CHANNEL MODELING AND ASSOCIATED INTER-CARRIER INTERFERENCE EQUALIZATION FOR OFDM SYSTEMS WITH HIGH DOPPLER SPREAD

Maxime Guillaud, Dirk T.M. Slock

Eurecom Institute
2229 route des Crêtes, B.P. 193, 06904 Sophia Antipolis Cedex, FRANCE
Tel: +33 4 9300 2647/2606; Fax: +33 4 9300 2627
{maxime.guillaud,dirk.slock}@eurecom.fr

ABSTRACT

In this paper, we address the problem of OFDM transmission over a time-varying, frequency-selective channel with high Doppler spread. This creates situations where the channel significantly evolves over the time span of one OFDM symbol. We analyze the CP-OFDM transmission mechanism, and the impairments due to the channel variations, using a decomposition of these variations over a base of sinusoid functions sampling the Doppler spectrum at subcarrier frequencies. On the one hand, this leads to a fairly parsimonious parameterization of the time-varying channel impulse response. On the other hand we show that, considering a whole CP-OFDM symbol, this leads to a duality between equalization of the delay spread of a time-varying channel in time domain, and the equalization of the Doppler spread of a frequency selective channel in frequency domain. Using this duality, we show that equalization in the frequency domain can benefit from all known time domain equalization methods.

1. INTRODUCTION

The problem of inter-carrier interference (ICI) in orthogonal frequency-division multiplexing (OFDM) transmission has been largely ignored untill recently, and the common assumption that the channel would not vary over one OFDM symbol has rarely been questioned. Recently, the need to consider fast-varying channels for OFDM transmission arose. The influence of uncompensated ICI was investigated by Li in [1], where bounds on the ICI power are derived from the Doppler spread of the channel. Attempts to combat the effects of ICI have been presented, such as Gorokhov and Linnartz's proposal to use the Taylor expansion of the Doppler effect for equalization [2].

Since the channel varies continuously, and since it can not be estimated at every instant in order to limit the amount of training information, the receiver has to make assumptions on the maximum variation speed of the channel coefficients (*i.e.* Doppler spread) and to rely on interpolation. This is used for instance by Stamoulis, Diggavi and Al-Dhahir in [3], where the channel state is linearly interpolated in time between samples over an OFDM symbol. They also proposed an associated time-varying filtering of high complexity to make the channel (almost) time-invariant over an OFDM symbol.

The transformation of time-varying parameters into time-invariant coefficients over an observation interval by expanding the parameter waveforms in a set of basis functions has been introduced by Yves Grenier in the early 80's for non-stationary ARMA models, and was reintroduced in 1996 by Tsatsanis and Giannakis. More recently still, Sayeed etal proposed in [4] to represent channel variations using a set of fixed basis functions, and to do all the processing in the new "canonical" coordinate system based on Doppler, multipath and direction of arrival. We'll show that these canonical coordinates for temporal channel variations are particularly suited for OFDM (or CP-SC) systems since it leads to a description of Doppler spreading as FIR filtering in frequency domain. We also show that the number of taps of this filter is proportional to the Doppler spread.

2. CHANNEL MODEL

Let us consider the transmission of a complex signal over a fast-varying channel. The signal is defined in discrete-time by the complex values s_k, $k = -\infty \ldots +\infty$, and is transmitted at sample rate T_s.

2.1. Continuous time channel model

We consider a multipath, time-varying complex channel $h(t, \tau)$, where t is the time and τ the lag. Hence, $h(t, \cdot)$ is the channel impulse response as seen by the signal received at time t. For the sake of simplicity, we include in h the pulse-shaping filter at the TX and the receiver filter. The spectrum of $h(t, \tau)$ with respect to the time variable t (*i.e.*

the Doppler spectrum) has a finite support $[-B_D, \; B_D]$ for every τ. Its spectrum with respect to the lag variable τ has a finite effective bandwidth, due to the pulse-shaping filter. For the sake of simplicity, we assume in the sequel that synchronization is such that the channel is causal, starts at time 0 and has a finite delay spread τ_{max}. This model yields the received signal

$$r(t) = \sum_{k=-\infty}^{+\infty} s_k \, h(t, t - kT_s) + v(t) \; . \qquad (1)$$

2.2. Discrete time channel model

Let us discretize the received signal at the sample rate T_s (this typically satisfies Nyquist due to unused subcarriers) according to

$$r_p = r(t_0 + pT_s), \quad 0 \le t_0 \le T_s \; .$$

We also discretize h along both temporal dimensions (time t and lag τ) with the same step T_s and epoch t_0, and denote

$$h_{p,l} = h(t_0 + pT_s, t_0 + lT_s)$$

for integer p and l. Let $L = \lceil \tau_{max}/T_s \rceil$ be the number of non-zero coefficients in the discretized impulse response. Hence, letting $l = p - k$, (1) can be rewritten as

$$r_p = \sum_{k=-\infty}^{+\infty} s_k \, h_{p,p-k} = \sum_{l=0}^{L-1} s_{p-l} \, h_{p,l} + v_p \; . \qquad (2)$$

3. OFDM TRANSMISSION

For OFDM symbol d, we gather the N frequency-domain constellation symbols c_k to be transmitted in

$$\underline{c}_d = [c_{dN} \cdots c_{dN+N-1)}]^T \qquad (3)$$

The time-domain equivalent is obtained via the N-point inverse DFT

$$\underline{s}_d = F^{-1}\underline{c}_d \qquad (4)$$

In order to avoid interference between consecutive OFDM symbols, we assume that a cyclic prefix [5] of length P ($P \ge L - 1$) is used, *i.e.* the last P samples of $\underline{s}_d$ are prepended in the time-domain to the OFDM symbol itself before transmission. The cyclic prefix insertion is represented by

$$C = \begin{bmatrix} 0_{P \times N - P} & I_P \\ I_N \end{bmatrix}$$

It yields the transmitted signal

$$\underline{s}'_d = \begin{bmatrix} s_{d(N+P)-P} \cdots s_{d(N+P)+N-1} \end{bmatrix}^T = C\,\underline{s}_d \qquad (5)$$

and the corresponding received signal

$$\underline{r}'_d = \begin{bmatrix} r_{d(N+P)-P} \cdots r_{d(N+P)+N-1} \end{bmatrix}^T \; . \qquad (6)$$

In order to simplify the notations, and w.l.o.g., let us consider the OFDM symbol $d = 0$, and denote $\underline{s}' = \underline{s}'_0$ and $\underline{c} = \underline{c}_0$. The multipath channel is represented by

$$H' = \begin{bmatrix} h_{-P,0} & 0 & \cdots & 0 \\ \vdots & h_{1-P,0} & \ddots & \vdots \\ h_{L-1-P,L-1} & \vdots & \ddots & 0 \\ 0 & h_{L-P,L-1} & & \\ & & \ddots & h_{N-1,0} \\ \vdots & \ddots & & \vdots \\ 0 & \cdots & 0 & h_{N+L-2,L-1} \end{bmatrix}$$

which yields

$$\underline{r}' = H'\underline{s}' + \underline{v}' \; . \qquad (7)$$

Then, the cyclic prefix removal operation consists in discarding the first P values of the received signal, which in general contain interference from the previous OFDM symbol. The last $L - 1$ samples in $\underline{r}'$ are themselves interfering with the following OFDM symbol, and should be ignored as well. Therefore, in the sequel we will only consider

$$\underline{r} = D\underline{r}' \qquad (8)$$

where

$$D = \begin{bmatrix} 0_{N \times P} & I_N & 0_{N \times L-1} \end{bmatrix}$$

Note that $\underline{r}'$ is not exactly equal to $\underline{r}'_0$ since this representation does not take into account the interference from the previous OFDM symbol. Nevertheless, the interference only affects the first $L - 1$ values in $\underline{r}'$. Since the first P values are discarded by D, and since $P \ge L - 1$, the values in $\underline{r}$ are correct.

Finally, the frequency-domain equivalent of $\underline{r}$ is obtained by DFT

$$\underline{u} = F\underline{r} = FDH'CF^{-1}\underline{c} + \underline{w} \; . \qquad (9)$$

4. CANONICAL CHANNEL REPRESENTATION

Let us focus on the properties of the equivalent channel between $\underline{c}$ and $\underline{u}$. In particular, the effect of the pre- and postmultiplication by D and C respectively, is to create an equivalent square channel matrix $H = DH'C$ with the following

structure

$$H = \begin{bmatrix} h_{0,0} & & h_{0,L-1} & \cdots & & h_{0,1} \\ \vdots & h_{1,0} & & & \ddots & \vdots \\ \vdots & & & & & h_{L-2,L-1} \\ h_{L-1,L-1} & \vdots & & & & \\ & h_{L,L-1} & \ddots & & & \\ & & & \ddots & & \\ & & & & \ddots & \\ & & & & & h_{N-1,0} \end{bmatrix}$$

In a quasi-static environment (defined by $NB_DT_s \ll 1$), H is a circulant matrix, i.e. $\forall(p,l), h_{p,l} = h_{0,l}$. It is well known that the consequence of this is that the frequency-domain expression of the channel is a diagonal matrix : FHF^{-1} has non-zero values on its diagonal only, and their amplitude represents the channel gain on each frequency subband. Hence, in this case, equalization becomes a trivial operation.

Contrarily, we choose to focus on the case where the channel coefficients vary in time during one OFDM symbol. In this case, H is not circulant anymore, and a more involved analysis is necessary.

Let us now decompose the variations of each $h_{p,l}$ over one OFDM symbol along the lines exposed in [4], over a base of sinusoid functions. We choose to use F_s/N-spaced sinusoids, i.e. using the same spacing as the subcarriers. Since we need to represent the varying channel coefficients over a time period of NT_s, a sampling frequency in frequency domain of F_s/N allows to satisfy Nyquist. In order to cover all the Doppler spectrum, we define $M = \lceil NB_D T_s \rceil$ and decompose

$$h_{p,l} = \sum_{m=-M}^{M} a_{l,m} e^{j\frac{2\pi mp}{N}}, \qquad \begin{matrix} p = 0 \ldots N-1 \\ l = 0 \ldots L-1 \end{matrix} \qquad (10)$$

In this paper, we assume that the coefficients describing the channel variations ($a_{l,m}, 0 \le l \le L, -M \le m \le M$) are perfectly known.

Equivalently, in matrix form, this base can be represented by $\{Q^m, m = -M \ldots M\}$, where

$$Q = \text{diag}\left(1, e^{j\frac{2\pi}{N}}, \ldots, e^{j\frac{2\pi(N-1)}{N}}\right)$$

By defining the $N \times N$ circulant shift matrix

$$J = \begin{pmatrix} 0 & & & 1 \\ 1 & \ddots & & \\ & \ddots & \ddots & \\ & & 1 & 0 \end{pmatrix}$$

we can rewrite the equivalent channel matrix as

$$H = \sum_{l=0}^{L-1} \sum_{m=-M}^{M} a_{l,m} J^l Q^m . \qquad (11)$$

4.1. Time-Frequency Duality

Let us point out that the DFT has interesting properties on Q and J :

$$FQF^{-1} = J, \quad \text{and} \quad FJF^{-1} = Q^* \qquad (12)$$

(complex conjugate). Which yields

$$A = FHF^{-1} = \sum_{l=0}^{L-1} \sum_{m=-M}^{M} a_{l,m} Q^{l*} J^m \qquad (13)$$

Comparing equations (11) and (13), we see a duality between the time-domain and the frequency-domain representations of the channel. Both A and H are band matrices, with respectively L and $2M + 1$ non-zero diagonals. As we will see in the sequel, the number of non-zero diagonals has a direct influence on equalization complexity.

4.2. Approximation Error

Since a signal cannot be simultaneously finite duration and finite bandwidth, there will be some approximation error in the canonical representation that we shall analyze here. Introduce the Toeplitz matrices

$$R_l^h[n] = \begin{bmatrix} r_{n(N+P),l}^h & \cdots & r_{n(N+P)-N+1,l}^h \\ \vdots & \ddots & \vdots \\ r_{n(N+P)+N-1,l}^h & \cdots & r_{n(N+P),l}^h \end{bmatrix} \qquad (14)$$

where $r_{n,l}^h = E\, h_{p+n,l} h_{p,l}^*$. Let $V_m = [1 \cdots 1]Q^{m*}$ and

$$\underline{a}_l[n] = \begin{bmatrix} a_{l,-M}[n] \\ \vdots \\ a_{l,M}[n] \end{bmatrix} = \begin{bmatrix} V_{-M} \\ \vdots \\ v_M \end{bmatrix} \begin{bmatrix} h_{n(N+P),l} \\ \vdots \\ h_{n(N+P)+N-1,l} \end{bmatrix} \qquad (15)$$

or $\underline{a}_l[n] = V\, \underline{h}_l[n]$. Straight LS projection leads to the approximation error

$$\widetilde{\underline{h}}_l[n] = P_{V^H}^{\perp}\, \underline{h}_l[n] \qquad (16)$$

where $P_X^{\perp} = I - P_X$, $P_X = X(X^HX)^{-1}X^H$. The associated MSE is

$$\begin{aligned} E\|\widetilde{\underline{h}}_l[n]\|^2 &= \frac{1}{N} \sum_{i=M+1}^{N+M-1} E\, \|V_i \underline{h}_l[n]\|^2 \\ &= \frac{1}{N} \sum_{i=M+1}^{N+M-1} V_i R_l^h[0]\, V_i^H \\ &= \frac{1}{N} \sum_{i=M+1}^{N+M-1} S_l^h(f) * \left(\frac{\sin Nf}{N \sin f}\right)^2 \Bigg|_{f=\frac{i}{N}} \end{aligned} \qquad (17)$$

where $S_l^h(f)$ is the Doppler spectrum of $h_{p,l}$. The convolution of the Doppler spectrum with the spectrum of the Bartlett window leads to some bandwidth extension. The error level can be kept low though by increasing somewhat the number of terms M in the canonical basis expansion.

5. APPLICATIONS TO EQUALIZATION

Using the proposed channel model, the equalization becomes in the frequency domain recovering $\underline{c}$ from $\underline{u} = A \underline{c} + \underline{w}$. This fact alone is not particularly engaging, since A is matrix of size $N \times N$, and N is rather large for usual communications systems. Nevertheless, a closer look at the decomposition of A in equation (13) shows that its structure is a circulant banded matrix. This can be regarded as a $(2M + 1)$-tap frequency-varying cyclic FIR filter in the frequency domain. This situation should be compared to the equivalent equalization effort in the time domain, which would involve equalizing a length-L time-varying FIR channel.

All the known equalization methods that apply to this case in the time domain are useable. Let us mention, among other methods, Viterbi ML equalization. Decision-feedback equalization (DFE) can also play an interesting role in this framework, since one of its drawback in classical systems is the need for an initialization sequence. In actual OFDM systems, muted tones used as guard bands may prove to be convenient sets of initialization sequences for DFE-based systems. Whereas MMSE equalization methods would require matched filtering and spectral factorization, fairly simple iterative implementations of ZF equalizers are straightforward to derive. Indeed, the usual frequency domain equalizer of OFDM would produce $(\mathrm{diag}\,(A))^{-1} \underline{u}$ which would in the absence of ICI allow to recover $\underline{c}$ since the diagonal part of $(\mathrm{diag}\,(A))^{-1}A$ is I_N. However, due to ICI the non-diagonal elements are non-zero though typically small. This allows us to propose the following iterative implementation of a ZF LE: in iteration (i)

$$\underline{c}^{(i)} = (\mathrm{diag}\,(A))^{-1}\underline{u} + (I - (\mathrm{diag}\,(A))^{-1}A)\,\underline{c}^{(i-1)} \quad (18)$$

with $\underline{c}^{(0)} = 0$. An iterative implementation of a ZF DFE can be obtained by passing $\underline{c}^{(i-1)}$ through a decision device, soft or hard, with or without channel decoding/reencoding, before continuing to iteration i.

6. CHANNEL PARAMETER ESTIMATION

For training based channel estimation in slowly varying environments, one can afford to periodically insert complete OFDM training symbols. Such schemes are not appropriate here since due to the channel variation speed, virtually every OFDM symbol would have to be training. Instead, the OFDM symbol should contain a mix of data and pilot tones. Requiring that the pilot tones be sufficient to allow (time-varying) channel identifiablity on the basis of a single OFDM symbol might be asking a bit too much for systems with both large delay and Doppler spreads. The iterative equalization techniques just mentioned may also be applied for channel estimation on the basis of pilot tones. In general though, the pilot tone design problem would be similar to the training sequence design for the estimation of a time-varying channel with delay spread in the time domain.

Due to the large number of channel parameters and the uncertainty in delay and Doppler spreads, MMSE channel estimation should outperform deterministic estimation significantly. The prior info for MMSE estimation is composed of power delay and Doppler profiles. Inter OFDM symbol channel correlation allows for inter OFDM symbol prediction of the channel or more precisely its expansion coefficients $\underline{a}_l[n]$. Neglecting the delay spread in the pulse shape, we get that these coefficients are more or less decorrelated across l. The correlation between OFDM symbols is described by $R_l^a[n] = \mathrm{E}\,\underline{a}_l[m+n]\underline{a}_l^H[m] = V\,R_l^h[n]\,V^H$ which is very much dominated by its diagonal. As a result, each coefficient $a_{l,m}[n]$ can be predicted separately in time n. The predicted values form a priori information for the estimation in the current OFDM symbol period.

7. CONCLUSIONS

We proposed a new modeling scheme for time-varying, frequency-selective channels with high Doppler spread, that proves particularly convenient when used in conjunction with CP-OFDM modulation. We showed that this model enables the receiver to exploit the duality between Doppler spread in OFDM systems and delay spread in classical systems, and that virtually any equalization algorithm existing in the time domain can be transposed into the frequency domain.

8. REFERENCES

[1] Y. Li and L. J. Cimini Jr., "Bounds on the interchannel interference of OFDM in time-varying impairments," *IEEE Trans. Comm.*, vol. 49, no. 3, pp. 401–404,2001.

[2] A. Gorokhov and J.-P. Linnartz, "Robust OFDM receivers for dispersive time varying channels : Equalization and channel acquisition," in *Proc. ICC*, 2002, vol. 1, pp. 470–474.

[3] A. Stamoulis, S. N. Diggavi, and N. Al-Dhahir, "Estimation of fast fading channels in OFDM," in *Proc. IEEE Wireless Communications and Networking Conference (WCNC)*, 2002, vol. 1, pp. 465–470.

[4] E. N. Onggosanusi, A. M. Sayeed, and B. D. Van Veen, "Canonical space-time processing for wireless communications," *IEEE Transactions on Communications*, vol. 48, no. 10, pp. 1669–1680, October 2000.

[5] Z. Wang and G. B. Giannakis, "Wireless multicarrier communications: Where Fourier meets Shannon," *IEEE Signal Processing Magazine*, vol. 17, no. 3, pp. 29–48, May 2000.