Selective Multi-User Diversity

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Abstract – This paper considers the problem of multi-user wireless scheduling using the so-called multi-user diversity (MUDiv) concept, which requires instantaneous channel feedback from from all users to the base station. We ask ourselves whether this feedback is justified in view of the increased capacity post scheduling. We revisit the MUDiv concept and propose a technique allowing to dramatically reduce the feedback (by up to 90%) needs while preserving the essential of the scheme performance. The technique is analyzed both analytically and through simulations.

1. INTRODUCTION

Multi-user diversity was introduced first by Knopp and Humblet [1], then recently extended by Tse [2, 3], as a means to provide diversity against channel fading in multi-user packet-switched wireless networks. Assuming a reasonably large number of users are active in a given cell, and assuming they experience independent time-varying fading conditions, the network scheduler extracts a diversity gain by granting access to the channel only to those users which are close to a peak in terms of transmission quality (signal-to-noise ratio or normalized capacity). In the case of stationary users, channel time selectivity can be artificially introduced through random multi-antenna combining [4]. Interestingly, MUDiv is often seen as a competing approach to MIMO diversity (e.g. space time codes [5]) with MUDiv having the advantage of riding on the users's SNR peaks rather than solely eliminating the SNR fades.

In order to manage the priorities among the subscribers, the scheduler requires the channel quality information of all users at all times. In non-reciprocal wireless links (in fact most systems today, e.g. FDD networks such as 3G-FDD) this information must be fed back regularly by all users and as often as the channel changes (up to 200HZ for vehicular applications) via dedicated or contention-based uplink channels. The spectrum resource that must be provisioned to carry this amount of feedback with an acceptable error and latency performance makes true MUDiv hardly practical when the number of active users is high [6].

In this paper we revist the concept of MUDiv and ask ourselves how much feeback is the capacity gain given by MUDiv really worth? Also, can we reduce the amount of feedback and still preserve the scheduler performance?

We make the point that, thanks to the multi-user diversity effect, the user to be scheduled is bound to have a "good enough" channel, thus giving the opportunity to greatly limit the range of feedback. Hence we introduce a new and simple scheduling technique, referred to as *Selective Multi-user Diversity (SMUD) scheduling* in which each user compares its channel quality to a threshold only those who fall above it are allowed to request access and feedback their achievable downlink transmission rate, the others remaining silent. Beyond this contribution we also optimize the threshold to attain a prescribed level of scheduling outage and analyze the network feedback load (average and standard deviation) as function of the chosen threshold. Finally, we study the system average capacity and outage capacity as function of the feedback load analytically and show that a dramatic reduction in the feedback is possible while preserving most of the capacity performance.

2. SYSTEM AND SIGNAL MODELS

We consider a single cell with K active users served by one access point (AP). The scheduling is organized on a slot by slot basis. For the sake of exposition, we consider here a SISO system (i.e. single transmit single receive) so the channel quality of each user is reduced to its instantaneous signal to noise ratio. However, a MIMO extension can be easily devised.

2.1. SNR and Transmission Rate

Each user k, $k = 1..K$ is experiencing a fading link to/from the AP, with signal-to-noise ratio during slot s given by $\gamma_k(s)$ where $\gamma_k(s)$ is the SNR of a Rayleigh fading channel. For simplicity of exposition here, the average SNR is assumed to be the same for all users $\bar{\gamma} = E(\gamma_k(s))$, where the expectation operator $E(.)$ is also equivalent to a time average. For a SISO system, the Shannon capacity achievable by user k over time slot s, if that user was selected for transmission, is simply

$$C(k,s) = \log_2(1+\gamma_k(s)). \quad (1)$$

In what follows we follow the work of [3] in using the capacity in (1) as a measure of the transmission rate used by the scheduler.

3. MUDIV AND SCHEDULING ALGORITHMS

3.1. Proportional Fair Scheduling

In a MUDiv framework, the scheduler grants access to the channel to the user with the best relative conditions in terms of rate at any given slot s. To maintain fairness over a given finite time horizon the scheduler can exploit normalized metrics that takes into account the accumulated throughput up to slot s. An example of this is given by the PFS technique below [2, 3]:

At slot s, we schedule user $k_*(s)$ with maximum normalized capacity, i.e. such that

$$k_*(s) = \text{argmax}_{k=1..K}\{\frac{C(k,s)}{R(k,s)}\}, \quad (2)$$

where $R(k,s)$ is the actual transmission throughput of user k over the link up to slot s. The throughputs are updated on a per slot basis according to:

$$\begin{aligned} R(k,s+1) &= R(k,s)(1-1/t_c) \quad k \neq k_* & (3)\\ R(k_*,s+1) &= R(k_*,s) + C(k_*,s)/t_c, & (4) \end{aligned}$$

where t_c is a time constant adjusted to maintain fairness over a pre-determined time horizon, i.e. The larger t_c, the longer the horizon, the less stringent the fairness constraint.

3.2. Max SNR Scheduling

Importantly, when all users experience the same SNR distribution, a simpler scheduler giving access to the user such that:

$$k_*(s) = \text{argmax}_{k=1..K}\{C(k,s)\} = \text{argmax}_k\{\gamma_k(s)\} \quad (5)$$

will, by symmetry, also maintain fairness over a "long enough" horizon. In that case, the scheduling strategy (5) above is approximately equal to (2) with large enough t_c.

4. SELECTIVE MULTI-USER DIVERSITY

4.1. SNR thresholding

Because the user to be scheduled for transmission is the one with best relative channel conditions it is unlikely that, for a reasonable total number of users K, a user with bad relative signal-to-noise ratio will be selected by the scheduler. To exploit this, we propose to consider a subset of users, coined *feedback users* by only considering those for which the channel quality is greater than a prescribed threshold, resulting in what is referred to here as Selective Multi-User Diversity (SMUD). Users decide *locally* to attempt to access the channel and send feedback to the AP or not. In the negative case, they remain silent for that slot. We may for instance define the threshold in terms of the instantaneous SNR γ_{th}. At slot s, user k will feedback its channel quality to the AP if and only if

$$\text{Feedback condition: } \gamma_k(s) \geq \gamma_{th}.$$

4.2. Scheduling outage

Let $P(s)$ be the number of feedback users at slot s, defined by

$$P(s) = \text{card}\{k, \text{ such that } \gamma_k(s) \geq \gamma_{th}\},$$

where card is the cardinal operator. When $P(s) > 0$, the scheduler performs the selection as in (2) or (5) but this time within the set of feedback users only. The throughput is updated normally as in (3),(4).

In the opposite case, $P(s) = 0$ and no user feeds back any information to the AP, in which case we declare a *scheduling outage*. In the event of an outage, the scheduler reverts to a conventional 'blind' fair selection mode (e.g. round robin, random user pick etc..).

5. PERFORMANCE ANALYSIS OF SELECTIVE MU DIVERSITY

There are several metrics of interest in assessing the performance of the SMUD scheme described above. In what follows we proceed with some analytical derivations of key metrics in view of optimizing the threshold.

5.1. System Capacity

In this subsection, we first characterize the statistics of the post-scheduling SNR. We then use these results to obtain the average system capacity.

5.1.1. CDF and PDF of SNR

For large values of the time constant and for the case $\bar{\gamma}_k = \bar{\gamma}$, the PFS becomes equivalent to scheduling the user with maximum SNR $\gamma_k(s)$. Hence we take:

$$\begin{aligned} \gamma_{k_*}(s) &= \max\{\gamma_1(s), \gamma_2(s), .., \gamma_K(s)\} \text{ if } P(s) > 0\\ \gamma_{k_*}(s) &= \text{rand}\{\gamma_1(s), \gamma_2(s), .., \gamma_K(s)\} \text{ if } P(s) = 0 \end{aligned}$$

where rand is the random pick operator.

For simplicity of notation, let γ^* denote the SNR post scheduling. The CDF of γ^*, denoted by $P_{\gamma^*}(\gamma)$ is by definition the probability that γ^* falls below γ. Two cases have to be considered. First when $\gamma \leq \gamma_{th}$ then $P_{\gamma^*}(\gamma)$ is equal to the probability that all users SNRs are below γ_{th} and that the randomly picked user SNR is below γ.

$$P_{\gamma^*}(\gamma) = (P_\gamma(\gamma_{th}))^{K-1} \ P_\gamma(\gamma), \qquad \gamma \leq \gamma_{th}, \quad (6)$$

where $P_\gamma(\gamma)$ is the CDF of the users' SNR and is for example given by $P_\gamma(\gamma) = 1 - e^{-\gamma/\gamma_{th}}$ in the Rayleigh fading case. For the second case (i.e., $\gamma > \gamma_{th}$), $P_{\gamma^*}(\gamma)$ is given by

$$P_{\gamma^*}(\gamma) = \sum_{k=0}^{K} \binom{K}{k} (P_\gamma(\gamma_{th}))^{K-k} \ (P_\gamma(\gamma) - P_\gamma(\gamma_{th}))^k . \quad (7)$$

Taking the derivative of $P_{\gamma^*}(\gamma)$ in (6) and (7) with respect to γ, we obtain the PDF of γ^*, $p_{\gamma*}(\gamma)$ as

$$\begin{aligned} p_{\gamma*}(\gamma) &= (P_\gamma(\gamma_{th}))^{K-1} \ p_\gamma(\gamma), \qquad \gamma \leq \gamma_{th} \\ p_{\gamma*}(\gamma) &= \sum_{k=1}^{K} \binom{K}{k} (P_\gamma(\gamma_{th}))^{K-k} \ k \ p_\gamma(\gamma) \\ &\quad \times (P_\gamma(\gamma) - P_\gamma(\gamma_{th}))^{k-1}, \qquad \gamma > \gamma_{th}, \quad (8) \end{aligned}$$

where $p_\gamma(\gamma)$ is the PDF of the SNR, given by $p_\gamma(\gamma) = \frac{1}{\bar{\gamma}} e^{-\gamma/\bar{\gamma}}$ in the Rayleigh fading case.

5.1.2. Average Capacity

The system average capacity is given by

$$E(C_*) = \int_0^\infty \log_2(1 + \gamma*) \ p_{\gamma*}(\gamma*) \ d\gamma*, \quad (9)$$

where $p_{\gamma^*}(\cdot)$ was obtained in (8). For the Rayleigh fading case, it can be shown using integration by part, the binomial expansion, and equations [7, Eqs. (3.352.1) and (3.352.2)], that the average system capacity is expressible in terms of first order exponential integral functions $E1(x) = \int_1^\infty e^{-xt}/t \ dt$ as [8]:

$$\begin{aligned} E(C_*) &= \log_2(e) \left(1 - e^{-\gamma_{th}/\bar{\gamma}}\right)^{K-1} \\ &\left[e^{1/\bar{\gamma}} \left(E_1\left(\frac{1}{\bar{\gamma}}\right) - E_1\left(\frac{1+\gamma_{th}}{\bar{\gamma}}\right)\right) - e^{-\gamma_{th}/\bar{\gamma}} \ \ln(1+\gamma_{th})\right] \\ &+ \log_2(e) \sum_{k=1}^{K} \binom{K}{k} k \left(1 - e^{-\gamma_{th}/\bar{\gamma}}\right)^{K-k} \\ &\sum_{n=0}^{k-1} \binom{k-1}{n} (-1)^n \frac{e^{(-k+1+n)\gamma_{th}/\bar{\gamma}}}{n+1} \\ &\left[e^{-(n+1)\gamma_{th}/\bar{\gamma}} \ \ln(1+\gamma_{th}) + e^{\frac{n+1}{\bar{\gamma}}} \ E_1\left(\frac{(n+1)(1+\gamma_{th})}{\bar{\gamma}}\right)\right]. \end{aligned} \quad (10)$$

5.2. Scheduling Outage Probability

Here we give an expression for $P_o = \text{Prob}(P(s) = 0)$. This event corresponds to the probability that all users fail to exceed the predetermined threshold γ_{th}, i.e.,

$$P_o = \text{Prob}(\gamma_k(s) < \gamma_{th}, \text{ for all } k = 1 \cdots K).$$

Assuming again that all users experience i.i.d. fading with the same average SNR $\bar{\gamma}$ then we have

$$P_o = (P_\gamma(\gamma_{th}))^K,$$

which for the Rayleigh fading case can be written as

$$P_o = \left(1 - e^{-\gamma_{th}/\bar{\gamma}}\right)^K. \quad (11)$$

5.3. Feedback Load

We are interested in quantifying the reduction in the feedback load obtained by adopting the SMUD scheme instead of the classical full feedback MUDiv algorithm.

The normalized average feedback load $\bar{F}$ is defined as the ratio of the average load per time slot by the total number of users K. Mathematically this can be simply written as

$$\bar{F} = \frac{E(P(s))}{K}. \quad (12)$$

The conditional probability that k out of K users are pre-selected during a particular time slot is equal to the conditional probability that the SNRs of these k users equal or exceed the threshold γ_{th} (and of course the remaining $K-k$ users SNRs do not), i.e. $(1 - P_\gamma(\gamma_{th}))^k \ (P_\gamma(\gamma_{th}))^{K-k}$. Thus, for i.i.d. fading among users, the probability that $P(s) = k$ during time slot s is equal to

$$\text{P}_k = \binom{K}{k} (1 - P_\gamma(\gamma_{th}))^k \ (P_\gamma(\gamma_{th}))^{K-k}. \quad (13)$$

Therefore the normalized average load is given in these conditions by

$$\bar{F} = \frac{1}{K} \sum_{k=0}^{K} k \binom{K}{k} (1 - P_\gamma(\gamma_{th}))^k \ (P_\gamma(\gamma_{th}))^{K-k} \quad (14)$$

which simplifies to

$$\bar{F} = 1 - P_\gamma(\gamma_{th}), \quad (15)$$

and reduces to

$$\bar{F} = e^{-\gamma_{th}/\bar{\gamma}} \quad (16)$$

in the Rayleigh fading case.

In [8], one interesting result is obtained, showing that the feedback load actually converges to a fixed deterministic (lower) value in the large K region, helping the provisioning of the feedback channel in practice.

5.4. Threshold Choice

Various strategies are possible for optimizing the threshold γ_{th}, including choosing it to reach a pred-determined scheduling outage probability P_o. For instance in i.i.d. fading environment, inverting (11) leads to

$$\gamma_{th} = -\overline{\gamma} \ \ln\left(1 - P_o^{1/K}\right).$$

If the feeback channel is narrow, it may be also of interest to choose the threshold γ_{th} in order to meet a certain normalized average feedback load specification. In i.i.d Rayleigh fading this can be obtained from (16) as

$$\gamma_{th} = -\overline{\gamma} \ \ln(\bar{F}).$$

6. SIMULATIONS

We compare the performance of SMUD with a full feedback scheme, both using the PFS algorithm with a time constant of $t_c = 500$ (slots). We give comparison with the analytical results shown earlier (SNR of 5dB).

Fig. 1 shows the average system capacity versus number of users for the SMUD scheme with γ_{th} given successively by (from top down): 0dB, 3dB, 6dB, 9dB, 12dB, 15dB, 18dB. We give the performance of the full feedback MUDiv scheme for comparison. Relatively little is lost in performance below 9dB threshold. The analytical results are shown in dashed curves, showing the quality of the prediction.

Fig. 2 shows the feedback load ($\bar{F}$) as function the threshold γ_{th}. At 9dB theshold, the load is less than 10% of what it is with the original PFS algorithm.

Fig. 3 shows the system capacity in relation to the required feedback load, for various number of users. This confirms that, for K above 25 or so, a feeback load greater than 10% results in very little aditional gain in terms of capacity and is therefore unnecessary.

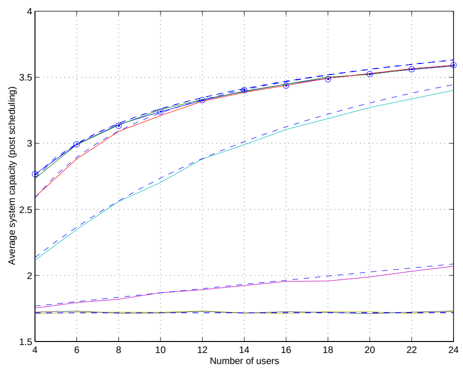

Figure 1: Average capacity for thresholds $\gamma_{th} = 0, 3, 6, 9, 12, 15, 18dB$ (from top down). Dotted is the full feedback PFS algorithm. Dashed is the theoretical result.

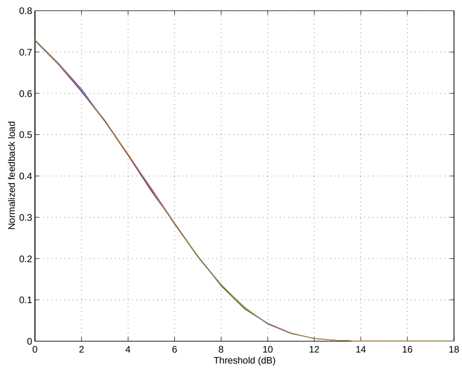

Figure 2: Average feedback load vs. threshold γ_{th}. Dashed (superposed) is the theoretical result.

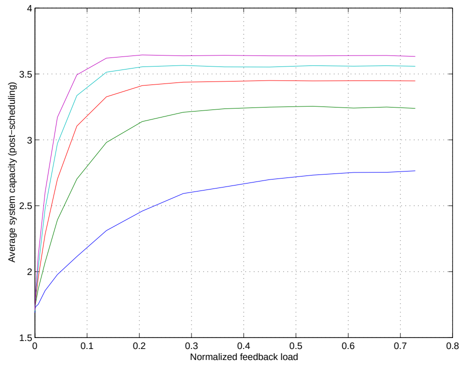

Figure 3: Average capacity for $K = 4, 10, 16, 22, 28$ (bottom to top).

7. CONCLUSION

Selective multi-user diversity reduces dramatically (by as much as 90%) the feedback required by the traditional MUDiv scheme while preserving most of its performance. Ongoing extensions of the analysis include cases of unequal SNRs and MIMO cases.

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