CAPACITY COMPLYING MIMO CHANNEL MODELS

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ABSTRACT

In [1][1], a unified framework for constructing MIMO models based on the principle of maximum entropy was provided. Several MIMO directional and double directional models were derived and in each case, the mutual information was shown to have asymptotically a Gaussian behavior. Interestingly, the results were shown to be accurate, through simulations, in the finite regime with an astonishing small number of antennas. In this paper, the models are analyzed and validated[2] based on a wideband outdoor measurement campaign carried out in Oslo during summer 2002. The measurements were performed at a center frequency of 2.1 GHz with a bandwidth of 100 MHz in three different urban scenarios: a regular street grid scenario, an open city place and an indoor cell site.

1. INTRODUCTION

In [1], several models have been developed based on the principle of consistency. A general framework using the information available was provided to model the MIMO link in the following general frequency representation:

$$Y(f) = \sqrt{\frac{\rho}{n_t}}\mathbf{H}_{n_r \times n_t}(f,t)X(f)+N(f) \qquad (1)$$

ρ is the received SNR, f, t, n_t and n_r represent respectively frequency, time, the number of transmitting and receiving antennas, $Y(f)$ is the $n_r \times 1$ received vector , $X(f)$ is the $n_t \times 1$ transmit vector , $N(f)$ is an $n_r \times 1$ additive standardized white Gaussian noise vector. The general double directional representation was shown to be given by (see figure 1):

$$\mathbf{H}(f,t) = \frac{1}{\sqrt{ss_1}}\Phi_{n_r \times s}\mathbf{P^r}\Theta_{s \times s_1}\mathbf{P^t}\Psi_{s_1 \times n_t} \qquad (2)$$

Here, s and s_1 represent the scatterers respectively at the receiving and transmitting side. $\Phi_{n_r \times s}$ represents the matrix of steering directions of arrival vectors with respective powers $\mathbf{P^r}$ whereas $\Psi_{s_1 \times n_t}$ represents the matrix of steering directions of departure vectors with respective powers $\mathbf{P^t}$. $\Theta_{s \times s_1}$ is a $s \times s_1$ matrix with i.i.d Gaussian matrix. The model is consistent in the sense that this model incorporates the i.i.d, DoA ($\Psi_{s_1 \times n_t} = \mathbf{F}_{n_t}$ is $n_t \times n_t$ Fourier transform and $\mathbf{P^t} = \mathbf{I}_{n_t}$) and DoD ($\Phi_{n_r \times s} = \mathbf{F}_{n_r}$ is $n_r \times n_r$ Fourier transform and $\mathbf{P^r} = \mathbf{I}_{n_r}$) based models as special cases from an information theoretic point of view. The goal of this paper is to validate model (2) with recent measurements performed at 2.1 Ghz in Oslo . But how to validate the different models, in other words how to choose between a set $\{M_0, M_1, ..., M_K\}$ of K models (note that M specifies only the type of model and not the parameters of the model i.e DoA, DoD or Double Directional for example)? A common misconception affirms that we can always fit the model to the data as long as one takes enough parameters for the model. In the work of Jaynes [2] and Bretthorst [3], this idea is infirmed and it is explicitly shown how probability theory can be used to rank several models. The reader can find a useful discussion in [4]. Although of limited use and as a first step, we will only, in this contribution, analyze the mutual information compliance criteria. This is motivated by the fact that in usual communications systems, engineers are interested in models that fulfill only a certain criteria. The fact that the model is adequate or not is not an issue as long as it reproduces in an accurate manner the same performance as measurements in the simulated chain. The criteria range from BER, Signal to interference ratio to mutual information. The mutual information is an interesting criteria from a network planning perspective.

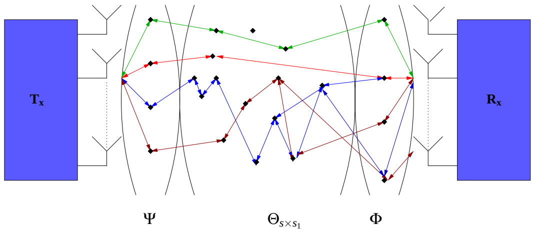

Fig. 1. Double directional based model.

[1]This work is part of the European FLOWS (Flexible Convergence of Wireless Standards and Services) project and related documents can be downloaded at http://www.flows-ist.org.

[2]The notion of model validation will be defined in this contribution.

2. MUTUAL INFORMATION COMPLYING MODELS

In [1], the mutual information distribution of many models has been provided and the cumulative distribution function of the mutual information was shown to have a Gaussian behavior of the form:

$$F(I^M) = 1 - Q(\frac{I^M - n_t\mu}{\sigma})$$

In each case (i.i.d, DoA based, DoD based and double directional), expressions of μ and σ have been provided.
For a given frequency f, a model will be called mutual information complying if it minimizes:

$$\int_0^{\infty} \mid F(I^M) - F_{\text{empirical}}(I^M, f) \mid^2 dI^M \qquad (3)$$

Here $F_{\text{empirical}}(I^M, f)$ is the empirical CDF given by measurements[3]. In general (except for the i.i.d Gaussian case where there is nothing to do), for minimizing the criteria in the directional cases, one has to optimize criteria (3) with respect to the steering directions. This is not an easy task. However, since we are interested in mutual information issues, only the non-correlated scatterers (called here the dominant scatterers) scale the mutual information and therefore we can use (as a first approximation) scatterers on Fourier directions (which is equivalent to Sayeed's Virtual Representation [5]). In this case, let us review the asymptotic parameters of each model (ρ is the SNR) given in [1]:
I.I.D Gaussian model. There is no optimization to perform in this case and μ and σ are equal to:

$$\mu = \frac{n_r}{n_t}\ln(1+\rho-\rho\alpha) + \ln(1+\rho\frac{n_r}{n_t}-\rho\alpha) - \alpha$$

$$\sigma^2 = -\ln[1 - \frac{n_t\alpha^2}{n_r}]$$

$$\alpha = \frac{1}{2}[1+\frac{n_r}{n_t}+\frac{1}{\rho} - \sqrt{(1+\frac{n_r}{n_t}+\frac{1}{\rho})^2 - 4\frac{n_r}{n_t}}]$$

DoA based model. In this case, one has to optimize criteria (3) with respect to the number of scatterers s for which the expression of μ and σ are given by:

$$\mu = \frac{s}{n_t}\ln(1+\rho\frac{n_r}{s}-\rho\frac{n_r}{s}\alpha) + \ln(1+\rho\frac{n_r}{n_t}-\rho\frac{n_r}{s}\alpha) - \alpha$$

$$\sigma = -\ln[1 - \frac{n_t\alpha}{s}]$$

$$\alpha = \frac{1}{2}[1+\frac{s}{n_t}+\frac{1}{\rho\frac{n_r}{s}} - \sqrt{(1+\frac{s}{n_t}+\frac{1}{\frac{n_r}{s}\rho})^2 - 4\frac{s}{n_t}}]$$

Since s will depend on the frequency, we will define as in [6] the richness spectrum as: $R_{\text{doa}} = \frac{s(f)}{n_r}$
DoD based model. In this case, one has to optimize criteria (3) with respect to the number of scatterers s_1 for which the expression of μ and σ are given by:

$$\mu = \frac{s_1}{n_t}\ln(1+\rho\frac{n_r}{s_1}-\rho\alpha) + \frac{n_r}{n_t}\ln(1+\rho-\rho\alpha) - \frac{s_1}{n_t}\alpha$$

$$\sigma = -\ln[1 - \frac{s_1}{n_r}\alpha]$$

$$\alpha = \frac{1}{2}[1+\frac{n_r}{s_1}+\frac{1}{\rho} - \sqrt{(1+\frac{n_r}{s_1}+\frac{1}{\rho})^2 - 4\frac{n_r}{s_1}}]$$

Since s_1 will also depend on the frequency, we can also define the richness spectrum as: $R_{\text{dod}} = \frac{s_1(f)}{n_t}$
Double Directional model. In this case, one has to optimize criteria (3) with respect to s (scatterers at the receiving side) and s_1 (scatterers at the transmitting side) for which the expressions of μ and σ are given by:

$$\mu = \frac{s}{n_t}\ln(1+\rho\frac{n_r}{s}-\rho\frac{n_r}{s})) + \frac{s_1}{n_t}\ln(1+\rho\frac{n_r}{s_1}-\rho\frac{n_r}{s}\alpha) - \frac{s_1}{n_t}\alpha$$

$$\sigma = -\ln(1 - \frac{\alpha^2 s_1}{s})$$

$$\alpha = \frac{1}{2}\left[1+\frac{s}{s_1}+\frac{s}{\rho n_r} + \sqrt{(1+\frac{s}{s_1}+\frac{s}{\rho n_r})^2 - 4\frac{s}{s_1}}\right]$$

As previously, the richness spectra can be defined independently on both sides. Note that having a model that gives the same mutual information as measurements does not validate at all the model but gives a model tool for simulating a capacity network. If the criteria changes and one focuses on BER, the model may be completely inadequate even though it complies with mutual information measurements.

3. MEASUREMENT SET-UP

In this section[4], we describe the wideband outdoor measurement campaign carried out in Oslo during summer 2002 [9]. The measurements were performed at a center frequency of 2.1 GHz with a bandwidth of 100 MHz in three different

[3]Note that we could also minimize the Kullback distance between the two distributions:

$$D(P, P_{\text{empirical}}) = \int P(I^M)\log\left(\frac{P(I^M)}{P_{\text{empirical}}(I^M)}\right) dI^M \qquad (4)$$

where P and $P_{\text{empirical}}$ are respectively the theoretical and empirical probability distribution of the mutual information.

[4]A comprehensive introduction to the measurement set-up can be found in [7] and [8].

urban scenarios: a regular street grid scenario, an open city place and an indoor cell site. In each scenario, many routes have been measured: at 2.1 GHz, 150 routes have been measured (and in each case, many time snapshots) . The measurements performed at 2.1 GHz are relevant for UMTS.

- The street grid scenario is in Oslo downtown and corresponds to Concrete/Brick buildings. The buildings are around 20-30 m high. In this scenario, two different receiver positions were tested, one high position on a roof terrace and the other one, a low position on street level. The area is often referred as ”Kvadraturen”.
- The urban open place is also in downtown Oslo and corresponds to an almost quadratic open market square of approximatively 100×100 meters. In Oslo, the square is called ”Youngstorget”. The square is partly filled with market stalls especially during the summer months. The surrounding buildings are of variable size. In this scenario, the receiver was placed above some arcades.
- For the indoor scenario, the measurements were performed in a modern office building with open indoor areas. The building (Telenor headquarters building at Fornebu) has a irregular structure and is mostly of glass and steel. Measurements were taken in two different parts of the building. Inside a work zone and in a common area called the ”Atrium”.

In all the measurement set-up, a wideband channel sounder with synchronized switching between transmitter and receiver was used. The transmitter was placed arbitrarily and used as the mobile part, mounted on a trolley. Both transmitter and receiver antennas are broadband patch arrays with integrated switching networks. The transmitter is an 8 element uniform linear array (ULA) while the receiver antenna is an 8×4 planar array, i.e two dimensional with 8 elements horizontally and 4 vertically, giving a total of 32 elements. In all the cases, the receiver acted as the base station and only 8 elements were used. The transmitter antenna was connected using the 4 center elements with both polarizations. The main channel sounder specification are listed on the following Table (1). The sounder was manufactured by SINTEF Telecom and Informatics in Trondheim, Norway, on assignment from Telenor.

In all the following figures, the SNR will be fixed at 10 dB.

4. RESULTS

4.1. Are the measured mutual information Gaussian?

Before trying to see if the models derived within this paper are mutual information complying, one has to verify that

Measurement frequency	2.1 GHz
Measurement bandwidth	100 MHz
Delay resolution	10ns
Sounding signal	linear frequency chirp
Transmitter antenna	8 element ULA
Element spacing	71.4 mm (0.5 λ)
Receiver antenna	32 (8×4) element
Element spacing	73.0 mm (0.51 λ)

Table 1. Channel sounder specification at 2.1 GHz.

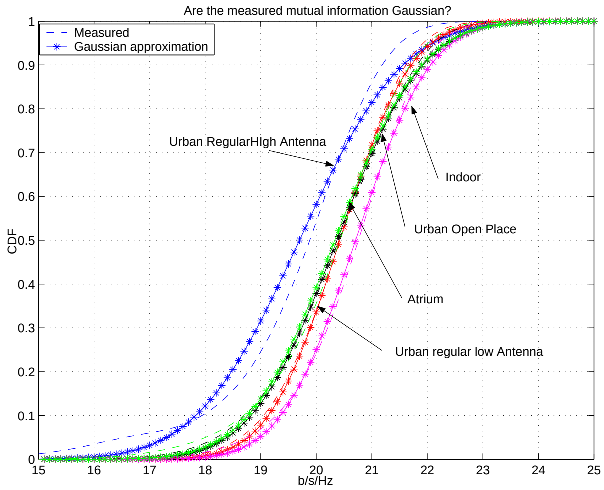

Fig. 2. Are the measured mutual information Gaussian

the measured mutual information has a Gaussian behavior. In Figure 2, we have plotted respectively the measured mutual information for the scenarios of interest, namely the urban open place, the urban regular low antenna position, the urban regular high antenna, the indoor and the Atrium scenario. We have also plotted the Gaussian pdf of each scenario based on the first and second measured moment i.e if $\mu_{\text{empirical}}$ and $\sigma_{\text{empirical}}$ are respectively the measured mean and variance then for each scenario:

$$P(I^M) = \frac{1}{\sqrt{2\pi\sigma^2_{\text{empirical}}}} e^{-\frac{(I^M - \mu_{\text{empirical}})^2}{2\sigma^2_{\text{empirical}}}}$$

As one can see, the mutual information has a Gaussian behavior (except for the Urban Regualr High Antenna case) and therefore, the model (2) can be considered as a good candidate for the mutual information compliance criteria [5]. In the following section, we will see how close are the measured capacities from the maximum entropy models.

[5]Actually, from all the 150 routes available in [7], only 8 routes did not have a Gaussian behavior. We don't know if this is due to measurements errors or something else.

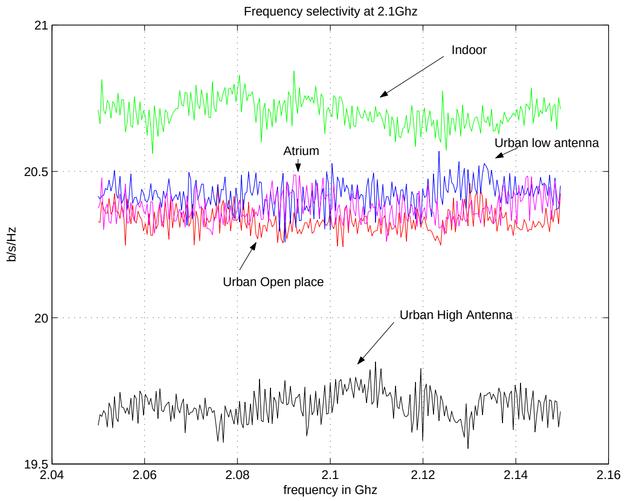

Fig. 3. Frequency selectivity for many scenarios at 2.1 GHz

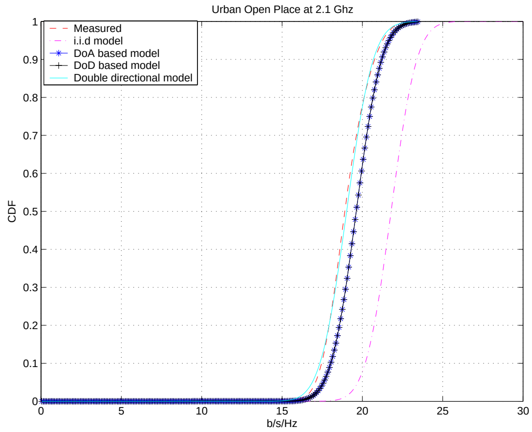

Fig. 4. Urban Open Place at 2.1 GHz.

4.2. What about frequency selectivity?

In [1], it was argued that frequency selectivity does not affect the mutual information. In Figure 3, we have plotted the mutual information for various frequencies (ranging from 2.05 to 2.15 GHz) in the urban open place scenario, the urban regular low antenna position, the urban regular high antenna position, the indoor and Atrium scenario[6]. As one can observe, for the different frequencies, the mutual information does not really vary which is adequate with our model structure: the highest variation occurs for the urban regular high antenna position and is about 0.3 b/s/Hz which makes a relative variation of ($\frac{19.85-19.55}{19.55} = 0.015$) around 1.5%.

Note that the mutual information is smaller in the urban regular high antenna position than in the low antenna scenario. This maybe due to the fact that there is less scattering objects when the antenna is high. Note also that the mutual information in the indoor scenario is slightly higher than the outdoor case due to a possibly higher number of scattering objects.

4.3. Are the models mutual information complying?

In Figure 4, we have plotted the measured cdf of the urban open place scenario with respect to the optimized DoA, DoD and double directional models. The double directional model fits accurately the data with a number of scatterers equal to $s = 7$ and $s_1 = 3$. It seems that the equal power case is sufficient to comply with the mutual information measurements. Therefore, the urban open place scenario can be fully described by a double directional model. One can observe that the number of scatterers is quite high. This may be explained by the fact that the open place site has quite a diverse building topography. Note that the Gaussian i.i.d model is too optimistic and overestimates the achievable rate. An average gap with measurements of 3 b/s/Hz exists.

In Figure 5, we have plotted the measured cdf of the Urban Regular Low Antenna Position scenario with respect to the optimized DoA, DoD and double directional models. The double directional model gives similar results as the DoA based model with a number of scatterers equal to $s = 5$ and $s_1 = 4$. Moreover, one can observe that there are more scatterers on the receiving side than the transmitting side ($s > s_1$). This is maybe due to the fact that the receiving antenna is low and therefore, many reflections occur at the receiving side. Note that one would get a better fitting curve if the power of the steering vectors are taken into account. Here also, the i.i.d Gaussian model does not at all represent this scenario and a gap of more than 4 b/s/Hz is revealed.

In Figure 6, we have plotted the measured cdf of the indoor scenario with respect to the optimized DoA, DoD and double directional models. The best performance is obtained in the double directional case for $s = 7$ and $s_1 = 4$. As previously, the power of the steering vectors should be taken into account to achieve better results. For the i.i.d case, a gap of more than 1 b/s/Hz is revealed.

In Figure 7, we have plotted the measured cdf of the Atrium scenario with respect to the optimized DoA, DoD and double directional models. In this case, The double directional model gives the same performance as the DoA based. The best fitting is obtained for $s = 7$ and $s_1 = 4$. For the i.i.d case, a gap of more than 1 b/s/Hz is revealed with the measurements.

[6] Note that the cdf has been averaged over different time snapshots but at the same frequency.

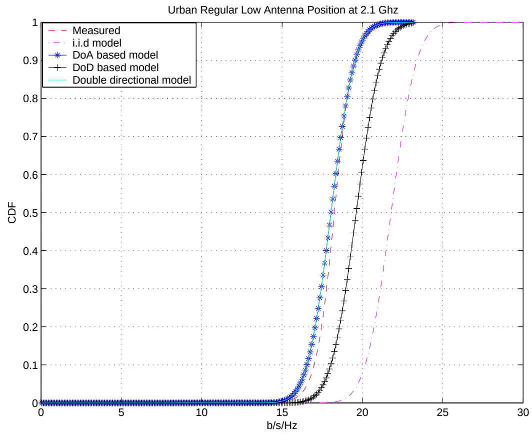

Fig. 5. Urban Regular, Low Antenna Position at 2.1 GHz.

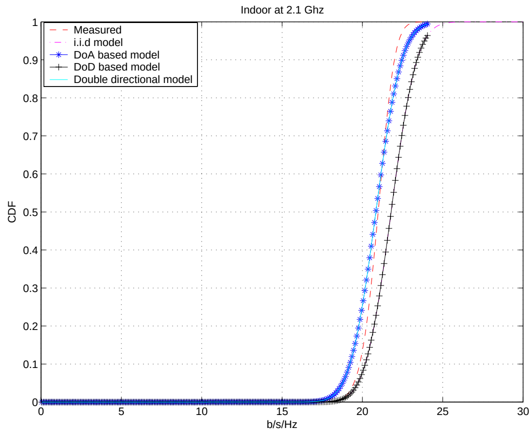

Fig. 6. Indoor scenario at 2.1 GHz.

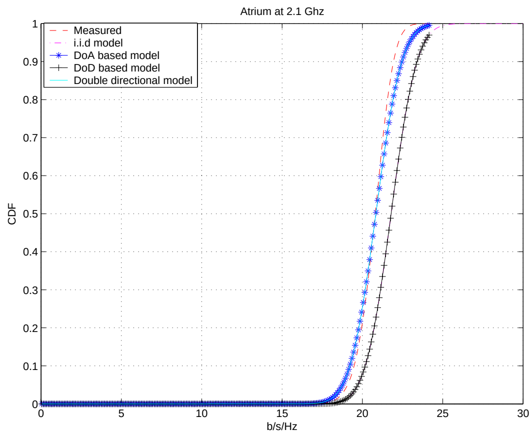

Fig. 7. "Atrium" scenario at 2.1 GHz.

5. CONCLUSION

The maximum entropy based model has been proved to be mutual information complying and is a good candidate to model the MIMO link based on other criteria such as BER[7]. Note that recent measurements at 5.2 GHz [4] have shown that the maximum entropy model with zero mean and equal power on the steering directions is not so accurate. The power of the steering directions should be taken into account.

6. REFERENCES

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