

The effect of line of sight components on the asymptotic capacity of MIMO systems

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Abstract

The asymptotic theoretic capacity of a MIMO system is derived when considering Rice distribution entries. Although the Rice distribution is well known to enhance the capacity performance with respect to the Rayleigh distribution in the SISO case, this assumption is seriously put into doubt in the MIMO case. Indeed, the conditions under which line of sight improves or not the capacity are still not well understood. This contribution determines the parameters of interest for analyzing Rice MIMO models and gives an explicit theoretic answer to the previous question.

I. INTRODUCTION

The problem of analyzing channel models is crucial for the efficient design of wireless systems [1]. Unlike the Additive White Gaussian channel, the wireless channel suffers for constructive/destructive interference signaling [2]. This yields a randomized channel with arbitrary statistics. Recently [3], [4], the need to increase spectral efficiency has motivated the use of multiple antennas at both the transmitter and the receiver side. Hence, in the case of the i.i.d Gaussian model and channel knowledge at the receiver, it has been proved [5] that the ergodic capacity increase is $\min(r,t)$ bits per second per hertz for every 3dB increase (r is the number of receiving antennas and t is the number of transmitting antennas) at high SNR¹. However, for other channel models, results are still unknown and may seriously put into doubt the MIMO hype. In particular, the effect of line of sight on the overall performance has still not been analyzed theoretically. Even though recent papers [6] have shown that the Rice distribution may incur a loss with respect to the i.i.d Rayleigh case, under what conditions this result is always true is still an open problem as recently put into question in [7]. Before going further, let us introduce the model if interest.

A. Channel Model

We assume that the transmission takes places between a mobile transmitter and receiver (see figure 1). The transmitter has t antennas and the receiver has r antennas. Moreover, we assume that the input transmitted signal goes through a time invariant linear filter channel. Finally, we assume that the interfering noise is additive white Gaussian. The transmitted signal and received signal are therefore related as:

$$y(t) = \sqrt{\frac{\rho}{t}} \int \mathbf{H}_{r \times t}(\tau) x(t - \tau) d\tau + n(t)$$

and

$$Y(f) = \sqrt{\frac{\rho}{t}} \mathbf{H}_{r \times t}(f) X(f) + N(f)$$

ρ is the received SNR, $Y(f)$ is the $r \times 1$ received vector (Fourier transform of the time signal $y(t)$), $X(f)$ is the $t \times 1$ transmit vector (Fourier transform of the time signal $x(t)$), $N(f)$ is an $r \times 1$ additive standardized white Gaussian noise vector (Fourier transform of $n(t)$). In all the following, without loss of generality, we will consider a channel with real entries. We will suppose that the average energy power of the channel is normalized such as: $\frac{1}{rt} \mathbb{E}(\text{trace}(\mathbf{H}\mathbf{H}^T)) = 1$ and use the notation $\bar{C} = \frac{1}{\sqrt{t}} C$ for any matrix C .

B. Statement of the problem

Although a Rice distribution is well known to enhance the performance with respect to the Raleigh one in the SISO case, these results cannot be straightforwardly extended to the MIMO case. Indeed, suppose that the channel matrix is deterministic with equal entries 1 (this is a limiting case of a Rice distribution with variance 0). In this case, the mutual information per receiving antenna with input Gaussian entries and covariance matrix $\mathbb{E}(X X^H) = \mathbf{I}$ is given by: $C = \frac{1}{r} \log_2 \det(\mathbf{I}_r + \frac{\rho}{t} \mathbf{H}\mathbf{H}^T) = \frac{1}{r} \sum_{i=1}^r \log_2(1 + \frac{\rho}{t} \lambda_i)$. In this case, since $\mathbf{H}\mathbf{H}^T$ is rank one, it has one single eigenvalue equal to rt and the capacity tends to:

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¹In the single antenna additive Gaussian channel, 1 bit per second per hertz can be achieved with every 3dB increase at high SNR

$C = \frac{1}{r} \log_2(1 + \rho r) \rightarrow 0$ when $r \rightarrow \infty$. This result shows that the line of sight component has a dramatic effect on the mutual information since it is well known that in the independent Rayleigh fading case, the capacity per antenna is constant.

However, suppose now that the mean of the different line of sight components are such as the matrix $\mathbf{H}\mathbf{H}^T = t\mathbf{I}_r$ then: $C = \frac{1}{r} \sum_{i=1}^r \log_2(1 + \rho) = \log_2(1 + \rho)$ which is non-zero.

Although we have only taken two extreme cases (the variance of the Rice distribution has not been taken into account), these trivial examples show that a more profound analysis should be conducted for determining the parameters governing the performance of the Rice distribution with respect to the i.i.d zero mean Gaussian case (i.i.d zero mean Gaussian entries).

II. MUTUAL INFORMATION FOR A RICE MODEL

A. Rice Model

We suppose in this section line of sight components in the MIMO transmission scheme. The frequency paths h_{ij} are assumed to have different mean $\mu_{i,j}$ (zero or not) but the same variance σ^2 .

In this case, the channel can be written as $H = A + B$ where A is the deterministic line of sight component part of the matrix such as each entry $a_{ij} = \alpha_{ij}$ and B is a gaussian zero mean i.i.d matrix such as each entry b_{ij} has a variance of σ^2 . In order to derive the asymptotic channel mutual information, we will make the following assumption.

Assumption 1: The matrix size $\frac{1}{t}AA^T$ grows large with $\beta = \frac{r}{t}$ remaining fixed such as the empirical eigenvalue distribution $F_{r,t}^{\overline{AA^T}}$ of $\frac{1}{t}AA^T$ converges in distribution to a fixed $F^{\overline{AA^T}}$.

In this case, let us express the average energy power of the channel:

$$\begin{aligned} \frac{1}{rt} \mathbb{E}(\text{trace}(\mathbf{H}\mathbf{H}^T)) &= \frac{1}{rt} \mathbb{E}(\text{trace}(\mathbf{A}\mathbf{A}^T + \mathbf{B}\mathbf{B}^T + \mathbf{A}\mathbf{B}^T + \mathbf{B}\mathbf{A}^T)) \\ &= \frac{1}{rt} \mathbb{E}(\text{trace}(\mathbf{A}\mathbf{A}^T + \mathbf{B}\mathbf{B}^T)) \\ &= \frac{1}{rt} \sum_{i,j} \alpha_{ij}^2 + \sigma^2 \rightarrow \int \lambda dF^{\overline{AA^T}}(\lambda) + \sigma^2 \end{aligned}$$

In order to compare the performance with the i.i.d Gaussian case, the following constraint is put on σ^2 :

$$\sigma^2 = 1 - \int \lambda dF^{\overline{AA^T}}(\lambda) \geq 0$$

B. Result

In the case of the previous Rice Model, the following theorem holds:

Theorem 1: As $t \rightarrow \infty$ with $r = \beta t$, the asymptotic mutual information with Gaussian input entries is given by:

$$C_{\text{Rice}} = \int_0^\rho \frac{1}{2\ln(2)\rho} \left(1 - \frac{1}{\rho} m_{\overline{HH^T}}\left(-\frac{1}{\rho}\right) \right) d\rho$$

with

$$\begin{aligned} m_{\overline{HH^T}}\left(\frac{-1}{\rho}\right) &= \frac{\psi - 1}{\beta\sigma^2} \\ \psi &= 1 + \sigma^2 \rho \psi \beta \int \frac{dF^{\overline{AA^T}}(\lambda)}{\rho\lambda + \psi^2 + \rho\sigma^2(1 - \beta)\psi} \\ \sigma^2 &= 1 - \int \lambda dF^{\overline{AA^T}}(\lambda) \end{aligned}$$

The proof of this theorem is provided in section VI and is based on results due to Girko [8]. This theorem is quite useful as it highlights that only the limiting distribution of the mean matrix $\frac{1}{t}\mathbf{A}\mathbf{A}^T$ matters (and not at all the explicit values of the mean). Note that the formula is general enough to incorporate the asymptotic capacity of the Rayleigh channel as a special case (by letting $dF^{\overline{AA^T}}(\lambda) = \delta(\lambda)$). Note also that the previous formula is also valid in the complex case.

III. SIMULATIONS

In all the following, we assume that $r = t$. Many scenarios of the mean matrix can be taken into account. As an example, let us assume the best scenario for the line of sight components i.e the deterministic line of sight component \mathbf{A} has equal entries $a_{ij} = (-\alpha, \alpha)$ such as the columns of matrix \mathbf{A} are orthogonal. In this case, $dF^{\overline{\mathbf{A}\mathbf{A}^T}}(\lambda) = \delta(\lambda - \alpha^2)$ and since $\beta = 1$, we have:

$$\Psi = 1 + \frac{(1 - \alpha^2)\rho\Psi}{\rho\alpha^2 + \Psi^2}$$

$$m^{\overline{\mathbf{H}\mathbf{H}^T}}\left(-\frac{1}{\rho}\right) = \frac{\rho((1 - \alpha^2)m^{\overline{\mathbf{H}\mathbf{H}^T}}\left(-\frac{1}{\rho}\right) + 1)}{\rho\alpha^2 + ((1 - \alpha^2)m^{\overline{\mathbf{H}\mathbf{H}^T}}\left(-\frac{1}{\rho}\right) + 1)^2}$$

and the asymptotic mutual information is solution of:

$$C_{\text{Rice}} = \int_0^\rho \frac{1}{2\ln(2)\rho} \left(1 - \frac{1}{\rho} m^{\overline{\mathbf{H}\mathbf{H}^T}}\right) d\rho$$

In figure 2, we have plotted the mutual information versus α for an 8×8 complex MIMO system with an orthogonal mean matrix at an SNR of 10dB. As one can see, the theoretical formula matches the asymptotic curves with a quite small number of antennas. The best performance in this case is obtained when $\alpha = 1$. However, one should note that for $0 \leq \alpha \leq 0.5$, orthogonal Rice fading has nearly no impact on the performance and behaves as complete zero mean i.i.d entries. In other words, orthogonal Rice fading achieves a significant gain only when the mean is superior to the variance.

IV. CONCLUSIONS

In this contribution, we have studied the influence of line of sight components on the overall performance of MIMO systems. Although in the SISO case, it is well acknowledged that the capacity of Rice fading outperforms Rayleigh fading, in the MIMO case, this result does not hold: the capacity in this case depends only on the limiting behavior of the eigenvalues of the mean matrix through an implicit equation given by theorem 1².

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VI. PROOF OF THEOREM 1

To derive the proof, we will use the following theorem due to Girko [8]:

Theorem 2: Assume that the random entries $h_{ij}^{(t)}$, $i = 1, \dots, r_t$ and $j = 1, \dots, t$, of the matrix $\mathbf{H}_{r_t \times t} = (h_{ij}^{(t)})_{i=1, \dots, r_t}^{j=1, \dots, t}$ are independent for any t ,

$$\mathbb{E}(h_{ij}^{(t)}) = \alpha_{ij}^{(t)}, \quad \text{Var}(h_{ij}^{(t)}) = (\sigma_{ij}^{(t)})^2, \quad 0 < \liminf_{t \rightarrow \infty} \frac{r_t}{t} < \limsup_{t \rightarrow \infty} \frac{r_t}{t} < \infty,$$

$$\sup_t \max_{\substack{i=1, \dots, r_t \\ j=1, \dots, t}} \left[\sum_{j=1}^t (\sigma_{ij}^{(t)})^2 + \sum_{i=1}^{r_t} (\sigma_{ij}^{(t)})^2 \right] < \infty, \quad (1)$$

$$\sup_t \max_{\substack{i=1, \dots, r_t \\ j=1, \dots, t}} \left[\sum_{j=1}^t |\alpha_{ij}^{(t)}| + \sum_{i=1}^{r_t} |\alpha_{ij}^{(t)}| \right] < \infty, \quad (2)$$

Lindeberg's condition is supposed to be satisfied, i.e., for every $\tau > 0$,

$$\lim_{t \rightarrow \infty} \max_{\substack{i=1, \dots, r_t \\ j=1, \dots, t}} \left[\sum_{j=1}^t \mathbb{E}[h_{ij}^{(t)} - \alpha_{ij}^{(t)}]^2 \chi\{|h_{ij}^{(t)} - \alpha_{ij}^{(t)}| > \tau\} \right. \\ \left. + \sum_{i=1}^{r_t} \mathbb{E}[h_{ij}^{(t)} - \alpha_{ij}^{(t)}]^2 \chi\{|h_{ij}^{(t)} - \alpha_{ij}^{(t)}| > \tau\} \right] = 0 \quad (3)$$

²Note that the conclusion here differ with respect to [7] as we constrain ourselves in all the study to a power limited channel

$$\mu_{r_t}\{x, \mathbf{H}_{r_t \times t} \mathbf{H}_{r_t \times t}^T\} = r_t^{-1} \sum_{k=1}^{r_t} \chi\{\omega : \lambda_k < x\}, \quad (4)$$

and $\lambda_1 \geq \dots \geq \lambda_{r_t}$ are the eigenvalues of the random matrix $\mathbf{H}_{r_t \times t} \mathbf{H}_{r_t \times t}^T$.

Then with probability one,

$$\lim_{t \rightarrow \infty} |\mu_{r_t}\{x, \mathbf{H}_{r_t \times t} \mathbf{H}_{r_t \times t}^T\} - F_{r_t}(x)| = 0, \quad (5)$$

where $F_{r_t}(x)$ is the random distribution function whose Stieltjes transform is given by the formula

$$\int_0^\infty (x-z)^{-1} dF_{r_t}(x) = r_t^{-1} \text{Tr}[\mathbf{C}_1 + \mathbf{A}_{r_t \times t} \mathbf{C}_2^{-1} \mathbf{A}_{r_t \times t}^T]^{-1}, \quad (6)$$

$z = u + iv$, $v \neq 0$, $\mathbf{A}_{r_t \times t} = (\alpha_{ij}^{(n)})_{i=1, \dots, r_t}^{j=1, \dots, t}$, and $\mathbf{C}_1 = (c_{1i} \delta_{ij})_{i,j=1}^{r_t}$ and $\mathbf{C}_2 = (c_{2i} \delta_{ij})_{i,j=1}^t$ are diagonal matrices with

$$c_{1p} = -z + \sum_{j=1}^t (\sigma_{pj}^{(t)})^2 \{[\mathbf{I}_{t \times t} - z^{-1} \mathbf{H}_{r_t \times t} \mathbf{H}_{r_t \times t}^T]^{-1}\}_{jj}, \quad p = 1, \dots, r_t,$$

$$c_{2k} = 1 + \sum_{j=1}^{r_t} (\sigma_{jk}^{(t)})^2 \{[-z \mathbf{I}_{r_t \times r_t} + \mathbf{H}_{r_t \times t} \mathbf{H}_{r_t \times t}^T]^{-1}\}_{jj}, \quad k = 1, \dots, t,$$

$p \lim_{t \rightarrow \infty} \{c_{1p} - \varphi_p\} = 0$, and $p \lim_{t \rightarrow \infty} \{c_{2k} - \psi_k\} = 0$, $p = 1, \dots, r_t$, $k = 1, \dots, t$, where the variables φ_p and ψ_k satisfy the following system of canonical equations

$$\varphi_p = -z + \sum_{j=1}^t (\sigma_{pj}^{(t)})^2 \{[(\delta_{ij} \psi_i)_{i,j=1}^t + \mathbf{A}_{r_t \times t}^T [\delta_{ij} \varphi^{-1}]_{i,j=1}^{r_t} \mathbf{A}_{r_t \times t}]^{-1}\}_{jj}, \quad p = 1, \dots, r_t, \quad (7)$$

$$\psi_k = 1 + \sum_{j=1}^{r_t} (\sigma_{jk}^{(t)})^2 \{[(\delta_{ij} \varphi_i)_{i,j=1}^{r_t} + \mathbf{A}_{r_t \times t} [\delta_{ij} \psi^{-1}]_{i,j=1}^t \mathbf{A}_{r_t \times t}^T]^{-1}\}_{jj}, \quad k = 1, \dots, t. \quad (8)$$

There exists a unique solution of the previous system of the canonical equations in the class of analytic functions

$$K = \{\text{Im} \varphi_p(z) < 0, \text{Im} \psi_k(z) > 0, \text{Im} z > 0, k = 1, \dots, t, p = 1, \dots, r_t\}.$$

Applying Theorem 2 to the case of a matrix $\overline{\mathbf{H}}$ with constant variances $(\sigma_{ij}^{(t)})^2 = \frac{\sigma^2}{t}$, we can derive the Stieltjes transform of the matrix $\frac{\mathbf{H}_{r_t \times t} \mathbf{H}_{r_t \times t}^T}{t}$ when the dimensions of the matrix tend to infinity with constant ratio. Condition (1) of Theorem 2 is always verified thanks to the assumption that $\text{Var}(\overline{h}_{ij}^{(n)}) = \frac{\sigma^2}{t}$. Condition (2) follows from the hypothesis that the eigenvalue distribution of matrix $\frac{1}{t} \mathbf{A}_{r_t \times t} \mathbf{A}_{r_t \times t}^T$ converges weakly to a deterministic function and the constraint: $\int \lambda dF^{\overline{\mathbf{A}}^T}(\lambda) \leq 1$. Condition (3) always is satisfied for Gaussian entries with variance $\frac{\sigma^2}{t}$. In fact, $\forall \tau, \varepsilon > 0$ there exists a $t(\varepsilon, \tau)$ such that

$$L = \frac{\sigma^2}{t(\varepsilon, \tau)} \left(\sum_{j=1}^t \text{Prob}\{|\overline{h}_{ij}^{(t)} - \alpha_{ij}^{(t)}| > \tau\} + \sum_{i=1}^{r_t} \text{Prob}\{|\overline{h}_{ij}^{(t)} - \alpha_{ij}^{(t)}| > \tau\} \right) \quad (9)$$

$$= \frac{\sigma^2(t(\varepsilon, \tau) + r_t(\varepsilon, \tau))}{t(\varepsilon, \tau)} \times \left(\text{Prob}\{\overline{h}_{ij}^{(t)} - \alpha_{ij}^{(t)} > \tau\} + \text{Prob}\{\overline{h}_{ij}^{(t)} - \alpha_{ij}^{(t)} < -\tau\} \right) \quad (10)$$

$$= \frac{2\sigma^2(t + r_t)}{t} \times \sqrt{\frac{t}{2\pi\sigma^2}} \int_\tau^\infty e^{-\frac{x^2}{2\sigma^2}} dx \quad (11)$$

$$= \frac{2\sigma^2(t(\varepsilon, \tau) + r_t(\varepsilon, \tau))}{t(\varepsilon, \tau)} Q\left(\frac{\tau \sqrt{t(\varepsilon, \tau)}}{\sigma}\right) \quad (12)$$

$$< \varepsilon. \quad (13)$$

Let Φ^{r_t} and Ψ^t denote the diagonal matrices $(\delta_{ij}\varphi_i)_{i,j=1}^{r_t}$ and $(\delta_{ij}\psi_i)_{i,j=1}^t$ respectively. Then the canonical system of equations (7) and (8) can be rewritten as follows:

$$\begin{aligned}\varphi_p &= -z + \frac{\sigma^2}{t} \sum_{j=1}^t \left\{ [(\delta_{ij}\psi_i)_{i,j=1}^t + \frac{1}{t} \mathbf{A}_{r_t \times t}^T [\delta_{ij}\varphi^{-1}]_{i,j=1}^{r_t} \mathbf{A}_{r_t \times t}]^{-1} \right\}_{jj}, \quad p = 1, \dots, r_t, \\ &= -z + \frac{\sigma^2}{t} \text{trace} \left[\Psi^{(t)} + \frac{1}{t} \mathbf{A}_{r_t \times t} (\Phi^{(r_t)})^{-1} \mathbf{A}_{r_t \times t}^T \right]^{-1}, \quad k = 1, \dots, r_t.\end{aligned}\quad (14)$$

and

$$\begin{aligned}\psi_k &= 1 + \frac{\sigma^2}{t} \sum_{j=1}^{r_t} \left\{ [(\delta_{ij}\varphi_i)_{i,j=1}^{r_t} + \frac{1}{t} \mathbf{A}_{r_t \times t} [\delta_{ij}\psi^{-1}]_{i,j=1}^t \mathbf{A}_{r_t \times t}^T]^{-1} \right\}_{jj}, \quad k = 1, \dots, t, \\ &= 1 + \frac{\sigma^2}{t} \text{trace} \left[\Phi^{(r_t)} + \frac{1}{t} \mathbf{A}_{r_t \times t} (\Psi^{(t)})^{-1} \mathbf{A}_{r_t \times t}^T \right]^{-1}, \quad k = 1, \dots, t.\end{aligned}\quad (15)$$

It is apparent that φ_p , $p = 1, \dots, r_t$ are all equal and the same holds for ψ_k , $k = 1, \dots, t$. Let us denote their value respectively by φ and ψ . Then the canonical system of equations (14) and (15) is reduced to a system of two equations

$$\begin{aligned}\varphi &= -z + \frac{\sigma^2}{t} \text{trace} \left[\psi \mathbf{I}_t + \varphi^{-1} \frac{1}{t} \mathbf{A}_{r_t \times t}^T \mathbf{A}_{r_t \times t} \right]^{-1} \\ &= -z + \frac{\sigma^2 \varphi}{t} \text{trace} \left[\varphi \psi \mathbf{I}_t + \frac{1}{t} \mathbf{A}_{r_t \times t}^T \mathbf{A}_{r_t \times t} \right]^{-1} \\ &= -z + \sigma^2 \varphi K_1(\varphi, \psi)\end{aligned}\quad (16)$$

and

$$\begin{aligned}\psi &= 1 + \frac{\sigma^2}{t} \text{trace} \left[\varphi \mathbf{I}_{r_t} + \Psi^{-1} \frac{1}{t} \mathbf{A}_{r_t \times t} \mathbf{A}_{r_t \times t}^T \right]^{-1} \\ &= 1 + \frac{\sigma^2 \psi}{t} \text{trace} \left[\varphi \psi \mathbf{I}_{r_t} + \frac{1}{t} \mathbf{A}_{r_t \times t} \mathbf{A}_{r_t \times t}^T \right]^{-1} \\ &= 1 + \sigma^2 \psi K_2(\varphi, \psi)\end{aligned}\quad (17)$$

where $K_1(\varphi, \psi) = \frac{1}{t} \text{trace} [\varphi \psi \mathbf{I}_t + \frac{1}{t} \mathbf{A}_{r_t \times t}^T \mathbf{A}_{r_t \times t}]^{-1}$ and $K_2(\varphi, \psi) = \frac{1}{t} \text{trace} [\varphi \psi \mathbf{I}_{r_t} + \frac{1}{t} \mathbf{A}_{r_t \times t} \mathbf{A}_{r_t \times t}^T]^{-1}$. It is easy to verify that the following relation $K_1(\varphi, \psi) = K_2(\varphi, \psi) + \frac{1-\beta}{\varphi\psi}$ holds. From Equation (17) we obtain

$$K_2(\varphi, \psi) = \frac{\psi - 1}{\sigma^2 \psi}.\quad (18)$$

φ can be derived from Equation (16) and Equation (18) as function of ψ . Indeed,

$$\begin{aligned}\varphi &= -z + \sigma^2 \varphi \left(\frac{\psi - 1}{\sigma^2 \psi} + \frac{1 - \beta}{\varphi \psi} \right) \\ \varphi \left(1 - \sigma^2 \frac{\psi - 1}{\sigma^2 \psi} \right) &= -z + \sigma^2 \frac{1 - \beta}{\psi} \\ \frac{\varphi}{\psi} &= -z + \sigma^2 \frac{1 - \beta}{\psi} \\ \varphi &= -z \psi + \sigma^2 (1 - \beta)\end{aligned}\quad (19)$$

The fixed point equation of theorem 1 in the unknown variable ψ can be derived from Equation (19) and Equation (17) through:

$$\psi(z) = 1 + \sigma^2 \frac{\psi}{t} \frac{r_t}{r_t} \text{trace} \left[(-z\psi^2 + \sigma^2(1 - \beta)\psi) \mathbf{I}_{r_t} + \frac{1}{t} \mathbf{A}_{r_t \times t} \mathbf{A}_{r_t \times t}^T \right]^{-1}.\quad (20)$$

Asymptotically, as $r_t = \beta t \rightarrow \infty$, we obtain

$$\psi(z) = 1 + \sigma^2 \psi \beta \int \frac{dF^{\overline{AA}^T}(\lambda)}{\lambda - z\psi^2 + \sigma^2(1 - \beta)\psi}.\quad (21)$$

The Stieltjes transform $m_{\overline{\mathbf{H}\mathbf{H}^T}}(z)$ can be derived using Equation (6) which yields:

$$m_{\overline{\mathbf{H}\mathbf{H}^T}}(z) = \frac{\psi K_2(\psi)}{\beta} \quad (22)$$

$$= \frac{\psi(z) - 1}{\beta\sigma^2} \quad (23)$$

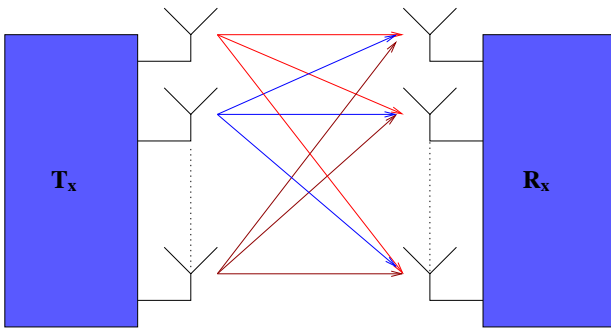
The capacity is given by: $C_{\text{Rice}}(\rho) = \int \log_2(1 + \rho\lambda) dF_{\overline{\mathbf{H}\mathbf{H}^T}}(\lambda)$

$$\begin{aligned} \frac{dC_{\text{Rice}}}{d\rho} &= \frac{1}{2\ln(2)} \int_0^\infty \frac{\lambda}{1 + \rho\lambda} dF_{\overline{\mathbf{H}\mathbf{H}^T}}(\lambda) \\ &= \frac{1}{2\ln(2)\rho} \left(1 - \int_0^\infty \frac{1}{1 + \rho\lambda} dF_{\overline{\mathbf{H}\mathbf{H}^T}}(\lambda) \right) \\ &= \frac{1}{2\ln(2)\rho} \left(1 - \frac{1}{\rho} m_{\overline{\mathbf{H}\mathbf{H}^T}}\left(-\frac{1}{\rho}\right) \right) \end{aligned}$$

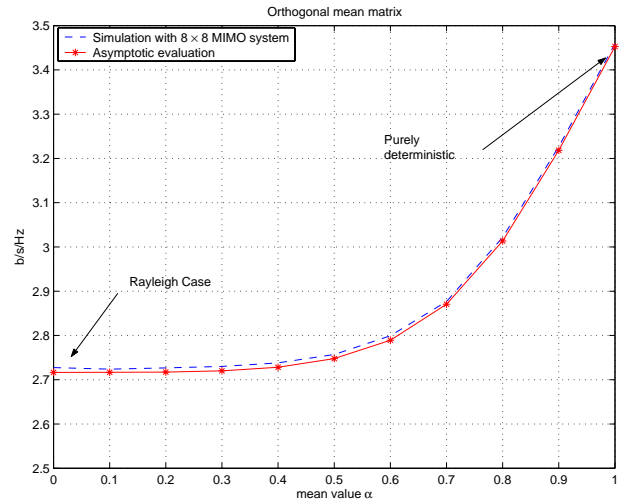
with

$$m_{\overline{\mathbf{H}\mathbf{H}^T}}\left(-\frac{1}{\rho}\right) = \frac{\psi\left(-\frac{1}{\rho}\right) - 1}{\beta\sigma^2}$$

$$\psi\left(-\frac{1}{\rho}\right) = 1 + \sigma^2\rho\psi\beta \int \frac{dF_{\overline{\mathbf{A}\mathbf{A}^T}}(\lambda)}{\rho\lambda + \psi^2 + \rho\sigma^2(1 - \beta)\psi}$$



1. MIMO channel representation



2. Mutual information with orthogonal mean matrix at 10dB

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