LDPC Coding for Interference Mitigation at the Transmitter

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Abstract

We consider a Gaussian additive-noise channel with interference known to the transmitter but unknown to the receiver. A precoding technique based on lattice quantization is able to achieve Costa's "dirty-paper" limit for arbitrary interference signal, when this is known non-causally. When interference is known causally, the same technique yields asymptotically optimal achievable rates for high-SNR. We provide explicit code constructions based on low-density parity-check codes and on M-PAM modulation able to closely approach the rates achievable by the lattice precoding scheme for causal known-interference.

1 Background

Memoryless channels with input X, output Y and state-dependent transition probability $P_{Y|X,S}$ where the channel state S is i.i.d., known to the transmitter and unknown to the receiver, date back to Shannon [1], who considered the case of state sequence known causally, and to Kusnetsov and Tsybakov [2], who considered the case of state sequence known non-causally. Gel'fand and Pinsker [3] proved the capacity formula

$$C = \sup_{P_{T|S}} \{I(T;Y) - I(T;S)\} \tag{1}$$

for the non-causal case, where T is an auxiliary random variable with conditional distribution $P_{T|S}$ and X is a deterministic function of S and T. This yields Shannon's capacity formula [1]

$$C = \sup_{P_T} I(T;Y) \tag{2}$$

for the causal case with i.i.d. state sequence, by restricting the supremization in (1) to T independent of S.

In the case where the channel is $Y = X + S + Z$, with $Z \sim \mathcal{N}(0, \sigma^2)$, $E[|X|^2] \leq \mathcal{E}$ and the interference S is also Gaussian and known non-causally at the transmitter, Costa [4] proved that the capacity in (1) is equal to the standard AWGN capacity $\frac{1}{2}\log_2(1 + \mathcal{E}/\sigma^2)$, as if interference was not present. From the title of Costa's paper, coding strategies for the non-causal known interference case are nicknamed "Dirty-Paper" coding, and by analogy, coding strategies for the causal case are nicknamed "Dirty-Tape" coding [5, 6].

While in early works such problems were motivated mainly by applications to data storage on defective media [2], more recently "Dirty-Paper" coding gained renewed attention because it arises as the main tool in several important settings such as broadcast vector Gaussian channels [7, 8, 9, 10, 11, 12], precoding for ISI channels [13, 6] and data hiding [14, 15, 16].

From the information theoretic point of view, Gel'fand, Pinsker and Costa's results were generalized in many ways (see [6] and references therein). However, efficient coding strategies able to approach the theoretical results are still not a common practice (preliminary results can be found in [14, 15]) although, based on random coding arguments, it can be shown that sequences of good lattice codes, mimicking Gel'fand and Pinsker random binning scheme, do exist and can approach Costa's result in the additive interference AWGN case [17].

In this paper we review the lattice precoding scheme of [5, 6], and we provide explicit code constructions based on low-density parity-check (LDPC) codes and scalar quantization demonstrating their ability to approach the theoretical limits for the “Dirty-Tape” problem (the causal case). Numerical results show that the proposed schemes are very effective. Moreover, they can be generalized to the non-causal case, where the transmitter knows the interference sequence with anticipation of k symbols. The anticipation k might be regarded either as a problem constraint (given by nature) or as a complexity parameter (set by the designer), to tradeoff optimality and encoding/decoding complexity. The case of $k > 1$ is currently under consideration in the on-going work [18].

2 Inflated lattice precoding

In [5, 6] a deterministic coding strategy for the AWGN channel with additive interference is proposed. This technique, referred to as *inflated lattice precoding*, is based on lattice dithered quantization and on a scaling operation of the received signal (inflation). Dithering makes the performance of inflated lattice precoding independent of the interference signal, and allows the extension of Costa’s result to any interference statistics and even to arbitrary interference sequences [6], making it an efficient coding approach for the arbitrary varying channel with states known to the transmitter [19].

The inflated lattice precoding scheme is illustrated in Fig. 1. Let k be an integer dividing the block length n, and consider a k-dimensional lattice Λ with fundamental Voronoi cell $\mathcal{V}$, with mean zero and normalized second-order moment $\mathcal{E}$ (i.e., such that $\int_{\mathcal{V}} \mathbf{x} d\mathbf{x} = 0$ and $\frac{1}{V(\Lambda)} \int_{\mathcal{V}} |\mathbf{x}|^2 d\mathbf{x} = k\mathcal{E}$).

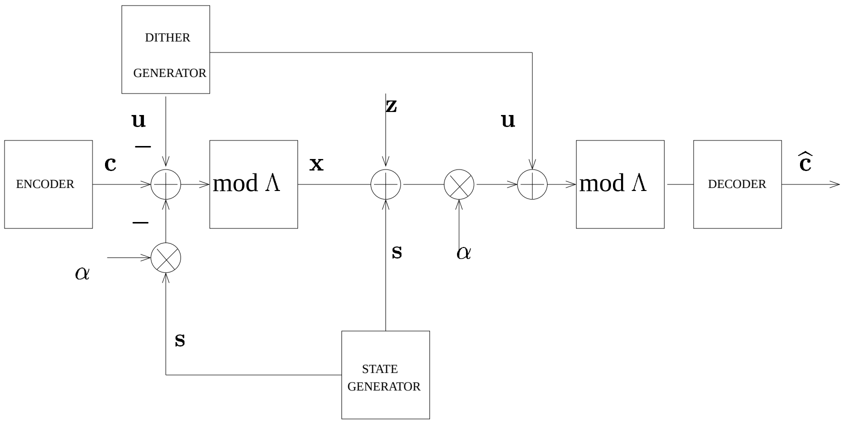

Figure 1: The inflated lattice precoding scheme.

A code $\mathcal{C}$ of rate R and block length n is constructed according to a uniform distribution over $\mathcal{V}$, so that

$$\mathcal{C} \subseteq \underbrace{\mathcal{V} \times \cdots \times \mathcal{V}}_{n/k \text{ times}}$$

Furthermore, the transmitter and the receiver share common randomness and can generate the same *dither* signal $\mathbf{u}$, uniformly distributed over $\mathcal{V}^{n/k}$.

For an n-dimensional vector $\mathbf{v}$, the vector $\mathbf{v} \bmod \Lambda^{n/k}$ is defined as

$$\mathbf{v} \bmod \Lambda^{n/k} = \mathbf{v} - \boldsymbol{\lambda}(\mathbf{v}) \tag{3}$$

where

$$\boldsymbol{\lambda}(\mathbf{v}) \stackrel{\Delta}{=} \arg \min_{\boldsymbol{\lambda} \in \Lambda^{n/k}} |\mathbf{v} - \boldsymbol{\lambda}|^2 \tag{4}$$

In other words, reducing $\mathbf{v}$ modulo $\Lambda^{n/k}$ consists of quantizing $\mathbf{v}$ by using $\Lambda^{n/k}$ as a lattice quantizer, and computing the total quantization error vector.

Let $\mathbf{c} \in \mathcal{C}$ be the codeword to be transmitted. After observing the interference signal $\mathbf{s}$, the transmitter produces the channel input sequence

$$\mathbf{x} = [\mathbf{c} - \alpha \mathbf{s} - \mathbf{u}] \bmod \Lambda^{n/k} \tag{5}$$

where $\alpha \in [0, 1]$ is a scaling coefficient (to be optimized), and sends $\mathbf{x}$. Thanks to the dither signal $\mathbf{u}$, $\mathbf{x}$ is uniformly distributed on $\mathcal{V}^{n/k}$ and its average energy per symbol is $\mathcal{E}$, so that the power input constraint is satisfied.

After receiving $\mathbf{y} = \mathbf{x} + \mathbf{s} + \mathbf{z}$, the receiver computes

$$\mathbf{y}' = [\alpha \mathbf{y} + \mathbf{u}] \bmod \Lambda^{n/k} \tag{6}$$

It can be shown [5, 6] that the channel from the encoder output to the decoder input (see Fig. 1) is equivalent to the additive modulo$-\Lambda^{n/k}$ noise channel

$$\mathbf{y}' = [\mathbf{c} + \mathbf{z}'] \bmod \Lambda^{n/k} \tag{7}$$

where $\mathbf{z}'$ is distributed as $[(1-\alpha)\mathbf{u} + \alpha \mathbf{z}] \bmod \Lambda^{n/k}$.

Finally, the decoder computes (or approximates) the ML decision

$$\widehat{\mathbf{c}} = \arg \max_{\mathbf{c} \in \mathcal{C}} p_{Z'}(\mathbf{y}' - \mathbf{c}) \tag{8}$$

where $p_{Z'}$ denotes the pdf of $\mathbf{z}'$ defined above.

For $k = n$, the inflated lattice precoding scheme applies to the non-causal "Dirty-Paper" case and, by choosing a sequence of good n-dimensional lattices, it can achieve the AWGN capacity $\frac{1}{2}\log_2(1+\text{SNR})$ in the limit for $n \to \infty$. However, the same scheme can be applied in the general case of anticipation k, where the transmitter at time i knows only the interference signal components $s_i, \ldots, s_{i+k-1}$. In particular, for $k = 1$ the inflated lattice precoding scheme yields an effective coding strategy for the causal "Dirty-Tape" problem. It can be shown that the inflated lattice precoding scheme for $k = 1$ is asymptotically optimal for the dirty-tape problem in the limit for high SNR [6] and that the maximum penalty incurred by knowing the interference causally only is the shaping gain, i.e., 0.254 bit/symbol [5].

Notice also that, even in the case where $\mathbf{s}$ is known non-causally, we might choose $k < n$ for complexity reasons. For example, in [15, 14] inflated lattice precoding with $k = 1$ is proposed for data-hiding, even if in that context the host signal is clearly known non-causally. The advantage of this scheme is that lattice quantization (4) reduces to standard one-dimensional uniformly-spaced quantization.

3 Codes for "writing on dirty tape"

For $k = 1$, we have $\Lambda = \Delta\mathbb{Z}$, with $\Delta = \sqrt{12\mathcal{E}}$, and the Voronoi region $\mathcal{V}$ is the interval $[-\frac{\Delta}{2}, \frac{\Delta}{2}]$. In this case, it is natural to construct the code $\mathcal{C}$ over the equally-spaced M-PAM signal set

$$\mathcal{A} = \left\{ \frac{\Delta}{2M}(2m - M + 1) \ : \ m = 0, \ldots, M-1 \right\}$$

The first-order transition pdf of the corresponding modulo-noise channel (defined in general by (7)) is given by

$$p_{Z'}(z) = \sum_{k \in \mathbb{Z}} \frac{P_Z\left(\frac{z + k\Delta + (1-\alpha)\Delta/2}{\alpha}\right) - P_Z\left(\frac{z + k\Delta - (1-\alpha)\Delta/2}{\alpha}\right)}{(1-\alpha)\Delta} \tag{9}$$

where $P_Z(z)$ denotes the cdf of the noise of the original channel. In this work we consider AWGN, therefore $P_Z(z)$ is the Gaussian distribution $\mathcal{N}(0, \sigma^2)$.

The rate achievable by the inflated lattice precoding scheme is given by

$$R^{\text{prec.}} = \max_{\alpha \in [0,1]} \left\{ \log_2 \Delta - h(Z') \right\} \tag{10}$$

The optimization of the inflation coefficient α must be obtained numerically. Fig. 2 shows the optimal α as a function of the SNR $\mathcal{E}/\sigma^2$. The Costa's value $\alpha = \mathcal{E}/(\sigma^2 + \mathcal{E})$ [4], which is optimal for $k = n \to \infty$, is shown for comparison.

We consider two code constructions for $\mathcal{C}$: LDPC multilevel coding and direct design of LDPC-coded modulation.

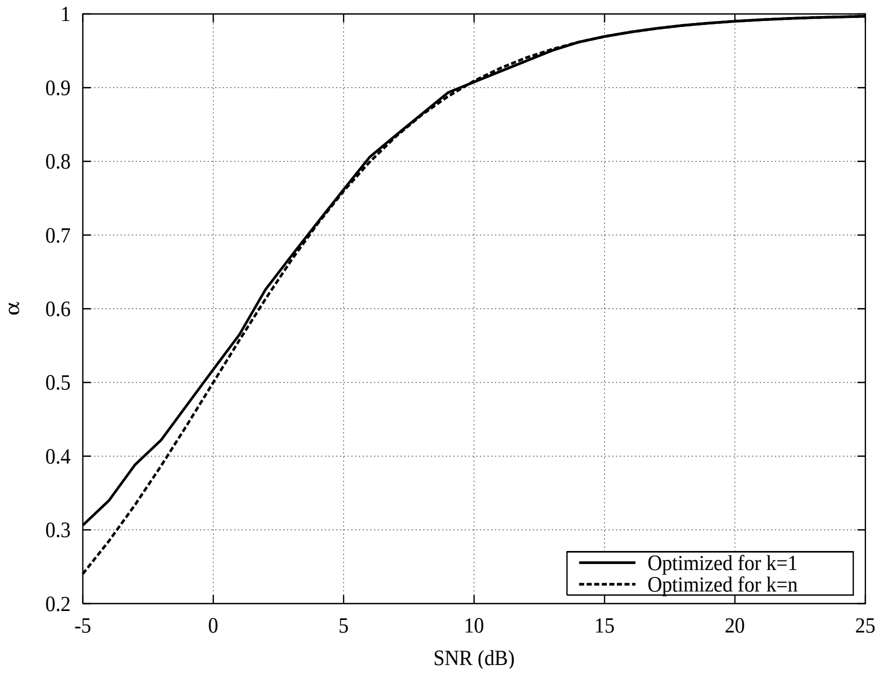

Figure 2: Optimal inflation factor for $k = 1$ and $k = n \to \infty$.

3.1 Multilevel coding

Multilevel coding (see [20] and references therein) is a general method for constructing coded modulation schemes. Fig. 3 shows a block diagram of the multilevel encoder, where m binary codes produce codewords $\mathbf{c}_1, \dots, \mathbf{c}_m$ of length n. These are arranged as rows of a $m \times n$ matrix, and a binary labeling function $\phi : \mathbb{F}_2^m \to \mathbf{A}$ is applied columnwise, in order to form the corresponding codeword of $\mathcal{C}$.

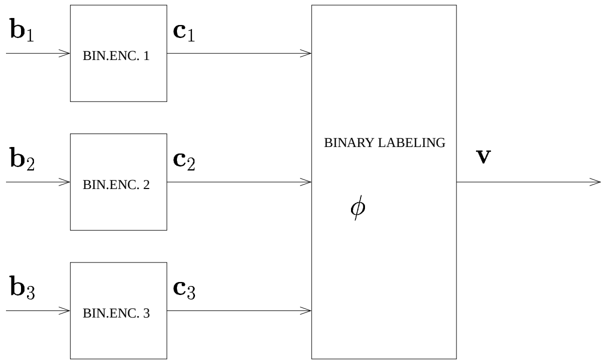

Figure 3: Multilevel coded modulator with three levels.

As binary component codes we choose LDPCs from the database of LDPC ensembles optimized for the binary-input AWGN channel provided in [21]. Following [20], the choice of the component code rates is dictated by the mutual information chain rule. Namely, let A be uniformly distributed on $\mathcal{A}$ and let $(b_1, \dots, b_m)$ be binary uniform random variables, then

$$I(A; Y') = I(b_1, \dots, b_m; Y') = \sum_{i=1}^{m} R_i$$

where we define the rate at level i as

$$
\begin{aligned}
R_i &= I(Y'; b_i | b_1, \ldots, b_{i-1}) \\
&= E\left[\log_2 \frac{\sum_{a \in \mathcal{A}(b_1,\ldots,b_i)} p_{Z'}(Y'-a)}{\sum_{a' \in \mathcal{A}(b_1,\ldots,b_{i-1})} p_{Z'}(Y'-a')}\right]
\end{aligned}
\quad (11)
$$

where $\mathcal{A}(b_1, \ldots, b_i)$ denotes the subset of points of $\mathcal{A}$ whose label first i positions are given by $(b_1, \ldots, b_i)$. The rates R_i are achievable by a multistage decoder that considers the levels in sequence, by decoding each level i assuming that the decoding outcomes at previous levels $1, \ldots, i-1$ are correct, and by treating the levels $i+1, \ldots, m$ as a random nuisance. The multistage decoder is analogous to the well-known successive decoding used in standard Gaussian multiaccess and broadcast channels.

The binary labeling ϕ affects the level rates R_i but, as long as ϕ is a one-to-one mapping, the total mutual information is independent of ϕ. Fig. 4 shows an example of the set-partitioning labeling [22] considered in this work for 8-PAM codes and Fig. 5 shows the corresponding achievable rate of the 8-PAM modulo-noise channel, with the level rates R_i.

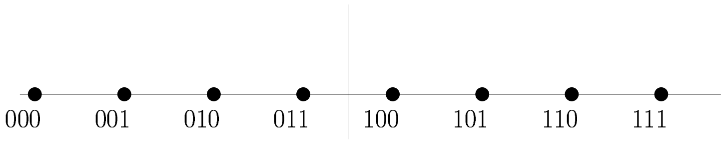

Figure 4: Set-partitioning labeling of 8-PAM.

Levels are decoded from right to left of the labels. We notice that for high SNR only the first level need coding at rate $R_1 < 1$, while the other two can be transmitted uncoded (i.e., $R_2 = R_3 = 1$). This provides a very simple scheme requiring a single decoding stage followed by symbol-by-symbol detection of the remaining stages. Hence, multilevel coding with set-partitioning labeling is particularly attractive in the high-SNR region. Incidentally, this is also the region where the inflated lattice precoding scheme with $k = 1$ pays the smallest relative penalty with respect to the full non-causal case (as already noticed, the shaping gain 0.254 bit/symbol).

3.2 LDPC-coded modulation

Our second approach consists of directly optimizing the ensemble of LDPC-coded modulation schemes constructed over the M-PAM alphabet $\mathcal{A}$, by exploiting the *density-evolution* method developed to analyze LDPC codes under message-passing decoding in the limit of infinite block length [23, 24]. Fig. 6 shows the Tanner graph of the code, where the bitnodes are partitioned into groups of m nodes, each of which is associated to the m label positions of a M-PAM symbol. The super-nodes corresponding to modulation symbols will be referred to as "$\mathcal{A}$-nodes".

We say that a $\mathcal{A}$-node has *type* $(d_1, \ldots, d_m)$ if its i-th label bitnode has degree d_i. We enumerate the $\mathcal{A}$-node types in lexicographic order, and let $d_{t,i}$ be the degree of the i-th bitnode in the $\mathcal{A}$-nodes of type t. We let $\lambda_{t,i}$ denote the fraction of edges connected with $\mathcal{A}$-nodes of type t in position i. For a graph with e edges, the number of $\mathcal{A}$-nodes of type t is given by $n_t = e\lambda_{t,i}/d_{t,i}$, therefore $\lambda_{t,i}/d_{t,i}$ must not depend on i.

As in standard LDPC notation, we let ρ_j be the fraction of edges connected to checknodes of degree j. The ensemble optimization consists of finding, for any given SNR, the set of $\mathcal{A}$-node types, the left degree distribution $\{\lambda_{t,i}\}$ and the right degree distribution $\{\rho_j\}$ such that the coding rate is maximized subject to the constraint that the density evolution (DE) converges to zero bit-error probability as the number of iterations goes to infinity (see [23, 25] for the details). Since optimizing the degree distributions based on the exact DE is computationally very intensive, we propose a design method based on a one-dimensional approximation of DE, obtained by approximating the message densities by Gaussian pdfs (see [24] and especially [26] for similar approaches in different contexts).

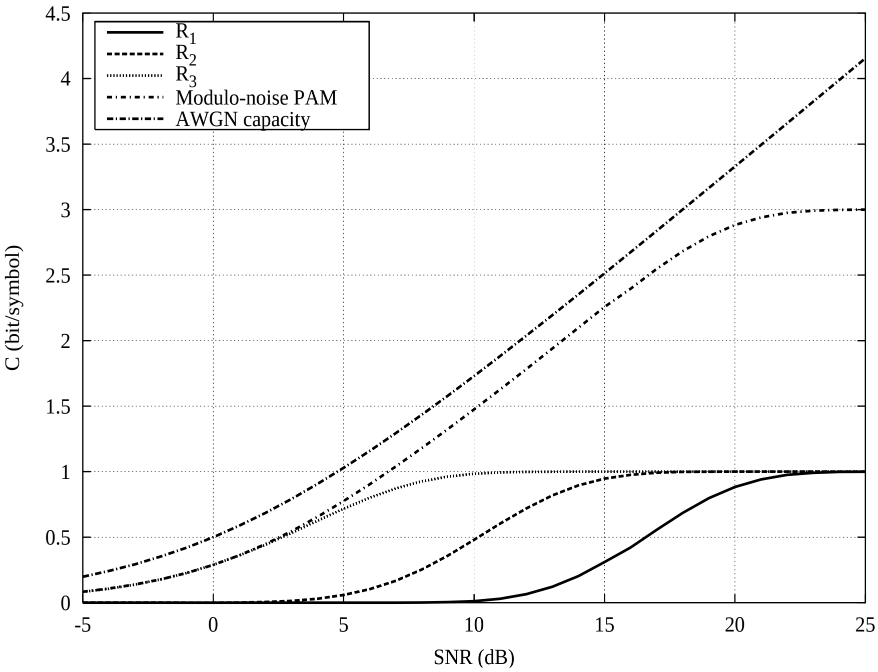

Figure 5: Total and per-level rates for 8-PAM with set-partitioning labeling in the modulo-noise channel.

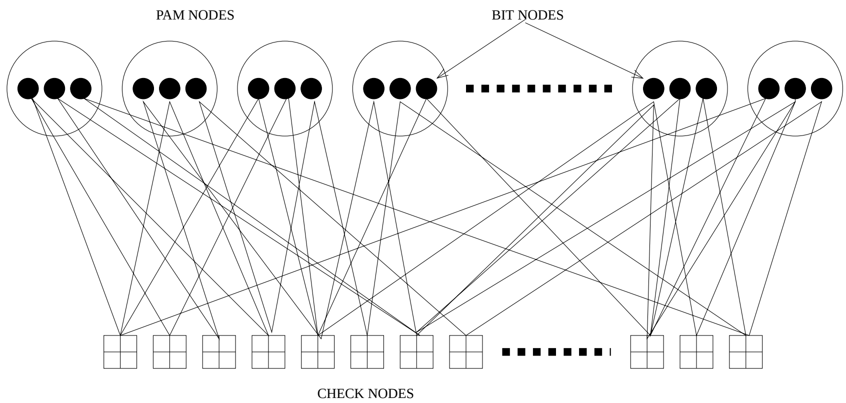

Figure 6: Tanner graph of an LDPC-coded modulation scheme.

From $\sum_{t,i} \lambda_{t,i} = 1$ we obtain the constraint $\sum_t \lambda_{t,1} \sum_{i=1}^m \frac{d_{t,i}}{d_{t,1}} = 1$. The design coding rate is given by

$$R = \log_2 M - \frac{\sum_j \rho_j / j}{\sum_t \lambda_{t,1}/d_{t,1}} \quad \text{bit/symbol} \tag{12}$$

For given right degree sequence $\{\rho_j\}$, we wish to obtain an optimization problem in the variables $\{\lambda_{t,1}\}$.

Consider an $\mathcal{A}$-node of type $(d_1, \dots, d_m)$ and consider an edge o connected in position i. The message-passing transformation of the iterative belief-propagation decoder associated to the message output by the node onto edge o is given by

$$\mathcal{L}_{i,o}^{\text{out}} = \log \frac{\sum_{a \in \mathcal{A}_0^i} p_{Z'}(y' - a) \exp\left(-\sum_{j=1}^m b_j \sum_{u=1}^{d_j} \mathcal{L}_{j,u}^{\text{in}}\right)}{\sum_{a \in \mathcal{A}_1^i} p_{Z'}(y' - a) \exp\left(-\sum_{j=1}^m b_j \sum_{u=1}^{d_j} \mathcal{L}_{j,u}^{\text{in}}\right)} - \mathcal{L}_{i,o}^{\text{in}} \tag{13}$$

where $\mathcal{L}_{j,u}^{\text{in}}$ denotes the input message from edge u connected to the bitnode in position j, and $\mathcal{A}_0^i$ (resp. $\mathcal{A}_1^i$) denotes the signal subset of all points of $\mathcal{A}$ having symbol 0 (resp. 1) in label position i and where y' denotes the channel output corresponding to the given $\mathcal{A}$-node.

It has been shown in [25] that the message pdfs generated by the belief propagation algorithm at each iteration satisfy a *symmetry condition*. If the pdfs are Gaussian, the symmetry condition imposes that the variance must be twice the mean. Assuming that all input messages are Gaussian i.i.d. $\sim \mathcal{N}(\mu, 2\mu)$, we can obtain by Monte Carlo simulation of (13) the distribution of $\mathcal{L}_{i,o}^{\text{out}}$ for every i and node type t, parameterized in the input mean value μ.

Following [26], we shall replace the pdf $f_{\mathcal{L}}(z) \stackrel{\Delta}{=} \frac{d}{dz} \Pr(\mathcal{L} \le z | b = 0)$ of a message $\mathcal{L}$ relative to a bitnode b by the value of the mutual information functional $I(b; \mathcal{L})$ which, for symmetric pdfs, is given by

$$I(b; \mathcal{L}) = 1 - \int_{-\infty}^{\infty} \log_2 \left(1 + e^{-z}\right) f_{\mathcal{L}}(z) dz \tag{14}$$

Then, from the Gaussian assumption of the input messages and the explicit mapping (13), for any $\mathcal{A}$-node type and bitnode position i we can find (numerically) a mutual information transfer function

$$\mathsf{y} = \Gamma_{t,i}(\mathsf{x}) \tag{15}$$

where $\mathsf{x} \stackrel{\Delta}{=} I(b; \mathcal{L}^{\text{in}})$ and $\mathsf{y} \stackrel{\Delta}{=} I(b; \mathcal{L}^{\text{out}})$.

Again as a consequence of the message pdf symmetry, the mutual information between a message $\mathcal{L}^{\text{out}}$ and its associated bitnode value b on a randomly selected graph edge, chosen with probability $\lambda_{t,i}$, is given by

$$\mathsf{y} = \sum_{t,i} \lambda_{t,i} \Gamma_{t,i}(\mathsf{x}) \tag{16}$$

Fig. 7 shows the functions $\Gamma_{t,i}$ for an 8-PAM-node of type $(2, 5, 10)$.

For the message-passing mapping at the checknodes we use the approximated duality property [27], stating that the mutual information transfer function of a checknode is closely approximated by the mutual information transfer function of a bitnode with the same degree, by applying the mapping $\mathsf{x} \mapsto 1 - \mathsf{x}$ to the input and $\mathsf{y} \mapsto 1 - \mathsf{y}$ to the output. Assuming the input messages i.i.d. Gaussian $\sim \mathcal{N}(\mu, 2\mu)$ and by defining the binary-input Gaussian mutual information functional

$$J(\mu) \stackrel{\Delta}{=} 1 - \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-z^2} \log_2 \left(1 + e^{-2\sqrt{\mu} z - \mu}\right) dz, \tag{17}$$

the mutual information transfer function of a checknode of degree j is given by

$$\mathsf{y} = 1 - J\left((j-1) J^{-1}(1 - \mathsf{x})\right)$$

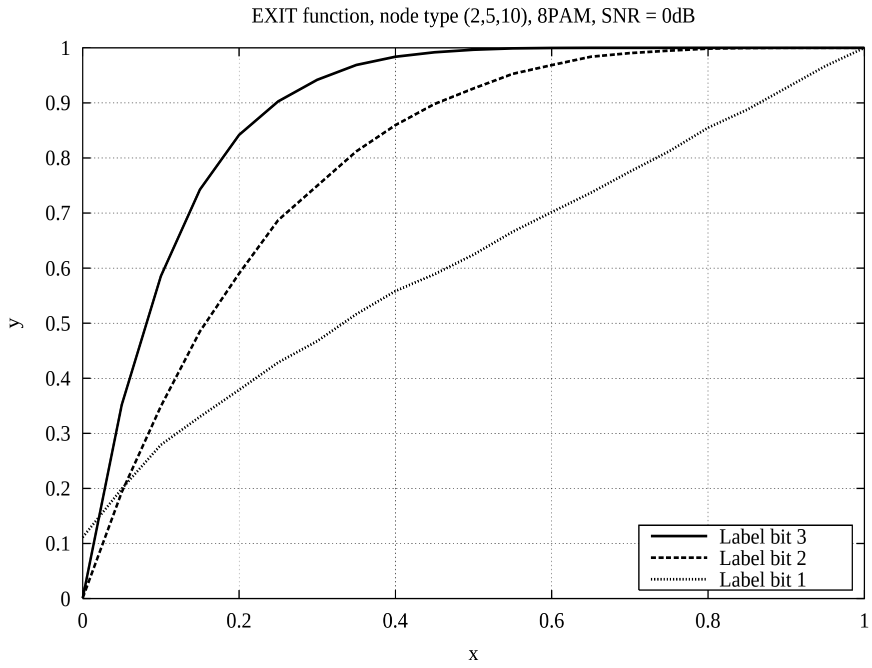

Figure 7: Mutual information transfer function for a 8-PAM node of type $(2, 5, 10)$ for SNR= 0 dB.

Eventually, the one-dimensional DE approximation is given by the recursion

$$\mathsf{x}^{(\ell)} = \sum_{t,i} \lambda_{t,i} \Gamma_{t,i} \left(1 - \sum_j \rho_j J \left((j-1) J^{-1} \left(1 - \mathsf{x}^{(\ell-1)} \right) \right) \right) \tag{18}$$

for $\ell = 1, 2, \ldots$, with initial condition $\mathsf{x}^{(0)} = 0$.

The recursion (18) has a unique stable fixed point in $\mathsf{x} = 1$ (corresponding to vanishing bit-error probability), if and only if

$$\mathsf{x} > \sum_{t,i} \lambda_{t,i} \Gamma_{t,i} \left(1 - \sum_j \rho_j J \left((j-1) J^{-1} (1 - \mathsf{x}) \right) \right), \quad \forall\, \mathsf{x} \in [0, 1) \tag{19}$$

Hence, we can sample the above equation for x taking on values in a fine grid of points in the interval $[0, 1)$ and for each point we obtain a linear constraint in the variables $\lambda_{t,1}$ (recall that $\lambda_{t,i} = \frac{d_{t,i}}{d_{t,1}} \lambda_{t,1}$, therefore the only independent variable of the optimization problem are $\{\lambda_{t,1}\}$). Since both the constraints and the objective function (12) are linear in the $\{\lambda_{t,1}\}$, the solution is readily obtained by linear programming.

3.3 Results and conclusions

Fig. 8 shows the performance of some multilevel LDPC codes and LDPC-coded M-PAM codes obtained according to the methods described above, for the dirty-tape channel with AWGN. In all cases, the block length (in PAM symbols) is 20000 and the points correspond to bit-error probability $\leq 10^{-4}$ with a maximum of 100 decoder iterations.

Points labeled by "2PAM" correspond to standard binary LDPC, points labeled by "MCM" correspond to multilevel coded modulation and points labeled by "LDPC" correspond to LDPC-coded modulation.

We observe that the theoretical achievable rate obtained by the inflated lattice precoding scheme with $k = 1$ and with coding over the interval $[-\Delta/2, \Delta/2]$ (an infinite alphabet) can be closely approached by

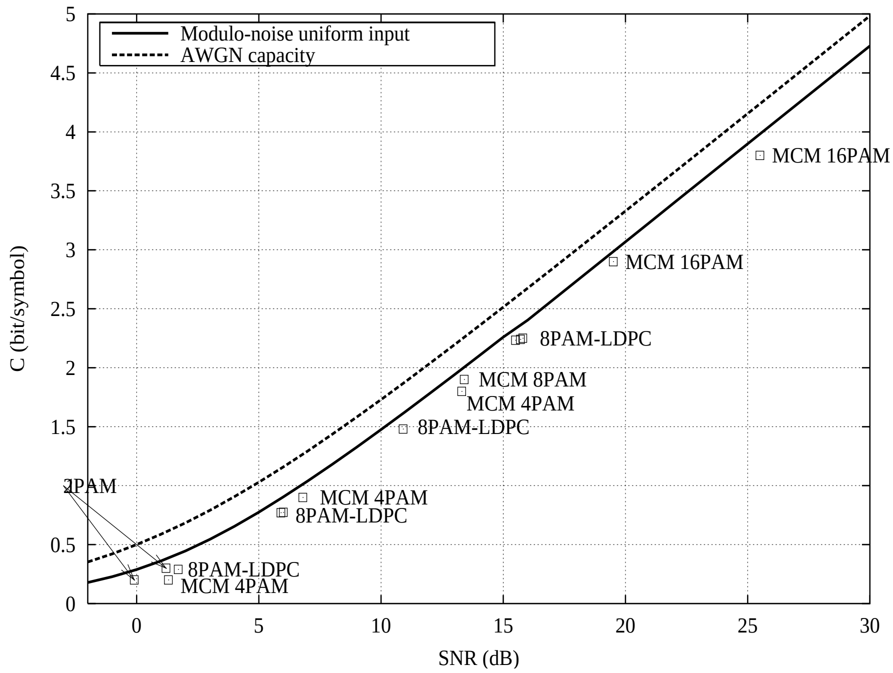

Figure 8: Performance of LDPC-coded modulation schemes for the dirty-tape channel with AWGN.

our LDPC-coded PAM modulation. For low SNR, $M = 2$ suffices to approach the limit, while for high SNR the multilevel construction with one or two coding levels and the remaining levels left uncoded proves to be both efficient and simple in terms of complexity. There exists a region of intermediate SNR where it is indeed worthwhile to construct explicitly optimized LDPC-coded modulation. In our examples we considered codes over the 8-PAM alphabet, but the coding design ideas apply immediately to other cases.

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