A Low-Complexity Approach to Space-Time Coding for Multipath Channels

Mari Kobayashi and Giuseppe Caire

Institut EURECOM,
2229 Route des Cretes, B.P. 193,
06904 SOPHIA ANTIPOLIS CEDEX, France
E-mail: {mari.kobayashi, giuseppe.caire}@eurecom.fr

Abstract

We consider a low-complexity single-carrier space-time coding scheme for frequency-selective channels based on the concatenation of trellis-coded modulation (TCM) with time-reversal orthogonal space-time block coding (TR-STBC). The TR-STBC turns the frequency-selective multi-input multi-output channel into a standard single-input single-output channel affected by inter-symbol interference. The decoder is based on reduced-state joint equalization and decoding, where a minimum mean-square-error decision-feedback equalizer is combined with a Viterbi decoder operating on the TCM trellis, and where tentative decisions are found on the survivors (per-survivor processing). In this way, the decoder complexity is independent of the channel memory and of the constellation size. Nevertheless, simulations show that the proposed scheme yields similar/superior performance with respect to the best previously proposed schemes at much lower complexity.

1. Introduction

The design of space-time codes (STCs) for single carrier transmission over frequency selective multi-input multi-output (MIMO) channels has been investigated in a number of recent contributions [1–4]. Maximum-Likelihood (ML) decoding in MIMO frequency-selective channels is generally too complex for practical channel memory and modulation constellation size. Hence, research has focused on sub-optimal low complexity schemes. We may group these approaches into two classes. The first approach is based on mitigating inter-symbol interference (ISI) by some MIMO equalization technique, and then designing a space-time code/decoder for the resulting flat fading channel. For example, the use of a linear minimum mean-square-error (MMSE) MIMO equalizer combined with Alamouti's space-time block coding (STBC) has been investigated in [4]. However, this scheme does not achieve in general the maximum diversity order offered by the channel. The second approach is based on designing the STC by taking into account the ISI channel and then performing joint equalization and decoding. For example, trellis coding and bit-interleaved coded modulation (BICM) with turbo equalization have been proposed in [3, 5, 6]. Time reversal orthogonal STBC (TR-STBC) [1] (see also [2]) converts the MIMO frequency-selective channel into a standard single-input single-output (SISO) channel with ISI, to which conventional techniques developed for ISI channels can be applied. The problem with these schemes is that turbo-equalization needs a soft-in soft-out MAP decoder for the ISI channel, whose complexity is exponential in the channel length and in the constellation size. For example, MAP symbol-by-symbol detection implemented by the BCJR algorithm runs on a trellis with M^{L-1} states, where M denotes the constellation size and L denotes the channel length expressed in symbol.

In this work we consider the concatenation of TR-STBC with an outer trellis coded modulation (TCM) scheme [7]. We shall provide an information-theoretic justification for the proposed scheme in terms of achievable rate and diversity. At the receiver, we apply a reduced-state sequence detection based on MMSE decision-feedback equalizer (MMSE-DFE) and per-survivor processing (PSP). Since the PSP detector/decoder works on the trellis of the underlying TCM code, the receiver complexity is independent of the channel length and of the constellation size. This makes our scheme applicable in practice, while the schemes proposed in [2, 3, 6] are not. Another advantage is that TCM can easily implement adaptive modulation (it is sufficient to expand the constellation, add uncoded bits and increase the number of parallel transitions in the TCM trellis, following [7]).

Notations: i) $\mathbf{v} \sim \mathcal{N}_{\mathbb{C}}(\overline{\mathbf{v}}, \mathbf{R})$ means that the random vector $\mathbf{v}$ is complex Gaussian with circular symmetry, with mean $E[\mathbf{v}] = \overline{\mathbf{v}}$ and covariance $E[\mathbf{v}\mathbf{v}^H] - \overline{\mathbf{v}}\overline{\mathbf{v}}^H = \mathbf{R}$. ii) $\mathbf{I}_n$ denotes the $n \times n$ identity matrix. iii) Superscripts ^{T} and ^{H} indicate transpose and Hermitian transpose, respectively. iv) $\circ\mathbf{x} = (x_{N-1}^*, x_{N-2}^*, \ldots, x_0^*)^T$ indicates time reversal complex conjugate of a vector $\mathbf{x}$.

2. Channel Model

We consider a frequency-selective multi-input single-output (MISO) channel with N_t transmit and one receive antenna. The extension to MIMO channel is straightforward. The multipath channel from the i-th transmit antenna to the receiving antenna is assumed to be random but time-invariant for a sufficiently large number of symbols (quasi-static fading assumption). The overall i-th channel impulse response is given by the convolution of the physical channel with the transmit shaping filter. After sampling at the symbol rate $1/T_s$, we obtain discrete-time overall impulse response $\mathbf{g}_i$ of length

$$L = \lfloor \Delta/T_s \rfloor + \kappa \tag{1}$$

where T_s represents a symbol duration, Δ is the delay spread and κ is the length of the shaping filter impulse response expressed in symbol intervals. We assume that a block $\mathbf{s}_i$ of N symbols is sent from the i-th antenna, followed by $L-1$ zeros (zero-padding), in order to eliminate inter-block interference. Let $\mathbf{G}_i$ denote the $(N-1+L)\times N$ tall Toepliz matrix whose j-th column is given by

$$(\underbrace{0,\ldots,0}_{j},\mathbf{g}_i^T,\underbrace{0,\ldots,0}_{N-1-j})^T$$

Then, the discrete-time received signal for a transmitted block of length N is given by

$$\mathbf{r} = \sum_{i=1}^{N_t} \mathbf{G}_i \mathbf{s}_i + \mathbf{w} \tag{2}$$

where $\mathbf{w} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, N_0 \mathbf{I}_{N-1+L})$ is AWGN.

3. Concatenated TCM TR-STBC

3.1. Time Reversal STBC

TR-STBC [1] is the natural extension of orthogonal STBC based on generalized orthogonal designs (GOD) to the frequency-selective channels, in the sense that the optimal receiver is formed by matched filtering and combining and that detection of the data streams sent by the antennas is perfectly decoupled. However, while for orthogonal STBC in frequency-flat fading post-combining symbol-by-symbol detection is optimal, for TR-STBC in frequency-selective fading the resulting channel after combining is an ISI channel, for which sequence detection or equalization is required.

A $[T, N_t, k]$ GOD is defined by a $T \times N_t$ matrix $\mathbf{S}(\mathbf{x})$ whose entries are elements of $\mathbf{x}$ or their conjugates, where $\mathbf{x}$ is a vector of k symbols from a complex constellation, and such that $\mathbf{S}(\mathbf{x})^H\mathbf{S}(\mathbf{x}) = |\mathbf{x}|^2\mathbf{I}_k$ [8]. Given a $[T, N_t, k]$ GOD with the property that any row contains either elements of $\mathbf{x}$ or elements of $\mathbf{x}^*$, we construct the corresponding TR-STBC by replacing each symbol x_i by a vector of N symbols $\mathbf{x}_i$ followed by $L-1$ zeros, and each symbol x_i^* by the time-reversal vector $\circ\mathbf{x}_i$, also followed by $L-1$ zeros.

The received signal is passed through a bank of filters matched to $\mathbf{g}_1, \ldots, \mathbf{g}_{N_t}$. The N_t matched filter outputs are combined in order to obtain k separate SISO channels with ISI in the form

$$y_{i,n} = \sum_{l=-(L-1)}^{L-1} \gamma_l x_{i,n-l} + \nu_{i,n} \tag{3}$$

where $\gamma_l = \sum_{i=1}^{N_t} \int g_i(t) g_i^*(t-\ell T_s)dt$ is the sum of the channel autocorrelations of all N_t transmit antennas, and where $\{\nu_{i,n}\}$ is a complex Gaussian noise sequence with autocorrelation function $N_0\gamma_l$.

3.2. Concatenation

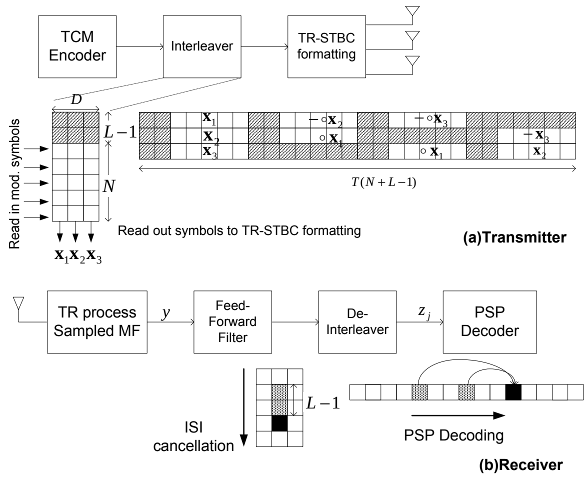

Fig.1 Block diagram of our scheme for $N_t = 3$

Next, we consider the concatenation of TR-STBC with an outer TCM scheme. Namely, the blocks of symbols $\mathbf{x}_i$ are produced by interleaving the output of a TCM encoder. Fig.1(a) shows the block diagram of the proposed concatenated scheme for $N_t = 3$, based on the rate-3/4 STBC with block length $T = 4$ and the following transmit array

$$\mathbf{S}(x_1, x_2, x_3) = \begin{bmatrix} x_1 & x_2 & x_3 \\ -x_2^* & x_1^* & 0 \\ -x_3^* & 0 & x_1^* \\ 0 & -x_3 & x_2 \end{bmatrix}$$

The sequence generated by the TCM encoder, formed of symbols from an M-ary complex constellation (e.g., PSK or QAM), is arranged by rows into a $N \times D$ row-column interleaver. The resulting D vectors of length N are mapped onto the $T(N+L-1)\times N_t$ TR-STBC transmit array (this is shown transposed in Fig.1(a) where the shadowed areas correspond to zeros). The spectral efficiency of the resulting concatenated scheme is given by $\eta = \frac{N}{N+L-1}R_{\text{stbc}}R_{\text{tcm}}$, where R_{stbc} is the rate [symbol/channel use] of the underlying STBC, and R_{tcm} is the rate [bit/symbol] of the outer TCM code.

4. Joint equalization and decoding

ML detection of the overall concatenated scheme is too complex, since it would require running a Viterbi Algorithm (VA) on an expanded trellis, where the number of states depends on the channel length, on the constellation size and on the additional memory introduced by the interleaver. To overcome this problem, we propose a reduced-state joint equalization and decoding approach via PSP. The block diagram of the receiver is shown in Fig.1(b). Since the PSP deals with the causal ISI by using the survivors of the trellis of VA, a pre-filter is necessary to eliminate the anti-causal ISI in (3). Therefore, we introduce the forward filter of the MMSE-DFE prior to VA. Once the ISI channel becomes causal, decoding of the TCM code is done jointly with the feedback filter of the MMSE-DFE by using the tentative decisions found on the survivors of the VA. In this way, the VA operates on the trellis of the TCM scheme, irrespectively of the channel length and of the constellation size.

4.1. MMSE-DFE forward filter

In order to compute the MMSE-DFE forward filter with linear complexity in the channel length L and in the TR-STBC block size N, we utilize the block formulation based on Cholesky factorization of [9]. For any of the i-th blocks, we rewrite (3) in block form as

$$\mathbf{y} = \mathbf{\Gamma x} + \boldsymbol{\nu} \tag{4}$$

where $\mathbf{x} = (x_{N-1}, \dots, x_0)^T$, $\mathbf{y} = (y_{N-1}, \dots, y_0)^T$, $\boldsymbol{\nu} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, N_0\mathbf{\Gamma})$ and $\mathbf{\Gamma}$ is a $N \times N$ square Toeplitz matrix where $[\mathbf{\Gamma}]_{j,j+l} = \gamma_l$. The Cholesky factorization gives

$$N_0\mathbf{I} + \mathbf{\Gamma} = \mathbf{B}^H\mathbf{\Delta}\mathbf{B} \tag{5}$$

where $\mathbf{B}$ is an upper triangular with unit diagonal (causal and monic) and $\mathbf{\Delta} = \mathrm{diag}(\sigma_{N-1}, \dots, \sigma_0)$ is a diagonal matrix with positive real diagonal elements. The feedback filter matrix is equal to $\mathbf{B} - \mathbf{I}$, which is strictly causal. Schur algorithm computes this factorization with linear complexity in L and N by considering the banded Toeplitz structure of $\mathbf{\Gamma}$ where each row of contains at most $2L - 1$ non-zero elements. The MMSE-DFE forward filter is given by

$$\mathbf{F} = \mathbf{\Delta}^{-1}\mathbf{B}^{-H} \tag{6}$$

The output of this filter can be obtained efficiently by applying back-substitution to $\mathbf{y}$, yielding linear complexity in L and N.

4.2. Per-Survivor Processing

Let z_j be the sequence of symbol-rate samples obtained after forward filtering and deinterleaving. Due to the structure of the interleaver, the decisions in the decision-feedback section of the equalizer can be found on the survivors of a VA acting on the original TCM trellis (i.e., without state expansion!). Hence, the branch metric at the j-th trellis step extending from state s is given in the form

$$m_j(s,x) = \left| z_j - \left(1 - \frac{1}{\sigma_n}\right) x - \sum_{\ell=1}^{L'} b_{n,\ell}\widehat{x}_{j-\ell D}(s) \right|^2 \tag{7}$$

where $n = \lfloor j/D \rfloor$, $L' = \min(L, n)$, x is the constellation symbol labeling the trellis branch, $\widehat{x}_{n-\ell D}(s)$ are the tentative decisions found on the surviving path terminating in state s, and $(b_{n,1}, \dots, b_{n,L})$ are the coefficients of the MMSE-DFE feedback filter where $b_{n,\ell} = [\mathbf{B}]_{N-1-n,N-1-n+l}$.

The proposed scheme is motivated by the following result:

Proposition 1. Under Gaussian random coding and in the limit for $D \to \infty$ (infinite interleaving depth), for a given channel realization $\{\gamma_\ell\}$, the proposed joint equalization and decoding scheme achieves the information rate

$$I = \int_{-1/2}^{1/2} \log_2 \left(1 + \frac{\mathcal{E}}{N_0}\Gamma(f)\right) df \tag{8}$$

where $\Gamma(f) \triangleq \sum_\ell \gamma_\ell e^{-j2\pi\ell f}$. □

The information rate I in (8) is the maximum achievable rate for the ISI channel (3) under the constraint that the input signal has constant power spectral density equal to $\mathcal{E}$.

As far as the achievable diversity is concerned, we have the following result:

Proposition 2. Under Gaussian random coding and in the limit for $D \to \infty$, the concatenated TCM TR-STBC scheme achieves maximum diversity N_tL_p, where L_p is the number of separable paths of the underlying physical channel. □

We hasten to say that Proposition 2 does not imply in general that the proposed scheme for a finite practical value of D and a given TCM scheme over some discrete alphabet achieves maximum diversity. Unfortunately, the analysis of the PSP decoding technique for finite D is very hard, if not impossible. Nevertheless, our numerical results demonstrate that, somewhat surprisingly, in all cases considered it is sufficient to choose D equal to a small integer in order to obtain a frame-error rate (FER) curve with high-SNR behavior $O(\mathrm{SNR}^{-N_tL_p})$, i.e., with maximum diversity.

5. Frame Error Rate Analysis

In this section, we provide two approximations to FER of the proposed TR-STBC TCM scheme with PSP reduced state decoding. These approximations are based on the following idea: suppose that a genie helps the PSP decoder by removing some ISI. Let the decoder input after genie-aided interference removal be modeled by

$$z_j = \sqrt{\beta} x_j + w_j \tag{9}$$

where $E[|w_j|^2] = 1$, $E[|x_j|^2] = \mathcal{E}$ and where $\beta\mathcal{E}$ is the signal to interference plus noise ratio (SINR). We make a Gaussian approximation of the residual interference plus noise w_j and we use the union bound to the FER of the TCM scheme. We find [10]

$$P_w \leq \mathrm{E}_\beta \left[\min \left\{ 1, K \sum_d A_d Q \left(\sqrt{\frac{\mathcal{E} d^2 \beta}{2}} \right) \right\} \right] \tag{10}$$

where K denotes the frame length in trellis steps and A_d is the average number of *simple* error events at squared Euclidean distance d.

Matched Filter Bound (MFB)

If we assume that the genie removes all ISI, then $\beta = \gamma_0$, and the Gaussian approximation on w_j is exact. This is the so-called matched-filter bound. It is not an exact lower bound since in (10) we use the union bound (an upper bound of a lower bound yields just an approximation). However, extensive simulation results indicate that the MFB yields always a lower bound to the FER.

Genie-Aided MMSE-DFE Bound (GAB)

If we assume that the genie provides exact decisions on each survivor (i.e., the MMSE-DFE works under the ideal feedback assumption), then

$$\beta = \exp \left\{ \int_{-1/2}^{1/2} \ln \left(1 + \frac{\mathcal{E}}{N_0} \Gamma(f) \right) df \right\} - 1. \tag{11}$$

where $\Gamma(f)$ was defined before. In this case, since the term w_j in (9) contains both noise and anti-causal ISI, the Gaussian Approximation is not exact. Quite surprisingly, simulations show that this approximation is very tight and predicts very accurately the FER performance of our scheme with actual PSP (i.e., without ideal decision feedback).

6. Numerical Results

In order to evaluate the performance of the proposed system, simulations were performed in the following conditions. Two ISI channel models are considered: a symbol-spaced P-path channel with the equal strength paths and the pedestrian channel B [11] for the TD-SCDMA 3G standard [12]. The Ungerboeck TCM codes are used for different modulation so as to implement adaptive modulation easily. By ignoring the loss due to zero-padding, SNR is defined as $SNR \triangleq \frac{N_t \mathcal{E}}{N_0} = R_{stbc} \frac{E_b}{N_0}$ where we used $E_b \eta = N_t R_{stbc} \mathcal{E}$. Fig.2 compares our proposed system with other schemes for $\eta = 2$[bit/ch.use]. We have $N_t = 2$ and 2-path ISI channel. The corresponding outage probability is shown for comparison. Similar performance is obtained by the ST-BICM schemes of [2, 6] and the trellis STC scheme of [5] with higher complexity. Both these schemes use an iterative space-time turbo equalizer consisting of two soft-input soft-output MAP decoders. One for the ISI channel with 8-, 16- states for [2], [5, 6] and another for the trellis code (16-, 32-states for [2, 5], [6] respectively). The complexity of BCJR algorithm is roughly twice as high as a Viterbi decoder and 5 turbo iterations are performed. Therefore, our scheme with the 64-state TCM outperforms the iterative schemes with far less complexity.

Fig.3 shows the performance of our PSP scheme compared to the outage probability for different modulation schemes over 4-path ISI channel for $N_t = 2$. 4-state Ungerboeck TCM is used. Two approximate bounds as well as the simulation results of PSP and genie-aided-detector with perfect feedback assumption are provided. The PSP plots are found between the MFB and the GAB and they coincide with the genie-aided-detector plots. This means that with an interleaver depth $D = 4$ the decision from survivors is sufficiently reliable so that PSP could eliminate perfectly the ISI. We can observe also that for any modulation the gap between the outage probability and the PSP plots remains constant, which shows the optimality of our TR-STBC TCM scheme with two transmit antennas in the sense of a tradeoff between diversity and multiplexing [13].

Finally, Fig.4 shows a $N_t = 4$ antenna system over the pedestrian channel B. To construct a TR-STBC array, we concatenated 16-state Ungerboeck TCM and the rate-3/4 GOD [8, 4, 6] [14]. Even on a realistic channel model where the number of separable path L_p is smaller than the number of physical paths, the PSP has almost the same slope as the outage probability at high SNR, which shows that the maximum diversity of $N_t L_p$ is achieved. However, unlike the result in Fig.3, the gap between the outage probability and PSP increases as a constellation becomes larger. This fact is well-known and it is due to the non-optimality of GODs for $Nt > 2$ [13].

7. Conclusion

In this paper, we proposed a concatenated TR-STBC TCM scheme for single-carrier frequency selective fading channel. Thanks to a joint equalization and decoding approach, our scheme achieves much lower complexity with similar/superior performance than other proposed schemes with the same spectral efficiency. As an application of our TR-STBC TCM scheme, we can envisage a MISO down link scenario such as TD-SCDMA. This system is based on slotted quasi-synchronous CDMA at 1.28 Mchip/s ($\sim$

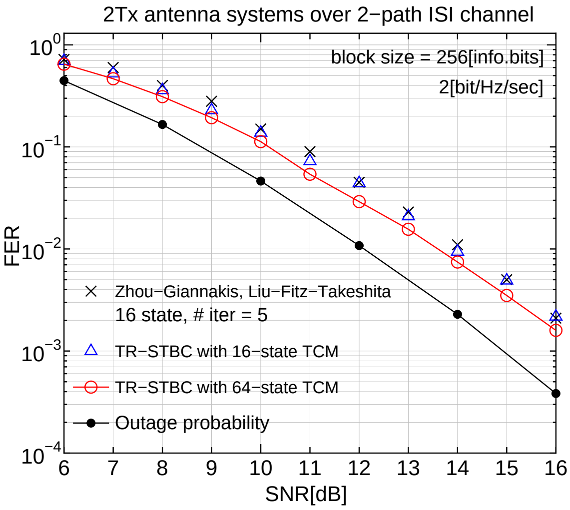

Fig.2 Comparison with more complex schemes

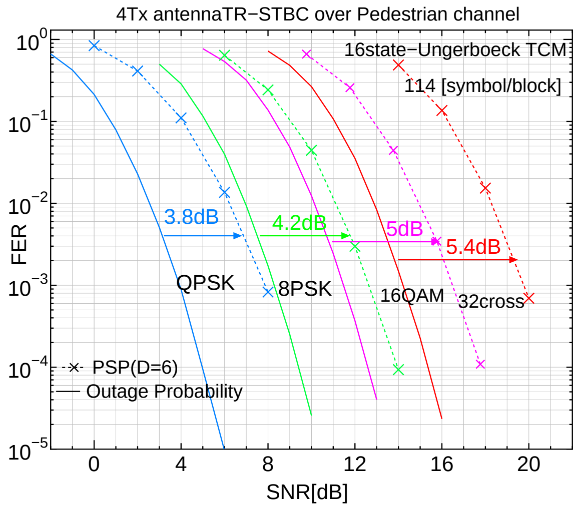

Fig.4 transmit antenna system over Pedestrian channel

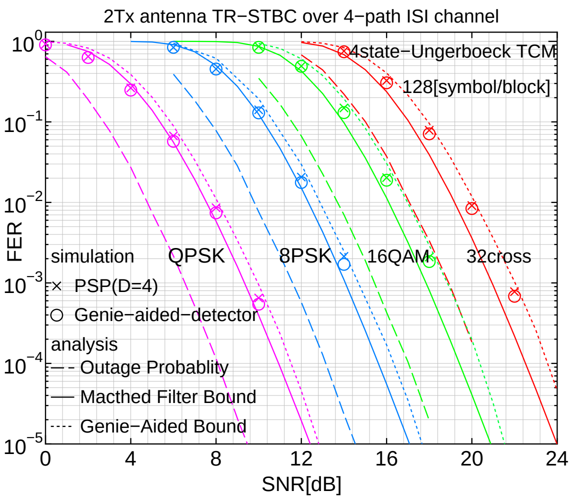

Fig.3 Comparison with outage probability

2MHz bandwidth). A slot structure (675 μs) is following: two blocks of 352 chips are separated by 144 chips midamble for channel estimation. At the end of the second block, 16 chips of guard interval (GI) are added for slot separation. With 128 chips plus 16 chips of GI (total 144 chips) we can estimate easily 4 channels of length 16 chips in the frequency domain, using an FFT of length 128. For example, we can apply rate-3/4 TR-STBC for 4 antennas with a 8-PSK TCM code. Using $N = 76$, $L - 1 = 16$, and $R_{tcm} = 2$, the resulting spectral efficiency is: $\eta = \frac{3}{4}\frac{76\times 8}{864}R_{tcm} = 1.056$ [bit/chip]. This yields 1.28 Mb/s on a single carrier. On three carriers (equivalent to the 5 MHz European UMTS), we obtain 3.84 Mb/s, well beyond the target of 2Mb/s. We can conclude that our TR-STBC TCM scheme can be a good candidate for the downlink scenario of TD-SCDMA.

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