ON THE ACHIEVABLE RATES OF UWB SYSTEMS

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***Abstract*—In this work we study the achievable rates of memoryless signaling strategies adapted to ultraWideBand (UWB) multipath fading channels. We focus on strategies which do not have explicit knowledge of the instanteneous channel realization, but may have knowledge of the channel statistics. We evaluate the average mutual information of the general binary flash-signaling rates as a function of the channel statistics and derive expressions for the achievable information rates of non-coherent detection using m-ary PPM.**

I. INTRODUCTION

In this work, we consider achievable rates for transmission strategies suited to *Ultra-wideband (UWB)* systems and focus non-coherent receivers (i.e. those which do not perform channel estimation, but may have prior knowledge of the second-order channel statistics). Here we take a UWB system to be loosely defined as any wireless transmission scheme that occupies a bandwidth between 1 and 10 GHz and more than 25% of it's carrier frequency in the case of a passband system.

The most common UWB transmission scheme is based on transmitting information through the use of short-term impulses, whose positions are modulated by a binary information source [1]. This can be seen as a special case of *flash signaling* coined by Verdu in [6]. Similar to direct-sequence spread-spectrum, the positions can further be modulated by an m-ary sequence (known as a *time-hopping sequence*) for mitigating inter-user interference in a multiuser setting [2]. This type of UWB modulation is a promising candidate for military imaging systems as well as other non-commercial sensor network applications because of its robustness to interference from signals (potentially from other non-UWB systems) occupying the same bandwidth. Based on recent documentation from the FCC it is also being considered for commercial adhoc networking applications based on peer-to-peer communications, with the goal be to provide low-cost high-bandwidth connections to the internet from small handheld terminals in both indoor and outdoor settings. Proposals for indoor wireless LAN/PAN systems in the 3-5 GHz band (802.15.3) are also considering this type of transmission scheme.

In this work, we focus on the case of non-coherent detection since it is well known [7] [8] that coherent detection is not required to achieve the *wideband* AWGN channel capacity, $C_\infty = \frac{P_R}{N_0 \ln 2}$ bits/s, where P_R is the received signal power in watts, and N_0 is the noise power spectral density. In [8] Telatar and Tse showed this to be the case for arbitrary channel statistics in the limit of infinite bandwidth and infinite carrier frequency. Their transmission model was based on frequency-shift keying (FSK) and it was shown that channel capacity is achieved using very impulsive signals.

In [6] Verdu addresses the spectral efficiency of signaling strategies in the wideband regime under different assumptions regarding channel knowledge at the transmitter and receiver. The characterization is in terms of the minimum energy-per-bit to noise spectral density ratio $(E_b/N_0)_{\min}$ and the wideband slope $\mathcal{S}_0$. The latter quantity is measured in bits/s/Hz/3dB and represents growth of spectral efficiency at the origin as a function of E_b/N_0. Verdu's work is fundamental to our problem since is shows that approaching C_∞ with non-coherent detection is impossible for practical data rates (>100 kbit/s) even for the vanishing spectral efficiency of UWB systems. This is due to the fact that $\mathcal{S}_0$ is zero at the origin for non-coherent detection. To get an idea of the loss incurred, consider a system with a 2GHz bandwidth and data rate of 20 Mbit/s (this would correspond to a memoryless transmission strategy for channels with a 50ns delay-spread) yielding a spectral-efficiency of .01 bits/s/Hz. For Rayleigh statistics the loss in energy efficiency is on the order of 3dB, which translates into a factor 2 loss in data rate compared to a system with perfect channel state information at the receiver. The loss becomes less significant for lower data rates and/or higher bandwidths.

The main goal of this work is to examine under what conditions different non-coherent signaling strategies can approach the wideband channel capacity with perfect channel knowledge at the receiver subject to a large but finite bandwidth constraint and different propagation conditions. Section II deals with the underlying system model for transmission and reception as well as the channel model. In section III we evaluate expressions for the achievable information rates of non-coherent detection using random codebooks and m-ary PPM. Finally in section IV we examine a quasi-coherent detection scheme using noisy channel estimates.

II. SYSTEM MODELS

We restrict our study to strictly time-limited memoryless real-valued signals, both at the transmitter and receiver. The time-limited and memoryless assumptions are made possible due to the virtually unlimited bandwidth of UWB signals. The transmitted pulse, of duration T_p, is passed through a linear channel, $h(t,u)$, representing the response of the channel at time t to an impulse at time u. We assume that the impulse response of the channel is of duration $T_d \gg T_p$. The channel

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is further assumed to be a zero-mean process(i.e. non line-of-sight communications), motivated by the fact that scattering off objects causes 180 degree phase reversals in the impinging components of the wavefront.

The received signal bandwidth W is roughly $1/T_p$, in the sense that the majority of the signal energy is contained in this finite bandwidth. The received signal is given by

$$r(t) = \int_0^{T_p} x(u)h(t,u)du + z(t) \quad (1)$$

where $z(t)$ is white Gaussian noise with power spectral density $N_0/2$. The channel is further assumed to satisfy

$$\int_0^{T_d+T_p} \int_0^{T_p} h^2(t,u)dtdu < \infty \quad (2)$$

which rules out impulsive channels and practically models the bandlimiting nature of analog transmit and receive chains. The transmitted signal is written as

$$x(t) = \sum_{k=0}^{N} s(u_k)\sqrt{E_s}p\,(t - kT_s) \quad (3)$$

where k is the symbol index, T_s the symbol duration, $E_s = PT_s$ the transmitted symbol energy, $u_k \in \{1, \ldots, m\}$ is the transmitted symbol at time k, $p(t)$ and $s(u_k)$ are the assigned pulse and amplitude for symbol u_k. For all k, $p(t)$ is a unit-energy pulse of duration T_p. The considered model encompasses modulation schemes such as flash signaling, m-ary PPM, amplitude, and differential modulations. A guard interval of length T_d is left at the end of each symbol (from our memoryless assumption) so that $T_s \geq T_p + T_d$. From the point of view of spectral efficiency, we have that $\frac{E_b}{N_0} = \frac{P}{N_0 C(P/N_0)}$, where $C(P/N_0)$ is the average mutual information of the underlying signaling scheme as a function of the SNR.

The large bandwidths considered here (>1GHz) provide a high temporal resolution and enable the receiver to resolve a large number of paths of the impinging wavefront. Providing that the channel has a high diversity order (i.e. in rich multipath environments), the total channel gain is slowly varying compared to its constituent components. It has been shown [3–5] through measurements that in indoor environments, the UWB channel can contain several hundreds of paths of significant strength. We may assume, therefore, that for all practical purposes, the total received energy should remain constant at its average path strength, irrespective of the particular channel realization. Variations in the received signal power will typically be caused by shadowing rather than fast fading. For all the following developments we will assume that the total channel gain is constant and, without loss of generality, equal to 1.

The finite-energy random channel may be decomposed as

$$h(t,u) = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} h_{i,j}\theta_j(u)\phi_i(t) \quad (4)$$

where $h_{i,j}$ are the projections of the channel on the the input and output eigenspaces, $\{\theta_j(u)\}$ is the set of eigenfunctions (for $L^2(0,T_p)$) of the transmit pulse and $\{\phi_i(t)\}$ is the set of eigenfunctions (for $L^2(0,T_p+T_d)$) of the received signal. Since the input in (3) is one-dimensional, the most appropriate choice for $p(t)$ is the one which maximizes the expected energy of the channel output

$$p(t) = \underset{f(t)}{\operatorname{argmax}}\, \mathrm{E}\int_0^{T_d+T_p}\left(\int_0^{T_p} h(t,u)f(u)du\right)^2 dt = \theta_1(t) \quad (5)$$

where $\theta_1(t)$ is the eigenfunction corresponding to the maximum eigenvalue, μ_1, of the input cross-correlation function

$$R_i(u,u') = \mathrm{E}\int_0^{T_s} h(t,u)h(t,u')dt = \int_0^{T_s} R_h(t,t;u,u')dt \quad (6)$$

and $R_h(t,t';u,u') = \mathrm{E}h(t,u)h(t',u')$. The use of this input filter is conditioned on the emmision requirments of UWB systems, and thus it may not be possible to satisfy the maximal energy solution in practice.

The above decomposition allows us to write (1), for each symbol k, as the equivalent channel

$$r_{k,i} = \sqrt{\mu_1}h_i\sqrt{E_s}s(u_k) + z_i, i = 1, ..., \infty \quad (7)$$

where z_i is $\mathcal{N}(0, N_0/2)$. For notational convenience we have dropped the index corresponding to the input projection from h_{ij} since we are constrained one-dimensional inputs. Furthermore, if we choose the output eigenfunctions to be the solution to

$$\lambda_i\phi_i(t) = \int_0^{T_d+T_p}\int_0^{T_p}\int_0^{T_p} R_h(t,u;t',u')\theta_1(u)\theta_1(u')\phi_i(t')dudu'dt' \quad (8)$$

we have that the h_i are uncorrelated and have variance λ_i.

Because of the bandlimiting nature of the channels in this study, the channel will be characterized by a finite number, D, of significant eigenvalues which for rich environments will be roughly equal to $1 + 2WT_d$, in the sense that a certain proportion of the total channel energy will be contained in these D components. Under our rich scattering assumption D is limited by bandwidth and not insufficient scattering and we may in some cases make the following approximation

$$\mu_1\sum_{i=1}^{D} h_i^2 \approx 1 \quad (9)$$

for all channel realizations. This assumption essentially says that the received signal energy is not impaired by signal fading due to the rich scattering environment.

For notational convenience, we will assume that the eigenvalues are ordered by decreasing amplitude.

III. Non-Coherent Detection

In this section we consider non-coherent receivers, that may or may not have access to the second-order channel statistics. The motivation for such a study is to derive alternative receivers that are reasonable from an implementation point of view. We particularly focus on solutions that can be implemented with analog technology.

We assume that the transmitter does not have any side information about the channel and that he is constrained to the use of flash-like signaling. We first numerically compute the average mutual information of a such a system, then we derive a lower bound on the achievable rates for m-ary PPM modulation with different receivers. This modulation can be seen as a specially-designed channel code for flash-signaling.

A. Average Mutual Information

From the results of Verdu in [6] we have in our rich scattering case that $(E_b/N_0)_{\min}$ is $\frac{\ln 2}{\mu_1 \mathrm{E} \sum_{i=1}^{D} h_i^2} \approx \ln 2$. In the case of vanishing spectral efficiency, binary flash-signaling is first-order optimal, and achieves this minimal E_b/N_0. Using the notation from the previous section, we express the binary flash signaling scheme as

$$u_k = \begin{cases} 1 & \text{with probability} \quad \eta \\ 0 & \text{with probability} \quad (1-\eta) \end{cases} \tag{10}$$

$s(0)=0$, $s(1)=\sqrt{\frac{E_s}{\eta}}$, and $T_s = T_d + T_p$. We assume that the h_i are Gaussian ergodic sequences, which implies that the system's temporal resolution is not fine enough to resolve all the degrees of freedom of the considered channel and that the projection of $h(t,u)$, on each of the kernel's directions, is the combination of a relatively large number of independent multipath components. Measurments of UWB channels [3] have shown that channel components can be considered to fade according to Rayleigh statistics, indicating that this assumption is quite reasonable. Then R is a zero-mean Gaussian vector with covariance matrix $E\left[RR^T\right] = \mathbf{diag}(s(u_k)E_s\lambda_i + \frac{N_0}{2})$. It is easily shown that

$$\begin{aligned} I(u_k;R) &= -\frac{1}{T_s} \underset{Y}{E}\left[\eta \log\left(\eta + (1-\eta)\sqrt{\prod_{i=1}^{D} 1 + \frac{2E_s\lambda_i}{\eta N_0}}\right.\right. \\ &\qquad \left. e^{\left(-\frac{1}{2}Y^T(\mathbf{diag}(\frac{N_0\eta}{2E_s\lambda_i}))^{-1}Y\right)}\right) \\ + &\quad (1-\eta)\log\left((1-\eta) + \frac{\eta}{\sqrt{\prod_{i=1}^{D} 1 + \frac{2E_s\lambda_i}{\eta N_0}}}\right. \\ &\qquad \left.\left. e^{\left(\frac{1}{2}Y^T(\mathbf{diag}(\frac{N_0\eta}{2E_s\lambda_i}+1))^{-1}Y\right)}\right)\right] \quad \text{bits/s} \end{aligned} \tag{11}$$

with Y a zero-mean gaussian random vector with covariance matrix $\mathbf{I}$. This is easily computed numerically.

B. m-ary PPM with Energy Detection

In this section we consider both the optimal non-coherent detector, for known second order channel statistics, and a suboptimal mismatched detector.

We use base-band m-ary PPM signals to transmit the information bits. Each PPM symbol corresponds to choosing one out of m symbol times in which to emit the transmit pulse $p(t)$, which is a special case of flash signaling with $\eta = 1/m$. Here, $T_s = T_p + T_d$, so that the channel can be considered to be memoryless.

The data is encoded using a randomly generated codebook $\mathcal{C} = \{C_1, C_2, \ldots, C_M\}$ of cardinality M and codeword length N. Each codeword C_l is a sequence $C_l = (c_{1,l}, c_{2,l}, \ldots, c_{N,l})$ corresponding to the emission timeslot indexes within each of the N symbol-times used for its transmission. With $c_{i,j} \in \{0, m-1\}$ and T_s is the spacing between two consecutive emission time slots. Let C_w be the transmitted codeword, using the notations of model (3) we have $u_k = c_{k,w}$, and $s(u_k) = 1$.

For all $n \in [1,N]$ and $k \in [1,m]$ let

$$\begin{aligned} R_{n,k} &= [i \in [0,D], \quad < r(t), \phi_i(t-(k-c_{n,w})T_s) >] \\ &= S_{n,k} + Z_{n,k} \end{aligned} \tag{12}$$

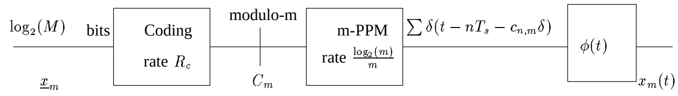

Fig. 1. Transmitter block diagram.

where $S_{n,k}$ and $Z_{n,k}$ are respectively the signal and noise components of $R_{n,k}$, and w denotes the index of the transmitted codeword.

$$Z_{n,k} = [i \in [0,D], \quad < z(t), \phi_i(t-(k-c_{n,w})T_s) >] \tag{13}$$

$Z_{n,k}$ is a Gaussian random vector with mean zero and covariance matrix $\mathbf{K_z} = \frac{N_0}{2}\mathbf{I}$. Using the same reasoning as in the previous section, we will assume that $S_{n,k}$ has a gaussian distribution.

1) Optimal detector: The Maximum likelyhood non-coherent detector can be written as

$$\hat{k} = \underset{k}{\operatorname{argmax}} \quad \Pr(r(t)/w=k) = \underset{k}{\operatorname{argmax}} \quad \frac{1}{N}\sum_{n=1}^{N} q_{n,k} \tag{14}$$

with $q_{n,k} = R_{n,k}Q^{-1}R_{n,k}^T$ and $Q = \mathbf{diag}(\frac{N_0}{2}\left(1 + \frac{N_0}{2E_s\lambda_i}\right))$ [1].

We use an equivalent receiver, the decoder forms the decision variables

$$q_k = \frac{1}{N}\sum_{n=1}^{N} q_{k,n} \tag{15}$$

and uses the following assymptoticaly optimal (for infinite length codewords) threshold rule to decide on a message: if q_k exceeds a certain threshold ρ for exactly one value of k, say $\hat{k}$, then it will declare that $\hat{k}$ was transmitted. Otherwise, it will declare a decoding error. This is the same sub-optimal decoding scheme considered in [8].

An upper bound of the decoding error probability is then given by the following theorem

Theorem 1: The probability of codeword error is upper bounded by

$$\begin{aligned} \Pr[\text{error}] \quad \leq \quad & M \min_{t>0} \\ & \exp -N\left[t\rho - \ln\left((1-p)\prod_{i=1}^{D}\frac{1}{\sqrt{1-\frac{2t}{1+\frac{N_0}{2E_s\lambda_i}}}}\right.\right. \\ & \left.\left. +p\prod_{i=1}^{D}\frac{1}{\sqrt{1-\frac{4E_s\lambda_i t}{N_0}}}\right)\right] \end{aligned} \tag{16}$$

with $\rho = (1-\epsilon)$, and $p = 1/m$ the probability of a "collision" at a slot n between the sent codeword C_w and a candidate codeword C_k.

Proof: The decision variable for the transmitted codeword C_w is given by

$$\frac{1}{N}\sum_{n=1}^{N}\left(S_{n,w} + Z_{n,w}\right)Q^{-1}\left(S_{n,w} + N_{n,w}\right)^T \tag{17}$$

[1]this detector is equivalent to the classical estimator-correlator [9]

by the ergodicity of the noise process, this time-average will exceed the threshold with probability arbitrarily close to 1 for any $\epsilon > 0$ as N gets large. For all $k \neq w$ We bound the probability $\Pr[q_k \geq \rho]$ using a Chernoff bound

$$\begin{aligned} \Pr[q_k \geq \rho] &= \Pr[Nq_k \geq N\rho] \\ &\leq \min_{t>0} e^{-tN\rho} \prod_{n=1}^{N} E\left[e^{tq_{n,k}}\right] \end{aligned} \tag{18}$$

We have that for all $c_{n,k} = c_{n,w}$

$$\mathrm{E}\left[e^{tq_{n,k}}\right] = \prod_{i=1}^{D}\left(1 - \frac{4E_s\lambda_i t}{N_0}\right)^{-\frac{1}{2}} \tag{19}$$

and for all $c_{n,k} \neq c_{n,w}$

$$\mathrm{E}\left[e^{tq_{n,k}}\right] = \prod_{i=1}^{D}\left(1 - \frac{2t}{1+\frac{N_0}{2E_s\lambda_i}}\right)^{-\frac{1}{2}} \tag{20}$$

Let l be the number of collisions between codewords C_w and C_k, then we have that

$$\begin{aligned} \Pr[q_k \geq \rho] \leq \min_{t>0} \Bigg[& e^{-Nt\rho}\left(\prod_{i=1}^{D}\left(1-\frac{4E_s\lambda_i t}{N_0}\right)^{-\frac{1}{2}}\right)^{l} \\ & \left(\prod_{i=1}^{D}\left(1-\frac{2t}{1+\frac{N_0}{2E_s\lambda_i}}\right)^{-\frac{1}{2}}\right)^{N-l}\Bigg] \end{aligned} \tag{21}$$

Averaging over all the realizations of the randomly generated codebook we obtain

$$\begin{aligned} & \underset{C}{E}\left[\Pr[q_k \geq \rho]\right] \\ \leq\ & \underset{C}{E}\left[\min_{t>0} e^{-Nt\rho}\prod_{i=1}^{D}\frac{1}{\left(1-\frac{4E_s\lambda_i t}{N_0}\right)^{\frac{l}{2}}\left(1-\frac{2t}{1+\frac{N_0}{2E_s\lambda_i}}\right)^{\frac{N-l}{2}}}\right] \\ \overset{(a)}{\leq}\ & \min_{t>0}\underset{C}{E}\left[e^{-Nt\rho}\prod_{i=1}^{D}\frac{1}{\left(1-\frac{2t}{1+\frac{N_0}{2E_s\lambda_i}}\right)^{\frac{(N-l)}{2}}\left(1-\frac{4E_s\lambda_i t}{N_0}\right)^{\frac{l}{2}}}\right] \\ =\ & \min_{t>0} e^{-Nt\rho}\left(\sum_{l=0}^{N}\binom{l}{N}p^l(1-p)^{N-l}\right. \\ & \left.\prod_{i=1}^{D}\left(1-\frac{2t}{1+\frac{N_0}{2E_s\lambda_i}}\right)^{-\frac{N-l}{2}}\left(1-\frac{4E_s\lambda_i t}{N_0}\right)^{-\frac{l}{2}}\right) \\ =\ & \min_{t>0} e^{-Nt\rho} \\ & \left((1-p)\prod_{i=1}^{D}\frac{1}{\sqrt{1-\frac{2t}{1+\frac{N_0}{2E_s\lambda_i}}}}+p\prod_{i=1}^{D}\frac{1}{\sqrt{1-\frac{4E_s\lambda_i t}{N_0}}}\right)^{N} \end{aligned} \tag{22}$$

note that in (a) we perform a looser minimization operation for the sake of feasibility of the analytical developments. Using a union bound we obtain the desired result. ■

The decoding error probability in equation (16) decays to zero exponentially in N as long as the transmission rate R satisfies

$$\begin{aligned} R &= \frac{1}{mNT_s}\log(M) \\ &\leq \max_{t>0}\frac{1}{T_s}\left(t\rho - \ln\left((1-p)\prod_{i=1}^{D}\left(1-\frac{2t}{1+\frac{N_0}{2E_s\lambda_i}}\right)^{-\frac{1}{2}}\right.\right. \\ &\quad \left.\left. +p\prod_{i=1}^{D}\left(1-\frac{4E_s\lambda_i t}{N_0}\right)^{-\frac{1}{2}}\right)\right) \end{aligned} \tag{23}$$

Due to the finite cardinality of the symbol alphabet our information rate is bounded by

$$R \leq \frac{1}{T_s}\text{bits/s} \tag{24}$$

2) Mismatched non-coherent detector: We now consider the case where the receiver does not have access to channel statistics and/or is constrained to use a time-invariant front-end filter because of implementation considerations. The received signal is first filtered by the time-limited unit-energy filter $q(t)$,this allow us to reduce the number of degrees of freedom of the received signal while capturing the majority of its information bearing part. [2]

$$\begin{aligned} y(t) = (r * q)(t) &= ((s+z)*q)(t) \\ &= s_q(t) + z_q(t) \end{aligned} \tag{25}$$

then for each potential emission position $t_{n,k} = (n-1)mT_s + kT_s$ we compute the received energy on the interval from $t_{n,k}$ to $t_{n,k} + T_d + T_p$

$$q_{n,k} = \int_{t_{n,k}}^{t_{n,k}+T_d+T_p} y^2(t)\,dt \tag{26}$$

Let $\{\phi_{n,1},\ldots,\phi_{n,D}\}$ be the kernel of the Karhunen-Loeve expansion of the noise part of the signal $z_q(t)$ on the interval from t_n to $t_n + T_d + T_p$, then $< z_q(t), \phi_{n,i}(t) >= Z_{n,k}(i)$ where $Z_{n,k}$ is a Gaussian random vector with mean zero and covariance matrix $\mathbf{diag}(\nu_i)$. Moreover

$$\begin{aligned} \sum_{i=1}^{D}\nu_i &= E\left[z_q^2(t)\right] \\ &= \frac{N_0}{2} \end{aligned} \tag{27}$$

Let $\sqrt{E_s\lambda_i}s_{n,i} =< s_q(t), \phi_{n,i}(t) >$ which for $k = w$ yields

$$\begin{aligned} E_s\sum_{i=1}^{D}\lambda_i s_{n,i}^2 &= \int_{t_{n,w}}^{t_{n,w}+T_d+T_p} s_q^2(t)\,dt \\ &= \alpha E_s \end{aligned} \tag{28}$$

α is the porportion of signal energy captured by the frontend filter $q(t)$.

Theorem 2: The probability of codeword error is upper bounded by

$$\Pr[\text{error}] \leq M\min_{t>0} e^{-N\left[t\rho+\frac{D}{2}\ln(1-N_0t)-\ln\left((1-p)+pe^{\frac{t}{1-N_0t}\alpha E_s}\right)\right]} \tag{29}$$

with $\rho = (1-\epsilon)\alpha E_s + D\frac{N_0}{2}$ and $p = 1/m$. Note that this result is valid for arbitrary channel statistics.

Proof: The proof uses the same technique as in (1). ■

[2] $q(t) = p(t)$ is the intuitive choice for such a filter

IV. Quasi-Coherent Detection

A question of significant importance is whether coherent detection schemes are robust to channel estimation imperfections. In this section we adress the problem of characterizing the degradation of coherent detection performance due to an additive channel estimation noise. This will be achieved through the derivation of an upper bound on the decoding error probability.

We consider the case where a noisy estimate of the channel $\tilde{h}(t,u) = h(t,u) + n(t,u)$ is available at the receiver side, where $n(t)$ is a white gaussian zero-mean random process with variance $\sigma^2_{\tilde{h}}$. We use the same system described in (III-B.1) Projecting $\tilde{S}$ over the same decomposition kernel as in (7) we obtain

$$\begin{aligned} \tilde{S_n} &= [i \in [0,D], \; < \tilde{s}(t), \phi_{i,n}(t)) >] \\ &= S_{n,w} + N_{n,w} \end{aligned} \tag{30}$$

The received signal is then correlated against the channel estimate for all the possible pulse emission positions at each signal frame

$$\begin{aligned} q_{n,k} &= R_{n,k} \tilde{S_n}^T \\ &= (S_{n,k} + Z_{n,k})\,(S_{n,w} + N_{n,w})^T \end{aligned} \tag{31}$$

An upper bound of the decoding error probability is given by the following theorem

Theorem 3: The probability of codeword error is upper bounded by

$$\Pr[\text{error}] \leq M \min_{t>0} e^{-N\left[t\rho + \frac{D}{2}\ln(1-\frac{N_0\sigma^2_{\tilde{h}}t^2}{2}) - \frac{\alpha E_s N_0 t^2}{4} - \ln((1-p)+pV)\right]} \tag{32}$$

with

$$V = \exp \frac{\alpha E_s \left(2t + \sigma^2_{\tilde{h}} \frac{N_0}{2} t^4\right)\left(\sigma^2_{\tilde{h}} + \frac{N_0}{2}\right)}{2\left(1 - \frac{N_0 \sigma^2_{\tilde{h}} t^2}{2}\right)} \tag{33}$$

and $\rho = (1-\epsilon)\alpha E_s$, $\alpha = \sum_{i=1}^{D} \lambda_i s^2_{n,i}$, and $p = 1/m$. Note that this result is valid for arbitrary channel statistics.

V. Discussion

In this work we assess the potential performance of non-coherent receivers for the detection of UWB signals. It has been shown that the achievable information rates for fairly simple modulation formats (such as m-PPM) can come close to asymptotically optimum (in terms of vanishing spectral efficiency) flash-signaling transmission. We show in Figs. 2, the numerical evaluation of the bounds in the previous sections, where the symbol alphabet size (i.e. m) has been optimized for each SNR. We see that the typical information rate losses are less than a factor 2 with reseasonably simple linear filter analog receivers. The optimization of the modulation size, as a function of the system operating SNR, leads to a constant received peak SNR (and outer code rate on the order of 1/2 irrespective of average received SNR. It is also shown that increasing the system bandwidth W degrades the performance in the low SNR region and that in practice system bandwidths should be on the order of 1, 2 GHz.

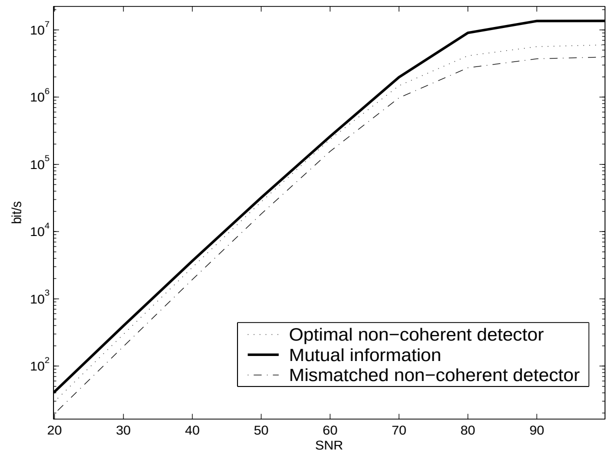

Fig. 2. Achievable rates of energy detection based receivers: Td=50 ns, W=1GHz

The networks which will likely employ UWB signaling, for example *Wireless Personal Area Networks*(WPAN), are characterized by requirements for adhoc and peer-to-peer communications and thus need to use signaling schemes robust to strong impulsive interference (from nerby interferers). We extend the results presented above to the multiple access case through the use of a single user erasure based energy detector and derive similar random coding bounds [10]. The theoretical high processing gain of UWB systems is shown to be effective against multiuser interference and results in information rates close to those of *genie aided* (i.e. known interfering codewords) multiuser receivers in the case of strong interference.

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