}(d_0)_{nl} &= M\log_2(\rho(d_0)/e) + \frac{M\epsilon_{nl}\log_2(e)}{2} \\ \bar{C}(d_0)_{nl} &= M\log_2(\alpha_{nl} d_0^{-\epsilon_{nl}} e^{\frac{\epsilon_{nl}}{2} - 1}) \end{aligned} \quad (19)$$

Interpretations: In view of (18) and (19), one can see that the NLOS will suffer from its higher path loss exponent in the low number of antenna case, resulting in smaller capacity compared to the LOS case. However, conform to intuition its capacity performance is linear in M in the high SNR case thanks to the multiplexing gain and will take over the LOS performance when M is larger than a certain threshold. This is contrast with the coverage performance where such a threshold does not always exist.

5. NUMERICAL EVALUATION

We evaluate analytically the coverage (km) and cell capacity performance of MIMO systems with various number of antennas ($N = M$) in both LOS (flat) and NLOS (urban) settings. We use the highly popular COST-HATA 231 path loss models [9] and assume a standard link budget for an outdoor wireless WAN scenario (P_t

is 1W, $G_t = 16dB, Gr = 6dB$). We find in this case $\alpha_l = 68$dB, $\epsilon_l = 2$ and $\alpha_{nl} = 27$dB, $\epsilon_{nl} = 3.8$.

We first look at a low rate system with a target rate of $C_{min} = 0.5$ Bits/Sec/Hz at cell edge. Fig. 1 shows the cell radius as function of M, showing clearly the huge disadvantage of NLOS for such a system regardless of how many antennas are used. In Fig. 2, we look at high rate systems with target rates of $C_{min} = 20, 24, 28$ Bits/Sec/Hz respectively, and measure the achievable cell radius as function of M. We observe three different behaviors: For $C_{min} = 20$, the LOS case is always the most favorable regardless of M, for $C_{min} = 28$, the LOS case is always the *least* favorable. Finally for the intermediate capacity target, the result depends on the number of antennas.

Finally, in Fig. 3, we fix the cell size to small values $d_0 = 1, 2, 3km$ and measure the cell capacity. As predicted, the NLOS scenario eventually leads to the best performance provided enough antennas are used.

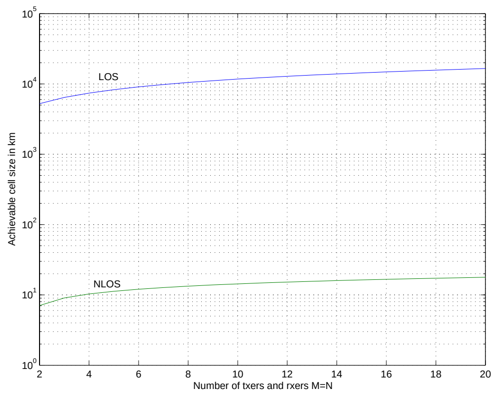

Figure 1: Coverage for low-rate system with target capacity 0.5 Bit/Sec/Hz. The LOS scenario is best regardless of how many antennas are used.

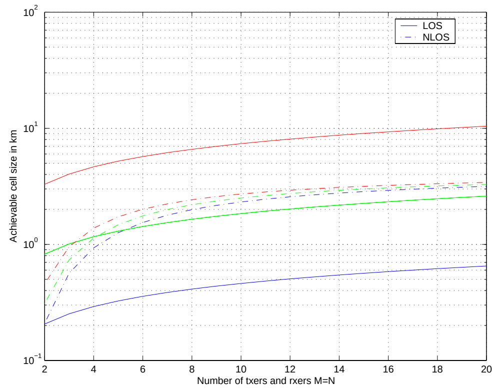

Figure 2: Coverage for high-rate system with target capacity 20,24,28 Bits/Sec/Hz (from top to bottom). LOS shown in solid, NLOS in dash-dotted. Which propagation scenario is best depends on the target capacity and M.

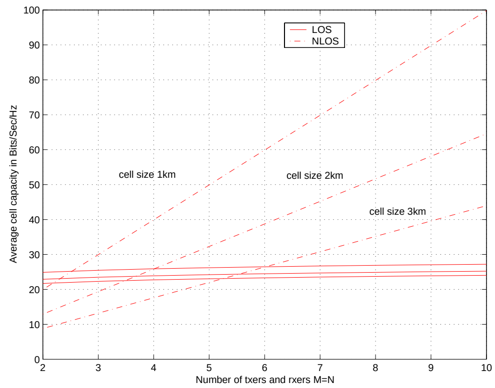

Figure 3: Cell capacity for high-rate system with cell radius 1,2,3 km (from top to bottom). LOS shown in solid, NLOS in dash-dotted. The NLOS scenario ends up being the most favorable once enough antennas are used.

6. CONCLUSION

For a high-rate system, a NLOS scenario provides the best capacity when the number of antenna exceeds a threshold depending on the power budget parameters. In terms of coverage, which propagation scenario is best depends on the required capacity level. For a very high-rate system, the NLOS is also the most favorable in terms of coverage. For a low rate coverage-oriented system, the LOS scenario is the most favorable regardless of how many antennas are used.

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