CAPACITY OF A DOWNLINK MC-CDMA MULTI-CELL NETWORK

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ABSTRACT

In this paper, a new approach to determine the number of cells for a given environment and coverage area is provided. Base stations are located on a square grid of surface A. Each base station covers an area of size αA. The goal is to determine the optimal parameter α in the case of a downlink MC-CDMA scheme where each user is equipped with a matched filter. Two types of codes are studied: random i.i.d and orthogonal (per cell) codes. We show that in the absence of path loss, the cellular architecture depends only on the second and fourth moment of the channel.

1. INTRODUCTION

Multi-carrier CDMA transmission schemes have been advocated recently as robust schemes combining the advantages of CDMA with the simple frequency equalization structure of OFDM [1]. In a MC-CDMA scheme using a Cyclic Prefix for preventing inter-block interference, the information is spread across all the carriers by a spreading code at the transmitter. An important problem that arises in the design of such a system concerns the deployment of an efficient architecture to cover the users. Increasing the number of cells in a given area yields indeed a better coverage but increases at the same time the inter-cell interference. The gain provided by a cellular network is not at all straightforward and depends on many parameters: path loss, type of codes used, receiving filter, channel characteristic. Although very complex, this contribution is a first step into analyzing the problem. Some answers are provided on the optimal cellular coverage in the case of a matched filter. In order to obtain interpretable expression, the problem is analyzed in the asymptotic regime (the total number of users N tends to infinity, the number of users K per cell tends to infinity but the ratio $\frac{K}{N} \rightarrow \alpha$ is constant) by using tools of free probability theory [2]. Previous studies have already studied the capacity of a CDMA multi-cell network in the uplink scenario [3] with Wyner's model or with simple interference models [4]. However, none has taken explicitly into account the impact of the code structure (orthogonality or not) and the multi-path channel characteristics. The remainder of the paper is organized as follows: in section 2, the MC-CDMA cellular model is described. In section 3, the capacity of the downlink MC-CDMA scheme is derived. Finally, in section 4, some conclusions are drawn on the the behavior of the capacity with respect to the spreading code structure.

2. MC-CDMA CELLULAR MODEL

2.1. Cellular Model

The base stations are supposed to be located at the center of a square grid of size αA (see figure 1). The total number of users, the number of users per base station[1], the density of the network and the size of the network are respectively denoted by N, K, d and A. For a fixed d, we suppose that $A = \frac{N}{d} \rightarrow \infty$ and $N \rightarrow \infty$, $K \rightarrow \infty$ but $\frac{K}{N} \rightarrow \alpha$ (called the load of the base station). The number of base stations in the network is given by: $\frac{A}{\alpha A} = \frac{1}{\alpha}$ (inverse of the load). The coordinates of the base station (p,q) are $m_{pq} = (p\sqrt{\alpha A}, q\sqrt{\alpha A})$.

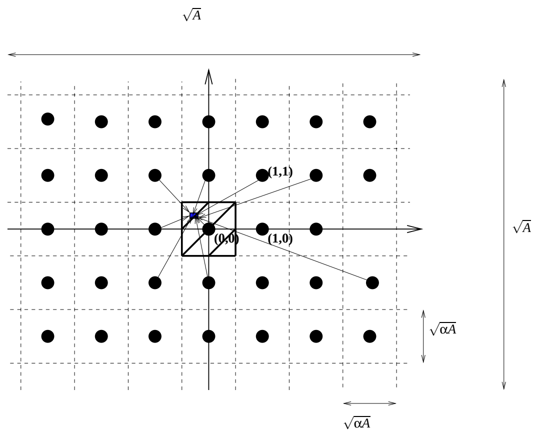

Fig. 1. Representation of the Cellular Scenario

[1]We assume that the users are uniformly distributed so that each base station has K users.

2.2. MC-CDMA Model

The transmission scheme for the downlink mobile communication is depicted in fig.2. The transmitter consists of a spreader and an OFDM multiplexer. Each user's information is encoded separately and modulated directly onto a spreading sequence of length N. Each column of $\mathbf{W}_{pq}$ represents the code allocated to each user. Each receiver comprises an OFDM demultiplexer, a demodulator and a de-spreader. We suppose that although synchronization be-

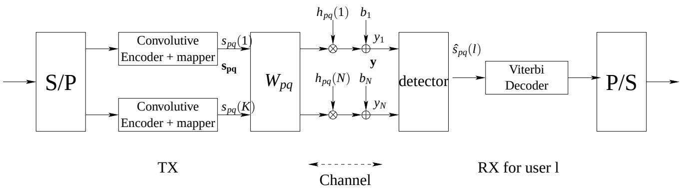

Fig. 2. MC-CDMA downlink scheme.

tween the base stations is not assured, the cyclic-prefix is long enough so that the receiving signal of user l in cell (p,q) can be written in the following form[2]:

$$\mathbf{y}(x_l,y_l) = \sum_{i,j} \sqrt{P_{ij}(x_l,y_l)} H_{ij} W_{ij} \mathbf{s_{ij}} + \mathbf{b} \qquad (1)$$

(x_l,y_l) are the coordinates of user 1 in cell (p,q). $\mathbf{y}(x_l,y_l)$ is the $N \times 1$ received vector, $\mathbf{s_{ij}}$ is the $K \times 1$ transmit vector of cell (i,j) $[s_{ij}(1),...,s_{ij}(K)]^T$, $\mathbf{b}$ is an $N \times 1$ noise vector $[b(1),...,b(N)]^T$ with zero mean unit variance Gaussian independent entries. $P_{ij}(x_l,y_l)$ represents the path loss between base station m_{ij} and the user (x_l,y_l) whereas the diagonal matrix $H_{ij} = diag([h_{ij}(1),...,h_{ij}(N)]$ represents the frequency channel (after OFDM demodulation) between user l and base station m_{ij}. Each base station has an $N \times K$ code matrix W_{ij}:$W_{ij} = [\mathbf{w_{ij}^1},...,\mathbf{w_{ij}^K}]$. The user l is subject to intra-cell as well as inter-cell interference.

In all the following, we will make the following ergodic assumption for the channel[3]. For each continuous bounded function $f : \mathbb{R} \to \mathbb{R}$, $\lim_{N\to+\infty} \frac{1}{N}\sum_{k=1}^{N} f(|h_k|^2) = \int f(t)p(t)\, dt$ almost surely. In other words, we assume that the empirical channel distribution converges weakly to the limiting distribution given by the probability density function $p(t)$.

3. PERFORMANCE ANALYSIS

In all the following, without loss of generality, we will focus on user l of cell (p,q). We assume that the user does not know the codes of the other cells as well as the codes of other users within the same cell. Moreover, the user is supposed to be equipped with the matched filter receiver: $\mathbf{g} = H_{pq}\mathbf{w_{pq}^l}$.

3.1. General capacity formula

The output of the matched filter is given by:

$$\begin{aligned}
\mathbf{g}^H\mathbf{y}(x_l,y_l) &= \sqrt{P_{pq}(x_l,y_l)}\mathbf{g}^H H_{pq}\mathbf{w_{pq}^l} s_{pq}(l) \\
&+ \sqrt{P_{pq}(x_l,y_l)}\mathbf{g}^H H_{pq} U_{pq} \begin{bmatrix} s_{pq}(1) \\ \vdots \\ s_{pq}(K) \end{bmatrix}_{(K-1)\times 1} \\
&+ \sum_{(i,j)\neq(p,q)} \sqrt{P_{ij}(x_l,y_l)}\mathbf{g}^H H_{ij} W_{ij}\mathbf{s}_{ij} + \mathbf{g}^H\mathbf{b}
\end{aligned}$$

where $U_{pq} = [\mathbf{w_{pq}^1},...,\mathbf{w_{pq}^{l-1}},\mathbf{w_{pq}^{l+1}},...,\mathbf{w_{pq}^K}]$. The output $\mathrm{SINR}(x_l,y_l,(p,q))$ of user l with coordinates (x_l,y_l) in cell (p,q) has the following expression:

$$\mathrm{SINR}(x_l,y_l,(p,q)) = \frac{\mathrm{S}}{I_1 + I_2 + \sigma^2\mathbf{g}^H\mathbf{g}} \qquad (2)$$

$$\begin{aligned}
S &= P_{pq}(x_l,y_l) \mid \mathbf{g}^H H_{pq}\mathbf{w_{pq}^l} \mid^2 \\
I_1 &= \sum_{(i,j)\neq(p,q)} P_{ij}(x_l,y_l)\mathbf{g}^H H_{ij} W_{ij} W_{ij}^H H_{ij}^* \mathbf{g} \\
I_2 &= P_{pq}(x_l,y_l)\mathbf{g}^H H_{pq} U_{pq} U_{pq}^H H_{pq}^* \mathbf{g}
\end{aligned}$$

We would like to quantify the number of bits/s/Hz the system is able to provide to all the users. It has been shown [5] that the interference plus noise can be considered as Gaussian when K and N are large enough. In this case, the mean spectral efficiency in cell (p,q) is given by:

$$\gamma^{(pq)} = \frac{1}{N}\mathbb{E}\left(\sum_{i=1}^{K} \log_2(1 + \mathrm{SINR}(x_i,y_i,(p,q))\right) \quad (3)$$

For a fixed K and N, it is extremely difficult to get some insight of expression (3). Therefore, in the next two sections, we will focus our analysis on certain type of codes.

- random i.i.d matrix: coefficients of the codes are modeled as centered variance $\frac{1}{N}$ i.i.d random variables. This choice is justified in order to get interpretable expressions of the SINR.

- random Haar distributed isometric matrix: We study isometric matrices obtained by extracting $K < N$ columns from a Haar unitary matrix[4]. Since perfect

[2]Note that even if this is not the case, the result holds as our analysis is asymptotical and therefore all Toeplitz matrices are diagonalizable in a Fourier Basis in that regime.

[3]The tools introduced can take into account the non ergodic case. However, the formulas have no simple interpretation.

[4]A $N \times N$ random unitary matrix is said to be Haar distributed if its probability distribution is invariant by right (or equivalently left) multiplication by deterministic unitary matrices.

synchronization is ensured within a cell, this case is more suited to our study[5].

We will show in particular that $\mathrm{SINR}(x_l, y_l, (p,q))$ converges almost surely to a deterministic value independent of the particular code chosen. In this case, the spectral efficiency can be rewritten:

$$\begin{aligned}\gamma^{(pq)} &= \frac{K}{N}\mathbb{E}\left(\log_2(1+\mathrm{SINR}(x,y,(p,q)))\right)\\ &= \frac{1}{A}\int_{-\frac{\sqrt{\alpha A}}{2}}^{\frac{\sqrt{\alpha A}}{2}}\int_{-\frac{\sqrt{\alpha A}}{2}}^{\frac{\sqrt{\alpha A}}{2}}\log_2(1+\mathrm{SINR}(x,y,(p,q))dxdy\end{aligned}$$

The total capacity of the network is given by: $C = \sum_{p,q}^{\frac{1}{\alpha}}\gamma^{(pq)}$.

3.2. Capacity with random i.i.d codes

In this case, the following proposition holds:

Proposition 1 *When N grows towards infinity and $K/N \rightarrow \alpha$, the capacity of downlink MC-CDMA with random i.i.d spreading codes and matched filter is given by:*

$$\begin{aligned}C(\alpha) &= \lim_{N\rightarrow\infty}\sum_{p,q}^{\frac{1}{\alpha}}\gamma^{pq}\\ \gamma^{pq} &= \frac{d}{N}\int_{-\sqrt{\frac{\alpha N}{4d}}}^{\sqrt{\frac{\alpha N}{4d}}}\int_{-\sqrt{\frac{\alpha N}{4d}}}^{\sqrt{\frac{\alpha N}{4d}}}\log_2\left(1+\frac{P_{pq}(x,y)(\mathbb{E}(\mid h\mid^2))^2}{I_1+I_2+\sigma^2\mathbb{E}(\mid h\mid^2)}\right)dxdy\\ I_1 &= \alpha(\mathbb{E}(\mid h\mid^2))^2\sum_{(i,j)\neq(p,q)}P_{ij}(x,y)\\ I_2 &= \alpha P_{pq}(x,y)\mathbb{E}(\mid h\mid^4)\end{aligned}$$

Proof 1 *Due to lack of space, we only give here the main steps of the proof. Three terms have to be derived separately in the SINR formula. Since $\mathbf{w_{pk}^l}$ is independent of W_{ij} for $(i,j)\neq(p,q)$, it is possible to show that [7]:*

$$\begin{aligned}\mathbf{g}^H H_{ij}W_{ij}W_{ij}^H H_{ij}^*\mathbf{g} &\overset{a.s}{\rightarrow} \frac{1}{N}\mathrm{Trace}(W_{ij}^H H_{ij}^* H_{pq}H_{pq}H_{pq}^* H_{ij}W_{ij})\\ &\overset{a.s}{\rightarrow} \alpha\mathbb{E}(\mid h\mid^2)\\ \mathbf{g}^H H_{pq}U_{pq}U_{pq}^H H_{pq}^*\mathbf{g} &\overset{a.s}{\rightarrow} \frac{1}{N}\mathrm{Trace}(H_{pq}^* H_{pq}U_{pq}U_{pq}^H H_{pq}^* H_{pq})\\ &\overset{a.s}{\rightarrow} \alpha(E)(\mid h\mid^4)\\ \mathbf{g}^H\mathbf{g} &\overset{a.s}{\rightarrow} \mathbb{E}(\mid h\mid^2)\end{aligned}$$

3.3. Capacity with Random Orthogonal Codes

In this case, the following proposition holds:

Proposition 2 *When N grows towards infinity and $K/N \rightarrow \alpha$, the capacity of downlink MC-CDMA with random orthogonal spreading codes and matched filter is given by:*

$$\begin{aligned}C(\alpha) &= \lim_{N\rightarrow\infty}\sum_{p,q}^{\frac{1}{\alpha}}\gamma^{pq}\\ \gamma^{pq} &= \frac{d}{N}\int_{-\sqrt{\frac{\alpha N}{4d}}}^{\sqrt{\frac{\alpha N}{4d}}}\int_{-\sqrt{\frac{\alpha N}{4d}}}^{\sqrt{\frac{\alpha N}{4d}}}\log_2\left(1+\frac{P_{pq}(x,y)(\mathbb{E}(\mid h\mid^2))^2}{I_1+I_2+\sigma^2\mathbb{E}(\mid h\mid^2)}\right)dxdy\\ I_1 &= \alpha(\mathbb{E}(\mid h\mid^2))^2\sum_{(i,j)\neq(p,q)}P_{ij}(x,y)\\ I_2 &= \alpha P_{pq}(x,y)\left(\mathbb{E}(\mid h\mid^4)-(\mathbb{E}(\mid h\mid^2))^2\right)\end{aligned}$$

Proof 2 *Due to lack of space, we only give here the main steps of the proof. The difference with the i.i.d case concerns the term $I_2 = {w_{pk}^l}^H H_{pq}^* H_{pq}U_{pq}U_{pq}^H H_{pq}^* H_{pq}w_{pk}^l$. Indeed, the proof must follow a different procedure as w_{pk}^l is not independent of U_{pq}. It can be shown [6]:*

$$\begin{aligned}I_2 &\overset{a.s}{\rightarrow} \frac{1}{N-K}\sum_{i=1}^{N}\mathrm{Trace}\left(H_{pq}^* H_{pq}U_{pq}U_{pq}^H H_{pq}H_{pq}(\mathbf{I}-U_{pq}U_{pq}^H)\right)\\ &\overset{a.s}{\rightarrow} \frac{\alpha}{1-\alpha}\mathbb{E}(\mid h\mid^4)-\frac{1}{1-\alpha}\int t^2 d\mu(t)\end{aligned}$$

μ is the limiting eigenvalue distribution of matrix $H_{pq}^* H_{pq}U_{pq}U_{pq}^H$. Using results from free probability, one can show that: $\int t^2 d\mu(t) = \alpha^2\mathbb{E}(\mid h\mid^4)+(1-\alpha)\alpha\left(\mathbb{E}(\mid h\mid^2)\right)^2$.

Note that the capacity formula in the orthogonal case is always superior to the i.i.d case.

4. DISCUSSION

In general, to determine the capacity of the network, one has to plot the previous formulas with respect to the path loss model as no explicit form can be obtained. However, in the case of no path loss ($P_{ij}(x,y) = P$ for all i, j), interesting conclusions can be drawn. Indeed, the previous capacity formulas yield in the i.i.d and orthogonal case respectively:

$$\begin{aligned}C_{\mathrm{i.i.d}}(\alpha) &= \log_2\left(1+\frac{P\left(\mathbb{E}(\mid h\mid^2)\right)^2}{I^{\mathrm{i.i.d}}}\right)\\ C_{\mathrm{ortho}}(\alpha) &= \log_2\left(1+\frac{P\left(\mathbb{E}(\mid h\mid^2)\right)^2}{I^{\mathrm{ortho}}}\right)\\ I^{\mathrm{i.i.d}} &= P\left(\mathbb{E}(\mid h\mid^2)\right)^2+\alpha P\left(\mathbb{E}(\mid h\mid^4)-\left(\mathbb{E}(\mid h\mid^2)\right)^2\right)\\ &+ \sigma^2\mathbb{E}(\mid h\mid^2)\\ I^{\mathrm{ortho}} &= I^{\mathrm{i.i.d}}-\left(\mathbb{E}(\mid h\mid^2)\right)^2\end{aligned}$$

Since $\left(\mathbb{E}(\mid h\mid^4)-\left(\mathbb{E}(\mid h\mid^2)\right)^2\right)\geq 0$, $C_{\mathrm{i.i.d}}$ is a decreasing function of α. The best capacity is obtained for an infinite number of cells. Therefore, the need to go cellular in the

[5] In [6], simulations show that these matrices provide similar performance as Walsh-Hadamard codes.

i.i.d case is due to the codes and not the path loss irrespective of the channel statistics. In figure 3, the theoretical capacity has been plotted in the case of independent frequency i.i.d Rayleigh fading. Simulations in the case of a scenario of N=1024 users have been provided and show the accuracy of our asymptotical results. In the case of orthogonal per cell codes, the results depend on the kurtosis $T = \frac{\mathbb{E}(|h|^4)}{\left(\mathbb{E}(|h|^2)\right)^2}$. If $T > 2$, one must have an infinite number of cells whereas if $T \leq 2$, one must have only one cell to accommodate all the users. In this case, the need to go cellular depends o how "peaky" is the channel. In figure 4 and 5, we have plotted respectively the capacity with $\mathbb{E}(\mid h \mid^4) = 1.5$ and $\mathbb{E}(\mid h \mid^4) = 3$ in the case of i.i.d and orthogonal per cell spreading. As one can see, orthogonal per cell spreading yields always a better capacity. In general, network providers have a predetermined cost $L = \frac{dC(\alpha)}{d(\frac{1}{\alpha})}$ (i.e each base station added must provide at least L bits/s/Hz). The intersection of the former curve with the capacity gives the optimum number of cells to be deployed.

5. CONCLUSION

In this paper, simple formulas have been provided to give rules for determining the number of cell in a downlink MC-CDMA scheme. The asymptotic capacity of a multi-cell network with matched filter has been derived and shown to fit simulations with reasonable matrix size. Moreover, the impact of the orthogonality of the spreading codes has been analyzed. Contrarily to past belief, the result show that the need to go cellular does not come from the path loss but from the type of codes used as well as the channel second and fourth moment. Extensions to other types of receivers can be found in [8].

6. REFERENCES

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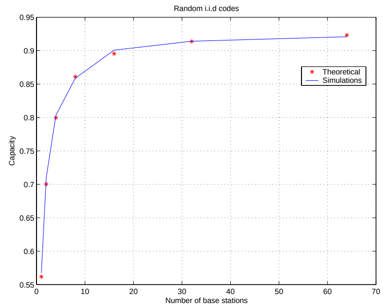

Fig. 3. Random i.i.d codes, Rayleigh fading, N=1024 users, No path loss

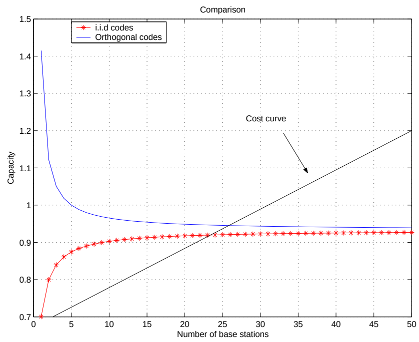

Fig. 4. Random i.i.d and orthogonal codes, $\mathbb{E}(\mid h \mid^2) = 1$ and $\mathbb{E}(\mid h \mid^4) = 1.5$, no path loss

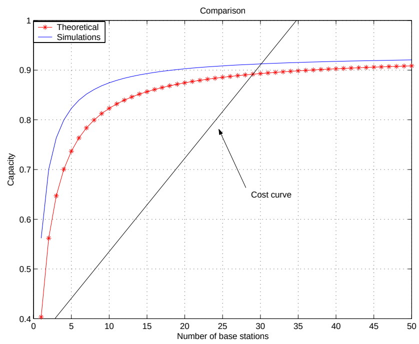

Fig. 5. Random i.i.d and orthogonal codes, $\mathbb{E}(\mid h \mid^2) = 1$ and $\mathbb{E}(\mid h \mid^4) = 3$, no path loss