ITERATIVE TECHNIQUES FOR CDMA AND ALGORITHMS FOR MIMO DETECTION

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Abstract

The explosive growth of mobile internet is a major driver of wireless telecommunications market today. In future years, the number of online wireless users will exhibit strong progression in every geographic region and devices will tend to support multimedia applications which need big transmission data rates and high quality. Third generation telecommunication systems, like UMTS in Europe, aim at rates approaching 2 Mbits/s in particular cellular environments, but they will initially be deployed in coexisting second generation systems, like GSM, which were conceived to support voice traffic, but are evolving to higher-data-rates versions like GPRS. These hybrid systems have the target to support globally data rates at least of 144 kbits/s and locally of 2 Mbits/s.

This thesis makes several contributions to the area of mobile advanced receivers for CDMA and approximate/exact maximum likelihood (ML) detection for multi-input multi-output (MIMO) systems. Advanced signal processing techniques are applied to increase the performance of the receiver by mitigating the distortion caused by radio propagation channel and the interference introduced by the multiple access to the wireless system.

The conventional receiver for DS-CDMA is matched filter. These receivers are interference limited. Another receiver called zero forcing (ZF) separates cochannel signals but at the expense of increase in signal-to-noise (SNR) at the output of the receiver and because of noise enhancement the performance of the ZF receiver degrades at low SNR. In order to improve the performance of the receiver a natural choice is to minimize the overall error, which results in minimum mean square error (MMSE) receiver. Better results can be obtained if some valid constraints are used for detection. The third type of receiver is ML receiver but unfortunately the computational complexity grows exponentially in the number of users, in case of CDMA and in the number of antennas in case of MIMO systems. An alternative to ML (by enumeration) technique is to increase the likelihood function iteratively until local/global maximum is reached. This iterative technique is called expectation maximization (EM) algorithm. The EM algorithm is a broadly applicable approach to the iterative computation of ML estimates, useful in variety of incomplete-data prob-

lems, where algorithms such as the Newton-Raphson method may turn out to be more complicated. On each iteration of the EM algorithm, there are two steps- called expectation step or E-step and maximization step or M-step. The basic idea of the EM algorithm is to associate with the given incomplete-data problem, a complete-data problem for which ML estimation is computationally more tractable.

In the first part of the thesis, we use EM algorithm to estimate the channel amplitudes blindly and compare the results with the Cramer-Rao bound (CRB). Furthermore, we find low complexity relaxed ML detection for the CDMA, and show its superior performance to the MMSE receiver.

The second part of the thesis concerns the detection problem in MIMO systems. As mentioned earlier, the ML method by enumeration for detection is computational complex. In the language of optimization theory, the ML problem is NP-hard. Recently, low complexity exact ML has been obtained by sophisticated method called sphere decoding. The sphere decoding searches the closest point in a lattice to a given received signal. Its computational complexity is polynomial (if the radius of the sphere is optimally chosen) for high SNR and at low SNR its complexity explodes. We are able to device an algorithm for exact ML detection using a discrete geometric approach. The advantage of this algorithm is that its performance is polynomial irrespective of the SNR and no heuristic is employed in our algorithm. An alternative way to ML problem is to devise low complexity algorithms whose performance is close to the exact ML. This can be done using semidefinite programming (SDP) approach. The computational complexity of the SDP approach is comparable to the average complexity of the sphere decoder but still it is quite complicated for large systems. We obtained low complexity (by reducing the number of the variables) approximate ML by second order cone programming (SOCP) approach.

In the above discussion the channel state information is assumed to be known at the receiver. We further looked into the problem of detection with no channel knowledge at the receiver. The result was the joint channel-symbol estimation. We obtained the results of joint channel-symbol estimation using EM algorithm and in order to reduce the complexity of the resulting EM algorithm, we used mean field theory (MFT) approach (a method vastly used in statistical physics). The MFT approach is used to approximate the posteriori MAI probabilities for MIMO system and the results are compared with exact ML for a known channel.

Résumé

Cette thèse apporte plusieurs contributions dans le domaine des récepteurs CDMA avancés, et des techniques de détection par maximum de vraisemblance (MV) exact et approché pour les systèmes multi-entrées multi-sorties (MEMS). Des techniques de traitement de signal élaborées sont utilisées pour améliorer les performances du récepteur en atténuant la distorsion causée par le canal radio, et l'interférence introduite par l'accès multiple.

Dans la première partie de cette thèse, nous utilisons l'algorithme d'Expectation-Maximization (EM) pour estimer en aveugle les amplitudes des coefficients du canal, et nous comparons les résultats avec la borne de Cramér-Rao. De plus, nous développons une version relaxée de la détection MV, à faible complexité, et montrons que ses performances surpassent celles du détecteur à erreur quadratique moyenne minimale (EQMM). La deuxième partie de cette thèse concerne le problème de la détection dans les systèmes MEMS. En effet, l'énumération inhérente au détecteur MV rend sa complexité rédhibitoire. En langage de théorie de l'optimisation, le problème de détection MV est NP-complet. Récemment, la détection MV exacte à faible complexité a été rendue possible par l'algorithme de décodage par sphère (sphere decoding). Le décodeur par sphère cherche le point d'un treillis le plus proche du signal reçu. Sa complexité à haut rapport signal-à-bruit est polynomiale (à condition que le rayon de la sphère soit choisi de manière optimale). La complexité explose lorsque le rapport signal-à-bruit diminue. Nous proposont un algorithme de détection MV exacte utilisant une approche géométrique discrète. L'avantage de cet algorithme est sa complexité polynomiale, quel que soit le rapport signal-à-bruit, et le fait qu'aucune méthode heuristique n'est employée. Une solution alternative au problème de la détection MV est de développer des algorithmes à faible complexité dont la performance est proche de celle du détecteur MV exact. Ceci peut être fait grâce à l'utilisation de la méthode de programmation semi-définie (en anglais, semi-definite programming). Cette approche offre une complexité comparable à celle du décodeur par sphère, qui reste assez élevée pour des systèmes de grande taille. Nous obtenons une méthode MV approchées à faible complexité par l'approche de la programmation en cône du deuxième ordre. Nous établissons également des bornes sur la performance de la méthode

de programmtion semi-définie.

Jusqu'ici, la connaissance de l'état du canal était considérée comme parfaite. Nous avons également exploré le problème de la détection sans connaissance du canal au récepteur. Le résultat est un algorithme d'estimation conjointe du canal et des symboles, basé sur l'algorithme EM, et la théorie du champ dynamique moyen (CDM, ou mean field theory, méthode largement utilisée en physique statistique) pour en réduire la complexité. L'approche du CDM est utilisée pour approximer les probabilités *a posteriori* d'interférence d'accès multiple. Les performances de la méthode proposée sont comparées à celles du détecteur MV exact pour un canal connu.

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Thanks a lot also to my family for always supporting me.

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Acronyms and Abreviations

ARQ	Automatic Repeat Request
AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
BLAST	Bell Laboratories Layered Space Time
BPSK	Binary Phase Shift Keying
CDMA	Code Division Multiple Access
DF	Decision Feedback
DS-CDMA	Direct Sequence CDMA
EM	Expectation Maximization
FA	Finite Alphabet
FDMA	Frequency-Division Multiple Access
FEC	Forward Error Correction
FH	Frequency Hopping
GML	Gaussian Maximum Likelihood
HMM	Hidden Markov Model
i.i.d	independent and identically distributed
ILSE	Iterative Least Square with Enumeration
ILSP	Iterative Least Square with Projection
KKT	Karush-Kuhn-Tucker
LOS	Line of Sight
LP	Linear Programming
LRT	Linear Response Theory
MAI	Multiple Access Interference
MAX-CUT	Maximum Cut
MF	Matched Filter

MFT	Mean Field Theory
MIMO	Multi-input Multi-output
MLSD	Maximum Likelihood Sequence Detection
MSE	Mean Square Error
MMSE	Minimum MSE
MRC	Maximum Ratio Combining
MUD	Multiuser Detection
NMFT	Naive MFT
NMSE	Normalized MSE
PCA	Principal Component Analysis
PSD	Positive Semidefinite
QPSK	Quarternary Phase Shift Keying
SAGE	Space-Alternating Generalized EM
SDMA	Space Division Multiple Access
SDP	Semidefinite Programming
SF	Spreading Factor
SIMO	Single Input Multiple Output
SISO	Single Input Single Output
SNR	Signal to Noise Ratio
SOCP	Second Order Cone Programming
s.t.	subject to
STBC	Space Time Block Coding
STTC	Space Time Trellis Coding
SVD	Singular Value Decomposition
TDMA	Time-Division Multiple Access
UMTS	Universal Mobile telecommunication System
ZF	Zero-Forcing

List of Symbols

I_N or I	$N \times N$ Identity matrix
j	$\sqrt{-1}$
$(\cdot)^*$	Complex conjugate operator
$(\cdot)^T$	Transpose operator
$(\cdot)^H$	Hermitian operator
$*$	Convolution operator
Tr	Trace operator
det	Determinant
vec	column vectorization of matrices
$\otimes$	Kronecker product
$R(.)$	real part of complex number
$I(.)$	imaginary part of complex number
$E\{.\}$	mathematical expectation

Chapter 1

Introduction

The continous growth of traffic and emergence of new services have begun to change the structure of the wireless networks. Future mobile communications systems will be characterized by high throughput, integration of services and flexibility. The advent of 3^{rd} generation systems will open up a range of possible services and will significantly increase the available data rates and decrease the Bit Error rate (BER). To achieve high data rates at such BERs, multiple access interference (the major impairment) cancelation will be required. 3^{rd} generation systems will use one form or another of Direct Sequence Code Division Multiple Access (DS-CDMA). In the first part of the thesis we study multiuser detection, which has the potential of improving DS-CDMA communications. In multiple access techniques different users share the same communication medium. The three basic multiple access techniques are, Frequency Division Multiple Access (FDMA), Time Division Multiple Access (TDMA) and the Code Division Multiple Access (CDMA). In CDMA system different codes are assigned to each user. In FDMA users are given a separate carrier frequency and in FDMA the users are multiplexed by orthogonal time slots. A CDMA scheme is one in which each user transmits his signal using a bandwidth much larger than the data rate [17]. Two major spreading schemes exist, namely the direct sequence and frequency hopping spreading [14]. In DS-CDMA all users use the same bandwidth, but each user is assigned a distinct code. In practical systems, a combination of the above three multiple access schemes is usually employed (taken two at a time traditionally). An example is the European Global system for Mobile Communications (GSM) or the North American IS-54 standard that are both based upon a combination of FDMA and TDMA multiple access strategies.Another example, Qualcomm Inc.'s IS-95, is di-

rect sequence CDMA based mobile cellular system, with user assigned distinct, pseudo-random (PN) spreading sequences in an otherwise frequency split system. The goal is to make different user signals look as noise-like for each other possible. Other methods of spreading the spectrum like frequency hopping (FH) CDMA, never really became popular for wireless systems.
Perhaps the foremost concern in the successful implementation of future cellular networks is capacity, and can be defined as the number of concurrent users that can be supported for a given total bandwidth. Consequently, a number of comparisons between the above multiple access methods have been carried out in recent years in order to establish the superiority of one over the other in terms of system capacity. However, no practical examples are available to make one believe that one system is better than the other.

1.1 Characterization of the propagation channel

The basic phenomena that influence radio propagation in wireless communication system are:

1. reflection, which occurs when a propagating electro-magnetic wave impinges on a smooth surface with a dimension several times larger the wavelength (λ),

2. scattering, which happens when the wave strikes a rough surface or a body whose dimensions are lesser or of the order of λ, thus causing the reflected energy to scatter all over,

3. diffraction,which arises when a dense obstructing body of dimension larger than λ lies in the path between transmitter and receiver; the electromagmetic wave rolls around the body and can reach the receiver even when there is no line of sight path.

Depending on the type of environment, i.e., urban, rural etc., one or several of these phenomena might occur. Therefore, channel models have been developed for a particular environments that take into account the effects of these mechanisms, and translate them into signal distortions like time-spreading and loss in signal-to-noise ratio (due to multi-path components). The transmitted signal can therefore be considered to be passing through a channel which has a certain impulse response.
There is another concern, called fading, which is the power variation at the receiver due to time varying channel. There are two types of fading:

1. large -scale fading, defined as the average signal power attenuation due to motion over large areas, occurring due to major contours (hills, buildings etc) between the transmitter and the receiver. The receiver is said to be shadowed by these imposing obstacles. Shadowing is statistically characterized as log-normally distributed random variable. If P is the power

transmitted to the kth mobile situated at a distance d_k from the base station, then the received power is given by

$$P_{k,dB} = P_{dB} - L_{k,dB}$$

$$L_{k,dB} = L_k(d_o)_{dB} + 10log_{10}(\frac{d_k}{d_o})^n + G_\sigma$$

where G_σ denotes a zero mean Gaussian random variable (in dB) with standard deviation σ (also in dB). The large scale fading mechanism is surrounding and distance dependent, i.e., even for vehicles moving at high speeds, the variation over time is rather slow. $L_k(d_o)_{dB}$ is the free space path loss at a reference distance d_o somewhere close to the transmitting antenna. Hence the estimate of the total path loss (in dB) including the mean path loss (nth power loss with distance) and the variations about the mean accounting for shadowing, can be obtained ($n = 2$) for free space. It can be smaller in the presence of a guided wave phenomenon in urban streets and larger when obstacles are present, e.g., when the mobile station is situated indoors.

2. Small scale fading manifests itself as rapid changes in amplitude and phase of the received signal. These variations are the result of a large number of multi-path components with uniformly distributed phases adding up over time. When the received signal is composed of multiple reflected rays plus a significant line of sight, the envelope amplitude due to small scale fading has Rician pdf. The fading in this instance is called Rician fading. Rayleigh fading occurs when there is no lone-of-sight component present between transmitter and the receiver.

   The worst case variations can be of the order of 20-30 dB. The variations are carrier frequency dependent.

The most important concept in describing the channel is channel coherence. Below we discuss several types of coherences that a wireless channel may exhibit.

1.2 Coherence versus selectivity

Fading is a general term used to describe a wireless channel affected by some type of selectivity. A channel has selectivity if it varies as a function of either time, frequency or space. The opposite of selectivity is coherence. A channel has coherence if it does not change as a function of time, frequency, or space over a specified window of interest.

Indeed the wireless channels may be functions of time, frequency, and space. The most fundamental concept in channel modeling is classifying the three possible channel dependencies of time, frequency, and space either coherent or selective. We will discuss each type of coherence briefly.

1.2.1 Temporal coherence

If the envelop of the modulated carrier wave does not change over a time window of interest, the wireless channel is said to have temporal coherence. Mathematically, we express this condition in terms of a narrow-band (no frequency dependence), fixed (no spatial dependence) channel, $\tilde{h}(t)$:

$$|\tilde{h}(t)| \approx V_o, \ \ for \ \ |t - t_o| \leq \frac{T_c}{2} \tag{1.1}$$

where V_o is some constant voltage, T_c is the size of the time window of interest, and t_o is some arbitrary moment in time. The largest value of T_c, on average, for which the above equation holds is called the coherence time and is the approximate time window over which the channel appears static. In the microwave and millimeter frequency regime the most common cause of temporal incoherence is the motion by either the transmitter or the principal scatterers in the propagation environment.

1.2.2 Frequency coherence

A wireless channel has frequency coherence if the magnitude of the carrier wave does not change over frequency window of interest. This window of interest is usually the bandwidth of the transmitted signal. Mathematically, we express the condition of frequency coherence in terms of the static (no time dependence), fixed channel, $\tilde{h}(f)$:

$$|\tilde{h}(f)| \approx V_o, \ \ for \ \ |f_c - f| \leq \frac{B_c}{2} \tag{1.2}$$

where V_o is some constant amplitude, B_c is the size of the frequency window of interest, and f_c is the center carrier frequency. The largest value of B_c for which equation (1.2) holds is called the coherence bandwidth and is the approximate range of frequencies over which the channel appears static.

The dispersion of multi-path propagation causes frequency coherence in the wireless communications system. Since each received multi-path wave has traveled a different path from the transmitter, the same transmitted signal will arrive at the receiver as a cluster of symbols, each with unique time delay. A dispersive channel, in time domain, introduces inter-symbol interference (ISI). In the frequency domain, a dispersive channel has peaks and valleys across the bandwidth of interest. This behavior in the frequency domain gives rise to two distinct classifications of fading in wireless communications. A wireless channel with a coherence bandwidth that is less than the bandwidth of the transmitted signal is said to have frequency-selective fading. A channel with coherence bandwidth that is greater than the transmitted signal bandwidth is said to have frequency flat fading.

1.2.3 Spatial coherence

Spatial coherence of a wireless channel is defined as the magnitude of the carrier wave that does not change over spatial displacement of the receiver. Once again, we express the condition of spatial coherence in terms of a static narrow-band channel, $\tilde{h}(r)$, which is function of one-dimensional (1D) space, r:

$$|\tilde{h}(r)| \approx V_o, \;\; for \;\; |r - r_o| \leq \frac{D_c}{2} \tag{1.3}$$

where V_o is some constant amplitude, D_c is the size of the position displacement, and r_o is an arbitrary position in space. The largest value of D_c for which the equation (1.3) holds is called coherence distance and is approximate distance that a wireless receiver can move with channel appearing to be static. Spatial coherence is the result of the multi-paths arriving at the receiver from different directions. These multi-path waves create pockets of constructive and destructive interference so that the received signal power does not appear to be constant over small changes in the receiver position. Thus, this type of channel exhibits spatial selectivity. If the distance traversed by a receiver is greater than the coherence distance of the channel, we say that the channel experiences small-scale fading (see [19] for more details on wireless channels).

1.3 Detection in CDMA system

Now we discuss briefly the detectors used in the CDMA system.

The conventional receiver used in the CDMA is the matched filter. It consists of parallel bank of K (the number of users) filters, each matched to its own code waveform. If only one user is active in the system, then this receiver is optimal for a particular user communicating over additive white Gaussian noise. If we accurately model interference from other users as an additive white Gaussian noise, the matched filter receiver works as Maximum Likelihood (ML) single user detector. One of the disadvantages of the conventional scheme is that it is severely affected by multiple access interference (MAI), making such system interference limited [12]. The conventional detectors also suffers from the near far problem in practice, which means that high power users destroys the communication of low power users, even if the code waveforms have low cross-correlations. Better results can be obtained if we jointly detect all users. For asynchronous transmission of L information bit per user using Binary Phase Shift Keying (BPSK)or Quadrature Phase Shift keying (QPSK) modulation and spreading, the ML detection is equivalent to minimizing Euclidean distance between actual and the received signal, assuming an additive white Gaussian noise channel. This problem is NP-hard and it is too complex for asynchronous CDMA even for a moderate number of users.

For certain special correlation matrices, it has been shown that the ML detection can be obtained in polynomial time [50, 61]. In [61], the authors identify a class of optimum multiuser detection problem which can be solved in polynomial complexity in the number of users. The identification is based on transforming a quadratic 0-1 programming problem into an equivalent problem in graph theory with positive edge weights. For synchronous CDMA, the result translates to designing a set of pseudo-random codes with the property that the cross correlation between every pair of codes in the set over one symbol period is non-positive. The authors also devised the method to construct codes with that property.

The exponential complexity of the ML detection has inspired a considerable effort over the past decade to device suboptimal low complexity receivers. Iterative methods to the ML problem have also been suggested in overwhelming articles. Without the ambition of being exhaustive, we refer to [34,48,114,113,112,111,42,43,49,47,59]. Based on iteratively maximizing the likelihood function, in [34] EM algorithm is applied to the CDMA detection problem by treating the bits of the interfering users as hidden data when updating the estimate for a given user's bit. Their derivation led to the iterative receivers that use soft-decisions for interference cancelation and/or sequential (rather than parallel) updates of estimates for users' data. Low complexity successive interference cancelation with SISO decoding is performed in [39]. The channel parameters are updated using the EM algorithm. The feature of their algorithm is that single user SISO decoders provide at each iteration an estimate of the a posteriori probabilities (APP) for the user code symbols, which are used to form soft estimate of interference to be subtracted from the received signal. In this way, the contribution of a user is effectively subtracted from the signal only if the symbol decision is sufficiently reliable. Using this scheme performance very close to single user performance is achieved after few iterations.

Another possibility to find low complexity solution is to relax the Finite Alphabet (FA) constraint and to solve the resulting constrained problem. In [88,89], the solution is allowed to lie in a closed convex set. One way [89], is to confine the solution vector to lie within a hypercube described by the data points. The algorithm [88] constrains the data estimate to lie within sphere.

To make a ML decision for a multiuser detection (MUD), we need to solve a binary constraint problem.

1.4 Outline of the thesis

The objective of the first part of this work is to use a constrained optimization for the CDMA and also use EM algorithm for the amplitude estimation in the synchronous CDMA case.
In [88], the authors considered the problem of maximizing the likelihood function over a sphere, i.e. confine the solution vector to lie within the sphere and project the solution vector on the sphere. This detector is ML under the assumption that the detected data vector is constrained to lie within a hypersphere. Based on the defining Karush-Kuhn-Tucker point, it is shown that the suggested detector is closely related to the MMSE detector. They analysed the convergence issues and gave an efficient implementation procedure. In fact, in the sphere constrained problem the solution vector lies on the sphere and not in the interior of the constraining sphere (as is done in [88]). The other problem with their method is that a small error in the solution vector can cause a large error when projected on to a sphere (provided the solution vector is well inside the sphere). In chapter 4, we constrain the solution vector to lie almost on (very close to) the sphere and we jointly estimate the complex channel coefficients and data vector. It is done as follows. In the objective function, we subtract/add the Kullback-Leibler (KL) distance function or euclidean distance function to keep the old parameter set close to the new ones. These distance functions can be considered as penalty terms. The above augmented cost function can be maximized/minimized subject to the constraint that the detected data vector lies on the sphere. In chapter 5, we further improved the result by solving exactly the sphere constraint problem (i.e solution vector lying on the surface of the sphere).
In [109], the authors used a Gaussian mixture formulation to model the synchronous CDMA and they used the EM algorithm to cope with the unknown amplitudes. They solved the problem by first projecting the received signal on the signal subspace to reduce the dimensionality of the problem. In chapter 3, we showed that the dimensionality reduction operation results in the failure of the EM algorithm when the number of users are moderate or small and discuss the convergence issues.
The results of chapter 3, chapter 4, and chapter 5 are published in [110], [98], and [116] respectively.

The second part of the thesis concerns detection of MIMO systems.
Digital communications using multiple-input-multiple output (MIMO), sometimes called ”volume to volume”wireless link, has emerged as one of the most significant technical breakthroughs in modern communications. The technology figures prominently on the list of recent technical advances with a chance of resolving the bottleneck of traffic capacity in future Internet-intensive

wireless networks. Perhaps even more surprising is that just a few years after its invention, the technology seems poised to penetrate large-scale standard-driven commercial wireless products and networks such as broadband wireless access systems, wireless local area networks (WLAN), 3G networks and beyond.

A MIMO system is simply the deployment of multiple antennas at the transmitter and the receiving end for a wireless system. New MIMO systems represent a huge change in how wireless communications systems are designed. This change reflects how we view multi-path in a wireless system:

The Old perspective: The ultimate goal of wireless communications is to combat the distortion caused by multi-path in order to approach the theoretical limit of capacity for band-limited channel.

The New respective: Since multi-path propagation actually represents multiple channels between a transmitter and receiver, the ultimate goal of wireless communications is to use multi-path to provide higher total capacity than the theoretical limit for a conventional band-limited channel. This philosophical reversal implies that many of the engineering design rules of thumb that were based on pessimistic worst-case scenario channel models have now become unrealistically optimistic. The idea behind MIMO is that the signals on the transmit antennas at one end and the receive antennas at the other end are ”combined” in such a way that the BER or data rate of the communication for each MIMO user will be improved. Such an advantage can be used to increase the network’s quality of service. However, reliable decoding in these systems requires very high complexity.

For a wide class of space-time transmission schemes, ML decoding requires to solve an integer least square problem, which is, in general, NP-hard. Practical methods to solve this employ approximations or heuristics. One of the suboptimal receivers used in MIMO systems is the zero forcing receiver, (i.e., invert the channel matrix and round to the closest integer and is called Babai estimate). The other more sophisticated but suboptimal receivers are nulling and cancelling (Decision feedback MIMO). They use the Babai estimate for one of the entries of symbol and assume that this symbol is known, subtract out its effect to obtain a reduced integer least square problem, then proceed similarly for the next symbol. Another receiver proposed in Bell laboratories is nulling and cancelling with optimal ordering, also called BLAST [3]. The basic principle of BLAST is to perform nulling/cancelling from the strongest to the weakest signal. However, BER performance of these receivers are inferior to those of exact ML methods. Exact methods that search over the entire Finite Alphabet(FA) require an exponential search. More sophisticated exact methods such as Kannan’s algorithm [74], the KZ algorithm [41], and the sphere decoding algorithm [3] attempt to reduce the search space. In the sphere decoding algorithm, we find the lattice points lying in a hypersphere centered around the received signal, and then we determine the closest lattice point to the received signal. The expected complexity of the sphere decoder when the radius is chosen correctly is $O(n^3)$ (for high SNRs). Choosing the optimal radius for the sphere decoder is NP-hard. Recently semidefinite relaxation has been successfully applied to the CDMA and MIMO systems. Using semidefinite relaxation very close to exact ML performance is obtained with complexity of $O(n^{3.5})$. Semidefinite programming (SDP) relaxation has been used for decoding in CDMA by [99,100,40,101]. The authors in [100] used SDP relaxation scheme to the synchronous CDMA and also showed that some existing detectors such as the decorrelator, the LMMSE detector, and a particular form of the modified SAGE detector can be considered as degenerate forms of the SDP relaxation ML. The SDP ML detector offers an attractive trade-off between BER performance and computational cost. Lattice reduction aided detector for the MIMO system was proposed in [73]. In [73], the authors used a lattice reduction technique for two transmit antennas and two receive antennas systems. They used a Gauss lattice basis reduction method to enhance the performance of the MIMO system. The work in [73] was extended for general MIMO systems in [72], using algorithm proposed by A. K. Lenstra, H. W. Lenstra, and Lovasz (“ LLL algorithm “) for lattice reduction, which is quite complex as compared to Gauss method (Gauss method works for 2×2 system). The objective of the second part of the thesis is to device low complexity algorithms for channel estimation and symbol detection. First of all, we assume that the channel state information is known at the receiver. As stated earlier the complexity of the SDP relaxation is $O(n^{3.5})$. Still for large system this could be computationally quite complex. In chapter 8, we propose to apply a second order cone programming (SOCP) approach to resolve large system problems, which offers substantial computational savings over SDP relaxation scheme and the sphere decoding, while maintaining the performance arbitrarily close to ML. In chapter 9, we derive exact ML detection scheme for MIMO system, when the number of receiving antennas is fixed, by maximizing the Euclidean distance function over zonotope. Using a classical theorem of discrete geometry, it is shown that vertices search can be done in polynomial time $O(n^m)$, where n, m are the number of transmit antennas and receive antennas respectively. This method is polynomial time irrespective of the SNR (as opposed to the sphere decoder whose complexity is exponential at low SNRs).

In the above chapters for MIMO detection, we assume that the channel is known at the receiver. In chapter 11, we propose to detect the symbols of each user and estimate the channel iteratively for a multiuser space time coding system. The channel gets estimated blindly via expectation maximization (EM) algorithm by formulating the problem as a Gaussian mixture model. The estimated channel is then used to detect the symbols for each user, which is also done in an iterative fashion, i.e., by user-wise detection. We consider FA for MAI, to simplify and to reduce the complexity of the resulting EM algorithm, we consider the introduction Mean Field methods for approximating the a posteriori MAI symbol probabilities. The idea of the Mean Field method is borrowed from the statistical physics community where it is extensively used for approximating probabilities in Ising model [97]. The BER using our method is very close to the exact ML, i.e., ML by

exhaustive search with exact channel state information at the receiver. In chapter 10, we use the EM algorithm to estimate the channel and to detect the information bits iteratively. Two cases for interfering users bits are considered, corresponding to Gaussian and discrete MAI priors. The algorithm iterates between channel estimates and symbol estimates until convergence. Simulations shows that BER quite close to the ML is achieved. In this chapter we also make use of the Mean Field method to simplify posteriori probability of interfering users bits while dealing the case of discrete MAI prior.
The results of the chapter 8, chapter 9, chapter 10, and chapter 11 will be published in [22], [20], [53], and [51] repectively.

Part I

Iterative estimation for CDMA

Chapter 2

CDMA fundamentals

In this chapter, we introduce the CDMA channel model and present optimal decisions rules.

2.1 System model

Let us consider a CDMA channel that is shared by K active users. Each user is assigned a signature waveform $p_k(t)$ of duration T, where T is the symbol interval. A signature waveform may be expressed as

$$p_k(t) = \sum_{n=0}^{N-1} a_k(n) p(t - nT_c), \quad 0 \le t \le T \tag{2.1}$$

where $\{a_k(n), \ 0 \le n \le N-1\}$ is a code sequence consisting of N chips that take values $\{\pm\}$, and $p(t)$ is the pulse of duration T_c, where T_c is the chip interval. Thus, we have N chips per symbol and $T = NT_c$. Without loss of generality we assume that K signature waveforms have unit energy.

The information sequence of the kth user is denoted by $\{d_k(m)\}$, where the value of each information symbol may be chosen uniformly from the set D. All data sequences are equally probable and each symbol is statistically independent of the other symbols and also between users. Let us consider the block of symbols of some arbitrary length, L. The corresponding equivalent low-pass waveform received over the channel for the kth user can be expressed as

$$s_k(t) = c_k(t) * \sum_{i=1}^{L} \sqrt{E_k(i)} d_k(i) p_k(t - iT), \tag{2.2}$$

for $0 \leq t \leq (L+1)T$, where $*$ denotes the convolution operator, $c_k(t)$ is the complex channel impulse response and $E_k(i)$ represents the energy per symbol. The composite transmitted signal for the K users may be expressed as

$$s(t) = \sum_{k=1}^{K} s_k(t - \tau_k) = \sum_{k=1}^{K} \sum_{i=1}^{L} \sqrt{E_k(i)} d_k(i) c_k(t) * p_k(t - iT - \tau_k), \tag{2.3}$$

where τ_k is the transmission delay for user k, which satisfy the condition $0 \leq \tau_k \leq T$ for $k = 1, 2, \ldots, K$. Without loss of generality, we assume that $0 \leq \tau_i \leq \tau_2 \leq \ldots, \leq \tau_K < T$. This is the model for the multiuser transmitted signal in asynchronous mode. For the synchronous case all time delays are zero.

For a frequency non-selective channel, the signal bandwidth is significantly smaller than the coherence bandwidth of the channel, and the multi-path components are not resolvable [14]. In this case, the received signal is the transmitted signal multiplied by a complex-valued random process representing the time variant characteristics of the channel. Furthermore, if we assume that the signal duration is significantly smaller than the coherence time of the channel, the channel is slowly fading and the channel parameters, attenuation and phase shift, are essentially constant for the duration of at least one symbol interval. When these assumptions are applied for all users, they experience the same simple AWGN channel. The transmitted signal is also assumed to be corrupted by AWGN. Hence the received signal may be expressed as

$$r(t) = s(t) + n(t) \tag{2.4}$$

where $n(t)$ is the noise, with variance σ^2.

2.2 Detection of signals in AWGN

In this section the matched filter (MF) output for DS-CDMA system with K users is derived. The sampled output y contains all data from the received continuous time signal $r(t)$, i.e., it is sufficient statistics for detection. The problem of detecting the signals can be viewed as an M hypothesis testing model, where M is the number of all possible combinations of the data symbols $d \in D^{LK}$. Each particular combination of d for hypothesis H_i is denoted by d_i. The hypothesis testing can be modeled as

$$H_i \;\; r(t) = s(t, d_i) + n(T), \quad -\infty < t < \infty, \;\; d_i \in D^{LK}, \;\; 1 \leq i \leq M, \tag{2.5}$$

where $s(t, d_i)$ and $n(t)$ represent the signal for hypothesis H_i with data symbols d_i and AWGN, respectively. The white noise has double-sided power spectral density $N_o/2$.

The problem is to observe $r(t)$ and to decide which hypothesis is true with minimum probability of error. The following idea will enable us to solve this problem easily. Our observation is a time continous random waveform. The first step is to reduce it to random variables collected in so called received vector. The method to obtain the received vector r is defined by series expansion [24].

$$r_k = \int_{-\infty}^{\infty} r(t)\phi_k^*(t)dt$$

$$r_k = \int_{-\infty}^{\infty} s(t)\phi_k^*(t)dt + \int_{-\infty}^{\infty} n(T)\phi_k^* dt \tag{2.6}$$

$$r_k = s_k + n_k \tag{2.7}$$

where r_k is the kth component of the vector r and $\phi_k^*(t)$ is the kth basis function for the series expansion. The vector s consisting of the components$\{s_k\}$ is called the signal vector and n is the noise vector.

A sufficient statistics of $r(t)$ is the sampled output of the matched filter of all the users for the whole interval. The MF output for the kth user during the ith signal interval is

$$y_k(i) = \int_{iT+\tau_k}^{(i+1)T+\tau_k} r(t)p_k(t - iT - \tau_k)dt, \;\; 1 \le k \le K, \;\; 1 \le i \le L,$$

$$= \int_{iT+\tau_k}^{(i+1)T+\tau_k} s(t)p_k(t - iT - \tau_k) + n(t)p_k(t - iT - \tau_k)dt \tag{2.8}$$

Using vector notation, the LK matched filter outputs can be expressed as

$$y = RCd + z \tag{2.9}$$

where $y = [y(1)\ldots y(L)]^H$, $y(i) = [y_1(1)\ldots y_K(L)]^H$, $d = [d(1)\ldots d(L)]^H$, $d(i) = [d_1(1)\ldots d_K(L)]^H$, $z = [z(1)\ldots z(L)]$, and $z(i) = [z_1(1)\ldots z_K(L)]^H$.

C is the diagonal matrix that contains the channel coefficients of the users, and R is the $LK \times LK$ correlation matrix of p_k. The Gaussian noise vector z has zero mean and autocorrelation matrix

$$E[zz^H] = \sigma^2 R \tag{2.10}$$

2.3 The optimum detector

The maximum a posteriori (MAP) detection criteria which minimizes the probability of error is based on maximizing the probability that d was transmitted given that y is received. Thus, it decides H_i if

$$P(H_i|y) > P(H_j|y) \;\; i \neq j$$

if the prior probabilities are equal then we have

$$P(H_i|y) = \frac{P(y|H_i)P(H_i)}{P(y)} = CP(y|H_i) \tag{2.11}$$

where C is the term which does not depend on which hypothesis is true. To maximize $P(H_i|y)$, we need only to maximize the likelihood function $P(y|H_i)$. Hence, for equal prior probabilities we decide H_i if

$$P(y|H_i) > P(y|H_j), \;\; i \neq j$$

This is known as the maximum likelihood (ML) decision rule. Since the noise z is Gaussian and the mean value of y conditioned on the transmitted vector d is, $E[y|d] = RCd$. This results in conditional pdf,

$$P(y|d) = \frac{1}{(\pi\sigma^2)^{KL/2}|R|^{1/2}} \exp(-\frac{(y-RCd)^H R^{-1}(y-RCd)}{\sigma^2}) \tag{2.12}$$

The negative loglikelihood function is

$$l(d) = (y-RCd)^H R^{-1}(y-RCd),$$

and to do ML detection for the symbols we have

$$\hat{d} = arg \;\; min_{d \in D^{LK}} \;\; d^H C^H RCd - 2Re\{y^H Cd\}. \tag{2.13}$$

The solution of the above equation requires a search over all the D^{LK} possible combinations of the components of the vector d. It is thus clear that the computational complexity increases exponentially with the number of users.

In the case of synchronous system, real channels and binary symbols, we can write the detection problem as the following optimization problem

$$\hat{d} = arg \;_{min_{d \in \{\pm 1\}^K}} \;\; d^T C^T RCD - 2y^T Cd, \tag{2.14}$$

and in the case of complex channels and QPSK symbols we have

$$\hat{d} = arg \;\; min_{d \in \{\pm 1 + \pm j\}^K} \;\; d^H C^H RCd - 2Re\{y^H Cd\} \tag{2.15}$$

which can be written as

$$\hat{d} = arg\ \ min_{\tilde{d}\in\{\pm 1\}^{2K}}\ \ \tilde{d}^T\tilde{R}\tilde{d} - 2\tilde{d}^T\tilde{y} \tag{2.16}$$

In the above equation we have converted complex quantities into twice larger real quantities.
In this chapter, we have developed the MAP and ML criteria for a CDMA channel. These criteria describe the decision rules based on the received signal. It is also shown that finding ML solution requires complexity that grows exponentially with the number of users.

Chapter 3

Iterative blind demodulation of synchronous CDMA

Multiuser detection is known to drastically increase the bandwidth efficiency of CDMA systems compared to conventional detection method using RAKE receiver.
Widely used techniques consist of removing the multiple access interference (MAI) from the received signal before making the data decision. In this chapter, iterative blind estimation of the complex amplitudes of the users is considered. A Gaussian mixture model formulation of the problem is introduced, and the Expectation Maximization (EM) algorithm is used for the estimation of the users' amplitudes. Simulation results compare the performance of the proposed algorithm with the Cramer-Rao bound.

3.1 Introduction

Code Division Multiple Access (CDMA) is one of the most common multiple access techniques for wireless communication systems. In CDMA, all users use the entire frequency band and are separated at the receiver by each user's quasi-orthogonal spreading codes in order to reduce inter-user interference. In recent years, various kinds of receivers have been proposed for the CDMA system. In this chapter, we consider the problem of estimating the received amplitudes of the users knowing only their spreading codes. Talwar, et al [81] proposed iterative least square with enumeration (ILSE). This method solves the problem by estimating the channel by short training sequence or from previous estimates and finds the data sequence over all possible data in the Finite Alphabet (FA). The authors also proposed iterative least square with projection (ILSP) which also initially estimates the channel with the same method as for ILSE but treats the problem as a continuous optimization problem, and projects the results onto the discrete alphabet. Iterative joint symbol detection and channel estimation for the CDMA using SAGE algorithm is proposed in [48]. In [34], the EM and SAGE algorithms are applied to derive various multiuser detectors for the white Gaussian noise channel. Monte Carlo simulations show near-far resistance of these schemes. In [114], the authors have proposed a class of nonlinear multiuser detectors. These "iterated-decission" multiuser detectors use optimized multi-pass algorithms to successively cancel MAI from the received data and generate symbol decisions whose reliability increases monotonically with each iteration. They significantly outperform decorrelating detectors and linear MMSE detectors, but have the same order of computational complexity. In [80,109], the authors considered the projection of the received signal on the signal subspace of the received signal autocorrelation matrix, and applied the Gaussian mixture formulation for amplitudes estimation. Their proposed algorithm is faced with two problems

1. Eigenvalue decomposition of the received signal autocorrelation matrix (a computationally complex operation), an other algorithm must be used for signal subspace tracking and also signal subspace mismatch can deteriorate estimation of the parameters.

2. The most important one is that by projecting the data vector onto signal subspace or any other matrix of lower dimension, it can be imagined (and this is borne out by experience with EM like techniques), that the result may not converge to true means of the Gaussians. This can be explained by the following reason: Let M be the number of Gaussians and P be the dimensionality of the data. If the dimension is decreased from P to q, the average Euclidean distance between any two means decreases as $\sqrt{q/P}$, and the probability that the means are separated by less than 2σ increases. The criterion for the separation of two Gaussian distributions in one dimension is that the distance between two means is greater than twice the standard deviation (2σ). Furthermore, Gaussians that are poorly separated in the original dimension will tend to become even more poorly separated as the dimension-

ality is decreased. Thus, it is very important that the Gaussians remain well separated after projection onto a lower dimensional space. If they are not, it will be difficult for the EM algorithm to recognize overlapping components as distinct Gaussian distributions, resulting in a total failure of the EM algorithm.

Our approach, considers directly the output of the channel (the received signal) as the mixture of a known number of Gaussian and estimates its parameters. This avoids the discussed problems, and by keeping the spreading factor not too high, the computational complexity is kept moderate (comparable to the case in which the projection is done) . Direct Maximum Likelihood (ML) estimation of parameters is complex, and therefore we use expectation maximization (EM) algorithm to find the parameters of our model. Mixture models, in particular mixtures of Gaussian, have been a popular tool for density estimation, clustering and unsupervised learning with wide range of applications. Mixture models are one of the most useful tools for handling incomplete data, in particular hidden variables. For Gaussian mixtures, the hidden variable indicate for each data point the index of the Gaussian that generated it. The EM technique is used to iteratively update the maximum likelihood estimate of the parameters of the mixture which are used to obtain the amplitudes of the users.

The rest of the chapter is organized as follows: The signal model for the problem is described in section 3.2. Section 3.3 is devoted to the principle of the EM algorithm. In section 3.4, 3.5, 3.6 and 3.7 EM formulation of the problem, convergence rate, simulations, performance are repectively analyzed. Some conclusions are finally drawn.

3.2 Signal model

We consider DS CDMA with K-users and a processing gain P. The output of the channel is chip matched filtered and sampled at the chip rate. The system is assumed to be synchronous. In a single data interval we have a P-dimensional vector x, given by

$$x = SAb + n \tag{3.1}$$

where S is $P \times K$ matrix whose columns are K users normalized spreading sequences:

$$S = [s_1|s_2|\ldots s_K]. \tag{3.2}$$

In eq (3.1), $A = \text{diag}(A_1, A_2, ...A_K)$, are the users' received amplitudes, $b = [b_1, b_2, \ldots, b_K]$ contains the symbols transmitted by the users, and n is a P dimensional Gaussian random vector for noise with covariance matrix given by $\sigma^2 I$, where I is identity matrix.

We assume that the symbols of the different users are independent i.e. $E[b_k b_l] = 1$ for $k = l$ and 0 otherwise.

We can write equation (3.1) as

$$x = Hb + n, \tag{3.3}$$

where $H = SA$ is $(P \times K)$ dimensional matrix.

Given model of equation (3.3) our goal is to estimate A (i.e. users signal amplitudes) from multiple independent observations of x.

3.3 EM framework for Maximum Likelihood estimation

First of all, we briefly describe EM algorithm.The Expectation-Maximization (EM) algorithm [77,79], is a broadly applicable approach to the iterative computation of maximum likelihood (ML) estimates, useful in a variety of incomplete-data problems. The EM algorithm is closely related to the ad hoc approach to estimation with missing data, where the parameters are estimated after filling in initial values for the missing data. The latter are then updated by their predicted values using these initial parameters estimates. The parameters are then re-estimated, and so on, proceeding iteratively until convergence. The development of the EM algorithm and the related methodology together with the availability of inexpensive and rapid computing power have made the analysis of data sets much more tractable than they were earlier.
EM algorithm is an iterative approach to Maximum Likelihood Estimation (MLE), originally formalized in (Dempster, Laird and Rubin, [78]). Each iteration is composed of two steps: an expectation (E) step and a maximization (M) step. The aim is to maximize the loglikelihood $l(\theta; D) = \log L(\theta; D)$, where θ are the parameters of the model and D are the data. Suppose that this optimization problem would be simplified by the knowledge of the additional variable χ, known as missing or hidden data. The set $D_c = D \cup \chi$ is referred to as the complete data set (in the same context D is referred to as incomplete data set). Correspondingly, the loglikelihood function $l_c(\theta; D_c)$ is referred to as complete data likelihood. χ is chosen such that the function $l_c(\theta; D_c)$ would be easily maximized if χ were known. However, since χ is not observable, l_c is a random variable and cannot be maximized directly. Thus, the EM algorithm relies on integrating over the distribution of χ, with the auxiliary function $Q(\theta, \hat{\theta}) = E_\chi[l_c(\theta; D_c|D, \hat{\theta}]$, which is the expected value of the complete data likelihood, given the observed data D and the parameter $\hat{\theta}$ computed at the previous iteration. Intuitively, computing Q corresponds to filling the missing data using the knowledge of the observed data and previous parameters. The auxiliary function is deterministic and can be maximized. An EM algorithm iterates the following two steps, for k=1,2,...., until a local or global maximum of the likelihood is found.

Expectation: Compute

$$Q(\theta; \theta^{(k)}) = E_{\chi}[l_c(\theta; D_c|D, \theta^{(k)}] \tag{3.4}$$

Maximization: Update the parameters as

$$\theta^{(k+1)} = arg\ _{max_\theta} Q(\theta; \theta^{(k)}), \tag{3.5}$$

Often in practice, the solution to the M step exists in closed form. In some cases, it is difficult to analytically maximize $Q(\theta; \theta^{(k)})$, as required by the M-step of the above algorithm, and we are only able to compute a new value $\theta^{(k+1)}$ that produces an increase of Q at each iteration, i.e., choose $\theta^{(k+1)}$ to increase the $Q(.)$ function $Q(\theta; \theta^{(k)})$ at each iteration. Hence the likelihood function increases after each iteration. In this case we have so called generalized EM (GEM) algorithm.

We have explained EM algorithm for ML estimation but it can also be used for maximum a posteriori estimation problems. The expectation step remains the same as for the ML estimation but the maximization step differs in that the objective function for the maximization is equal to $Q(\theta; \theta^{(k)})$ augmented by the log prior density, $\log p(\theta)$.

The EM algorithm has several appealing properties relative to other iterative algorithms such as Newton-Raphson and Fisher's scoring methods for finding MLEs. Some advantages compared to the other algorithms are as follows:

1. The EM algorithm is numerically stable with each EM iteration increasing the likelihood (except at the fixed point of the algorithm).
2. The EM algorithm has reliable global convergence under fairly general conditions.
3. The EM algorithm is generally easy to program, since no evaluation of the likelihood nor its derivatives are involved.
4. The EM algorithm requires small storage space. For instance, it does not have to store information matrix or its inverse at any iteration.

3.4 Formulation of EM for Gaussian mixture problems

We consider the BPSK case in which the transmitted data take on two possible values $\{-1, +1\}$ with all symbol vectors being equally likely.

In ML estimation problems we have a density function $P(x|\theta)$ that is governed by the set of parameters θ (e.g. P might be the set of Gaussians and θ could be the means and covariances). The data is of size N, supposedly drawn from this distribution, i.e $X = [x_1, ...x_N]$. That is, we assume that these data vectors are independent identically distributed (i.i.d) with distribution P. Therefore, the resulting density for the samples is

$$p(X|\theta) = \prod_{t=1}^{N} P(x_t|\theta) = L(\theta|X).$$

This function $L(\theta|X)$ is called the likelihood of the parameters given the data, or just the likelihood function. In the ML problem, our goal is to find θ that maximizes L. That is, we wish to find θ^* where

$$\theta^* = \arg\max_{\theta} L(\theta|X). \tag{3.6}$$

Assuming that the channel output x can be approximated by Gaussian distributions, i.e., $P(x/\theta)$ can be modeled as P-dimensional mixture of Gaussians. We can write

$$P(x|\theta) = \sum_{j=1}^{M} \alpha_j P(x|m_j, \Sigma_j), \tag{3.7}$$

where $M = 2^K$ and

$$P(x|m_j, \Sigma_j) = \frac{1}{(2\pi)^{(P/2)}|\Sigma_j|^{1/2}} \exp\left(-\frac{1}{2}(x - m_j)^T \Sigma_j^{-1}(x - m_j)\right), \tag{3.8}$$

with $\alpha_j \geq 0$, and $\sum_{j=1}^{M} \alpha_j = 1$. The parameter vector θ consists of mixing proportions α_j, the means vectors m_j, and the covariance matrices Σ_j. Given M and given N independent, i.i.d. samples $\{x_t\}_1^N$, we obtain the following likelihood

$$l(\theta) = \sum_{t=1}^{N} \log \sum_{j=1}^{M} \alpha_j P(x_t|m_j, \Sigma_j) \tag{3.9}$$

which is difficult to optimize because it contains the logarithm of a sum. If we consider X as incomplete, since we do not know which index j, within the mixture probability density function output has originated. Consider $\chi = \{x\}_{t=1}^{N}$ as the incomplete data, and we suppose the existence of unobserved data items $= \{y_i\}_{t=1}^{N}$ whose values indicate which component density generated each data item, then the likelihood expression is significantly simplified. That is, we assume that $y_t \in 1, \ldots, M$ for each i, and $y_t = k$ if the t^{th} sample was generated by the k^{th} mixture component. The complete data likelihood becomes.

$$\log(P(\chi, \mathcal{Y}|\theta)) = \sum_{t=1}^{N} \log(P(x_t|y_t)P(y)) = \sum_{t=1}^{N} \log(\alpha_{y_t} p_{y_t}(x_t|\theta_{y_t})) \tag{3.10}$$

The problem, of course, is that we do not know the values of $\mathcal{Y}$. We can proceed if we assume $\mathcal{Y}$ to be a random quantity.
First of all we derive the expression for the distribution of the hidden data. Let

$$\Theta^g = (\alpha_1^g, \ldots, \alpha_M^g, \theta_1^g, \ldots, \theta_M^g)$$

be the appropriate parameters for the likelihood $L(\Theta^g|\chi, \mathcal{Y})$. Given Θ^g, we can easily compute $p_j(x_t|\theta_j^g)$ for each t and j.The mixing proportion parameter α_j can be thought of as prior probabilities of each mixture component. Using Bayes's rule we have

$$p(y_t|x_t, \Theta^g) = \frac{\alpha_{y_t}^g p_{y_t}(x_t|\theta_{y_t}^g)}{p(x_t|\Theta^g)} = \frac{\alpha_{y_t}^g p_{y_t}(x_t|\theta_{y_t}^g)}{\sum_{k=1}^M \alpha_k^g p_k(x_t|\theta_k^g)}, \tag{3.11}$$

and

$$p(y|\chi, \Theta) = \prod_{t=1} p(y_t|x_t, \Theta^g), \tag{3.12}$$

where $y = (y_1, \ldots, y_N)$ is an instance of the hidden data independently drawn. Now we can start computing the E-step of EM algorithm.

$$Q(\Theta, \Theta^g) = \sum_{y \in \Omega} \log(L(\Theta|\chi, y))p(y|\chi, \Theta^g), \tag{3.13}$$

which can further be written as

$$Q(\Theta, \Theta^g) = \sum_{y_1=1}^{M} \cdots \sum_{y_N=1}^{M} \sum_{t=1}^{N} \log(\alpha_{y_t} p_{y_t}(x_t|\theta_{y_t}) \prod_{j=1}^{N} p(y_j|x_j, |\Theta^g). \tag{3.14}$$

The above equation can be further simplified as

$$Q(\Theta, \Theta^g) = \sum_{y_1=1}^{M} \cdots \sum_{y_N=1}^{M} \sum_{t=1}^{N} \sum_{l=1}^{M} \delta_{l,y_t} \log(\alpha_l p_l(x_t|\theta_l)) \prod_{j=1}^{N} p(y_j|x_j, \Theta^g) \tag{3.15}$$

$$Q(\Theta, \Theta^g) = \sum_{l=1}^{M} \sum_{t=1}^{N} \log(\alpha_l p_l(x_t|\theta_l)) \sum_{y_1=1}^{M} \cdots \sum_{y_N=1}^{M} \delta_{l,y_i} \prod_{j=1}^{N} p(y_j|x_j, \Theta^g) \tag{3.16}$$

After some simplification and using the fact that $\sum_{t=1}^{M} p(y_t|x_j, \Theta^g) = 1$, the above equation can be written in the following form

$$Q(\Theta, \Theta^g) = \sum_{l=1}^{M} \sum_{t=1}^{N} \log(\alpha_l p_l(x_t|\theta_l))p(l|x_t, \Theta^g) \tag{3.17}$$

$$Q(\Theta, \Theta^g) = \sum_{l=1}^{M} \sum_{t=1}^{N} \log(\alpha_l)p(l|x_t, \Theta^g) + \sum_{l=1}^{M} \sum_{t=1}^{N} log(p_l(x_t|\theta_l))p(l|x_t, \Theta^g). \tag{3.18}$$

The mixing proportions (α_j) and covariance matrices in our case are constant and are given by 2^{-K} and $\sigma^2 I$ respectively. To this end, we have for a d-dimensional Gaussian

$$p_l(x|\mu_l, \Sigma_l) = \frac{1}{(2\pi)^{d/2}|\Sigma_l|^{1/2}} e^{-1/2(x-m_l)^T \Sigma_l^{-1}(x-m_l)} \tag{3.19}$$

Taking log of the above equation, ignoring constant terms and plugging into eq. (3.19), we have

$$Q(\Theta, \Theta^g) = \sum_{l=1}^{M}\sum_{t=1}^{N}(-\frac{1}{2}\log(|\Sigma|) - \frac{1}{2}(x_t - m_l)^T \Sigma_l^{-1}(x_t - m_l))p(l|x_t, \Theta^q) \tag{3.20}$$

Differentiating eq (3.20) with respect to m_l and setting it to zero, we get

$$m_l = \frac{\sum_{t=1}^{N} x_t p(l|x_t, \Theta^g)}{\sum_{t=1}^{N} p(l|x_t, \Theta^g)}, \tag{3.21}$$

where the posteriori probabilities $p(l|x_t, \Theta^g)$ is defined as follows:

$$p(l|x_t, \Theta^g) = \frac{\alpha_l^g p_{y_i}(x_t|\theta_{y_i}^g)}{\sum_{k=1}^{M} \alpha_k^g p_k(x_t|\theta_k^g)}. \tag{3.22}$$

The algorithm works as follows. Firstly the posteriori probabilities are calculated using initial estimates of means. The posteriori probabilities tell us the probability that each received data belongs to each Gaussian. These posteriori estimates are used to find the update means of the mixture. These two steps are repeated until convergence. The convergence of the EM algorithm to a solution and the number of iterations depends on the tolerance, the initial parameters, the data set, etc. Using EM for Gaussian mixture, the amplitudes of the users are estimated as follows [109]

$$H^{g+1}.\sum_{i=1}^{2^K}\sum_{n=1}^{N} d_i d_i^T \frac{p_i(x(n)|H^g)}{p(x(n)|H^g)} = \sum_{i=1}^{2^K}\sum_{n=1}^{N} x(n) d_i^T \frac{p_i(x(n)|H^g)}{p(x(n)|H^g)} \tag{3.23}$$

where,

$$p(x|H) = \sum_{i=1}^{2^K} \frac{1}{2^K(2\pi\sigma^2)P/2} exp(\frac{-1}{2\sigma^2}(x - Hd_i)^T(x - Hd_i) \tag{3.24}$$

and

$$p_i(x|H) = \frac{1}{(2\pi\sigma^2)^{P/2}} exp(\frac{1}{2\sigma^2(x - Hd)^T(x - Hd)} \tag{3.25}$$

and $[d_i, i = 1, 2, \ldots, 2^K]$ is the set of all 2^K transmitted vectors. From the estimate of H, the amplitudes can be estimated as

$$A_k = \sqrt{(H^T H)_{k,k}} \tag{3.26}$$

3.5 Convergence rate of EM algorithm for Gaussian mixtures

Before deriving an expression for the convergence rate of the EM algorithm for the Gaussian mixture model, we state two results available in the statistical literature regarding the convergence of EM. First, it has been established that, under mild conditions EM is guaranteed to converge toward a local maximum of the loglikelihood function [79]. (Indeed the convergence is monotonic: $l(\Theta^{(k+1)}) \geq l(\Theta^{(k)})$, where $\Theta^{(k)}$ is the value of the parameter vector Θ at iteration k). Second, considering EM as a mapping $\Theta^{(k+1)} = M(\Theta^{(k)})$ with fixed point $\Theta^* = M(\Theta^*)$, we have

$$\Theta^{(k+1)} - \Theta^* \approx \frac{\partial M(\Theta^*)}{\partial \Theta^*}(\Theta^{(k)} - \Theta^*) \tag{3.27}$$

when $\Theta^{(k+1)}$ is near Θ^*, and thus

$$||\Theta^{(k+1)} - \Theta^*|| \leq ||\frac{\partial M(\Theta^*)}{\partial \Theta^*}||.||\Theta^{(k)} - \Theta^*||, \tag{3.28}$$

with

$$||\frac{\partial M(\Theta^*)}{\partial \Theta^*}|| \neq 0 \tag{3.29}$$

almost surely. That is, EM is a first order algorithm.
Now we prove the result for the convergence rate of EM algorithm for Gaussian mixture model.

Theorem: For Gaussian mixtures, the convergence rate r of the EM algorithm for means and hence for the channel is bounded by

$$r = \frac{\left\| m_j^{(k+1)} - m_j^* \right\|}{\left\| m_j^{(k)} - m_j^* \right\|} \leq \left\| I + P_{m_j}^* h_{mj}^* \right\| = \|A\| = \frac{\left\| H^{(k+1)} - H^* \right\|}{\left\| H^{(k)} - H^* \right\|} \tag{3.30}$$

where m_j, H are means, and channel coefficients respectively, and $()^*$ denotes the converged point. I and h_{mj}^* denotes the Identity matrix and Hessian of the likelihood function at m_j^* and $P_{m_j} = \frac{I}{\sum_{t=1}^{N} h_j^{(k)}(t)}$. The higher the values of $\|A\|$, the slower will be the convergence.

Proof: Xu and Jordan [82], showed that for each iteration the following relationship holds between the gradient of the loglikelihood and the EM update step:

$$m_j^{(k+1)} - m_j^{(k)} = P_{m_j}^{(k)} \frac{\partial l}{\partial m_j}|_{m_j = m_j^{(k)}}. \tag{3.31}$$

From the above equation, using Taylor expansion around the convergent point m_j^* for large k and noting that $P_{m_j}^{(k)} \frac{\partial l}{\partial m_j}|_{m_j = m_j^*} = 0$, we have

$$m_j^{(k+1)} = m_j^{(k)} + P_{m_j}^* h_{mj}^* (m_j^{(k)} - m_j^*), \tag{3.32}$$

which can be written as

$$m_j^{(k+1)} - m_j^* = m_j^{(k)} - m_j^* + P_{m_j}^* h_{mj}^* (m_j^{(k)} - m_j^*) \tag{3.33}$$

$$m_j^{(k+1)} - m_j^* = (I + P_{m_j}^* h_{mj}^*)(m_j^{(k)} - m_j^*). \tag{3.34}$$

The result follows after using Schwarz inequality. It has been proved by Ma et al.[83] that the asymptotic convergence rate of EM for Gaussian mixtures, locally around the true solution m_j^*, is $O(e^{0.5-\epsilon}(m_j^*))$ where $\epsilon > 0$ is an arbitrary small number, $O(x)$ means that it is higher order infinitisimal as $x \to 0$, and $e(m_j^*)$ is a measure of overlap of Gaussians in the mixture.
In other words, the large sample local convergence rate of the EM algorithm tends to be asymptotically super-linear when $e(m_j^*)$ tends to zero.

3.6 Simulations

The performance of the proposed method was evaluated as a function of SNR (signal to noise ratio) based on Monte Carlo simulations. The method was tested for 500 Monte Carlo trials per SNR point across range of SNR's. In each trial, the amplitude estimation error was recorded. Data block of 32 symbols were used in all simulations. The spreading gain was 32. The proposed method worked quite well for the two and three users case (due to the fact that there were only four and eight mixture of Gaussians respectively). In figure 3.1, the performance is compared with the approximate Cramer Rao bound which is not as tight as the Cramer-Rao bound (CRB). The difference between the simulations and the CRB can be explained by the fact that the initial parameter values for the EM algorithm were given as random numbers, i.e., initial values were not confined to be in the vicinity the true value of the parameter. This was done in order to show the results for EM in a more realistic way (because in reality it is very difficult to know a priori good starting points for an algorithm). Figure 3.2 compares the estimation error for three and four users. Beyond three users, the estimation error increased quite substantially (as is clear from figure 3.2). This effect can be explained from the fact that as the number of users increases, it is more probable for the EM algorithm to converge at false means of the mixture of Gaussians (if random initialization is done as in our case). Therefore, very good initialization is needed when number of users grows large. In figure 3.3 we show the failure of the EM algorithm for two closely spaced Gaussians. Figure 3.3 is the plot of three realizations of mixture of two Gaussians. The mean of the first Gaussian is $(-2, 2)$ and that of the second Gaussian is $(-1.7, 2.2)$. It is clear from the figure that EM did not converge to the true means. However, in figure 3.4 we have shown the plot of two well separated Gaussians. The mean of the first Gaussian is $(-2, 2)$ and that of the second Gaussian is $(-2, 0)$. Due to well separateness of the Gaussians, the EM converged almost to the true means.

3.7 Conclusions

In this chapter, we presented a Gaussian mixture formulation of the problem which consists of blindly estimating the users amplitudes for the synchronous CDMA system. We proposed an EM based algorithm to estimate the parameters of the mixture. The theoretical convergence rate for the means in the Gaussian mixture case was also presented. Simulation results show the usefulness of the method. The estimation error is compared with the approximate Cramer-Rao lower bound. The behavior of the convergence of the EM algorithm to the means of Gaussians is also discussed.

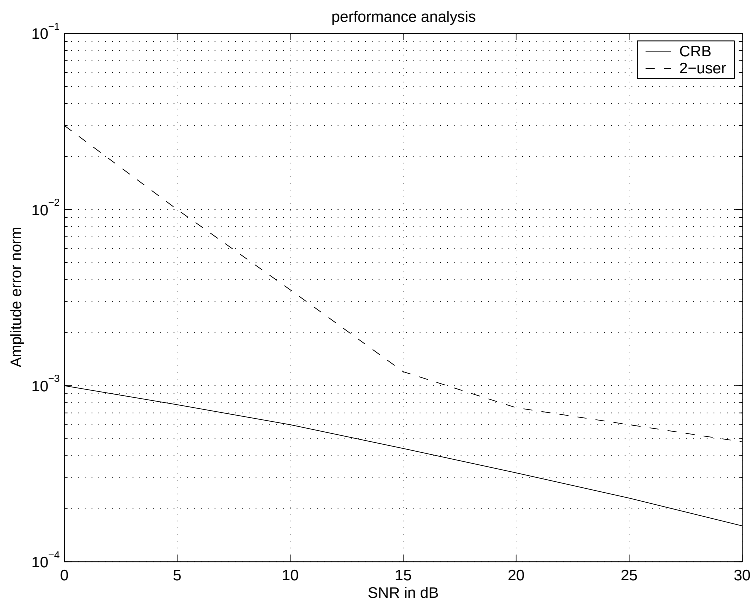

Figure 3.1: Amplitude estimation error.

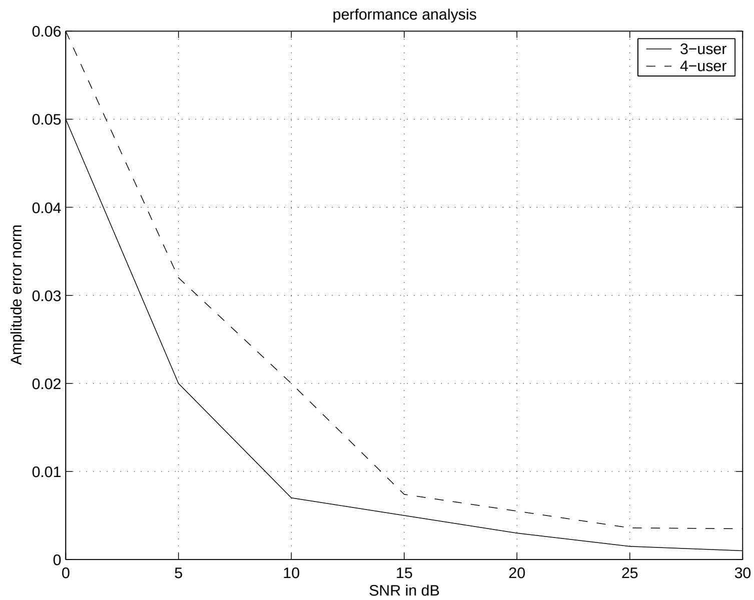

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Figure 3.2: Amplitude estimation error.

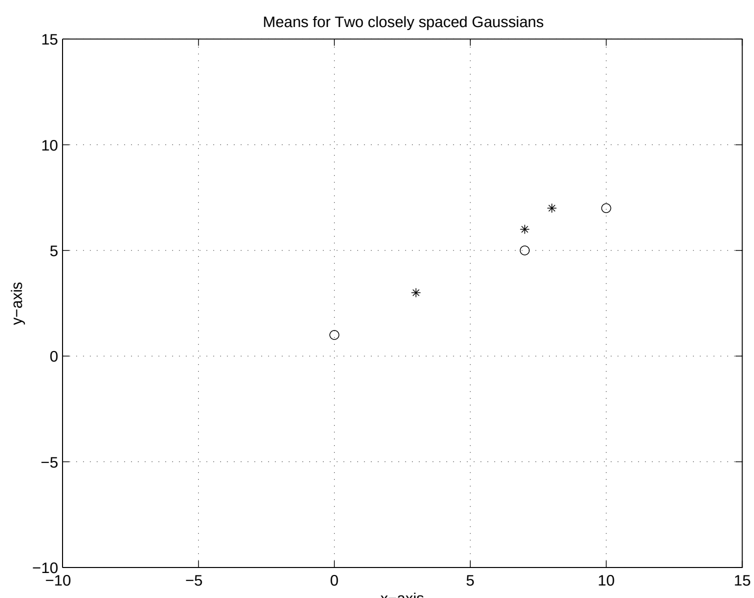

Figure 3.3: Convergence of EM for mean values of two closely spaced Gaussians

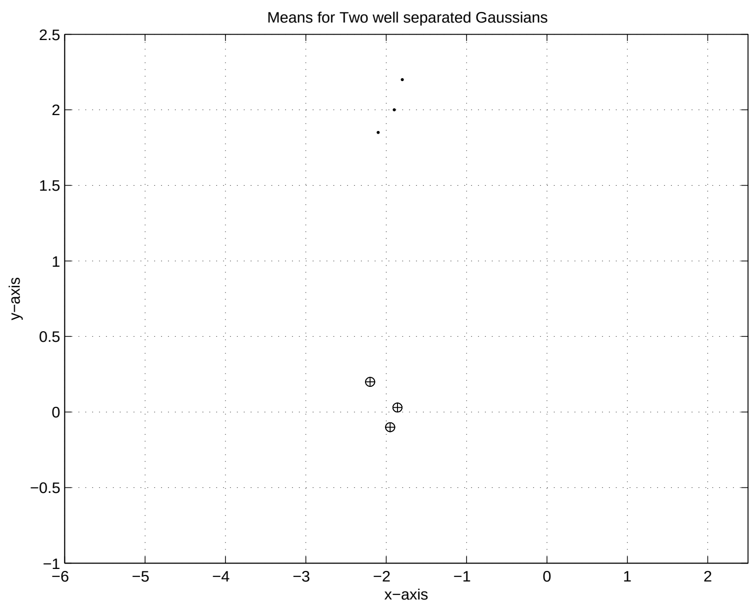

Figure 3.4: Convergence of EM for mean values of two well separated Gaussians

Chapter 4

Iterative constrained penalized likelihood estimation of parameters for CDMA

We describe in this chapter an iterative method for Maximum Likelihood (ML) parameter estimation corrupted by additive white Gaussian noise. In the objective function, we subtract/ add a Kullback-Leibler (KL) distance function or an Euclidean distance function to keep the old parameter set close to the new ones and can be considered as penalty term. The above augmented cost function can be maximized/minimized over the constraint that the detected data vector lies on the sphere. We simplify this constraint function by using a first order Taylor expansion at the old parameter value. The useful behavior of the proposed algorithm is verified by numerical experiments.

4.1 Introduction

In CDMA system, all resources are in principle available to all users simultaneously. The users are distinguished from each other by user specific signature sequences, modulating the transmitted data symbols using direct sequence spread spectrum techniques. In the past, many iterative techniques have been considered, see for example [81,92]. The unconstrained ML problem can be solved quite easily and is known as the decorrelating detector. In [99,40], the authors consider application of semidefinite programming (SDP) approach to the detection problem in CDMA, giving very close to ML performance. This method is however computationally complex for a large number of users. In [89], a constrained Maximum Likelihood problem was considered where the solution vector is constrained to lie with in a hypercube (“ Box constrained ML”). Special cases of this algorithm correspond to known, non-linear successive and parallel interference cancelation structures, using a clipped soft decision function for making tentative decisions. These structures are therefore ML under the assumption that the detected vector lies within a hypercube. In the same paper, the authors investigated the convergence issues and suggest an efficient implementation. Similarly, they also proposed a method of maximizing likelihood function over a sphere by confining the solution vector to lie within the sphere, and by projecting the solution vector on the sphere. This detector is ML under the assumption that the detected vector is constrained to lie within a hypersphere. Based on the Karush-Kuhn-Tucker point, it is shown by simulations that the suggested detector is closely related to the MMSE detector. The authors analyse also the convergence issues and give an efficient implementation. In fact, in the sphere constrained problem the solution vector lies on the sphere, and not in the interior of the constraining sphere (as is done in [88]). The other problem with the above method is that small error in the solution vector can cause large error when projected on to the sphere (provided the solution vector is well inside the sphere). In this chapter, we constrain the solution vector to lie almost (very close) on the sphere, and we jointly estimate the complex channel coefficients and data vector.
The rest of the chapter is organized as follows: The signal model for our problem is described in section 4.2. In section 4.3, we develop sphere constrained approximate penalized likelihood function. In section 4.4, we analyze the performance of the proposed method and simulations are presented.

4.2 Signal model

In this section, discrete-time baseband up-link signal model for CDMA communication system is described. We consider asynchronous CDMA with single path channels. The signal is corrupted by the presence of an additive white Gaussian noise (AWGN) with zero mean and variance $\frac{N_o}{2} = \sigma^2$. The number of users in the system are assumed to be K. The processing gain, $N = T_d/T_c$, where T_d is symbol duration and T_c is the chip duration. The users transmit binary information symbol streams $d_k(n) \in \{-1,1\}$, $n = 0,1,.....,L-1$ is the symbol index and L is the length of the data block. $s_k(n) = (s_k(nN+1).......s_k(n+1)N)^T$ where $s_k(i) \in (-1/\sqrt{N}, 1/\sqrt{N})$ is the spreading code of user k to modulate n^{th} bit. In mobile radio channels, each transmission path encounters temporal and spatial fading [9,11,16]. Furthermore, each user is transmitting at a specific power level. In our single path K-user system, this corresponds to each user being received with a random, time-dependent amplitude and phase, or equivalently, an arbitrary user k is affected by a random, time dependent complex channel coefficients, $c_k(i)$. The received baseband signal can be written as [95]

$$r = \sum_{i=0}^{L-1} \sum_{k=1}^{K} c_k(i) x_k(i) + n \tag{4.1}$$

$$r = \sum_{i=0}^{L-1} \sum_{k=1}^{K} c_k(i) d_k(i) \begin{pmatrix} 0_{iN+\tau_k} \\ s_k(i) \\ 0_{(L-i)N-\tau_k-1} \end{pmatrix} + n \tag{4.2}$$

Where τ_k is the kth user time delay. The convenient matrix notation is given by

$$r = SCd + n \tag{4.3}$$

where the symbol vector is given by $d = (d_1(0), d_2(0), ..., d_K(L-1))^T = (d_1, d_2, ..., d_{LK})^T$ and C is $LK \times LK$ diagonal matrix containing the physical channel parameters. The complex channel coefficients $c_k(i)$ contain all the fading and attenuation effects of the radio channel. S is the matrix of transmitted waveforms with column j expressed as

$$s_j = \begin{pmatrix} 0_{iN+\tau_k} \\ s_k(i) \\ 0_{(L-i)N-\tau_k-1} \end{pmatrix} \tag{4.4}$$

A minimal set of sufficient statistics of dimension LK is obtained through correlation, matched to the received signal. This also ensures the maximization of the SNR, i.e,

$$y = S^T r = S^T SCd + S^T n = RCd + z \tag{4.5}$$

where R is the spreading sequence correlation matrix and z is a zero mean Gaussian vector with covariance $\sigma^2 R$.

4.3 Sphere constrained approximate ML

Given the set of data $y \in R^{LK}$, our goal is to find parameters that maximize the $\log P(y|\theta)$ or minimize the negative of it. In iterative parameter estimation, given an old set of parameters θ_i we need to find a new set of parameters θ_{i+1} that improves the likelihood at each iteration. In our approach, we want the detected vector to lie close to the sphere, therefore we also require that the new parameter vector to stay "close" to the old set of parameters. In order to achieve it, we incorporate a distance function , which can also be thought of as a penalty function.The role of the distance function is to constrain the new parameter set to the old ones. We now search for new set of parameters θ_{i+1} that minimizes the distance function summed with the negative loglikelihood function subject to spherical constraint. We will call this function as "augmented log-likelihood". More formally, the update is found by setting $\theta_{i+1} = arg_{min_\theta} L(\theta)$ where

$$l(\theta) = -\log P(y|\theta) + dis(\theta, \theta_i) + \lambda(d^T d - LK). \tag{4.6}$$

Lagrange multiplier, λ [23,26] is used to enforce the spherical constraint on symbols. The distance function $dis(\theta, \theta_i)$ in our case is KL divergence but other distance function can also be used. The KL divergence is given by

$$dis(\theta, \theta_i) = \int_y P(y|\theta) \log \frac{P(y|\theta)}{P(y|\theta_i)} dy. \tag{4.7}$$

We approximate the sphere constraint by the first order Taylor expansion around d_i (old parameter set), i.e.,

$$d^T d - LK = (d^T d - LK)_{d_i} + (d - d_i)^T \nabla_d (d^T d - LK)|_{d=d_i}, \tag{4.8}$$

where d_i is the value of the symbol vector at iteration i. Substituting equation (4.7) and equation (4.8) in equation (4.6) we get

$$l(\theta) = -\log P(y|\theta) + dis(\theta, \theta_i) + \lambda((d - d_i)^T \nabla_d (d^T d - LK)|_{d=d_i}). \tag{4.9}$$

The first order approximation is valid because distance function (penalty function) will force the new parameters to remain close to the old ones at each iteration and hence the estimated vector d will always be close to the surface of the sphere. The KL divergence after bit of algebra can be written in the following form

$$dis(\theta, \theta_i) = \tfrac{LK}{2} + \tfrac{1}{2} trace(I) + \tfrac{1}{2\sigma^2}(m_\theta - m_{\theta_i})^T R^{-1} (m_\theta - m_{\theta_i}), \tag{4.10}$$

where I is identity matrix and m_θ is mean of the distribution. The above expression is a convex function. Plugging in values from the received signal and omitting constant terms gives

$$dis(\theta, \theta_i) = \frac{1}{2\sigma^2}(RCd - R(Cd)_i)^T (Cd - (Cd)_i), \tag{4.11}$$

and

$$\log P(y|\theta) = \tfrac{LK}{2}\log(2\pi) - \tfrac{1}{2\sigma^2}(y - RCd)^T R^{-1}(y - RCd), \tag{4.12}$$

which after permuting C and d gives

$$\log P(y|\theta) = \tfrac{LK}{2}\log(2\pi) - \tfrac{1}{2\sigma^2}(y - RDc)^T R^{-1}(y - RDc) \tag{4.13}$$

where D is a diagonal matrix with diagonal entries given by $(d_1(0), d_2(0)......d_k(L-1))$ and $c = \text{diag}(C)$ is vector composed of diagonal elements of matrix C. The log-likelihood equation can be further simplified as (after omitting constants)

$$-\log P(y|\theta) = \tfrac{1}{2\sigma^2}(y^T R^{-1} y - y^T Dc - c^T D^T y + c^T D^T RDc). \tag{4.14}$$

Taking the gradient with respect to c of the above function gives

$$-\nabla_c \log P(y|\theta) = \frac{1}{\sigma^2}(-D^T y + D^T RDc) \tag{4.15}$$

The distance function after permuting C and d is written as

$$dis(\theta, \theta_i) = \frac{1}{2\sigma^2}(RDc - R(Dc)_i)^T(Dc - (Dc)_i) \tag{4.16}$$

the subscript i indicates that the parameter is computed at ith iteration. Rearranging and taking gradient with respect to c gives

$$\nabla_c dis(\theta, \theta_i) = \frac{1}{\sigma^2}(D^T RDc - D^T R(Dc)_i) \tag{4.17}$$

Putting the above two gradients in the augmented loglikelihood equation and equating the resulting equation to zero gives

$$c = \frac{1}{2}(D^T RD)^{-1}(D^T y + D^T R(Dc)_i) \tag{4.18}$$

Similarly we take the gradient of the augmented loglikelihood function with respect to d and equating it to zero gives

$$d = (\frac{2}{\sigma^2}C^T RC)^{-1}(\frac{1}{\sigma^2}C^T y + \frac{1}{\sigma^2}C^T R(Cd)_i - 2\lambda d_i) \tag{4.19}$$

This expression is function of λ, i.e., Lagrange multiplier, which is given by

$$\lambda = \frac{h \pm \sqrt{h^2 - 4gj}}{2g}, \tag{4.20}$$

where

$$h = 4e^T U d_i, \tag{4.21}$$

$$j = e^T U e - LK, \tag{4.22}$$

$$g = 4 d_i^T U d_i, \tag{4.23}$$

and $U = X^T X$, $e = a + v$ where X is

$$X = (\frac{2}{\sigma^2} C^T R C)^{-1} \tag{4.24}$$

,

$$a = \frac{1}{\sigma^2} C^T y, \tag{4.25}$$

and

$$v = \frac{1}{\sigma^2} C^T R (Cd)_i, \tag{4.26}$$

We also calculated the formulas for C and d when Euclidean distance function is used instead of KL divergence function. The Euclidean distance between two parameters set is defined by

$$dis(\theta, \theta_i) = \frac{1}{2} \|\theta - \theta_i\|^2 \tag{4.27}$$

The Euclidean distance function after bit of simplification is written as

$$dis(\theta, \theta_i) = \frac{1}{2}(c^T c + d^T d - 2c^T c_i - 2d^T d_i + c_i^T c_i + d_i^T d_i) \tag{4.28}$$

where the subscript i denotes the value of the parameter at the ith iteration. In our case the parameter set is given by $\theta = (c, d)$, where c is vector composed of diagonal elements of C. With the same procedure as is done for KL distance case, i.e., taking gradient of the distance function with respect to c and d and also taking gradient of the loglikelihood function with respect to c and d and plugging the results into augmented loglikelihood function and imposing the spherical constraint.The update equations for c and d are given by

$$c = (\frac{1}{\sigma^2} D^T R D + I)^{-1} (\frac{1}{\sigma^2} D^T y + c_i), \tag{4.29}$$

where I is identity matrix. Similarly for d, we have

$$d = (\frac{1}{\sigma^2} C^T R C + I)^{-1} (\frac{1}{\sigma^2} C^T y + d_i - 2\lambda d_i), \tag{4.30}$$

where λ is given by

$$\lambda = \frac{m \pm \sqrt{m^2 - 4ln}}{2l}, \tag{4.31}$$

where $l = 4d_i^T U d_i$, $m = 4d_i^T UW$, $n = W^T UW - LK$ and $U = X^T X$. The expression for X and W are as follows

$$X = (\frac{1}{\sigma^2} C^T RC + I)^{-1}, \tag{4.32}$$

and

$$W = \frac{1}{\sigma^2} C^T y + d_i \tag{4.33}$$

The algorithm works as follows:

1) We start with the initial estimate of C_i and d_i,

2) We calculate C (the updated value) using eq. 4.29, the updated value of C is used to calculate λ. These values are in turn plugged into update expression for d, eq. 4.30 to get d updates. These two steps are continued until $\|vec(C_{i+1} - C_i)\| < \delta$, where δ is small number. Note that in the update equations for C and d (in case of KL distance), there are matrix inversions, i.e, we have to invert a matrix at each iteration which is computationally expensive. In the following lines we will derive a low complexity algorithm by eliminating matrix inversion. This is done by polynomial expansion of the signature correlation matrix, R, i.e.,

$$R^{-1} = (I + Q)^{-1} = \sum_{i=0}^{\infty} (-Q)^i \tag{4.34}$$

where Q is equal to matrix R with diagonal elements put to zero and $Q^0 = I$, where I is the identity matrix. If the elements of Q are small compared to one, i.e., low cross-correlation. The matrix R^{-1} can be approximated by a first order expansion (neglecting higher order terms), i.e.,

$$R^{-1} = I - Q \tag{4.35}$$

In this way matrix inversion is replaced by adding two simple matrices.

4.4 Simulations and conclusion

In this section we investigate the amplitude error and BER performance based on the simulations. The codes were selected at random and we considered two different scenarios. A lightly loaded case with six number of users as well as a highly loaded case with, $K = 24$. In both cases the processing gain was kept to 32. We plot the amplitude estimation error versus different values of SNR. As is clear from the figure (4.4), the estimation error decreases as the value of the SNR increases for both cases. However, the estimation error of the highly loaded case is more than the lightly loaded case. We also simulated for BER for lightly loaded case. It is clear from the figure (4.1) and figure (4.3) that our receivers (with KL distance function and Euclidean distance function) performs better than MMSE and the receiver proposed in [88] (they have the same performance). The MMSE receiver on average constrains the vector to lie within sphere [88]. In [88] the authors considered the constraint that the symbol vector lie within sphere. On the other hand as we approximate the spherical constraint with the first order Taylor expansion and we also do not let previously estimated vectors to be far from the new estimate (thanks to the distance function), therefore we are always close to the sphere. Hence, we can consider our constraint to be the shell region between two concentric hyper spheres, which is more constraining than a bowl constraint. We also plot the BER for approximate proposed receiver in figure (4.2). The approximation is done to reduce the complexity of the receiver. It is hoped that with the increase in the number of users, better results are expected owing to the fact that the first order approximation of the sphere will almost lie on the surface of the sphere, i.e., we will be almost on the surface of the sphere. In the figure (4.2), we also compared the low complexity version (approx. of the R^{-1}) of the algorithm with the MMSE. As is clear from the figure, it performs better than MMSE and the performance is almost identical with that of the exact proposed receiver. In all the simulations for the BER, the estimated values of the amplitudes were used. While in the case of MMSE, true amplitudes were used in the simulations. Figure 4.5 shows the two iterations of the proposed algorithm.

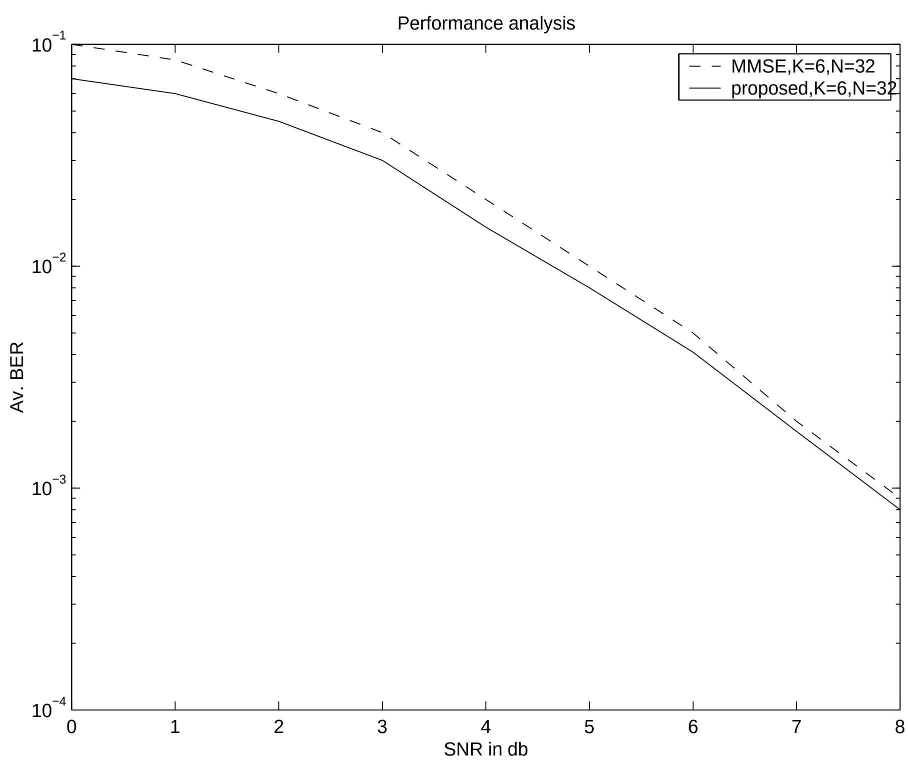

Figure 4.1: Average BER for MMSE and proposed receiver with KL distance function

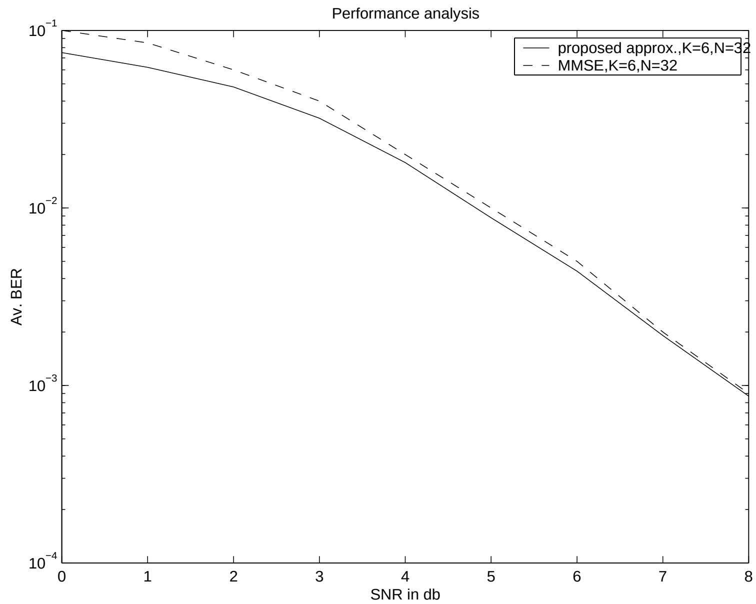

Figure 4.2: Average BER for MMSE and Approx. proposed receiver.

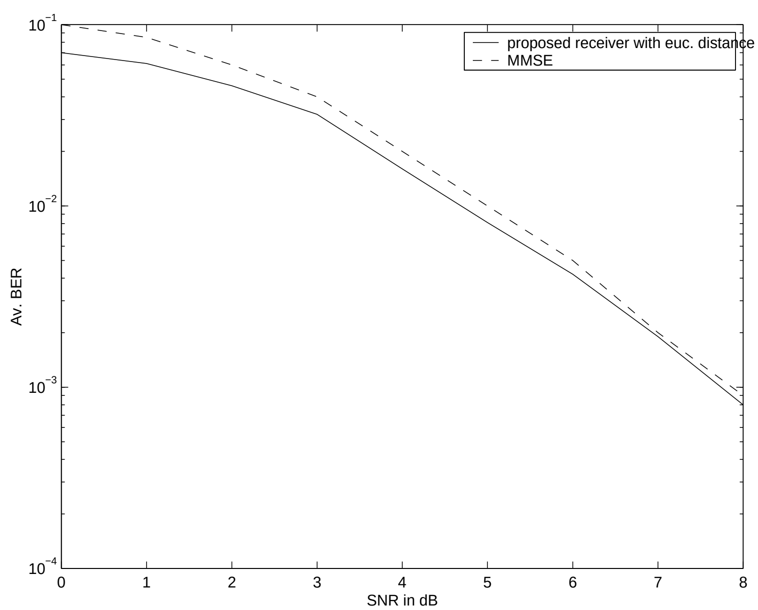

Figure 4.3: Average BER for MMSE and proposed receiver with Euclidean distance function.

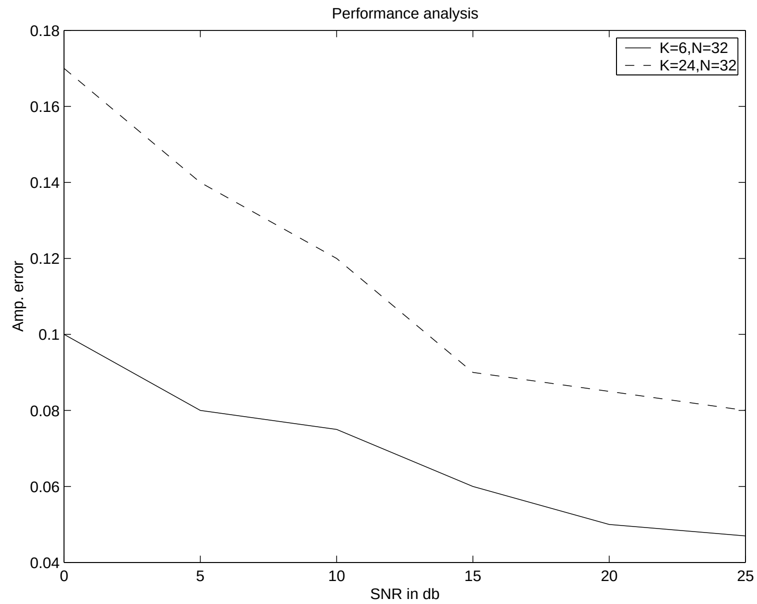

Figure 4.4: Root mean square amplitude error norm for proposed receiver.

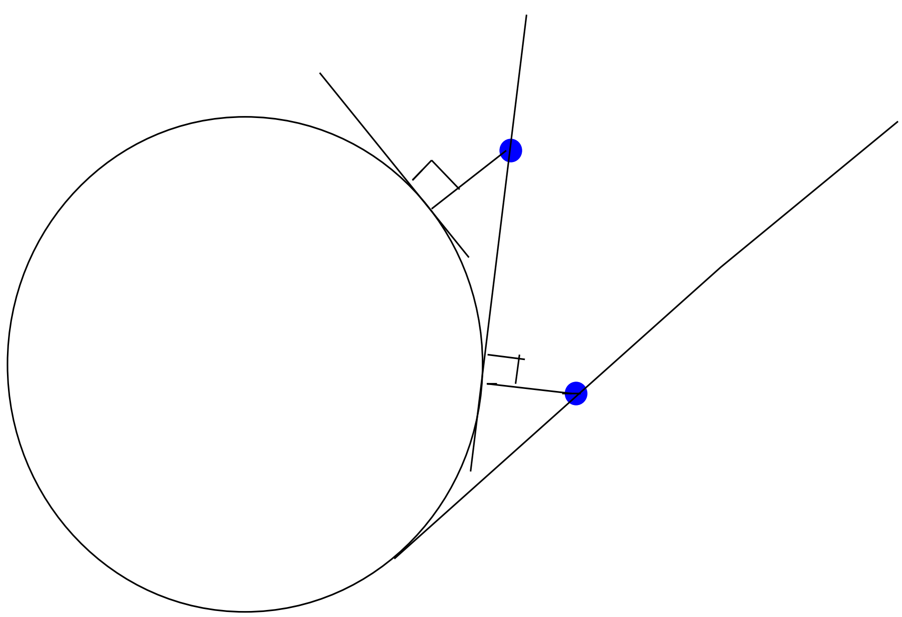

Figure 4.5: Two iterations of the proposed algorithm.

Chapter 5

Exact sphere constrained maximum likelihood detection of parameters for CDMA

We describe a method for Maximum Likelihood (ML) parameter estimation corrupted by an additive white Gaussian noise. The ML cost function is maximized over the constraint that the detected data vector lie on the sphere. The results are compared with MMSE and with [88]. Simulations results show superior performance in terms BER of our method comparing to both the methods. In chapter 4 we jointly estimated channel/symbols iteratively using approximate sphere constraint. In this chapter we detect symbol vector using exact sphere constraint assuming that the channel is known.

5.1 Introduction

The conventional receiver consists of a bank of a single-user matched filters followed by quantizers. It is reliable if the signature waveforms have low cross-correlations for all the possible delays, and if the power of all the users are not very different. Since these conditions are often difficult to satisfy in practice, several new multiuser detectors have been proposed. The linear decorrelating receiver is simple and can significantly outperform the conventional receiver in both synchronous and asynchronous CDMA system. This detector does not require estimation of the users powers, and achieves the optimal near-far resistance. However, the inversion of the channel performed by the decorrelating filter enhances noise. This creates a gap between the error probability of the decorrelator and the single user bound. Other recent approaches to the multiuser detection include multistage detectors. For example, in a two stage detector, decisions made by the first stage are used for interference cancellation in the second stage.
The linear MMSE detector achieves robustness against MAI by selecting the linear filter that minimizes the mean-square value of the output MAI plus noise. In [15] decision feedback and partial feedback detectors for asynchronous CDMA channels are introduced. The derivation of the feedback detector is based on spectral factorization which leads to a white noise channel model. In the same paper, the authors also described the implementation of the ML detector for this model.
The optimal detector is ML but prohibitively complex. However by relaxing the constraint, less complex approximate ML detectors can be obtained. In the previous chapter, we solved the ML problem by relaxing the sphere constraint (approximate sphere constraint). In this chapter, we will solve the ML problem with the exact sphere constraint, i.e., the ML cost function is maximized over the constraint that the detected data vector lie on the sphere and hence better results are expected. This is confirmed by simulation results. The rest of the chapter is organized as follows. In section 5.2 we describe signal model. Sphere constrained ML is given in section 5.3. In section 5.4 we show relationship between the sphere constraint and MMSE receiver. Simulations and conclusions are drawn in the last section.

5.2 Signal Model

In this section, discrete-time base band up-link signal model for CDMA communication system is described. We consider a asynchronous CDMA with single path channels. The signal is corrupted by the presence of an additive white Gaussian noise (AWGN) with zero mean and variance $\frac{N_o}{2} = \sigma^2$. The number of users in the system are assumed to be K. The processing gain, $N = T_d/T_c$, where T_d is symbol duration and T_c is the chip duration. The users transmit binary information symbol stream $d_k(n) \in \{-1, 1\}$, $n = 0, 1, ..., L-1$ is symbol interval index and L is the length of the data block. $s_k(n) = (s_k(nN+1).......s_k(n+1)N)^T$ with $s_k(i) \in (-1/\sqrt{N}, 1/\sqrt{N})$ is the spreading code of the user k to modulate n^{th} bit. The received base band signal can be written as,

$$r = \sum_{i=0}^{L-1} \sum_{k=1}^{K} d_k(i) \begin{pmatrix} 0_{iN+\tau_k} \\ s_k(i) \\ 0_{(L-i)N-\tau_k-1} \end{pmatrix} + n. \tag{5.1}$$

The convenient matrix notation is given by

$$r = Sd + n, \tag{5.2}$$

where the symbol vector is given by $d = (d_1(0), d_2(0), ..., d_K(L-1))^T = (d_1, d_2, ..., d_{LK})^T$. S is the matrix of transmitted waveforms with the column j expressed as

$$s_j = \begin{pmatrix} 0_{iN+\tau_k} \\ s_k(i) \\ 0_{(L-i)N-\tau_k-1}. \end{pmatrix} \tag{5.3}$$

Where τ_k is the time delay of kth user. A minimal set of sufficient statistics of dimension LK is obtained through correlation, matched to the received signal. This also ensures the maximization of the SNR, i.e.,

$$y = S^T r = S^T S d + S^T n = Rd + z, \tag{5.4}$$

where R is the correlation matrix and z is zero mean Gaussian vector with covariance $\sigma^2 R$.

5.3 Sphere constrained ML

Given the set of data $y \in R^{LK}$, our goal is to find parameters that maximize the $\log P(y|\theta)$ or minimize the negative of it. The negative loglikelihood function of y is given by

$$l(d) = d^T Rd - 2y^T d. \tag{5.5}$$

The sphere constrained ML problem for the asynchronous CDMA is then described as

$$d = arg_{min_d} d^T Rd - 2y^T d, \tag{5.6}$$

subject to $d^T d = LK$. The Lagrangian function associated with the above problem can be written as

$$L(d, \lambda) = d^T Rd - 2y^T d + \lambda(d^T d - LK). \tag{5.7}$$

To calculate the stationary points, we differentiate $L(d, \lambda)$ with respect to d and λ. The solution of the above problem is given by

$$d = (R + \lambda I)^{-1} y \tag{5.8}$$

Now the problem is to find the Lagrange multiplier. Using the quadratic constraint, we can write

$$y^T (R + \lambda I)^{-2} y = LK. \tag{5.9}$$

We proceed by computing computing the eigenvector decomposition of matrix R.

$$f(\lambda) = y^T (U \Sigma U^T + \lambda I)^{-2} - LK. \tag{5.10}$$

Which can further be written as

$$f(\lambda) = \sum_i \frac{z_i^2}{(\lambda + \sigma_i)^2} - LK = 0 \tag{5.11}$$

where $z_i = (U^T y)_i$, σ_i are the eigenvalues of R with $\sigma_1 < \ldots < \sigma_n$, and U are the eigenvectors of R. The zeroes of $f(\lambda)$ can be found numerically using Newton-Raphson method. We choose λ such that $R + \lambda I$ is positive definite. This selection of λ forces the $f(\lambda)$ to be convex. Now the problem is to find λ for which $f(\lambda)$ is zero. We find the bounds for λ in order to restrict our search to find the zeroes of $f(\lambda)$. The bounds can be straightforwardly obtained and are given by

$$\lambda \leq \frac{||y||}{LK} - \sigma_1, \tag{5.12}$$

and

$$\lambda \geq \frac{||y||}{LK} - \sigma_n. \tag{5.13}$$

A different approach to detect the data vector containing users' symbols is as follows. The likelihood function with its corresponding constraint can be written as

$$\bar{d}^T \bar{R} \bar{d}$$

$$subject\ \ to\ \ \bar{d}^T \bar{d} = LK + 1 = C \tag{5.14}$$

where

$$\bar{d} = \begin{pmatrix} d \\ 1 \end{pmatrix}, \tag{5.15}$$

and

$$\bar{R} = \begin{pmatrix} R & -y \\ -y^T & 0 \end{pmatrix}. \tag{5.16}$$

The solution to the above problem is given by

$$\bar{d} = V_{min}(\bar{R}), \tag{5.17}$$

where V_{min} is the eigenvector corresponding to the minimum eigenvalue of matrix $\bar{R}$. The minimum eigenvector is scaled such that the last term of this vector is 1.

5.4 Relationship between MMSE and sphere constraint

In this section we proof an analytic relationship which shows that the MMSE on the average imposes the sphere constraint on the symbols thus verifying the result shown through the simulations in [88]. The MMSE estimate of the symbols is given by [88]

$$d = (R + \sigma^2 I)^{-1} y, \tag{5.18}$$

where σ^2 is the noise variance. Having the above expression we can write

$$d^T d = Tr[(R + \sigma^2 I)^{-2} y y^T$$

$$= Tr[(R + \sigma^2 I)^{-2} (Rd + z)(d^T R + z^T)].$$

Assuming that d and z are independent and z is zero mean. Taking expectation of the above expression we have

$$E[d^T d] = Tr[(R + \sigma^2 I)^{-2} (R^2 + \sigma^2 R)], \tag{5.19}$$

where we have used the fact that $E[dd^T] = I$ and $E[zz^T] = \sigma^2 R$. The above expression can further be simplified as

$$E[d^T d] = Tr[R.(R + \sigma^2 I)^{-1}]. \tag{5.20}$$

Let $R = U \Sigma U^H$ be the eigen decomposition of the matrix R. Then we have

$$E[d^T d] = Tr[U \Sigma U^H.(U \Sigma U^H + \sigma^2 I)^{-1}],$$

$$= Tr[U \Sigma U^H.(U \Sigma U^H + \sigma^2 U U^H)^{-1}],$$

$$= Tr[U \Sigma U^H.(U(\Sigma + \sigma^2 I) U^H)^{-1}],$$

$$= Tr[U \Sigma U^H.(U(\Sigma + \sigma^2 I)^{-1} U^H)],$$

$$E[d^T d] = Tr[U(\Sigma(\Sigma + \sigma^2 I)^{-1}) U^H]$$

$$= Tr[U^H U(\Sigma(\Sigma + \sigma^2 I)^{-1})]$$

$$= Tr[\Sigma(\Sigma + \sigma^2 I)^{-1}] = \sum_{i=1}^{LK} \frac{\lambda_i}{\lambda_i + \sigma^2}, \tag{5.21}$$

where λ_i are the eigenvalues of the matrix R.

$$E[d^T d] = \sum_{i=1}^{LK} \frac{1}{1 + \sigma^2 \lambda_i^{-1}} \leq \sum_{i=1}^{LK} \frac{\lambda_1}{\lambda_1 + \sigma^2},$$

where λ_1 is the maximum eigenvalue of the matrix R. The above expression can be written as

$$E[d^T d] \leq \frac{\lambda_1}{\lambda_1 + \sigma^2} LK,$$

which can further be written as

$$E[d^T d] \leq LK, \tag{5.22}$$

hence establishing that the MMSE on average pose the sphere constraint.

5.5 Simulations and conclusion

In this section, we investigate BER performance based on the simulations. The codes were selected at random and we considered a lightly loaded case with six number of users. The processing gain was kept to 32. We simulated for BER using our constrained ML detector. It is clear from the figure that our receiver performs better than MMSE and the receiver proposed in [88] (they have the same performance). The MMSE receiver on the average constrains the data vector to lie within sphere [88]. In [88] the authors considered the constraint that the data vector lie within sphere, which is a loose constraint comparing to that of ours. Also we compared this algorithm with the one proposed in the previous chapter, which was approximation to the exact sphere constraint. It is clear from the figure that our algorithm outperforms the approximate constraint sphere receiver proposed in the previous chapter.

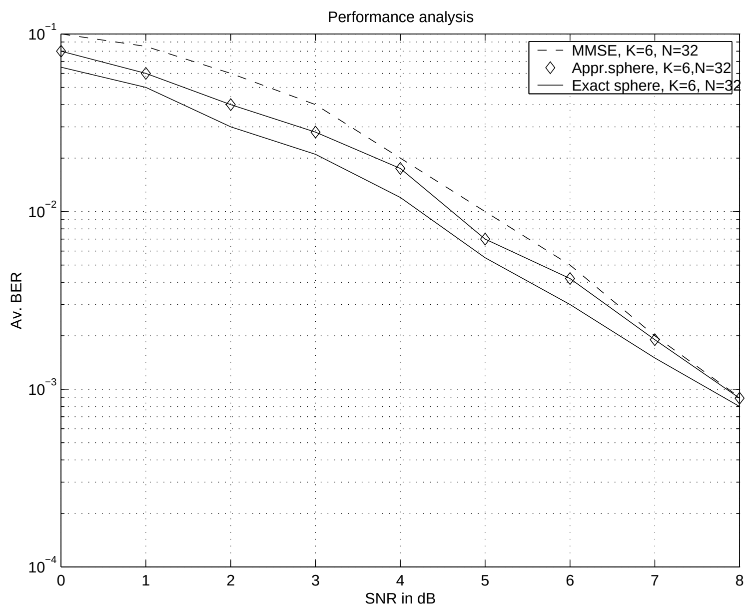

Figure 5.1: Av. BER of six users vs $SNR(dB)$.

Part II

Low Complexity MIMO Decoding

Chapter 6

Introduction to MIMO system

Multiple-input multiple-output (MIMO) systems are today regarded as one of the most promising research areas of wireless communications [21]. This is due to the fact that MIMO channels can offer a significant capacity gain over traditional single-input single-output (SISO) channels. The increase in spectral efficiency offered by MIMO systems is based on the utilization of space (or antenna) diversity at both the transmitter and the receiver. With MIMO systems, the data stream from a single user is demultiplexed into n separate sub-streams; n is equal to the number of transmit antennas. Each sub-stream is then encoded into channel symbols. It is common to impose the same data rate on all transmitters, but adaptive modulation rates can also be utilized on each sub-stream. The signals are received by m receive antennas.

With this transmission scheme, there is a linear increase in spectral efficiency compared to the logarithmic increase in more traditional systems utilizing receive diversity or no diversity. The high spectral efficiencies attained by MIMO systems are enabled by the fact that in a rich scattering environment, the signals from each individual transmitter appear highly uncorrelated at each of the receive antennas. When the signals are conveyed through uncorrelated channels between the transmitter and the receiver, the signals corresponding to each of the individual transmit antennas have attained different spatial signatures. The receiver can use these differences in the spatial signature to simultaneously and at the same frequency separate the signals that originated from different transmit antennas. MIMO systems offer diversity gain and multiplexing gain.

6.1 Diversity gain

Diversity is used in wireless systems to combat small scale fading caused by multi-path effects. The basic principle of diversity is that several replicas of the information signal are received through independent fading links (branches), then with high probability at least one or more of these links will not be in fade at any given instant and this probability will increase if the number of diversity branches increases. Diversity processing that reduces fading is a powerful tool to increase capacity and coverage of radio networks. The three main form of diversity traditionally exploited in the wireless systems are temporal diversity, frequency diversity and spatial (or antenna) diversity.

Temporal diversity: It is applicable in a channel that has time selective fading. The information is transmitted with spreading over a time span that is larger than the coherence time of the channel. The coherence time is the minimum time separation between independent channel fades. Time diversity is usually exploited via interleaving, forward error correction codes (FEC), and automatic repeat request (ARQ). One drawback of time diversity is the inherent delay incurred in time spreading.

Frequency diversity: It is effective when the fading is frequency selective. It can be exploited by spreading the information over a frequency span larger than the coherence bandwidth of the channel. The coherence bandwidth is the minimum frequency separation between independent channel fades and is inversely proportional to the delay spread of the channel. Frequency diversity cab be exploited through spread spectrum techniques or through interleaving and FEC in conjunction with multi-carrier modulation.

Spatial diversity: In space diversity we receive or transmit information signals from antennas that are spaced by more than coherence distance apart. The coherence distance is the minimum spatial separation of the antennas for independent fading and depends on the angle spread of the multi-paths signals arriving at or departing from antenna array. For example, if the multi-path signals arrive from all directions in azimuth, antenna spacing on the order of $0.4\lambda - 0.6\lambda)$ is adequate [11] for independent fading. On the other hand, if the multi-path angle spread is smaller, the coherence distance is larger. Empirical measurement show a strong coupling between antenna height and coherence distance for the base station antennas. Higher antenna heights imply larger coherence distances. At the terminal end, which is usually low and buried in scatterers, a $0.4\lambda - 0.6\lambda$ separation will be adequate. Receive diversity is well studied subject [9]. The use of multiple antennas at the base station in combination with the transmit diversity has become an active area of research in the past few years [93,96]. Transmit diversity in the case when the channel is known at the transmitter involves transmission such that the signals sent from the individual antennas arrive at the receive antennas in phase. In the case when the channel is not known at the transmitter side, transmit diversity require more sophisticated methods such as space-time coding which use coding across antennas (space) and time. The basic idea is to send information with different preprocessing (coding, modulation, delay, etc.) from different antennas such that the receiver can combine these signals to obtain diversity.

In a MIMO system both transmit and receive antennas combine to give a large diversity order. Let n and m be transmit and receive antennas respectively, a maximum of nm links are available and if all of these links fade independently, we get $nm - th$ order diversity.

Transmit and receive diversity are both similar and different in many ways. While receive diversity needs merely multiple antennas which fade independently, and is independent of coding/modulation schemes, transmit diversity needs special modulation/coding schemes in order to be effective. The receive diversity provides array gain, whereas transmit diversity does not provide array gain when the channel is unknown at the transmitter.

6.1.1 Multiplexing gain

Spatial multiplexing requires multiple antennas at both ends of the link [2]. The idea of spatial multiplexing is that the use of multiple antennas at the transmitter and the receiver in conjunction with the rich scattering in the propagation environment opens up multiple data pipes within the same frequency band to yield a linear (in the number of antennas) increase in capacity. This increase in capacity comes at no extra bandwidth or power consumption and therefore is very attractive. In spatial multiplexing the symbol stream to be transmitted is broken up into several parallel symbol streams which are then transmitted simultaneously and within the same frequency band from the antennas. Due to multi-path propagation, each transmit antenna induces a different spatial signature at the receiver. The receiver exploits these signatures differences to separate the individual data streams.

6.2 The MIMO channel model

A commonly used channel model in MIMO wireless communications is the block fading model, where the channel matrix entries are i.i.d. complex Gaussian (Rayleigh fading), constant during the block of symbols, and change in an independent fashion from one block to another. We assume that channel is only known at the receiver. The input-output relation of $m \times n$ matrix channel can be written as

$$r = Hs + n \tag{6.1}$$

where r is the size of number of receive antennas and s is size of number of transmit antennas and n is Gaussian noise vector. The elements of H are i.i.d. circularly symmetric complex Gaussian with zero mean and unit variance. Equivalently, each entry of H has uniformly distributed phase and Rayleigh distributed magnitude.

6.3 Capacity of MIMO channels

In this section we briefly give some capacity formulas for the MIMO systems. Assuming that the transmitted signal vector is composed of n statistically independent equal power components each with a circularly symmetric complex Gaussian distribution, the capacity of deterministic MIMO channel H is given by

$$C = \log_2[\det(I_m + \frac{\rho}{n} HH^H)] \ \ bps/Hz, \tag{6.2}$$

where ρ is the average transmit power. When $n = m$ and $H = I_n$, we get

$$C = n \log_2(1 + \frac{\rho}{n}) \rightarrow \rho/ln(2) \ \ as \ \ n \rightarrow \infty. \tag{6.3}$$

It is clear from the above equation that the capacity scales linearly with the increase in SNR. Now we give the equation of the capacity when the channel H is assumed to be random. The ergodic capacity is given by [1]

$$C = E_H\{\log_2[\det(I_m + \frac{\rho}{n} HH^H)]\} \ \ bps/Hz, \tag{6.4}$$

where E_H is the expectation with respect to random channel. Note that for fixed m as n gets larger $\frac{1}{n}HH^H \rightarrow I_m$ and hence the ergodic capacity in the limit of large n equals

$$C = m \log_2(1 + \rho) \ \ bps/Hz. \tag{6.5}$$

The ergodic capacity grows linearly with the number of receive antennas, which hints at significant capacity gains of MIMO fading channels. Now we discuss some receivers for the MIMO systems.

6.4 Zero forcing receiver

The zero forcing receivers simply inverts the channel transfer matrix, assuming that H is invertible. The estimate of data symbol vector s is obtained as

$$\hat{s} = H^{-1}r. \tag{6.6}$$

For ill-conditioned H the zero forcing receiver performs well in high SNR regime, whereas in the low SNR regime there will be significant noise enhancement.

6.5 Minimum mean-square error receiver (MMSE)

The zero forcing receiver yields perfect separation of the co-channel signals at the cost of noise enhancement. An alternative is MMSE receiver, which minimizes the overall error due to noise and mutual interference between co-channel signals. In this case, an estimate of s is obtained according to

$$\hat{s} = \frac{\rho}{n} H^H (\sigma_n^2 I_m + \frac{\rho}{n} H H^H)^{-1} r, \tag{6.7}$$

where we assume that $E\{ss^H\} = \frac{\rho}{n} I_n$ and $E\{nn^H\} = \sigma_n^2 I_m$. The MMSE receiver is less sensitive to noise at the cost of reduced signal separation quality. At high SNR the performance of the zero forcing and MMSE receiver is same.

6.6 V-BLAST receiver

In V-BLAST rather than jointly decoding all the transmit signals, we first decode the "strongest" signal, then subtract this strongest signal from the received signal, proceed to decode the strongest signal of the remaining transmit signals, and so on. The optimum detection order in such nulling and cancelling strategy is from the strongest to the weakest signal. Assuming that the channel is known at the receiver, the main steps of the V-BLAST algorithm can be summarized as follows:

1) Nulling: an estimate of the strongest transmit signal is obtained by nulling out all the weaker transmit signals (say using zero forcing criterion).

2) Slicing: the estimated signal is detected to obtain the data bit.

3) Cancelation: These data bits are remodulated and the channel is applied to estimate its vector signal contribution at the receiver. The resulting vector is then subtracted from the received signal vector and the algorithm returns to the nulling step until all transmit signals are decoded.

6.7 Maximum likelihood receiver

The receiver which yields the best performance in terms of error rate is maximum likelihood (ML) receiver. However, this receiver is computationally complex. Assuming that the channel state information is known at the receiver, ML receiver computes the estimate of data bits according to

$$\hat{s} = arg \;\; min_s \;\; ||r - Hs||^2, \tag{6.8}$$

where the minimization is performed over all possible codeword vectors s. Note that the complexity increase exponentially with the number of transmit antennas.

6.8 Transmit diversity

Transmit diversity is a technique which realizes spatial diversity gain in systems with multiple transmit antennas without requiring channel knowledge in the transmitter.

6.8.1 Indirect transmit diversity

In indirect transmit diversity we convert spatial diversity into time or frequency diversity, which can be readily exploited by the receiver. We shall discuss two techniques, namely delay diversity which converts spatial diversity into frequency diversity and intentional frequency offset diversity which converts spatial diversity into time diversity.

Delay diversity: Let us assume $n = 2$ and $m = 1$. In delay diversity the spatial diversity is converted into frequency diversity by transmitting the data bearing signal from the first antenna and a delayed replica from the second antenna. Assuming that the delay is one symbol interval, the effective SISO channel seen by the receiver is

$$H_e(e^{j2\pi\theta}) = h_o + h_1 e^{-j2\pi\theta} \tag{6.9}$$

where h_o and h_1 denote the channel gains between the transmit antennas 1 and 2, and the receive antenna respectively. The channel gains are assumed i.i.d. Gaussian. To show spatial diversity is converted into frequency diversity, we compute the frequency correlation function

$$R(e^{2\pi\nu}) = \frac{1}{2}E\{H_e(e^{j2\pi\theta})H_e^*(e^{j2\pi(\theta-\nu)})\}, \tag{6.10}$$

arranging the above two equations we get

$$R(e^{2\pi\nu}) = \frac{1}{2}(1 + e^{-j2\pi\nu}). \tag{6.11}$$

The function $|R(e^{2\pi\nu}|$ is fully decorrelated for $\nu = 0.5$. Such a channel looks exactly like a 2 paths channel with independent path fades and the same energy per path. Therefore Viterbi (ML) sequence detector will capture the diversity in the system.

Intentional frequency offset diversity: Let us assume two transmit and one receive antenna case. After coding and modulation, we transmit the signal from the first antenna and a frequency shifted (phase rotated) version thereof from the second antenna. The effective SISO channel seen by the receiver is

$$h_e[n] = h_o + h_1 e^{j2\pi n\theta_1}, \;\; n \in Z \tag{6.12}$$

where θ_1 with $|\theta_1| < 1/2$. Having the same assumptions as for delay diversity, the temporal correlation function of stochastic channel is $R[k] = \frac{1}{2}E\{h_e[n]h_e^*[n-k]\}$. Using the above equation we have

$$R[k] = \frac{1}{2}(1 + e^{j2\pi k\theta_1}), \tag{6.13}$$

if $|R[k]|$ is small data symbols spaced k symbols intervals apart undergo close to independent fading. The resulting temporal diversity can be exploited by using FEC in combination with time interleaving just as we may do in naturally time fading channels.

Below we describe two direct transmit diversity schemes, i.e., space-time block coding and space-time trellis coding which are popular now-a-days.

6.8.2 Space-time block coding

Space-time block coding has attracted much attention for practical applications [76,93]. A simple space-time block code known as Alamouti scheme performs very similar to maximum-ratio combining (MRC), a technique which realizes spatial diversity gain by employing multiple receive or transmit antennas (but needs channel knowledge at the transmitter for the later). We briefly review receive MRC for $n = 1$ and $m = 2$. The receive signal are given by

$$r_o = h_o s + n_o$$

$$r_1 = h_1 s + n_1,$$

where h_o and h_1 denote the Gaussian i.i.d. channel gains between the transmit antenna and receive antenna. n_o and n_1 are Gaussian noise samples. The receiver estimates the transmitted data symbols by forming the decision variable

$$y = h_o^H r_o + h_1^H r_1 = (|h_o|^2 + |h_1|^2)s + h_o^H n_o + h_1^H n_1. \tag{6.14}$$

Clearly, if either h_o or h_1 is not faded we have good channel. Thus we get second order diversity. Now we consider the case of two transmit antennas at the transmitter side and the receiver side is equiped with one receive antenna. In Alamouti scheme at a given period, two signals are simultaneously transmitted from two antennas. In the first time instant, the signal transmitted from antenna 1 is s_o and the signal transmitted from antenna 2 is s_1. In the next time instant $-s_1^H$ is transmitted from antenna 1 and s_o^H is transmitted from antenna 2. The received signal r_o and r_1 is given by

$$r_o = h_o s_o + h_1 s_1 + n_o$$

$$r_1 = -h_o s_1^H + h_1 s_o^H + n_1,$$

where h_o and h_1 denote the zero mean Gaussian i.i.d. channel gains between transmit antenna 1 and receive antenna and transmit antenna 2 and receive antenna respectively. The above equations can be re written in matrix form as

$$r = H_a s + n,$$

where $r = [r_o \;\; r_1^T]$ is the received vector,

$$H_a = \begin{pmatrix} h_o & h_1 \\ h_1^H & -h_o^H \end{pmatrix},$$

is the equivalent channel matrix, $s = [s_o \;\; s_1]^T$ and $n = [n_o \;\; n_1^H]$. Note that columns of H_a are orthogonal. On the receiving side (assuming perfect channel knowledge) the received vector r is multiplied by H_a^H which results

$$\hat{s} = H_a^H r.$$

The estimates of the symbols s_o and s_1 are given by

$$\hat{s_o} = (|h_o|^2 + |h_1|^2)s_o + \tilde{n_o}$$

$$\hat{s_o} = (|h_o|^2 + |h_1|^2)s_1 + \tilde{n_1},$$

where $\tilde{n_o} = h_o^H n_o + h_1 n_1^H$ and $\tilde{n_1} = h_1^H n_o - h_o n_1^H$. The symbols $\hat{s_o}$ and $\hat{s_1}$ are independently sent to an ML decoder.
The Alamouti scheme is a special case of the so-called space-time block codes.

6.8.3 Space-time trellis codes

Space-time trellis codes are extension of Trellis codes to the case of multiple transmit and receive antennas. In the space-time trellis coding the information stream to be transmitted is encoded by the space-time encoder into blocks of size $n \times T$, where T is the size of the burst over which the channel is assumed to be constant. One data burst therefore consists of T vectors $c_k(k = 0, \ldots, T-1)$ of size $n \times 1$ with the data symbols taken from finite complex alphabet chosen such that the average energy of the constellation element is 1. The $k-th$ receive symbol vector is given by

$$r_k = \sqrt{E_s} H c_k + n_k \;\; , k = 0, \ldots, T-1$$

where n_k is the complex valued Gaussian noise and E_s is symbol energy. When the channel is known to the receiver the ML decoder computes the estimated vector sequence as

$$\hat{c_k} = arg_{min_C} \sum_{k=0}^{T-1} ||r_k - \sqrt{E_s} H c_k||^2,$$

where $C = [c_o \ldots c_{T-1}]$ and the minimization is over all possible codeword matrices C. Here we review the design criteria of the STTC when the receiver knows the channel. We consider the pairwise error probability. Let $C = [c_o \ldots c_{T-1}]$ and $E = [e_o \ldots, e_{T-1}]$ be two different space-time codewords of size $n \times T$ and assume that C was transmitted. In case when SNR is high, the average probability (average over all channel realizations) that the receiver decides erroneously in favor of the signal E is upper bounded by

$$P(C \to E) \leq (\frac{E_s}{4})^{-r(B_{c,e})m} \prod_{i=0}^{r(B_{c,e})-1} \lambda_i(B_{c,e})^m, \tag{6.15}$$

where $B_{c,e} = (C - E)T(C_E)^H$ and $r(B_{c,e})$ denotes the rank of the matrix $B_{c,e}$ and $\lambda_i(B_{c,e})$ denotes its nonzero eigenvalues respectively.

The design criteria for STTC is follows:

1). The rank criterion: In order to achieve the maximum diversity nm, the matrix $B_{c,e}$ has to be full rank for every pairwise distinct codewords C and E.

2). The determinant criterion: If a diversity advantage of nm is the design target, the minimum of the determinant of $B_{c,e}$ taken over all pairs of distinct codewords C and E must be maximized. STTC offer better performance than STBC at the cost of increased decoding complexity.

Chapter 7

Fundamentals on semidefinite and second order cone programming

Semidefinite programming [60,66] is a special case of convex programming where the feasible region is an affine subspace of positive semidefinite matrices. There has been much interest in this area lately, partly because of applications in combinatorial optimization and in control theory and also because of the development of efficient interior-point algorithms [56]. Semidefinite programs are natural generalizations of linear programs.
The use of semidefinite programming in combinatorial optimization is not new though. Eigenvalue bounds have been proposed for combinatorial optimization since the late $60'$. *An explicit use of semidefinite programming in combinatorial optimization appeared in the seminal work of Lovasz [62] on the so-called theta function, and this lead Grotschel, Lovasz and Schrijver [115] to develop the only known (and non-combinatorial) polynomial-time algorithm to solve the maximum stable set problem for perfect graphs.*

7.1 Standard form of linear program

Let $c \in R^n$ and $b \in R^m$, $A \in R^{n\times m}$ with rows, $a_i \in R^n$, $i = 1, \dots, m$, $x \in R^n$ decision vector, 0 is the vector of zeros, appropriately dimensioned. The linear program is

$$\min\ c^T x,\ \text{subject to}\ a_i x = b_i,\ i = 1, \dots, m,\ x \geq 0 \tag{7.1}$$

7.2 Semidefinite program (SDP)

Here instead of vectors a_i we use symmetric matrices $A_i \in R^{n\times n}$, $i = 1, \dots, m$, $C \in R^{n\times n}$ and $X \in R^{n\times n}$ instead of cost and decision vectors c and x. The matrix X is symmetric (i.e., $X = X^T$ - this allows to make the assumption that C and A_i, $i = 1, \dots, m$ are also symmetric). We have $Tr(AB) = \sum_{i,j} A_{ij}B_{ij}$. The semidefinite program can be written as

$$\min\ Tr(CX), \text{subject to}\ Tr(A_i X) = b_i\ i = 1, \dots, m, X \geq 0 \tag{7.2}$$

7.3 Second order cone program (SOCP)

Let $A_j \in R^{m\times n_j}$, $c_j \in R^n$, $b \in R^m$, $j = 1, \dots, k$ and $x_j \in R^{n_j}$, $j = 1, \dots, k$ are decision variables. The general second order cone program is

$$\min\ c_1^T x_1 + \dots + c_k^T x_k$$

$$\text{subject to}\ A_1 x_1 + \dots + A_k x_k = b$$

$$x_j \geq_Q 0,\ j = 1, \dots, k, \tag{7.3}$$

where the relation $a \geq_Q$ is defined as $a_o \geq \sqrt{a_1^2 + \dots + a_n^2}$.

7.4 An overview on solving semidefinite and second order cone programs

Semidefinite programs can be solved very efficiently using interior point methods [56,58,70,52]. The most efficient variants solve the primal and the dual of the SDP and SOCP problems simultaneously.

7.4.1 Interior point methods

In nonlinear programming [23,71], one of the approaches to transform constrained optimization problems into unconstrained problems is to implement inequality constraints by adding a barrier term to the cost function. In the case of minimization problem, the value of the barrier term grows to infinity when the boundary is approached but it is small in the interior of the feasible region. The barrier prevents the line search [57] starting from the interior from leaving interior. Furthermore a close point to the boundary will have a descent direction which automatically will be directed away from the boundary. In order to produce the sequence of iterates that converges to the optimum which is usually located on the boundary, a mechanism that reduces the influence of the barrier term as the optimization process continues has to be incorporated. This is achieved by adding a weight to the barrier term and diminishing it successively. Under certain conditions, the minima of the sequence of the barrier problems can be shown to converge to an optimal solution of the original problem. This is known as sequential unconstrained minimization technique which interior point methods are based upon. Typically, the minima are not computed exactly but approximated by a few Newton's steps. Since Newton's method works particularly well on the class of barrier problems associated with SDP and SOCP, the algorithm converges very fast.
Here we introduce some barrier functions for SDP and SOCP and show how Newton's method can be used to solve optimization problems.

7.4.2 Barrier functions for SDP and SOCP

We now define the notion of barrier functions for convex sets in general and for SDP and SOCP, in particular. Then we explain how we can use barrier functions to find the optimal solutions of these problems. The formal definition of the barrier function is given below:
Barrier function: Let $C \subseteq R^n$ be a convex set with nonempty interior. Then the function $b: \ \mathrm{Int}(C) \rightarrow R$ is called a barrier function if it has the following properties.

1) b is convex

2) For each sequence of points $x_n \in \mathrm{Int}(C)$ such that $\lim_{n\rightarrow\infty} x_n$ exists and belongs to boundary, $\mathrm{Bd(C)}$, the $\lim_{n\rightarrow\infty} b(x_n) = \infty$

Note that since the domain of $b(x)$ is $\mathrm{Int}(C)$ and by the properties of $b(x)$, the minimum value of b is attained in the $\mathrm{Int}(C)$.

7.4.3 Barrier function for SDP

First let us define some notation. If $X \in R^{n\times n}$, define $\mathrm{vec}X \in R^{n^2}$. i.e., the vectorization of a matrix. Now consider the SDP:

$$\min \ Tr(CX), \ \ \text{s.t} \ \ Tr(A_iX) = b_i, \ \ \text{for} \ \ i = 1, \ldots, m, X \geq 0 \tag{7.4}$$

where $C, X, A_i \in S_n, \ i = 1, \dots, m$, S_n is the set of $n \times n$ symmetric matrices. Recall that $X \geq 0$ if , and only if $\lambda_i(X) \geq 0$ for all $i = 1, \dots m$, where $\lambda_i(X)$ is the i-th eigenvalue of matrix X. As the eigenvalues are positive so as their product and hence logarithm of the product of the eigenvalues is defined. We can use the following function as a barrier for SDP:

$$\sum_i \frac{1}{\lambda_i(X)}, \text{and} \ \ \sum_i -\log \lambda_i(X) = -\log \det X$$

Before forming the Lagrangian function and the equations which we shall use to find the critical points of it, let us slightly change the form of these equations. Let $A \in R^{m \times n^2}$ whose ith row is $\text{vec}^T A_i$ and $b \in R^m$ whose ith entry is b_i. Since

$$Tr(A_i X) = b_i = \text{vec}^T A_i \text{vec} X \tag{7.5}$$

we can write the set of equations $A_i X = b_i$, for $i = 1, \dots, m$ as $A\text{vec}X = b$. Applying the barrier function we get the following SDP

$$min \ \ Tr(CX) - \mu \log \det X \ \text{ s.t } \ A\text{vec}X = b \tag{7.6}$$

Then the Lagrangian function of the above problem is:

$$L(X, y) = Tr(CX) - \mu \log \det X + \sum_{i=1}^{m} y^T(b - A\text{vec}X) \tag{7.7}$$

where $\mu \in R$ and $y \in R^m$. Equating to zero the derivative of the Lagrangian we have

$$\nabla_X L = \text{vec}^T C - \mu \text{vec}^T X^{-1} - y^T A = 0 \tag{7.8}$$

$$\nabla_{y_i} l = b - A\text{vec}X = 0 \tag{7.9}$$

Let $Z = \mu X^{-1}$. Then we can write the above equations as,

$$A\text{vec}X = b$$

$$C - Z - \sum_i y_i A_i = 0$$

$$Z - \mu X^{-1} = 0 \tag{7.10}$$

The third equation can be written as $X - \mu Z^{-1}$ or $XZ = \mu I$ where I is $n \times n$ identity matrix. It can be shown that for X and Z positive semidefinite, $XZ = \mu I$, if and only if $XZ + ZX = 2\mu I$.

7.4.4 Barrier function for SOCP

We now define a suitable barrier function for SOCP and calculate the derivatives of its Lagrangian function. For simplicity let us consider the following single block SOCP:

$$\min\ c^T x\ \text{ s.t }\ Ax = b,\ \ x \geq_Q 0 \tag{7.11}$$

where $A \in R^{m\times n}$, $c \in R^n$, $b \in R^m$, and $x \in R^n$ are the variables. The relation $a \geq_Q 0$ is defined as $a_o \geq \sqrt{a_1^2 + \ldots + a_n^2}$. Let's write the constraints $x_o \geq ||\bar{x}||$ as $x_0^2 - x_1^2 - x_2^2 - \ldots - x_{n-1}^2 \geq 0$. Let $x = (x_o, x_1, x_2, \ldots, x_{n-1})^T$. We can write

$$x_0^2 - x_1^2 - x_2^2 - \ldots - x_{n-1}^2 = x^T B x \tag{7.12}$$

A suitable barrier function for SOCP is:

$$\log(x_o^2 - ||\bar{x}||^2) \tag{7.13}$$

By similar procedure as for SDP, the SOCP problem is replaced by

$$\min\ c^T x - \mu \log(x_o^2 - ||\bar{x}||^2),\ \ Ax = b \tag{7.14}$$

for $\mu > 0$. Therefore the Lagrangian function of this problem is:

$$L(x,y) = c^T x - \mu \log(x_o^2 - ||\bar{x}||^2) + y^T(b - Ax) \tag{7.15}$$

$$\nabla_x L = c^T - \frac{2\mu}{x_o^2 - ||\bar{x}||^2}(x_o, -x_1, -x_2, \ldots, -x_{n-1}) - y^T A = 0$$

$$\nabla_y L = b - Ax = 0 \tag{7.16}$$

Introducing the slack variable z, this system is equivalent to

$$Ax = b$$

$$A^T y + z = c$$

$$z - \frac{2\mu}{x_o^2 - ||\bar{x}||^2} Bx = 0 \tag{7.17}$$

where $z = \frac{2\mu}{x_o^2 - ||\bar{x}||^2} Bx$. This in turn can be written equivalently as:

$$x^T z = 2\mu,\ \ x_o z_i + x_i z_o = 0\ \text{ for }\ i = 1, 2, \ldots, n \tag{7.18}$$

The first equation can be obtained by multiplying from the left the equation $z - \frac{2\mu}{x_o^2 - ||\bar{x}||^2} Bx = 0$ by x^T and noting that $x^T BX = x_o^2 ||\bar{x}||^2$. The second set of equation arise from observing that

$$\frac{x_i}{z_i} = \frac{x_o}{z_o} - \frac{2\mu}{x_o^2 - ||\bar{x}||^2} \tag{7.19}$$

Thus just like SDP, applying logarithmic barrier function to SOCP problems results in getting primal and dual feasibility and a relaxed form of complementarity conditions.

7.5 Newton's method

So far we have been trying to find equations by using barrier functions. Now we wish to solve these systems. For SDP and SOCP the logarithmic barrier function resulted in a system of equations which contained primal and dual feasibility (a set of linear equations) and a relaxed form of complimentary conditions (a set of nonlinear equations). To handle the nonlinear equations, the main tool is using the Newton's method. The general approach is as follows:
We start with an estimate of the solution (x, y, z). Next we seek a direction $(\Delta x, \Delta y, \Delta z)$ such that moving in that direction with an appropriate step length will bring us closer to the solution of the system. The Newton's method replaces, (x, y, z) with $(x + \Delta x, y + \Delta y, z + \Delta z)$, and plugs it into system of equations. Then, noting that $(\Delta x, \Delta y, \Delta z)$ are unknowns, it removes any nonlinear terms in $\Delta's$ and solves the remaining system of linear equations (see [56,58] for more details).

Chapter 8

Maximum likelihood detection of a MIMO system using a second order cone programming approach

Multiple antenna systems are capable of providing high data rate transmission over wireless channels. To secure reliability of the data transmission, special attention has to be paid to the design of the receiver. The optimum receiver structure is the maximum likelihood sequence estimation (MLSE). However, the computational complexity of the ML decoding requires us to solve an Integer Least Square (ILS) problem, which is, in general, NP-hard. Recently,a semidefinite programming (SDP) relaxation approach has been proposed to approximately solve NP-hard problems in polynomial time [101]. Its worst case computational complexity is $O(n^{3.5})$. Even the SDP approaches are computationally expensive for large systems. The other approach currently used for ML decoding is the sphere decoding scheme [3]. The average case complexity of this scheme is $O(n^3)$, when the radius, r, is correctly chosen (which is itself a NP-hard problem). Also, at low SNRs, the complexity of the sphere decoder explodes. In this chapter, we propose to apply a second order cone programming (SOCP) approach [69] to resolve large system problems, which offers substantial computational savings over the SDP relaxation and sphere decoding schemes, while maintaining the performances arbitrarily close to ML.

8.1 Introduction

Multiple antenna wireless communication systems are capable of providing data rates at potentially very high level [1,13]. However,a reliable decoding in these systems requires a very high complexity. For a wide class of space-time transmission schemes [2], ML decoding requires us to solve ILS problem, which is, in general, NP-hard. Practical methods to solve this problem are to employ heuristics, or some approximation to the original problem. For example, MIMO Decision Feedback Equalization(DFE), nulling and cancelling with optimal ordering (BLAST), are some of these. However, the performance in terms of bit error rate is inferior to that of the exact ML methods.

One of the methods recently proposed is sphere decoding. In the sphere decoding algorithm, we find lattice points lying in the hypersphere centered around the received signal and then determine the point closest to the received signal. This algorithm has a polynomial complexity for high SNRs and when the radius is optimally chosen. Choosing the optimal radius is however, NP-hard. Moreover at low SNRs the complexity of sphere decoding is exponential.

In this chapter, we propose a novel method for data detection. The detection is based on transforming a quadratic 0-1 programming problem [64,55,54] into an equivalent problem in graph theory, called weighted Maximum Cut (MAX-CUT for short) in a graph. A MAX-CUT problem [5,25,67,68] can be solved in polynomial time for some classes of graphs, e.g, planar graphs, with positive edge weights. But if the graph contains negative weights too, the MAX-CUT problem becomes NP-hard. A standard way to solve a NP-hard combinatorial optimization problem is to first formulate it as a mathematical programming problem, and then to relax some of its constraints in order to solve it in polynomial time. A MAX-CUT problem (with negative and positive edge weights) can be solved to a very good approximation using a SDP relaxation, and the worst case result is $0.878 MAXCUT edges$ [4]. Still the MAX-CUT using SDP relaxation can be computationally complex for large sizes. We propose to use SOCP to the problem at hand for computational reduction. SOCP takes the advantage of reducing the number of variables in the problem; hence, optimization can be done much faster comparing with the SDP. In the following lines we briefly describe the MAX-CUT problem and the Goemans-Willianson approach to MAX-CUT on an undirected graph. In section 8.2 we describe MAX-CUT on a graph. Goemans-Williamson approach for the MAX-CUT problem is described in section 8.3. Section 8.4 and section 8.5 are devoted to signal model and second order cone programming for ML decoding respectively. Simulations and conclusions are drawn in the last section.

8.2 The MAX-CUT on an undirected graph

Let $G = (V, E)$ be an undirected graph, where $V = \{1, \ldots, n\}$ are the nodes and E is the set of edges, with cardinality, $|E| = m$ are the edges. Let w_{ij} be the weights given to an edge connecting the node i to the node j. The maximum cut entails partitioning, V into $S \subset V$ and $\bar{S} = V - S$, such that, sum of the weights on the edges from the subset S of $\bar{S}$ is maximized. In other words,

$$W(C_{opt}) = \max_{S \subset V} \sum_{\{(i,j) \in E | i \in S, j \in \bar{S}\}} w_{ij}. \tag{8.1}$$

Such a partition is known as a cut. Before the $90's$, two approaches were known for the MAX-CUT problem. These approaches are discussed below.

8.2.1 Two approaches

One approach, the Greedy Approach, works as follows: choose a node arbitrarily, and add it to S. Then for each remaining node n, if the sum of the weights from n to the nodes in the current S are less than the sum of the weights from n to the nodes in the current $\bar{S}$, add n to S; otherwise, add it to $\bar{S}$. Let $W(C_{greedy})$ denote the value of this cut. It can be shown that:

$$W(C_{greedy}) \geq \sum_{(i,j) \in E} w_{ij}/2 \geq W(C_{opt})/2. \tag{8.2}$$

A second approach, the Randomized Approach, works by randomly assigning nodes to S and $\bar{S}$. Let us denote the value of the cuts generated in this fashion by $W(C_{rand})$. It is shown that

$$E[W(C_{rand})] = \sum_{(i,j)} w_{ij}/2 \tag{8.3}$$

Until the early $90's$, it was not known whether this factor of 2 could be improved.

8.3 The Goemans-Williamson approach

Goemans-Williamson [4] approached the problem as follows: $\forall i \in V$, let

$$x_i = \begin{cases} -1 & \text{if } i \in S \\ +1 & \text{otherwise} \end{cases}$$

Then we have,

$$1 - x_i x_j = \begin{cases} 2 & \text{if } i \in S, j \in \bar{S} \text{ or vice-versa} \\ 0 & \text{otherwise} \end{cases}$$

So then the objective becomes to maximize $\sum_{i,j} \frac{w_{ij}}{2}(1 - x_i x_j)$ with $x_i^2 = 1$ This problem is NP-Hard. In order to deal with less complex problem, Goemans and Williamson considered the following relaxation. Associate to each vertex $i \in V$ a vector $v_i \in R^n$, they considered

$$\max \sum_{i<j} \frac{w_{ij}}{2}(1 - v_i v_j) \text{subject to } v_i.v_i = 1, \forall i \in V. \tag{8.4}$$

Because of the constraint $v_i.v_i = 1$, the vectors $v_1, \ldots, v_n$ are constrained to lie on n-dimensional sphere, S_{n-1}. Now there are two questions remaining: 1) How to solve this? 2) How good is the solution?

8.3.1 How to solve this?

Consider the $V \in R^{n \times n}$, with form

$$V = [v_1, \ldots, v_n]$$

Let $Y = V^T V$, and let us index Y with $y_{ij}, i = 1, \ldots, n, \; j = 1, \ldots, n$. Thus, $y_{ij} = v_i^T v_j$. The following formulation is then obtained

$$\max \sum_{i,j} \frac{w_{ij}}{2}(1 - y_{ij}) \text{subject to } y_{ii} = 1, \forall i \in N \; and \; Y \geq 0 \tag{8.5}$$

where the notation $A \geq 0$ indicates that the matrix A is positive semidefinite.

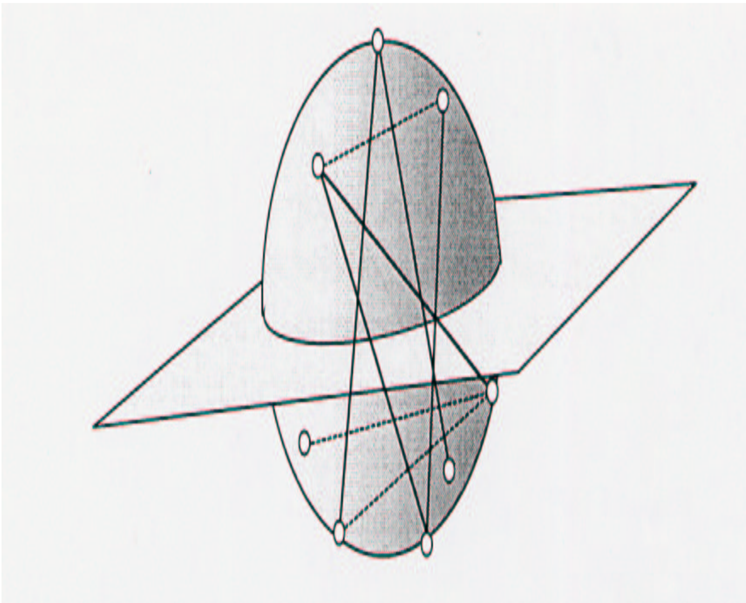

Figure 8.1: A cut in a graph given by random hyperplane

8.3.2 Randomized rounding algorithm

We now present the algorithm for MAX-CUT. For convenience, let us assume that we have an optimal solution to the above equation. the slight inaccuracy in solving it can be absorbed into approximation factor. Let $a_i, \ldots, a_n$ be the optimal solution, and let OPT_v be the corresponding objective function value. These vectors lie on the n-dimensional unit sphere S_{n-1}. We need to obtain a cut $(S, \bar{S})$ whose weight is a large fraction of OPT_v.

Let θ_{ij} denote the angle between vectors a_i and a_j. The contribution of this pair of vectors to OPT_v is $\frac{w_{ij}}{2}(1 - cos\theta_{ij})$. Clearly, the closer θ_{ij} is to π, the larger this contribution will be. In turn, we would like vertices v_i and v_j to be separated if θ_{ij} is large. The following method accomplishes precisely this. Pick r to be a uniformly distributed vector on the unit sphere S_{n-1} and let $S = \{v_i | a_i.r \geq 0\}$. Observe now that the probability that the vectors v_i and v_j are on the opposite sides of the hyperplane is exactly the proportion of the angle between v_i and v_j to π, i.e., $\arccos(v_i.v_j)/\pi$. Let the cut generated by the random hyperplane be $W(C_{randSDP})$ (see figure). Then by linearity of expectations

$$E[W(C_{randSDP})] = \sum_{i,j} \frac{\arccos(v_i^T v_j)}{\pi} \tag{8.6}$$

Since the expected value of the given cut is at most as large as the optimal cut, and since the expected value of the optimal cut is less than the value of the semidefinite relaxation, we have

$$\sum_{i,j} \frac{\arccos(v_i^T v_j)}{\pi} \leq W(C_{OPT}) \leq W(C_{SDP}) = \sum_{i,j} \frac{w_{ij}}{2}(1 - v_i^T v_j) \tag{8.7}$$

Now, the following question arises: does there exists $0 \leq \alpha \leq 1$ such that $\alpha W(C_{SDP}) \leq W(C_{random})$?. If such an α exists then by comparing term by term the sums we can write

$$\sum_{i,j} \alpha \frac{w_{ij}}{2}(1 - v_i^T v_j) \leq \sum_{i,j} w_{ij} \frac{\arccos(v_i^T v_j)}{\pi}. \tag{8.8}$$

It follows that we seek an α that is at most as large as $\frac{2\arccos(v_i^T v_j)}{\pi(1-v_i^T v_j)}$
by renaming $y = v_i^T v_j$, we see that such an α at most equals

$$\alpha = \min_{-1 \leq y \leq 1} \frac{2\arccos(\mathrm{y})}{\pi(1-y)} \tag{8.9}$$

This minimum is approximately 0.878 as shown in [4].

8.4 Signal Model

We assume a discrete time block fading multiple antenna channel model with N transmit and M receive antennas, also we assume that the receiver has perfect channel knowledge. The received signal at an instant is

$$x = Hs + n, \tag{8.10}$$

where $H \in R^{M\times N}$ is a known channel matrix, and $n \in R^{M\times 1}$ is the i.i.d. zero mean white Gaussian noise with variance σ^2. The fading coefficients are i.i.d. Gaussian with zero mean and unit variance. Under the aforementioned assumptions, the ML criterion requires us to find $s \in R^{N\times 1}$, where $s_i \in \{-1, 1\}$, which minimizes $\|x - Hs\|^2$. Now we develop SOCP method for the ML detection.

8.5 SOCP ML decoder

The problem can be written (after neglecting constant term) as

$$f(s) = \max_{s\in\{-1,1\}^N} \quad s^T Js + 2c^T s, \tag{8.11}$$

where $J = -H^T H$

and $c = Hs$. Since

$$s_i^2 = s_i, \forall i, \tag{8.12}$$

are the diagonal entries in J, can be absorbed in the vector c. Thus without loss of generality, we can assume that all diagonal entries of J are zeros. The above equation is equivalent to

$$f(s) = max_{y\in\{-1,1\}^{N+1},\ y_{N+1}=1} \ \ y^T \bar{L}y \tag{8.13}$$

where $\bar{L}$ is of the form

$$\bar{L} = \begin{pmatrix} J & c \\ c^T & 0 \end{pmatrix} \tag{8.14}$$

Since this cost function is symmetric, $y_{N+1} = 1$ need not be maintained explicitly. It can be shown that the MAX-CUT problem is equivalent to ML detection with BPSK [67]. As mentioned earlier that the MAX-CUT problem is NP-hard, therefore we will use some relaxation scheme to find near optimum solution. In this respect, note that

$$x_j = 1 or - 1 \Rightarrow x_j^2 = 1 \Leftrightarrow x_j^2 \leq 1, \ \ x_j^2 \geq 1 \tag{8.15}$$

$$x^T e_j e_j^T x \leq 1, \ \ x^T(-e_j e_j^T)x \leq -1 \tag{8.16}$$

where e_j is all zero vector except 1 at position j. Maximizing $x^T \bar{L}x$ implies minimizing θ subject to $-x^T \bar{L}x \leq \theta$

In order to minimize the last expression, we convert MAX-CUT problem into following quadratic problem

$$\begin{aligned} &\text{minimize } \theta \\ \text{subject to } & -x^T\bar{L}x - \theta \le 0 \\ & x^T e_j e_j^T x \le 1, \ \ j = 1, \dots, n \\ & x^T(-e_j e_j^T)x \le -1, \ \ j = 1, \dots, n \end{aligned} \tag{8.17}$$

A non-convex quadratic constraint problem is given by

$$\text{minimize } c^T x \text{subject to } x^T Q_p x + q_p^T x + r_p \le 0, \ \ p = 1, \cdots, m \tag{8.18}$$

where $Q_p \in S(n), \ \ c \in R^n$, $q_p \in R^n$ and $r_p \in R$. $S(n)$ is the set of real symmetric matrices. The above problem can be written as

$$\begin{aligned} & minimize \ c^T x \\ subject \ to \ & Tr(Q_p X) + q_p^T x + r_p \le 0, \ \ p = 1, \dots, m \\ & X = xx^T = rank(1) \end{aligned} \tag{8.19}$$

Tr(.) is the trace operator. This problem is NP-hard because of the last constraint. By imposing the constraint

$$X \ge xx^T \tag{8.20}$$

instead of

$$X = xx^T \tag{8.21}$$

we get a SDP relaxation.

The second relaxation using SOCP (for definition of SOCP see, e.g. [7]) proposed in [6] is as follows. First, suppose that we are given

$$C \subseteq S(n)^+, \tag{8.22}$$

where $S(n)^+$ denotes the set of $n \times n$ positive definite matrices. It is easy to see that for $Z \in S(n)$

$$Z \ge 0 \Rightarrow \forall C \in Com, Tr(CZ) \ge 0 \tag{8.23}$$

using this relaxation we relax the constraint (9.43) to

$$Tr((X - xx^T)C) \ge 0 \tag{8.24}$$

for $C \in Com$, which are convex quadratic constraints. If $C = S(n)^+$ then the right hand side of the above equation also implies the left hand side. A convex quadratic constraint cannow be easily transformed into second order cone constraints. To do this, for $C \in Com$ we first decompose $C = UU^T$, where

$$U \in R^{n \times k} \tag{8.25}$$

and $k = rank(C)$. Such a decomposition is always possible, as C is symmetric and positive semidefinite. The constraint

$$Tr(CX) \geq x^T Cx \tag{8.26}$$

is equivalent to

$$x^T UU^T x \leq Tr(CX). \tag{8.27}$$

It is known that any

$$w \in R^n, and \;\; \eta, \;\; \xi \in R, \tag{8.28}$$

$$w^T w < \eta\xi, \;\; \eta \geq 0, \;\; \xi \geq 0 \tag{8.29}$$

can be written as [6,7],

$$\left\| \begin{pmatrix} \xi - \eta \\ 2w \end{pmatrix} \right\| \leq \xi + \eta \tag{8.30}$$

Where the norm is Euclidean. If we take

$$w = U^T x, \;\; \xi = 1, \;\; \text{and} \;\; \eta = Tr(CX), \tag{8.31}$$

we can convert the inequality,

$$x^T UU^T x \leq Tr(CX), \tag{8.32}$$

into the following linear inequality with an additional second order cone condition [6],

$$\begin{pmatrix} 1 + Tr(CX) \\ 1 - Tr(CX) \\ 2U^T x \end{pmatrix} \in \mathcal{K}(k+2) \tag{8.33}$$

where

$$\mathcal{K}(r) = x \in R^r | (x_1 \geq \sqrt{\sum_{j=2}^{r} x_j^2}) \tag{8.34}$$

is the second order cone. This is the basic idea of the SOCP relation for the non-convex quadratic programming. The final form of SOCP is as follows.

$$\begin{aligned} &\text{minimize } c^T x \\ \text{subject to } & Tr(Q_p X) + q_p^T x + r_p \leq, \ \ p = 1, \dots, m \\ & \begin{pmatrix} 1 + Tr(CX) \\ 1 - Tr(CX) \\ 2U^T x \end{pmatrix} \\ & \in \mathcal{K}(rank(C) + 2), \\ & C \in Com, \ \ C = UU^T, \end{aligned} \tag{8.35}$$

The above SOCP has $O(n^2)$ variables. Now we use the technique exposed in [6] to further relax the constraints. We now describe their method. From now for the sake of simplicity we omit the subscript p and consider the linear inequality

$$Tr(QX) + q^T x + r \leq 0 \tag{8.36}$$

Let

$$Q = \sum_{j=1}^{n} \lambda_j u_j u_j^T \tag{8.37}$$

be the spectral decomposition of Q, where λ_j are the eigenvalues and u_j are the corresponding eigenvectors. Without loss of generalit,y we assume that

$$\lambda_1 \geq \dots \geq \lambda_l \geq 0 > \lambda_{l+1} \geq \dots \geq \lambda_n, \tag{8.38}$$

and put

$$Q^+ = \sum_{j=1}^{l} \lambda_j u_j u_j^T, \tag{8.39}$$

we can further write

$$x^T Q^+ x - Tr(Q^+ X) \leq 0 \tag{8.40}$$

$$x^T u_j u_j^T x - Tr(u_j u_j^T X) \leq 0, \ \ j = l+1, \dots, n. \tag{8.41}$$

Summing then up eq (8.36) and eq (8.40), we produce a new (weaker) inequality

$$x^T Q^+ x + Tr(\sum_{j=l+1}^{n} \lambda_j u_j u_j^T X) + q^T x + r \leq 0 \tag{8.42}$$

If (x, X) satisfies eq (8.36) and eq (8.40), then it also satisfies eq (8.42), but the converse is not generally true. Putting

$$z_j = Tr(u_j u_j^T X), \tag{8.43}$$

we obtain the following convex quadratic constraints

$$x^T Q^+ x + \sum_{j=l+1}^{n} \lambda_j z_j + q^T x + r \leq 0 \tag{8.44}$$

$$x^T u_j u_j^T x - z_j \leq 0, \;\; j = l+1, \ldots, n \tag{8.45}$$

where inequality (8.45) are the relaxed version of $n - l$ equalities.

The advantage of this method is that we can reduce the number of variables from $O(n^2)$ to the total number of the $n - l$ smallest eigenvalues of $Q_p s$. On the other hand, the inequality eq (8.44) and eq (8.45) are weaker than the original constrainst, i.e., eq (8.36), eq (8.40) and eq (8.41). In fact, if we do not impose an upper bound on z_j, than any x can satisfy eq (8.44) and eq (8.45) with large $z_j s$ (note that $\lambda_j < 0 \; \forall j > l$). Therefore, we require some restriction on z_j in advance. Using the technique described above, we can convert the MAX-CUT problem, eq (8.17), into SOCP for MAX-CUT as

$$\begin{aligned} &\text{minimize } \theta \\ \text{subject to} &\frac{-x^T L^+ x}{4} + \sum_{j=l+1}^{n} \lambda_j z_j - \theta \leq 0 \\ &x^T q_j q_j^T x - z_j \leq 0, \;\; j = l+1, \ldots, n \\ &x^T e_j e_j^T x \leq 1, \;\; j = 1, \ldots, n \\ &\sum_{j=l+1}^{n} z_j \leq n^2 \end{aligned} \tag{8.46}$$

where

$$L^+ = \sum_{j=1}^{l} \lambda_j q_j q_j^T, \tag{8.47}$$

$$L = \sum_{j=1}^{n} \lambda_j q_j q_j^T, \tag{8.48}$$

with

$$\lambda_1 \geq, \ldots \geq \lambda_l \geq 0 > \lambda_{l+1} \geq, \ldots, \geq \lambda_n. \tag{8.49}$$

It is straight forward to derive the last constraint in (8.46) bound from expression (8.43)

$$z_j = Tr(q_j q_j^T X). \tag{8.50}$$

The above SOCP MAX-CUT can be solved very efficiently by primal-dual interior point method [56].

8.6 Conclusions

In this chapter, we proposed a new ML detection method for the MIMO channel. This method is based on relaxing the MAX-CUT problem and solving the resulting system by SOCP. We have shown that the MAX-CUT (equivalently ML detection) problem can be formulated into SOCP problem. The advantage of the proposed method over the SDP relaxation is that the difference in the number of variables between SDP relaxation and SOCP increases as n becomes large. Hence MAX-CUT SOCP is numerically more efficient. Although, in general, the lower bound computed by SOCP relaxation is not as good as a bound obtained with SDP relaxation in theory, they are often close to each other [6]. The proposed method has three significant advantages over the sphere decoding technique.

1) The proposed method is polynomial time irrespective of the value of the SNR as opposed to sphere decoding which has exponential complexity for low SNR values.

2) The SOCP has no heuristic parameters to set (unlike the sphere decoder in which the optimal radius has to be selected in a heuristic way).

3) The sphere decoder complexity, $O(n^3)$, is at each time instant, while there is no such problem in SOCP approach.

Moreover our approach does not require number of the receive antennas to be greater than number of the transmit antennas as the BLAST receiver does. Simulations results in figure 8.2 show that SOCP gives very close approximation to the exact ML and that there is almost no difference between the performance of the SDP and SOCP method. Simulations are performed for two transmit and two receive antennas. In general for large n the saving of CPU time gets larger for SOCP in comparison to SDP method.

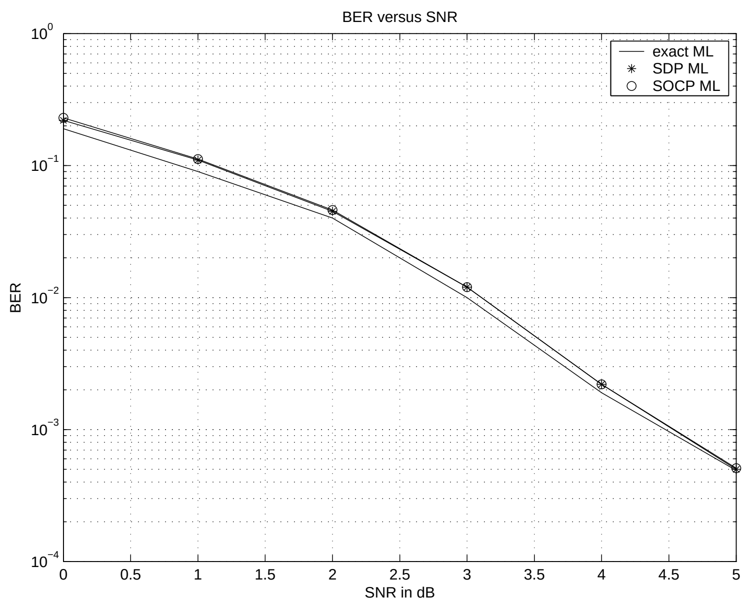

Figure 8.2: Av. BER for N=2, M=2 vs $SNR(dB)$.

Chapter 9

A polynomial time algorithm for exact maximum likelihood decoding of MIMO channels

The information capacity of wireless communication systems may be increased dramatically by employing multiple transmit and receive antennas. Often the optimal receiver in Maximum likelihood sequence detector (MLSD), which is considered to be practically infeasable due to high computational complexity. Therefore, in practice one often settles with less complex suboptimal receiver structure. In this chapter, we propose a polynomial time algorithm for exact maximum likelihood (ML) decoding for MIMO channels. The problem is posed as maximizing a quadratic form in N binary variables (BPSK case) with the vertices of a hypercube as constraint. We consider M receive antennas and N transmit antennas. We assume that $M < N$, and that M is fixed. The maximization of ML cost function with vertices of hypercube, i.e., $\{-1, 1\}^N$, as constraints, translates to having a symmetric matrix in quadratic form with fixed rank, M and with the hypercube constraint. With singular value decomposition (SVD)[8] of the symmetric matrix and suitable affine transformation of the hypercube constraint one ends up with maximizing a quadratic function (Euclidean distance) over extreme points (vertices) of zonotope (definition of zonotope will be given later). Using a classical theorem of discrete geometry, it is shown that the vertices search can be done in polynomial time $O(N^{M-1})$. The overall complexity of the algo-

rithm is the complexity to find extreme point of zonotope plus the complexity of the SVD operation plus the evaluation of the objective function at the vertices. To find the vertices of zonotopes, an efficient algorithm called reverse search algorithm can be employed [102,106,107]. Our approach has potential benefits over currently popular sphere decoding scheme [3]. The average case complexity of sphere decoding scheme is $O(N^3)$ *plus the complexity to perform QR decomposition* $(\frac{2}{3}N^3)$ *of the channel, when radius, r, is correctly chosen (which is NP-hard problem). Also at low SNRs the complexity of the sphere decoder explodes. The other problem with sphere decoding is that some form of the heuristics is used to choose the radius of hypersphere. On the contrary, the proposed method has polynomial complexity independent of SNR and also no heuristic is used in the algorithm.*

9.1 Introduction

Multiple antenna wireless communication systems are capable of providing data rate at potentially very high rates. To secure high reliability of the data transmission special attention has to be payed to the design of the receiver. In many communications systems the optimal receiver structure is maximum likelihood sequence detector (MLSD). However, computational complexity of the traditional MLSD often prohibits its practical implementation. Thus often one settles with suboptimal receivers like, MIMO-DFE, BLAST, are some of them. Recently, sphere decoding has gained quite popularity due to its average polynomial complexity (at high SNR) in the number of variables (antennas). The sphere decoder has average complexity of $O(N^3)$, when the radius of the hypersphere is optimally chosen (which is NP-hard). But it has exponential complexity for low SNRs. The other problem with the sphere decoder is that at each time instant it has average complexity, $O(N^3)$, which can be computationally very complex for large tramsmitted data blocks. In this chapter, we focus on MLSD. It is assumed that the receiver has perfect knowledge of the channel. We propose a novel exact method for data detection using some beautiful results in discrete geometry. The detection is based on maximizing ML function subject to vertices of zonotope (special polytope) constraint. We will show that optimal solution to the problem is polynomially bounded in N. The chapter is organized as follows. Signal model is described in section 9.2. In section 9.3 discrete geometric approach for ML is explained. Conclusions are drawn in the last section.

9.2 Signal Model

We assume a discrete time block fading multiple antenna channel model with N transmit and M receive antennas, we have assumed that the receiver has perfect channel knowledge. The received signal at an instant is

$$x = Hs + n \tag{9.1}$$

where

$$H \in R^{M \times N}, \tag{9.2}$$

is a known channel matrix,

$$n \in R^{M \times 1} \tag{9.3}$$

is the i.i.d. zero mean white Gaussian noise with variance σ^2. The fading coefficients are i.i.d. Gaussian with zero mean and unit variance. Under the aforementioned assumptions the ML criterion requires us to find $s \in R^{N \times 1}$, where $s_i \in \{-1, 1\}$, which minimizes $\|x - Hs\|^2$. The problem can be written (after neglecting constant terms) as

$$f(s) = max_{s \in \{-1,1\}^N} \;\; s^T J s + 2c^T s, \tag{9.4}$$

where

$$J = -H^T H \;\; and \;\; c = Hs. \tag{9.5}$$

The above equation, i.e., $f(s)$, is equivalent to

$$f(s) = max_{y \in \{-1,1\}^N, \; y_{N+1}=1} \;\; y^T \bar{L} y, \tag{9.6}$$

where $\bar{L}$ is

$$\bar{L} = \begin{pmatrix} J & c \\ c^T & 0 \end{pmatrix} \tag{9.7}$$

Since this cost function is symmetric, $y_{N+1} = 1$ need not been maintained explicitely.

9.3 Discrete geometric approach to MLSD

A basic problem in discrete optimization consists in optimizing a quadratic over some hypercube. This type of problem is NP-hard, and it is still considered a computational challenge to solve general modest size problems of this type to optimality. Quadratic programming (QP) over vertices of cube appears in various equivalent formulations in the literature. Our problem (MLSD) is maximization of a quadratic function over vertices of hypercube. Before delving into the solution of this problem we define some geometrical objects.

Polytope:

A polytope (convex polytope) is a convex hull of finite set of points in R^d (which are always bounded) or as bounded intersection of finite set of half spaces. Polytope can also be defined as a finite region of d-dimensional space enclosed by a finite number of hyperplanes.

Zonotope [103, pp. 198-199]:

Zonotopes are special polytopes that can be viewed in various ways : for example, as projections of cubes, as Minkowski sums of line segments, and set of bounded linear combinations of vector configurations. Each of these description gives different insight into the combinatorics of zonotopes. A zonotope is the image of a cube under an affine projection, $Z \subseteq R^d$ of the form [103],

$$Z = Z(V) = VC_p + z = \{Vy + z : y \in C_p\} \tag{9.8}$$

$$Z = Z(V) = \{x \in R^d : x = z + \sum_{i=1}^{p} x_i v_i, \quad -1 \leq x_i \leq 1\} \tag{9.9}$$

for some matrix (vector configuration),

$$V = [v_1, \ldots, v_p] \in R^{d \times p}. \tag{9.10}$$

Equivalently, since every d-cube C_d, is a product of line segments

$$C_d = C_1 \times \ldots \times C_1, \tag{9.11}$$

we get that every zonotope is the Minkowski sum of a set of line segments. Infact, if A_{map} is linear we get

$$\begin{aligned} Z(V) &= A_{map}(C_1 \times \ \ldots \times C_1) \\ &= A_{map}(C_1) + \cdots + A_{map}(C_1) \\ &= [-v_1, v_1] + \ldots [-v_p, v_p] \end{aligned} \tag{9.12}$$

and thus

$$Z(V) = [-v_1, v_1] + \cdots + [-v_p, v_p] + z, \tag{9.13}$$

for an affine map given by

$$A_{map}(y) = Vy + z. \tag{9.14}$$

Having defined polytope/zonotope, we can proceed with our problem. First, we begin by spectral factorization of

$$\bar{L} = UU^T, \tag{9.15}$$

where $U \in R^{M \times N}$ is the matrix composed of suitably scaled eigenvectors. We can write eq (9.6) as

$$\begin{aligned} \Psi(y) = y^T \bar{L} y = \|U^T y\|^2 \\ subject \ \ to \ \ y \in \{-1, 1\}^{N+1} \end{aligned} \tag{9.16}$$

Consider affine map [103 pp. 199],

$$R^N \longrightarrow R^M : \ \ z = U^T y. \tag{9.17}$$

This linear transformation maps a $\{-1, 1\}^{N+1}$ (hypercube) into a symmetric zonotope. For every extreme point $\bar{z}$ of zonotope there exists an extreme point $\bar{y} \in \{-1, 1\}^{N+1}$ such that $\bar{z} = U^T \bar{y}$ and thus eq (9.6) can be written in the following form

$$\Psi(y) = max_{z \in Z_{extreme}} \|z\|^2 \tag{9.18}$$

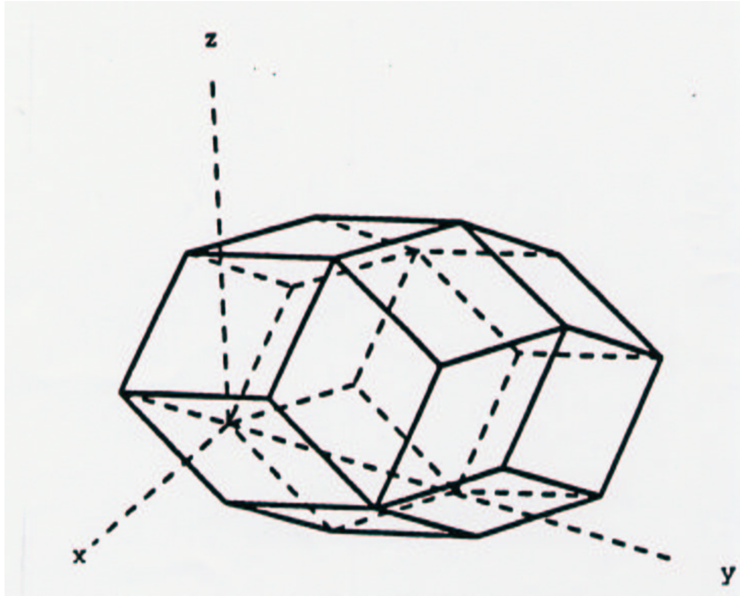

Figure 9.1: 3 dimentional zonotope with 5 generators.

From the above equation it is clear that our objective function and constraint both are symmetric and some of the extreme points of hypercube will correspond to some points lying inside or on the facets of zonotope. Observe that extreme points which lie inside or on the facets of the zonotope cannot be candidate for the maximization of our objective function. Therefore the maximum is attained at some vertex $\bar{z}$ of Z. Thus MLSD is thus reduced to the enumeration of vertices of zonotope Z. Now the problem is to calculate the number of facets and vertices of a M dimensional zonotope given by $N+1$ generators. Let $f_0(Z)$ and $f_{M-1}(Z)$ denote the number of vertices (extreme points) and facets of Z, repectively. The answer to the above question is given by the following classical theorem in discrete geometry:

Theorem:

Let Z be M dimensional zonotope given by $N+1$ generators $(N > M)$. Then

$$f_0(Z) \leq 2\sum_{i=0}^{M-1} \begin{pmatrix} N \\ i \end{pmatrix} , f_{M-1}(Z) \leq 2 \begin{pmatrix} N+1 \\ M-1 \end{pmatrix} \tag{9.19}$$

where

$$\begin{pmatrix} m \\ n \end{pmatrix} = \frac{m!}{(m-n)!n!} \tag{9.20}$$

Furthermore, the equalities are attained by certain zonotopes and therefore the bounds are best possible. From the above theorem it is clear that the upper bound of f_{M-1} is $O((N+1)^{M-1}) \approx$

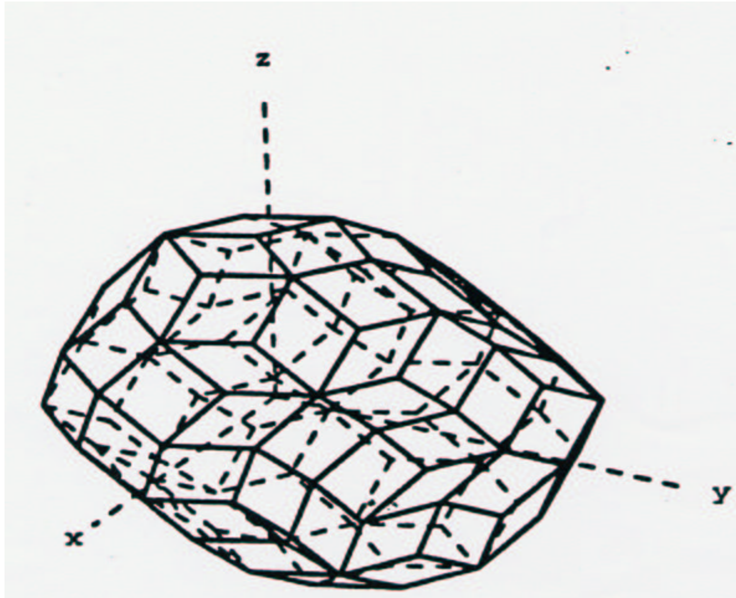

Figure 9.2: 3 dimensional zonotope with 10 generators.

$O(N^{M-1})$, for large N. The bound on the number of vertices is the expression

$$f_0(Z) \leq 2 \sum_{i=0}^{M-1} \binom{N}{i} \tag{9.21}$$

and the dominating one is the last term

$$\binom{N}{M-1} \tag{9.22}$$

which is of $O(N^{M-1})$ for large N. Thus the number of vertices are polynomially bounded and there exists efficient algorithm to generate extreme points of Z as given by the following theorem.

Theorem[105] :

Given $N+1$ generators of a zonotope there is $O((N+1)^M) \approx O(N^M)$, for large N, time algorithm to generate extreme points of zonotope for $M \geq 2$.

Uptil now we know the bound on the vertices but we do not know an algorithm to generate them. In order to explain it, we need a relationship between arrangements of hyperplanes and zonotopes.

Arrangements of hyperplanes [104, pp.4]:

A finite set of hyperplane in E^d defines a dissection of E^d into connected pieces of various dimensions. We call this dissection the arrangement $A(H)$ of H. For example, a finite set of lines in two dimensions dissects the plane into connected pieces of dimensions two, one and zero. It has been shown [104, pp. 20-26] that a zonotope in E^d corresponds to an arrangement in E^{d-1}. For example a two dimensional zonotope has corresponding one dimensional arrangement. In

[104,105] an algorithm is given to construct arrangements. The overall structure of the algorithm is given below.

The overall structure:

The construction of an arrangement proceeds incrementally, that is, the arrangement is built by adding hyperplane one at a time to the already existing arrangement. The order in which the hyperplanes are added is irrelevant. Let H denote the set of hyperplanes $H = [h_1, \ldots, h_n]$ in E^d and define

$$H_i = \{h_1, \ldots, h_i\} \tag{9.23}$$

for $1 \leq i \leq n$. $D(H)$ denotes the data structure to be described that represents the arrangement $A(H)$. It is assumed that the normal-vector of hyperplanes in H span E^d. Let the normal vectors of $h_1, \ldots, h_d$ span E^d. Construct $D(H_d)$.

For i running from $d+1$ to n, construct $D(H_i)$ from $D(H_{i-1})$ by insertion of h_i. Finally, $D(H) = D(H_n)$. Unfortunately, this algorithm may not be very practical because it has to store in memory the list of all vertices and faces generated before. This means that only the storage of vertices is of size $O(N^{M-1})$. In order to alleviate the complexity there exits an efficient algorithm, known as reverse search algorithm [102,106,107], for generating full dimensional regions. The advantage of this algorithm is that it can be highly parallelized and is also space and time efficient. In order to explain the basic idea of reverse search, let G be a connected graph and suppose we have some objective function to be maximized over these vertices. A local search algorithm on G is a deterministic procedure to move from any vertex to some neighboring vertex which is larger with respect to objective function until there exists no better neighboring vertex. A vertex without a better neighboring vertex is called local optimal. The algorithm is finite for any starting vertex, it terminates in finite number of steps. Simplex algorithm is an example of local search algorithms.

Let us imagine the simple case that we have finite search algorithm and there is only one local optimum vertex x^* (which is also optimal solution). Consider the directed graph T with same vertex set as G and the edges which are all ordered pair $(x, x^{'})$ of consecutive vertices x and $x^{'}$ generated by local search algorithm. It should be clear that T is a tree spanning all vertices with the only sink x^*. Thus if we trace this graph T from x^* systematically, say by depth first search, we can enumerate all vertices. The major operation here is tracing each edge against its orientation which corresponds to reversing the local search, while the minor work of backtracking is simply performing the search algorithm itself. We do not have to store any information about visited vertices for this search because T is itself a tree. Observe that for each vertex x, every vertex y below x in T (those y such that there is directed path from y to x) has no larger objective value and whenever it detects a vertex with lower objective value, then abandon going lower in the tree.

The advantage of the reverse search algorithm are:

1) Time complexity is proportional to the size of the output times a polynomial in the size of input

2) Space complexity is polynomial in the size of input,

3) Parallel implementation is straight forward. We believe that we can exploit the symmetry of our problem to further reduce the complexity, i.e., time and memory, of the reverse search algorithm. Having found the vertices with the help of reverse search algorithm, the task remaining is to calculate the value of the objective function at those vertices. We need to evaluate the objective function only on half the number of vertices (thanks to symmetry of our constraint) resulting in overall complexity of $O(N^M) + 6N^3$ for the proposed method. $6N^3$ is the complexity to calculate SVD of a symmetric $N \times N$ matrix. Having found the vertex of zonotope that maximizes our objective function, the corresponding vertex of hypercube can be found by, $z = U^T y$.

9.4 Conclusions

In this chapter, we have shown theoretically a polynomial time algorithm to decode exactly a MIMO system when the number of receiving antennas is fixed. By posing the problem as maximization of quadratic form over zonotope and using some classical theorems of discrete geometry, we were able to solve the problem in polynomial time. Comparing our method with the sphere decoding, we have the following three advantages over the later:

1) The sphere decoder has exponential complexity at low SNRs while there is no such problem in our method (it is independent of SNR).

2) The sphere decoder has polynomial complexity (at high SNRs) at each time instant (because the received signal point moves from one point to another in lattice at each time instant) for flat fading or for block fading channels, while no such problem exists in our method.

3) No heuristic is employed in our algorithm, where as in the sphere decoder radius of the sphere is chosen heuristically and choosing the optimum radius of the sphere is NP-hard.

The disadvantage of our algorithm is that the degree of the polynomial increases as the number of receive antennas, M, increases. Although, we have assumed perfect channel knowledge but the same analysis applies for noisy channel estimates too.

Chapter 10

Blind iterative receiver for multiuser MIMO systems

The information capacity of wireless communication systems may be increased dramatically by employing multiple transmit and receive antennas. In this chapter, we consider multiuser wireless communication system, employing multiple transmit and receive antennas. We estimate jointly channel and symbols user-wise by Maximum Likelihood approach (ML) approach. Two models are considered for the symbols of the interferers, corresponding to Gaussian and discrete priors. In the latter case, in which the finite alphabet gets exploited for the MAI symbols, a simplification for the posterior MAI symbol probabilities is introduced based on Mean Field Theory.

10.1 Introduction

Multiple Input Multiple Output (MIMO) system has gained much interest recently [44]. Deploying multiple antennas at both, the base station and the remote stations increase capacity of the wireless channels. The gain in capacity is because of diversity, spatial multiplexing, interference rejection and array gain. In order to fully exploit the advantages of an antenna array, one must know the channel that will distort the signal as well as well as interfering noise. In [45] joint channel estimation and decoding for linear MIMO systems has been carried out assuming short training sequance for channel estimation. In this chapter, we consider the problem of estimation of channel-symbols user-wise (i.e., considering other users as interferers) blindly (without training sequence). We use two approaches for the parameter estimation. In the Gaussian prior case [94], only the Multiple Access Interference (MAI) are modeled as stationary (white) sequences. We use ML formulation that gets implemented via Expectation Maximization (EM) algorithm to alternate between channel and the User of Interest (UoI) symbols estimates. Alternatively, we consider exploiting the finite alphabet for the MAI symbols, leading to significant MAI reduction capability. To simplify and to reduce the complexity of the resulting EM algorithm in the second approach, we consider the introduction of Mean Field methods for the approximation of the posterior MAI symbol probabilities. The chapter is organized as follows: In section 10.2, we define the communication model. Section 10.3 describes user-wise channel-symbol estimation with Gaussian prior on MAI. In section 10.4, we describe user-wise channel-symbol estimation procedure using discrete prior on MAI symbols. Conclusions are drawn in section 10.5.

10.2 Communication model

We model a wireless communication system with K users. Each user is equipped with N transmit antennas. The base station has M receive antennas. We assume flat fading between each transmit-receive pair. We denote $\alpha_{m,n}$ as complex fading gain from the n^{th} transmitter antenna to the m^{th} receive antenna, where $\alpha_{m,n} \sim N_c(0,1)$ is assumed to be zero mean circularly symmetric complex Gaussian random variable with unit variance. This is equivalent to the assumption that signals transmitted from different antennas undergo independent Rayleigh fades. It is also assumed that the fading gains remain constant over the entire signal frame, but they may vary from one frame to another. The received discrete time signal at instant t can be written as

$$x_t = Hd_t + n_t, \tag{10.1}$$

where $d_t = [d_{1t}^T d_{2t}^T \cdots d_{Kt}^T]^T$, is the symbol vector. $x_t = [x_{1t} x_{2t} \cdots x_{Mt}]^T$, is the received signal, $n_t = [n_{1t} n_{2t} \cdots n_{Mt}]^T$ is a Gaussian noise vector. $d_{it} = [d_{it}^1 d_{it}^2 \cdots d_{it}^N]^T$ is a vector consisting of symbols transmitted from N transmit antennas at an instant t. $d_{it}^{j} \in \{-1, 1\}$. $(.)^T$ is the transpose operator. Channel matrix H is given by

$$H = [H_1 H_2 \cdots H_K], \tag{10.2}$$

where H_i is as follows

$$H_i = \begin{pmatrix} \alpha_{1,1}^i & \alpha_{2,1}^i & \cdots & \alpha_{N,1}^i \\ \alpha_{1,2}^i & \alpha_{2,2}^i & \cdots & \alpha_{N,2}^i \\ \vdots & \vdots & \ddots & \\ \alpha_{1,M}^i & \alpha_{2,M}^i & \cdots & \alpha_{N,M}^i \end{pmatrix} \tag{10.3}$$

10.3 User-wise channel-symbols estimation with Gaussian MAI prior

The received signal is given by the equation (10.1). Each user channel is modeled as Gaussian vector which might be correlated in space, i.e., between antennas, but are assumed independent between users. The channel vector for user i can be written as $h_j^i \in N(0, R_{h_j h_j})$. In the first approach we assume that the interfering symbols as Gaussian i.i.d. random variables with known variance σ_r^2. Given T snapshot, i.e., $\{x_t\}_1^T$, we are now ready to define the complete data set. The complete data set is chosen as $\{x, d_r, H_r\}$, where d_r is the group of the interfering users' information bits transmitted at all time instants and H_r is their channel matrix and x is composed of the received vector from time instant 1 to time instant T . Without loss of generality, we will detect user 1 first. The pdf of the complete data set is given by

$$f(x, d_r, H_r; H_1, d_1) = f(x|H, d) f(d_r, ; H_1, d_1) f(H_r; H_1, d_1), \tag{10.4}$$

d_1 vector is composed of user 1 transmitted data at all time instants, $f(x|H, d)$, $f(d_r; H_1, d_1)$ and $f(H_r; H_1, d_1)$ are given by

$$f(x|H, d) = K_1 exp(\frac{-1}{\sigma^2}(x - Hd)^H (x - Hd)), \tag{10.5}$$

where K_1 is constant not depending on parameters to be estimated, $(.)^H$ is the Hermitian transpose and

$$f(d_r; H_1, d_1) = K_2 exp(\frac{-1}{2\sigma_r^2} d_r^T d_r), \tag{10.6}$$

where K_2 is another constant. In the above equation we have assumed without loss of generality that the prior mean for the interfering users' symbols is zero and the variance σ_r^2 of the symbols is known.

Having the above equations we are now ready to evaluate the E-step of the algorithm. Since we are conditioning on the received data, we take expectations with respect to d_r (interfering users' symbols) and their channel, H_r.

$$Q(H_1, d_1; H_1^{(k)} d_1^{(k)}) = E\{\log f(x, d_r, H_r; H_1, d_1 | x; H_1^{(k)}, d_1^{(k)})\}, \quad (10.7)$$

where $(.)^{(k)}$ is the iteration index and E is the expectation operator.

Evaluating the expectations and dropping the terms that do not depend on the parameters the above equation can be written as

$$Q(H_1, d_1; H_1^{(k)}, d_1^{(k)}) = \sum_{i=1}^{T} E\{(x_i - H_1 d_{1i} - H_r d_{ri})^H (x_i - H_1 d_{1i} - H_r d_{ri}) | x; H_1^{(k)}, d_1^{(k)}\}, \quad (10.8)$$

d_{ri} are the symbols transmitted by interfering users (with N transmit antennas each) at time instant i, x_i is the received signal at instant i, d_{1i} is the transmitted data vector of user 1 at instant i, H_1 is the channel matrix for user 1 and H_r is the channel matrix for the interfering users. $H = [H_1 | H_r]$. The above equation can be further written as

$$Q(H_1, d_1; H_1^{(k)}, d_1^{(k)}) = \sum_{i=1}^{T} E\{(x_i - D_{1i} h_1 - D_{ri} h_r)^H (x_i - D_{1i} h_1 - D_{ri} h_r) | x; H_1^{(k)} d_1^{(k)}\}, \quad (10.9)$$

where $D_{1i} = d_{1i} \otimes I$ and $h_1 = vec(H_1)$ and I is identity matrix. Similarly, we can define $D_{ri} = d_{ri} \otimes I$, and $h_r = vec(H_r)$. We have used the property that $vec(ABC) = (C^T \otimes A) vec(B)$, $\otimes$ is Kronecker product. The symbols d_{1i} are obtained by maximizing eq (10.9) over BPSK. Differentiating the E-step equation with respect to h_1 yields

$$h_1 = \sum_{i=1}^{T} (D_{1i}^T D_{1i})^{-1} \sum_{i=1}^{T} (D_{1i}^T x_i - D_{1i}^T . \hat{H}_r \hat{d}_r) \quad (10.10)$$

where

$$\hat{d_{ri}} = E\{d_{ri} | x; H, d_1^{(k)}\}, \quad (10.11)$$

in addition to $\hat{d_{ri}}$ we also need second order moment of d_{ri}. and $\hat{H}_r$ is

$$\hat{H}_r = E\{H_r | x; H_1^{(k)}, d_1^{(k)}\}. \quad (10.12)$$

Now the problem is to derive the expressions for $\hat{d_{ri}}$ and $\hat{H}_r$, i.e., the conditional mean of the interfering users bit and the conditional mean of the interfering users channel. In order to accomplish this, we first write the pdf for the observed data

$$f(x; H, d_1) = K_3 exp(-x^H R_{xx}^{-1} x), \quad (10.13)$$

where K_3 is another constant and R_{xx} is given by

$$R_{xx} = H_1 d_1 d_1^H H_1^H + H_r d_r d_r^H H_r^H + \sigma^2 I, \quad (10.14)$$

where d_1 and d_r are the data vector composed of transmitted symbols at all time instants of user 1 and the rest of the users respectively. In deriving the above equation, we used the fact that $E\{d_r\} = 0$. From now we will omit the EM iteration index, i.e.,k. The conditional pdf of d_{ri} as a function of known pdfs is follows (using the fact that transmitted symbols at instant i results in received vector at the same instant),

$$f(d_{ri}|x; H, d_1) = f(x_i|H, d_i)f(d_{ri})/f(x_i; H, d_{1i}), \tag{10.15}$$

where d_i is the vector of symbols of all the users at instant i, x_i is the received vector at instant i, d_{ri} are the interfering users data vector transmitted at instant i, and H is the channel matrix. Substituting the corresponding expressions and rearranging gives

$$f(d_{ri}|x; H, d_1) = \tfrac{K_1 K_2}{K_3} exp(-\tfrac{1}{\sigma^2}(x_i - Hd_i)^H(x_i - Hd_i) - \tfrac{1}{2\sigma_r^2} d_{ri}^T d_{ri} + x^H R_{xx}^{-1} x). \tag{10.16}$$

Since the conditional pdf of $\hat{d_r}$ will be Gaussian, it is easy to show that

$$\hat{d_{ri}} = \frac{R_{dd}}{\sigma^2}(H_r^H x_i - H_r^H H_1 d_{1i}), \tag{10.17}$$

where

$$R_{dd}^{-1} = \frac{1}{\sigma^2} H_r^H H_r + \frac{I}{2\sigma_r^2}, \tag{10.18}$$

where I is identity matrix. Similarly the expression for $\hat{H_r}$ is as follows

$$\hat{h_r} = vec(\hat{H}_r) = R_{hh} \sum_{i=1}^{T} \frac{1}{\sigma^2}(D_{ri}^T x_i - D_{ri}^T H_1 d_{1i}), \tag{10.19}$$

where R_{hh}^{-1} is given by

$$R_{hh}^{-1} = \frac{1}{\sigma^2} \sum_{i=1}^{T} D_{ri}^T D_{ri} + R_{h_r h_r}^{-1}, \tag{10.20}$$

and $D_{ri} = d_{ri} \otimes I$.

The algorithm detects user-wise channel- symbols. First, user 1 channel-symbols are estimated from the above procedure. Then the contribution of that user is subtracted from the received signal to get more clean signal. Then the user second is detected. The same procedure is repeated for the other users. After convergence of the EM algorithm, the solution of d_{1i} from equation (10.9) is projected on finite alphabet to get the symbols estimate. The same process is done for the other users too. The overall algorithm works as follows: first we initialize H_1 and d_1, 2) We evaluate $\hat{d_{ri}}$ from equation (10.17) and $\hat{h_r}$ from equation (10.19). These values are plugged into equation (10.9) and equation (10.10) to get the channel-symbol update. These steps are repeated until convergence.

10.4 User-wise symbol estimation using discrete MAI prior

The steps for deriving the algorithm are essentially the same except that the conditional mean of d_{ri} will be different than previously discussed, i.e., Gaussian random variable for the priors, which will result in different channel-symbols estimates. The conditional mean for d_{ri} is given by

$$\hat{d_{ri}} = E\{d_{ri}|x; H, d_1^{(k)}\} = \sum_{d_{ri}} d_{ri} f(d_{ri}|x; H, d_1^{(k)}). \tag{10.21}$$

From now for the sake of simplicity we will omit the EM iteration index, i.e., k. In order to calculate the conditional mean we have to evaluate the above expression, which is summation of all interfering users' symbols at instant i multiplied by their corresponding pdfs, which is computationally very expensive. Mean Field (MF) methods [36,37,91], provide tractable approximations for the computation of high dimensional sums and integrals in the probabilistic models. By neglecting certain dependencies between the random variables, a closed set of equations for the expected values of these variables are derived which often can be solved in a time that grows polynomially in the number of variables [91, chapter.2]. The MF approximation is obtained by taking the approximating family of probability distribution by all product distribution, i.e., $Q(d_{ri}) = \Pi_j Q_j(d_{ri}^j)$. We now choose a distribution which is close to the true distribution, i.e., $f(d_{ri}|x; H, d_1)$. The parameter of the distribution is chosen so as to minimize Kullback-Leibler (KL) distance, i.e.,

$$KL(Q \| f(d_{ri}|x; H, d_1) = \sum_{d_{ri}} Q(d_{ri}) ln \frac{Q(d_{ri})}{f(d_{ri}|x; H, d_1)}, \tag{10.22}$$

where $Q(d_{ri}) = \Pi_{j=1}^{(K-1)N} Q_j(d_{ri}^j)$ and $d_{ri}^j \in \{-1, 1\}$.

$$f(d_{ri}|x; H, d_1) = \frac{f(x_i|H, d_i)}{\sum_{d_{ri}} f(x_i|H, d_i)} = \frac{exp(-H(d_i))}{Z}, \tag{10.23}$$

where Z is independent of d_{ri}, $f(x_i|H, d_i)$ has the Gaussian distribution and the d_i is the vector of symbols of all users at instant i. After some simplification $H(d_i)$ can be written as

$$H(d_i) = \frac{1}{\sigma^2}(-x_i^H H d_i - d_i^T H^H x_i + d_i^T H^H H d_i). \tag{10.24}$$

The above equation has the form

$$H(d_i) = \sum_{j,n} d_{ri}^j J_{jn} d_{ri}^n - 2 \sum_j d_{ri}^j \theta_j + C, \tag{10.25}$$

where C is a term independent of d_{ri}, $J_{jn} = \frac{1}{\sigma^2}(H^H H)_{j,n}$, and $\theta_j = real(\frac{1}{\sigma^2}(H^H x_i)_j)$. $(.)_j$ is the j^{th} element of the vector $H^H x_i$. The KL distance between Q and $f(d_{ri}|x; H, d_1)$ can be written as

$$KL(Q \| f(d_{ri}|x; H, d_1) = lnZ + V[Q] - S[Q], \tag{10.26}$$

where

$$S[Q] = -\sum_{d_{ri}} Q(d_{ri}) lnQ(d_{ri}), \tag{10.27}$$

is the entropy and

$$V[Q] = \sum_{d_{ri}} Q(d_{ri}) H(d_i), \tag{10.28}$$

is the variational energy. The most general form of probability distribution for our problem is

$$Q_j(d_{ri}^j; m_j) = \frac{1 + d_{ri}^j m_j}{2}, \tag{10.29}$$

where m_j is the variational parameter which corresponds to the mean, i.e., $m_j = E\{d_{ri}^j\}$. The entropy can be written as

$$S[Q] = -\sum_j \frac{1+m_j}{2} \ln \frac{1+m_j}{2} + \frac{1-m_j}{2} \ln \frac{1-m_j}{2}, \tag{10.30}$$

and similarly the variational energy can be written as

$$V[Q] = \sum_{j,n} J_{jn} m_j m_n - 2 \sum_j m_j \theta_j. \tag{10.31}$$

In order to evaluate m_j we have to minimize the variational free energy, i.e.,

$$F[Q] = V[Q] - S[Q] \tag{10.32}$$

Differentiating this equation with respect to $m_j^{'}s$ gives nonlinear fixed point equations, i.e.,

$$m_j = tanh(-\sum_n J_{jn} m_n + \beta_j), j = 1, 2 \cdots (K-1)N \tag{10.33}$$

In the matrix form we can write the above equation as

$$\mathbf{m} = \mathbf{tanh}(-\mathbf{Jm} + \beta), \tag{10.34}$$

where $\beta_j = 2\theta_j$. The huge computational task (complexity grows exponentially with the number of interfering users times the transmitted symbols per user) of exact averages over $f(d_{ri}|x; H, d_1)$ has been replaced by solving the above set of $(K-1)N$ nonlinear equations, which can be done in time that grows only polynomially. As the above equation is nonlinear there may be local minima or saddle points. In order to avoid it, the solution must be compared by their value of variational free energy $F[Q]$.

10.5 Simulations and conclusions

In this chapter, we derived two receivers for user-wise joint channel-symbols estimate. In the first approach, the Gaussian prior on the interfering users' symbols is assumed and the EM algorithm is used for user-wise channel- symbols estimation. In the second proposed receiver a discrete prior is assumed on the interfering users' bits. In the later case, the complexity of computing the posteriori probabilities grows exponentially in the number of interfering users times the symbols per user. We derived low complexity method to circumvent this problem. The exact posteriori probabilities are replaced by the approximate separable distributions. The distributions are calculated by MFT (variational approach). Simulation results are shown in figure 10.1. The simulations were performed by considering one transmit and four receive antennas. The number of the users were two in the system. We used the estimated channel for our proposed receiver and it shows very close performance in the terms of the BER to the exact ML (i.e., when the channel is exactly known) approach.

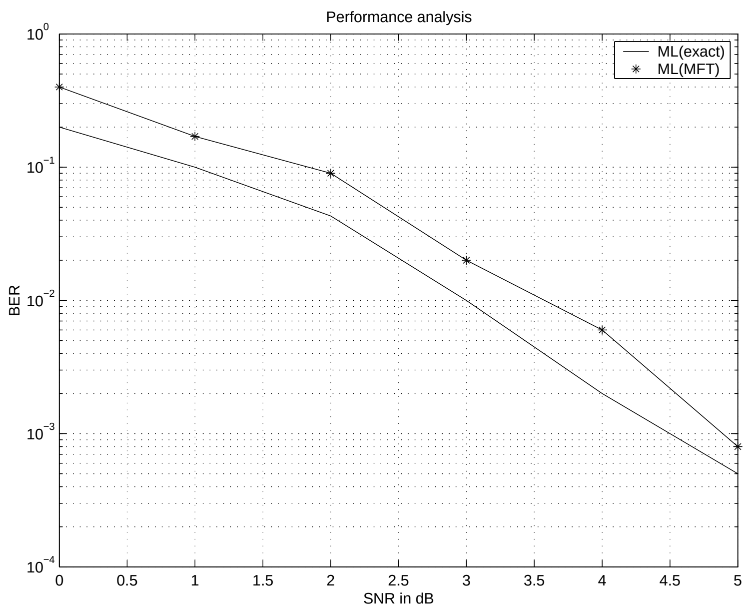

Figure 10.1: Av. BER of K=2 N=1, M=4 vs $SNR(dB)$.

Chapter 11

Blind iterative receiver for multiuser space-time coding systems

Space-time coding (STC) techniques, which combine antenna array signal processing and channel coding techniques, are very promising approaches to substantial increase in capacity in the wireless communications channels [10]. The goal of the system is to exploit this capacity in a practical way. In this work, by drawing analogies between a synchronous CDMA and an STC multiuser system, we apply multiuser detection techniques to the problem at hand. We use Gaussian mixture modeling to estimate the channel using the EM based approach, and we consider the Maximum Likelihood (ML) approach for detecting the symbols for all the users in an iterative fashion. Symbols are detected user-wise, considering the rest of the symbols as interfering symbols. Two models are considered for the symbols of the interferers, corresponding to Gaussian and discrete priors. In the latter case, in which the finite alphabet gets exploited for the MAI symbols, a simplification for the posterior MAI symbol probabilities is introduced based on Mean Field Theory [97].

11.1 Introduction

Deploying multiple antennas at both, the base station and the mobile stations, increase the capacity of wireless channels. The recently developed space-time coding (STC) techniques integrate the methods of transmitter diversity and channel coding, and provide significant capacity gains over the traditional communication systems in the fading wireless channels. Two types of Space-time coding schemes exist in the literature, i.e., Space-time block coding (STBC) and Space-time trellis coding (STTC).

Recently, iterative processing has attracted vast attention due to its successful applications in many areas of the coding and the signal processing.

In this work, we detect symbols of the each user and estimate the channel jointly. The channel gets estimated blindly via Expectation Maximization (EM) algorithm by formulating the problem as Gaussian mixture model. The estimated channel is than used to detect the symbols for each user, which is also done in iterative fashion, i.e., user-wise detection. The detection of symbols are done in two ways. In the first case of user-wise detection, the symbols of other users (interfering users) are modeled as Gaussian random variables. Alternatively, we consider exploiting the finite alphabet for the MAI symbols, leading to significant MAI reduction capability. To simplify and to reduce the complexity of the resulting EM algorithm in the second case, we consider the introduction of Mean Field methods for the approximation of the posterior MAI symbol probabilities. The chapter is organized as follows: In section 11.2, we define the signal model. Section 11.3, 11.4, describe Gaussian mixture model based estimation of the channel and the effect of the dimensionality reduction on the Gaussian mixture problem respectively. In section 11.5, we describe the detection procedure for our problem. In section 11.6 conclusions are drawn.

11.2 Signal model

We consider the Space-time block coding (STBC) system with K users. Each user is equipped with N transmit antennas. The base station has M receiving antennas. The kth user's STBC is defined by a $(P \times N)$ code matrix G_k, where P denotes the number of time slots for transmitting an STBC codeword or the temporal transmitter diversity order. A STBC encoder takes as input the code vector d_k, and transmits each row of symbols in G_k at P consecutive time slots. At each time slot, the symbols contained in an N-dimensional row vector of G_k are transmitted through N transmitter antennas simultaneously. For two antennas system the code matrix is given by

$$\mathbf{G} = \begin{pmatrix} d(1) & d(2) \\ -d^*(2) & d^*(1) \end{pmatrix} \tag{11.1}$$

where $(.)^*$ denotes the transpose and $d(i) \in \{1, -1\}$. We consider flat fading channel between each transmitter-receiver pair. The coefficient $\alpha_{i,j}$ is the path gain from transmit antenna i to the

receive antenna j at time t. The path gains $\alpha_{i,j}$ are modeled as samples of independent complex Gaussian random variables with mean zero and variance 1. This is equivalent to the assumption that signals transmitted from different antennas undergo independent Rayleigh fades. It is also assumed that the fading gain remains constant over the entire signal frame and vary from one frame to another (quasi-static fading).
The model for our problem is given by [92]

$$\underbrace{\begin{pmatrix} x^1 \\ x^2 \\ \vdots \\ x^M \end{pmatrix}}_{x=MP\times 1} = \underbrace{[H_1 H_2 \cdots H_K]}_{H=MP\times NK} \cdot \underbrace{\begin{pmatrix} d_1 \\ d_2 \\ \vdots \\ d_K \end{pmatrix}}_{NK\times 1} + \underbrace{\begin{pmatrix} n^1 \\ n^2 \\ \vdots \\ n^M \end{pmatrix}}_{MP\times 1} \tag{11.2}$$

In the above equation,

$$x^m = [x^m(1), x^m(2) \cdots x^m(P)]^*, \;\; m = 1, 2 \cdots M, \tag{11.3}$$

consist of the received signal from time slots 1 to P, at the mth receiver antenna. H_k denotes the channel response of the user k and

$$d_k = [d_k(1) d_k(2) \cdots d_k(N)]^* \tag{11.4}$$

is the code vector of the kth user, with $d_k(i) \in \{1, -1\}$; and

$$n^m = [n^m(1) n^m(2) \cdots n^m(P)]^H \tag{11.5}$$

is the additive noise vector at the mth receiver antenna.$(.)^*$ denotes transpose operator.
For single user, the Alamouti scheme, (two transmit antennas), STBC is given by

$$\begin{pmatrix} x^m(1) \\ x^m(2) \end{pmatrix} = \begin{pmatrix} d(1) & d(2) \\ -d^*(2) & d^*(1) \end{pmatrix} \begin{pmatrix} \alpha_{m,1} \\ \alpha_{m,2} \end{pmatrix} + \begin{pmatrix} n^m(1) \\ n^m(2) \end{pmatrix}$$

It can be further written as

$$\underbrace{\begin{pmatrix} x^m(1) \\ x^m(2)^H \end{pmatrix}}_{x^m} = \underbrace{\begin{pmatrix} \alpha_{m,1} & \alpha_{m,2} \\ \alpha_{m,2}^H & -\alpha_{m,1}^H \end{pmatrix}}_{H_1^m} \underbrace{\begin{pmatrix} d(1) \\ d(2) \end{pmatrix}}_{d_1} + \underbrace{\begin{pmatrix} n^m(1) \\ n^m(2)^H \end{pmatrix}}_{n^m} \tag{11.6}$$

where $(.)^H$ is the Hermitian transpose. From the above equation we can see the analogy between multiuser STBC signal model and synchronous CDMA signal model [109]. By stacking all x^m, we get the following equation for two transmit antenna system.

$$\underbrace{\begin{pmatrix} x^1 \\ x^2 \\ \vdots \\ x^M \end{pmatrix}}_{x=2M\times 1} = \underbrace{\begin{pmatrix} H_1^1 \\ H_1^2 \\ \vdots \\ H_1^M \end{pmatrix}}_{H_1=2M\times 2} \underbrace{\begin{pmatrix} d(1) \\ d(2) \end{pmatrix}}_{d_1=2\times 1} + \underbrace{\begin{pmatrix} n^1 \\ n^2 \\ \vdots \\ n^M \end{pmatrix}}_{n=2M\times 1} \tag{11.7}$$

11.3 Gaussian mixture model based channel estimation

The Gaussian mixture model was considered for the synchronous CDMA system in [109,110]. Due to analogy between the synchronous CDMA and Space time multiuser system, we can use the method used for the synchronous CDMA to the problem at hand. In ML estimation problem we have density function $P(x|\theta)$ that is governed by the set of parameters θ (e.g. P might be set of Gaussians and θ could be the means and covariances). The data is of size T, supposedly drawn from this distribution, i.e.,

$$X = [x_1, ...x_T]. \tag{11.8}$$

That is, we assume that these data vectors are independent identically distributed (i.i.d) with distribution P. Therefore the resulting density for the samples is

$$p(X|\theta) = \prod_{j=1}^{T} P(x_j|\theta) = L(\theta|X). \tag{11.9}$$

This function $L(\theta|X)$ is called the likelihood of the parameters given the data, or just the likelihood function. In the ML problem, our goal is to find θ that maximizes L. That is, we wish to find θ^* where

$$\theta^* = \arg\max_{\theta} L(\theta|X). \tag{11.10}$$

Assuming that the channel output, i.e., x can be approximated by Gaussian distributions, i.e., $P(x|\theta)$ can be modeled as MP-dimensional mixture of Gaussians. We can write

$$P(x|\theta) = \sum_{j=1}^{W} \alpha_j P(x|m_j, \Sigma_j), \tag{11.11}$$

where $W = 2^{NK}$ and

$$P(x|m_j, \Sigma_j) = \frac{1}{(2\pi)^{(MP/2)}|\Sigma_j|^{1/2}} \exp\left(-\frac{1}{2}(x - m_j)^H \Sigma_j^{-1}(x - m_j)\right), \tag{11.12}$$

$$\alpha_j \geq 0, \;\; and \;\; \sum_{j=1}^{W} \alpha_j = 1. \tag{11.13}$$

The parameter vector θ consists of mixing proportions α_j, the means vectors m_j, and the covariance matrices Σ_j. Given W and given T independent, i.i.d. samples $\{x_t\}_1^T$, we obtain the following likelihood

$$l(\theta) = \sum_{t=1}^{T} \log \sum_{j=1}^{W} \alpha_j P(x_t | m_j, \Sigma_j) \tag{11.14}$$

which is difficult to optimize because it contains logarithm of a sum. If we consider X as incomplete, since we do not know which index j, within the mixture probability density function resulted for a specific output. The complete ”data set” in this case is,

$$[x_1, \dots, x_T, i_1, \dots, i_T], \tag{11.15}$$

where i_n denotes the component of the pdf from which x_n is drawn. Using complete data set we can optimize our problem using EM algorithm.

The update for means is given by the following equation (for details see chapter 3 of this thesis).

$$m_j^{(k+1)} = \frac{\sum_{t=1}^{T} h_j^{(k)}(t) x_t}{\sum_{t=1}^{T} h_j^{(k)}(t)}, \tag{11.16}$$

where the posteriori probabilities $h_j^{(k)}(t)$ is defined as follows:

$$h_j^{(k)}(t) = \frac{\alpha_j^{(k)} P\left(x_t | m_j^{(k)}, \Sigma_j^{(k)}\right)}{\sum_{i=1}^{W} \alpha_i^{(k)} P\left(x_t | m_j^{(k)}, \Sigma_j^{(k)}\right)}. \tag{11.17}$$

The mixing proportions (α_j), and the covariance matrices in our case are of constant values and are given by 2^{-KN}, and $\sigma^2 I$ respectively. The channel gets estimated in similar fashion as is done in chapter 3, i.e., using eq. 3.23, eq. 3.24, eq. 3.25 and eq. 3.26.

11.4 Effect of dimensionality reduction on the Gaussian mixture problem

Dimensionality reduction has been subject of keen study for the past few decades [29,31,32]. In this section we will discuss the effect of dimensionality reduction on the Gaussian mixture problem.

In the following lines we will consider the effect of projection based on the principal component analysis (PCA) and random projections (to reduce the dimensionality of the problem) on the Gaussian mixture problem.

Lemma: ([108,46]). For any $c > 0$, pick a c-separated mixture of l Gaussians in R^n. Let $\delta, \epsilon \in (0,1)$ designate the confidence interval and accuracy parameters, respectively. Suppose the mixture is projected into a randomly chosen subspace of dimension

$$d \geq \frac{C_1}{\epsilon^2} ln \frac{l}{\delta}, \tag{11.18}$$

where C_1 is some universal constant. Then, with

$$probability > 1 - \delta \tag{11.19}$$

over the choice of subspace, the projected mixture in R^d will be $(c\sqrt{1-\epsilon})$-separated.
The c-separation for two Gaussians is defined by

$$||\mu_1 - \mu_2|| \geq c\sqrt{max[trace(\Sigma_1), trace(\Sigma_2)]} \tag{11.20}$$

where Σ_1 and Σ_2, are the covariance matrices of the two Gaussians respectively. A mixture of Gaussian is c-separated if its component Gaussians are pairwise c-separated. From the above lemma, it is clear that the separation between the Gaussians is reduced by the projection operation and the techniques like the EM works only well for well separated Gaussians (i.e., if they do not overlap too much).

11.4.1 Dimensionality reduction using PCA

Principal component analysis is an extremely important tool for data analysis which has found use in many experimental and theoretical studies. It defines a d-dimensional subspace of R^n which captures as much of the variation in the data set as possible.
The projection by PCA is quite easy to obtain. Let μ and Σ denote the mean and covariance of the high dimensional data S. The positive semidefinite matrix Σ can be written in the form $B^T DB$, where $D = diag(\lambda_1, \ldots, \lambda_n)$ is the diagonal matrix containing the eigenvalues of Σ, and B is the orthogonal $n \times n$ matrix. If the data points are rotated by B, the resulting data $Bx_1, \ldots, Bx_m$ has mean $B\mu$ and covariance

$$\frac{1}{m}\sum_{i=1}^{m}(Bx_i - B\mu)(Bx_i - B\mu)^T = B\Sigma B^T = D \tag{11.21}$$

Assume the eigenvalues are ordered so that $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n$. Then the direction of maximum variance of the rotated data is simply the first coordinate axis. Similarly, to select the d-dimensional subspace of maximum variance, simply pick the first d coordinate axis of the rotated space. In summary, PCA projects each point $x_i \in R^n$ to $x_i^* = P^T x_i$, where P^T is $d \times n$ projection matrix consisting of the first d rows of B. The projected data then has covariance $(\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_d)$. Now the question is that how much can PCA reduce the dimension of a mixture of k Gaussians?. It is quite easy to symmetrically arrange a group of k spherical Gaussians in $R^{k/2}$ so that a PCA projection to any smaller dimension will collapse some of the Gaussians together, and thereby decisively derail any hope of estimating the mean of the Gaussians. Thus PCA cannot in general be expected to reduce the dimension of a k Gaussians to below some critical value. Moreover, it is rather time-consuming process for high dimensional data. A much faster technique for dimensionality reduction is by random projection. Now we perform analysis of the random dimensionality reduction problem.

11.4.2 Random projection for dimensionality reduction

First we show that the Gaussians do not spread out (that is they are well separated) after random projection, that is, $\lambda_{max}(\Sigma^*) \leq \lambda_{max}(\Sigma_i)$. This is quite straightforward. Write the projection, say P^T, as a $d \times n$ matrix with orthogonal rows. P^Ttransforms Gaussian $N(\mu, \Sigma)$ in R^n to $N(P^T\mu, P^T\Sigma P)$ in R^d, and

$$\lambda_{max}(P^T\Sigma P) = max_{u \in R^d} \; \frac{u^T(P^T\Sigma P)u}{u^T u}$$

$$= max_{v \in R^n} \; \frac{(P^Tv)^T(P^T\Sigma P)(P^Tv)}{(P^Tv)^T(P^Tv)}$$

$$= max_{v \in R^n} \; \frac{(PP^Tv)^T\Sigma(PP^Tv)}{(PP^Tv)^T(PP^Tv)}$$

$$\leq max_{v \in R^n} \frac{v^T\Sigma v}{v^T v} = \lambda_{max}(\Sigma). \tag{11.22}$$

We have used the fact that $P^TP = I_d$.

Consider d identically distributed independent (i.i.d.) random variables which are components of the mean vector μ. The goal is to calculate the probability distribution of the sum of the squares of the difference of the two mean vectors X and Y, each of which is a vector with d components which are i.i.d. random variables, i.e.,

$$X = (X_1, X_2, \cdots X_d) \;\; and \;\; Y = (Y_1, Y_2, \cdots Y_d). \tag{11.23}$$

Let Z be the components of the difference of the means, i.e.,

$$Z = Y - X = (Z_1, Z_2, \cdots Z_d). \tag{11.24}$$

The probability density function of Z is follows

$$R(Z) = \int_{\infty}^{\infty} P(Z + X)U(X)dX \tag{11.25}$$

The joint probability density function,

$$J(Z_1, Z_2, \cdots Z_d), \tag{11.26}$$

is

$$J(Z_1, Z_2, \cdots Z_d) = R(Z_1)R(Z_2)\cdots R(Z_d). \tag{11.27}$$

Considering Gaussian probability density function for each component, the joint probability density function for means is:

$$J(Z_1, Z_2, \cdots Z_d) = \frac{1}{(2\pi 2\rho^2)^{\frac{d}{2}}} exp(-\sum_{i=1}^{d} \frac{Z_i^2}{4\rho^2}), \tag{11.28}$$

where we have assumed that the Gaussian is zero mean with variance ρ^2. This density function is spherically symmetric hence it can be written in the form of

$$\int\int\cdots\int f(r)dX^d = \int_0^{\infty} f(r)S_d r^{d-1}dr, \tag{11.29}$$

where S_d denotes the surface area of d-dimensional sphere. Let

$$z_i = \frac{Z_i}{\sqrt{2}\rho}. \tag{11.30}$$

The criterion for the separation of two Gaussian is that the distance between two means is greater than twice the standard deviation. We are interested to find the probability that

$$(\sum_{i=1}^{d} Z_i^2)^{\frac{1}{2}} < 2\rho, \tag{11.31}$$

i.e.,

$$(\sum_{i=1}^{d} z_i^2)^{\frac{1}{2}} < \frac{\sqrt{2}\sigma}{\rho}. \tag{11.32}$$

The solution to this problem takes the form of the standard distribution of the sum of the squares of d normally distributed random variables, $P(\chi^2|d)$. χ^2 distribution is one of the classic distribution of statistics. We can calculate the probability that

$$(\sum_{i=1}^{d} z_i^2)^{\frac{1}{2}} < \sqrt{\chi^2}, \tag{11.33}$$

as

$$S(d) = P(\chi^2|d) = \int_0^{\sqrt{\chi^2}} (\frac{1}{2\pi})^{\frac{d}{2}} e^{\frac{-r^2}{2}} S_d r^{d-1} dr, \tag{11.34}$$

which can be written as

$$S(d) = \frac{1}{2^{\frac{d}{2}}\Gamma(\frac{d}{2})} \int_0^{\chi^2} t^{\frac{d-2}{2}} e^{\frac{-t}{2}} dt, \tag{11.35}$$

where $\Gamma(.)$ is Gamma function. In order to calculate the minimum dimension for which the probability that

$$(\sqrt{\sum_{i=1}^{d} Z_i^2 < 2\sigma}) < \epsilon,$$

we solve the inequality

$$P(\frac{2\sigma^2}{\rho^2}|q_{min}) < \epsilon,$$

for the smallest value of q_{min}. If we have M mean vectors whose components are identically distributed independent random variables, then there are $\frac{M(M-1)}{2}$ difference vectors. The probability that no pair of means will be separated by less than 2σ is then,

$$P(\frac{2\sigma^2}{\rho^2}|q_{min}) < \frac{2\epsilon}{M(M-1)}, \tag{11.36}$$

therefore from eq (11.36) we have

$$S(q) = P(\frac{2\sigma^2}{\rho^2}|q_{min}) = \frac{1}{2^{\frac{q}{2}}\Gamma(\frac{q}{2})} \int_0^{\chi^2} t^{\frac{q-2}{2}} e^{\frac{-t}{2}} dt$$

$$\approx \frac{(\sigma/\rho)^{q/2}}{2^{q/2}\Gamma(q/2+1)} \tag{11.37}$$

Plot of the above equation (see fig 11.3) shows that when the data is projected into a space of smaller dimension, $q < d$, the probability of overlap between the means increases rapidly as q decreases.

11.5 User-wise symbol estimation using Gaussian prior on MAI

The discrete time received signal is given by equation (11.2), i.e., $x = Hd + n$. This equation can be further written as

$$x = H[d_1^* d_r^*]^*. \tag{11.38}$$

Where $(.)^*$ denotes the transpose operator, d_1, is the information bits of user 1 and d_r are the information bits of the rest of the users. Without loss of the generality, we detect the symbols for user 1 first, and for user K in the last, i.e., in the ascending order of the users.

In the first approach we assume the input interfering symbols as Gaussian i.i.d. random variables with variance σ_r^2, [94]. Given the above model we are now ready to define complete data set. We choose complete data set as $y = \{x, d_r\}$. The derivation of the algorithm is as follows: The pdf of the complete data set can be written as,

$$f(x, d_r; H, d_1) = f(x|H, d)f(d_r; H, d_1), \tag{11.39}$$

where $f(x|H, d)$ and $f(d_r; H, d_1)$ is given by

$$f(x|H, d) = K_1 exp(\frac{-1}{\sigma^2}(x - Hd)^H(x - Hd)), \tag{11.40}$$

where K_1 is constant not depending on parameters to be estimated.

$$f(d_r; H, d_1) = K_2 exp(\frac{-1}{2\sigma_r^2} d_r^* d_r), \tag{11.41}$$

where K_2 is another constant. In the above equation, we have assumed without loss of generality that the prior mean for the interfering users' symbols is zero and the variance σ_r^2 of the symbols is known. Having the above equations we are now ready to evaluate the E-step of the algorithm. Since we are conditioning on the received data, we take expectations with respect to d_r (interfering users' symbols).

$$Q(d_1; d_1^{(k)}) = E\{\log f(x, d_r; H, d_1|x; d_1^{(k)})\} \tag{11.42}$$

where $(.)^k$ is the iteration index. In the above equation, we will use the estimated value of the channel which is estimated by GMM approach and has fixed value. Evaluating the expectations, and dropping the terms that do not depend on the parameters, the above equation can be written as

$$Q(d_1; d_1^{(k)}) = E\{(x - H_1 d_1 - H_r d_r)^H (x - H_1 d_1 - H_r d_r)|x; d_1^{(k)}\}, \tag{11.43}$$

where

$$H = [H_1|H_r]. \tag{11.44}$$

The above equation can be further written as

$$Q(d_1; d_1^{(k)}) = E\{(x - D_1 h_1 - D_r h_r)^H (x - D_1 h_1 - D_r h_r)|x; d_1^{(k)}\}, \tag{11.45}$$

where

$$D_1 = d_1 \otimes I, \tag{11.46}$$

and

$$h_1 = vec(H_1), \tag{11.47}$$

I is identity matrix. Similarly, we can define

$$D_r = d_r \otimes I \ \ and \ \ h_r = vec(H_r). \tag{11.48}$$

We have used the property that

$$vec(ABC) = (C^T \otimes A)vec(B). \tag{11.49}$$

The estimate od d_1 can be obtained by maximizing the eq.(11.46) over finite alphabet, which is BPSK in our case. where

$$\hat{d_r} = E\{d_r|x; H, d_1^{(k)}\}. \tag{11.50}$$

In addition to $\hat{d_r}$ we need second order moment of d_r too. As the expressions for d_1 depend on the conditional mean and the second order moment, therefore now the problem is to find their expression. In order to accomplish this, we first write the pdf for the observed data

$$f(x; H, d_1) = K_3 exp(-x^H R_{xx}^{-1} x), \tag{11.51}$$

where K_3 is another constant and R_{xx} is given by

$$R_{xx} = H_1 d_1 d_1^H H_1^H + H_r d_r d_r^H H_r^H + \sigma^2 I. \tag{11.52}$$

In deriving the above equation we used the fact that $E\{d_r\} = 0$. The conditional pdf of d_r as a function of known pdfs is follows

$$f(d_r|x; H, d_1) = f(x|H, d) f(d_r)/f(x; H, d_1) \tag{11.53}$$

Substituting the corresponding expressions and rearranging gives

$$f(d_r|x; H, d_1) = \tfrac{K_1 K_2}{K_3} exp(-\tfrac{1}{\sigma^2}(x - Hd)^H (x - Hd) - \tfrac{1}{2\sigma_r^2} d_r^* d_r + x^H R_{xx}^{-1} x). \tag{11.54}$$

Since the conditional pdf of $\hat{d_r}$ will be Gaussian, it is easy to show that

$$\hat{d_r} = \frac{R_{dd}}{\sigma^2}(H_r^H x - H_r^H H_1 d_1), \tag{11.55}$$

where

$$R_{dd}^{-1} = \frac{1}{\sigma^2} H_r^H H_r + \frac{I}{2\sigma_r^2}, \tag{11.56}$$

where I is identity matrix. The algorithm detects user-wise symbols. First, symbols of user 1 are detected from the above procedure. Then the contribution of that user is subtracted from the received signal to get more clean signal. Then the same procedure is repeated for the other users. We can also vary the detection procedure by detecting user-wise bit by bit. For the later case, the algorithm will work as follows: First of all, bit number one will be detected by maximizing the Expectation equation with respect to that bit, considering all other bits (including other bits of the user to be detected) as the Gaussian random variables. Secondly, the contribution of this detected bit will be subtracted from the received signal. Then we estimate the second information symbol assuming the rest of the symbols as Gaussian random variables. We continue in this fashion until the last symbol of the user to be detected is estimated. By doing so the bit detection will improve because at each step Inter-symbol Interference (ISI) caused by the detected bit is subtracted from the received signal. The improvement will result provided that the bits are correctly detected and this also will improve detection for the rest of the symbols because at each step more clean signal will be processed. Solution of d_1 from equation (11.43) is projected on finite alphabet to get the symbols estimate .

11.5.1 User-wise symbol estimation using discrete MAI prior

The steps for deriving the algorithm are essentially the same except that the conditional mean of d_r will be different than previously discussed, i.e., the Gaussian random variable for the priors, which will result in different symbol estimates. The conditional mean for d_r is given by

$$\hat{d}_r = E\{d_r|x; H, d_1^{(k)}\} = \sum_{d_r} d_r f(d_r|x; H, d_1^{(k)}) \tag{11.57}$$

From now, for the sake of simplicity, we will omit the EM iteration index, i.e., k. In order to calculate the conditional mean we have to evaluate the above expression, which is summation of all interfering users' symbols multiplied by their corresponding pdfs, which is computationally very expensive. Mean Field (MF) methods [33,38], provide tractable approximations for the computation of high dimensional sums and integrals in the probabilistic models. By neglecting certain dependencies between the random variables, a closed set of equations for the expected values of these variables are derived which often can be solved in a time that grows polynomially in the number of variables [91, chapter.2]. The MF approximation is obtained by taking the approximating family of probability distribution by all product distribution, i.e.,

$$Q(d_r) = \Pi_j Q_j(d_{rj}). \tag{11.58}$$

We now choose a distribution which is close to the true distribution, i.e., $f(d_r|x; H, d_1)$. The parameter of the distribution is chosen so as to minimize Kullback-Leibler (KL) distance, i.e.,

$$KL(Q\|f(d_r|x; H, d_1) = \sum_{d_r} Q(d_r) ln \frac{Q(d_r)}{f(d_r|x; H, d_1)}, \tag{11.59}$$

where $Q(d_r) = \Pi_{j=1}^{(K-1)N} Q_j(d_{rj})$ and $d_{rj} \in \{-1, 1\}$.

$$f(d_r|x; H, d_1) = \frac{f(x|H, d)}{\sum_{d_r} f(x|H, d)} = \frac{exp(-H(d))}{Z} \tag{11.60}$$

where Z is independent of d_r and $f(x|H, d)$ has Gaussian distribution. After some simplification $H(d)$ can be written as

$$H(d) = \frac{1}{\sigma^2}(-x^H H d - d^H H^H x + d^H H^H H d). \tag{11.61}$$

The above equation has the form

$$H(d) = \sum_{i,j} d_{ri} J_{ij} d_{rj} - 2 \sum_{i} d_{ri} \theta_i + C, \tag{11.62}$$

where C is a term independent of d_r,

$$J_{ij} = \frac{1}{\sigma^2}(H^H H)_{ij}, \tag{11.63}$$

and

$$\theta_i = real(\frac{1}{\sigma^2}(H^H x)_i), \tag{11.64}$$

is the i^{th} element of the vector $H^H x$. The KL distance between Q and $f(d_r|x; H, d_1)$ can be written as

$$KL(Q \| f(d_r|x; H, d_1) = \ln Z + V[Q] - S[Q], \tag{11.65}$$

where

$$S[Q] = -\sum_{d_r} Q(d_r) \ln Q(d_r), \tag{11.66}$$

is the entropy and

$$V[Q] = \sum_{d_r} Q(d_r) H(d), \tag{11.67}$$

is the variational energy. The most general form of probability distribution for our problem is

$$Q_j(d_{rj}; m_j) = \frac{1 + d_{rj} m_j}{2}, \tag{11.68}$$

where m_j is the variational parameter which corresponds to the mean, i.e.,

$$m_j = E\{d_{rj}\}. \tag{11.69}$$

The entropy can be written as

$$S[Q] = -\sum_i \frac{1+m_i}{2} \ln \frac{1+m_i}{2} + \frac{1-m_i}{2} \ln \frac{1-m_i}{2}, \tag{11.70}$$

and similarly variational energy can be written as

$$V[Q] = \sum_{i,j} J_{ij} m_i m_j - 2 \sum_i m_i \theta_i \tag{11.71}$$

In order to evaluate m_i we have to minimize the variational free energy, i.e.,

$$F[Q] = V[Q] - S[Q]. \tag{11.72}$$

Differentiating this equation with respect to $m_i^{'}s$ gives nonlinear fixed point equations, i.e.,

$$m_i = tanh(-\sum_j J_{ij} m_j + \beta_i), i = 1, 2 \cdots (K-1)N \tag{11.73}$$

In the matrix form we can write the above equation as

$$\mathbf{m} = \mathbf{tanh}(-\mathbf{Jm} + \beta), \tag{11.74}$$

where

$$\beta_i = 2\theta_i. \tag{11.75}$$

11.5.2 Linear response theory

In approximating the posteriori probability $f(d_r|x; H, d_1)$, the correlations were neglected, when $Q(d_r)$ is chosen to factorize, i.e.,

$$E_{exact}\{d_{ri} d_{ri}\} \simeq E_Q\{d_{ri} d_{ri}\} = E_Q\{d_{ri}\} E_Q\{d_{ri}\}, \tag{11.76}$$

where $E_Q\{.\}$ stands for expectation with respect to distribution Q. A correction to the estimate is found by differentiating the following equation

$$E\{d_{ri}\} = Z^{-1} \sum_d d_{ri} e^{H(d)} \tag{11.77}$$

with respect to β_i to obtain linear response relation [97], i.e.,

$$\frac{\partial E\{d_{ri}\}}{\partial \beta_j} = E\{d_{ri} d_{rj}\} - E\{d_{ri}\} E\{d_{rj}\}. \tag{11.78}$$

The above relation is exact when expectation is taken according to exact probability distribution. However, if $E\{d_{ri}\}$ is reasonably well approximated with the mean field method, we can get the right hand side of the above equation by differentiating the left side of the equation with respect to β_j. In this way, we can improve the covariance and hence the second moment of the interfering users' bits which will result in improved channel estimates and symbol detection of the UoI as compared to Naive Mean Field Theory (NMFT). NMFT does not take into account correlations between random variables. This improvement is gained at the expense of very little additional complexity.

The huge computational task (complexity grows exponentially with the number of interfering users times the transmitted symbols per user) of exact averages over $f(d_r|x; H, d_1)$ has been replaced by solving the above set of $(K-1)N$ nonlinear equations, which can be done in time that grows only polynomially. As the above equation is nonlinear there may be local minima or saddle points. In order to avoid it, the solution must be compared by their value of the variational free energy $F[Q]$.

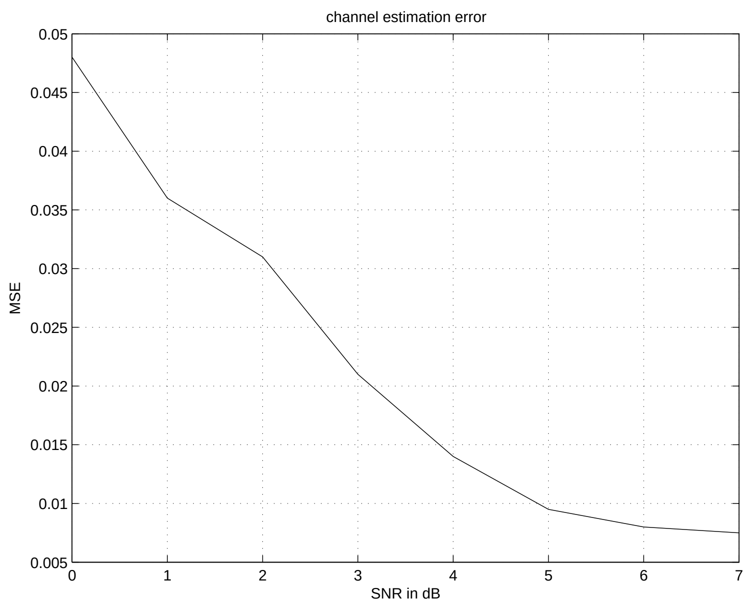

Figure 11.1: Channel estimate error vs $SNR(dB)$.

11.6 Conclusions and simulations

In this work, we proposed channel estimation and symbol detection for the Space-time block coded multiuser system. The channel is estimated blindly by formulating the STBC systems as the Gaussian mixture model. For the symbol detection two procedures are proposed. In the first approach, the Gaussian prior on the interfering users' symbols is assumed and the EM algorithm is used for user-wise symbol estimation. In the second proposed receiver a discrete prior is assumed on the interfering users' bits. In the later case, the complexity of computing the posteriori

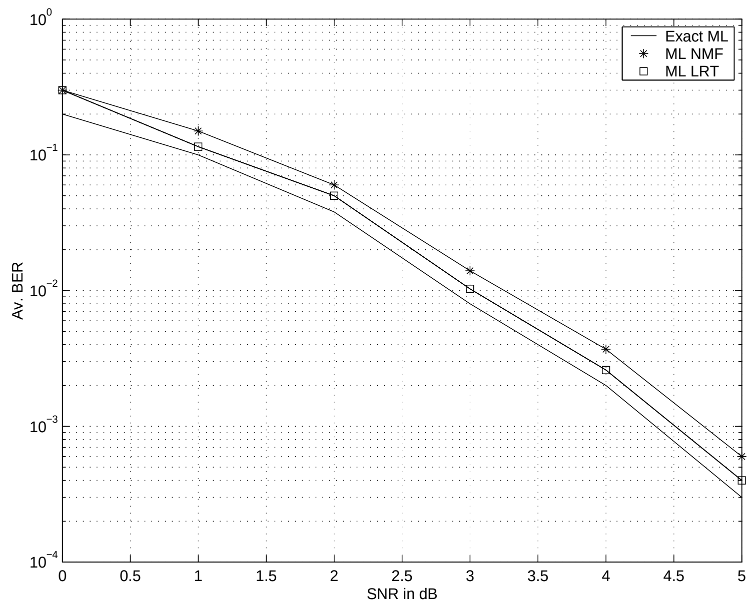

Figure 11.2: Av. BER of K=2, N=2, M=4 vs $SNR(dB)$.

probabilities grows exponentially in the number of interfering users times the symbols per user. We derived low complexity method to circumvent this problem. The exact posteriori probabilities are replaced by the approximate separable distributions. The distributions are calculated by MFT (variational approach). Figure 11.1 shows the channel estimation error for the system. Figure 11.2 shows the BER versus SNR for two users. We consider the case of two transmit and four receive antennas. It is clear from the figure for BER that the behavior of our algorithm is very close to the exact ML curve. It is also clear that we obtain better BER with the linear response theory in comparison to the naive mean field theory. In figure 11.3, the effect of projection on the Gaussian mixture problem. It is clear from the figure that the overlap between Gaussians increases rapidly as the dimensions decrease is less than ten. This means that the EM algorithm will not converge to true channel values when the dimension is reduced than ten.

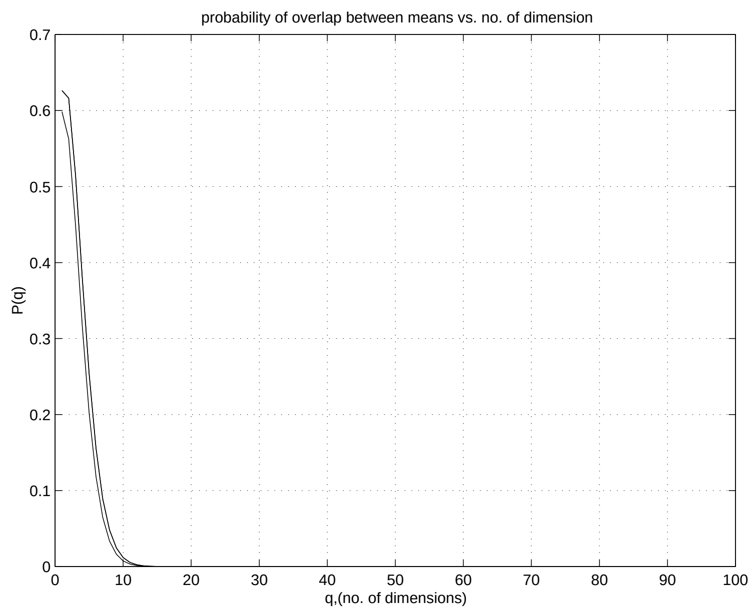
probability of overlap between means vs. no. of dimension
P(q)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
10
20
30
40
50
60
70
80
90
100
q,(no. of dimensions)

Figure 11.3:

Chapter 12

Conclusions

This thesis focused on iterative methods for CDMA and MIMO detection systems. Based on simulations results, the performance of the algorithms proposed in this thesis were compared with the existing methods for estimation/detection.
Most of the research in CDMA/MIMO detection has focused on developing new or improved suboptimal multiuser detection schemes that are more feasible to implement. The main goal of this thesis was to find algorithms which are capable of finding approximate/exact solution, given limited computational complexity.
After introduction to the CDMA, in chapter 3 we estimated the channel amplitudes using a Gaussian mixture formulation of the problem. The amplitudes were estimated blindly without training sequence using EM algorithm. In the same chapter the effect of projection on a lower dimension to reduce the dimensionality of problem was shown. It was clear from the simulations that for the EM algorithm, one cannot reduce the dimensionality to an arbitrary lower dimension (using random projection) because if the separation between Gaussians are not enough, the EM algorithm completely fails to converge to true parameter values.

In chapter 4, we evaluated the ML technique to estimate the parameters of the asynchronous DS-CDMA. The exact ML problem was relaxed to the ML problem with the constraint that the Euclidean norm of the symbol vector is a hypersphere. We relaxed the sphere constraint by using the first order Taylor expansion and in order that the current estimate is not far from the previous estimate we incorporated a distance function in the ML cost function, and iteratively estimate the amplitudes and detect symbol vector.

In chapter 5, the same problem was solved using an exact sphere constraint (as opposed to the linear approximation in the previous chapter) and we compared the results with the MMSE and the receiver proposed in chaper 4.

The second part of the thesis was focused on MIMO systems.

Recently, approximate algorithms for ML detection gained quite popularity. Specially, the SDP approach was shown to be very useful in the detection problem encountered in the communication systems. Still, the complexity of the SDP method is quite large for large systems and also the performance of the SDP approach is based on the simulations results only. We give an introduction of the MIMO systems and fundamentals of the semidefinite and the second order cone programming approach in chapter 6 and chapter 7 respectively. Chapter 8 was devoted to the approximate low complexity MIMO detection problem. First we formulated the ML detection as SOCP problem. The advantage of the SOCP approach is that it has less number of variables and hence numerically more efficient as compared to the SDP approach. This computational advantage is at the expense of very little performance loss.

In chapter 9, we solved the ML detection problem exactly in polynomial time for a fixed number of the receive antennas. The proposed method is based on minimizing a quadratic (ML cost function) subject to the constraint that the symbol vector lies on the vertices of the cube. This problem can be posed (after SVD operation) as maximizing the Euclidean distance subject to the constraint that the data vector lies on the vertices of a lower dimensional zonotope (a geometrical object). Using a classical algorithm in discrete geometry, the polynomial complexity of the method was shown. Our algorithm has potential benefits over the sphere decoder: in our algorithm same complexity is SNR independent, whereas the sphere decoder has exponential complexity at low SNRs.

In the first chapters of the second part, the perfect channel knowledge was assumed at the receiver. However, the receiver does not have a priori knowledge of the channel. In the following two chapters we iteratively, jointly estimated the channel and the symbols. In chapter 11 after finding analogy between the synchronous CDMA to the space-time coded system, we used the Gaussian mixture formulation to estimate the channel. We further showed theoretically the effect of the projection on the Gaussian mixture model and hence confirmed the results of chapter 3. After estimating the channel, the symbol detection is done in an iterative way. The complexity of detecting the users information bits was circumvented by using the mean field approaches, which are used vastly by the statistical physics community to approximate probabilities. In chapter 10, we used the EM algorithm to jointly estimate the channel-symbols. Here too, we used MFT to approximate the posteriori probabilities of the interfering users' symbols, and we compared the results with the ML solution found by exhaustive search.

In the following lines we give some directions for future works, which could be the extension of this thesis, as well as completely new directions. One proposed direction is to evaluate the performance of the MIMO detection schemes proposed in this thesis to the case when the receiver has noisy channel estimates.

In [73] lattice-reduction aided detectors for 2×2 antenna system has been analyzed and the authors showed the enhanced performance of MIMO systems when used in conjunction with the traditional linear and nonlinear detectors with quite low complexity. Their work was extended to arbitrary MIMO systems in [72], but at the expense of increased complexity. This complexity is due to the LLL algorithm [74] used for lattice reduction. New low complexity algorithms must be devised for lattice reduction so that it can be implemented in real-time for the communications system for decoding. Another possibility for research is to minimize ML cost function for detection (which is a quadratic form) over a lattice generated by the channel matrix. One should use Minkowski's convex body theorem/Geometry of numbers [27,28,30] to find a non-zero lattice point (information vector) which minimizes the value of the quadratic form.

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