On MIMO Capacity with Partial Channel Knowledge at the Transmitter

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Abstract

The maximum achievable capacity for a MIMO channel corresponds to the waterfilling solution provided that the transmitter has a perfect knowledge of the channel. In practice, the available knowledge may only be partial due to the time selectivity of the channel and delay of the feedback from the receiver. However, exploiting the partial knowledge leads to a significant improvement when compared to the capacity without any channel knowledge. In this paper we analyze the MIMO capacity with partial knowledge of the channel under practical frequency flat channel models.

1 Introduction

The introduction of Multi Input Multi Output (MIMO) systems leads to a significant increase in communication capacity. To take advantage of the use of MIMO systems, various space-time coding schemes have been proposed. These techniques assume the elements of the channel matrix to be i.i.d. In practice this assumption may not always be valid, since for physical reasons the channel components may be correlated [1]. This correlation corresponds to partial knowledge that can be fed back to the transmitter. When the partial channel knowledge is present at the transmitter, it is advantageous to use this information to optimize the precoder at the transmission [2, 3]. This precoder will basically be a cascade of space-time coder and a decorrelating beamformer.

In this paper, we investigate the achievable capacity given the available channel state information at the transmitter. We assume that, in addition to the channel correlations, the transmitter has more information about the channel: knowledge of slowly varying channel parameters, or knowledge of the channel up to the amplitude and phase shifts that arise when the roles of transmitter and receiver are reversed. We demonstrate how the partial knowledge of the channel leads to an improvement of the communication capacity when compared to the capacity without any channel knowledge. However, the additional improvement when compared to knowing only the channel correlations is demonstrated to be small. We note that similar results (for different channel models) have also been published in [2].

Throughout this article scalar quantities are denoted by regular lowercase letters. Lower case bold type faces are used for vectors and regular uppercase letters for matrices. Superscripts T and H denote the transpose and conjugate transpose, respectively. We use $\mathrm{diag}\{\boldsymbol{A}\}$ to denote the diagonal matrix of the diagonal elements of the matrix $\boldsymbol{A}$ and $\mathrm{tr}\{\boldsymbol{A}\}$ ($\det\{\boldsymbol{A}\}$) for the trace (determinant) of the matrix $\boldsymbol{A}$.

2 Channel models and assumptions

We consider a MIMO communication system with N receive and M transmit antennas. The received $N \times 1$ signal vector is given by

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{v}, \tag{1}$$

where $\boldsymbol{H}$ is an $N \times M$ random channel matrix, $\boldsymbol{x}$ is an $M \times 1$ transmitted signal vector and $\boldsymbol{v}$ is an $N \times 1$ noise vector, which is assumed to be complex circular Gaussian with covariance matrix $\sigma_v^2\boldsymbol{I}$. The channel covariance matrix at transmitter is defined as $\boldsymbol{\Sigma} = E\{\boldsymbol{H}^H\boldsymbol{H}\}$. We use normalization $\mathrm{tr}\{\boldsymbol{\Sigma}\} = 1$.

The ergodic capacity for the channel (1) is given by [5]

$$\mathcal{C} = E_H\left\{\log\det\left[I + \rho\boldsymbol{H}\boldsymbol{Q}\boldsymbol{H}^H\right]\right\}, \tag{2}$$

where $\rho = \frac{P}{\sigma_v^2}$ is the SNR and $P\boldsymbol{Q}$ is a covariance matrix of the transmitted Gaussian signals maximizing the above expression, under the power constraint $\mathrm{tr}\{\boldsymbol{Q}\} \le 1$. The expectation is calculated with respect to the channel distribution.

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2.1 Pathwise channel model

The pathwise model [4] for the channel matrix in the case of frequency flat fading is

$$\boldsymbol{H} = \sum_{l=1}^{L} c_l \boldsymbol{a}_l \boldsymbol{b}_l^T, \tag{3}$$

where L is the number of multipaths and c_l, $i = 1, \ldots, L$ denote the complex multipath amplitudes. We assume that the amplitudes c_l are i.i.d. circular symmetric complex Gaussian distributed with mean 0 and variance 1. The $N \times 1$ vectors $\boldsymbol{a}_i$ are the steering vectors of the receive antenna array and the M-vectors $\boldsymbol{b}_l$ are the steering vectors of the transmitting antenna array. Due to the i.i.d. assumption of the complex amplitudes, it is assumed that the multipath variances are included in the vectors $\boldsymbol{b}_i$. We also normalize $||\boldsymbol{a}_i||^2 = 1\ \forall i$. Generally all c_l, $\boldsymbol{a}_l$ and $\boldsymbol{b}_l$ are random variables. The complex amplitudes c_l model the fast fading channel parameters and the steering vectors model the slowly fading channel parameters.

The channel matrix may also be given as

$$\boldsymbol{H} = \boldsymbol{ACB}, \tag{4}$$

where $\boldsymbol{A} = [\boldsymbol{a}_1, \ldots, \boldsymbol{a}_L]$, $\boldsymbol{B} = [\boldsymbol{b}_1, \ldots \boldsymbol{b}_L]^T$ and $\boldsymbol{C} = \text{diag}\{c_1, \ldots, c_L\}$. If for every channel usage the receiver knows the realization of the channel and the slowly fading parameters remain constant over a sufficient time interval, the slowly fading parameters may be obtained at the receiver [6], and fed back to the transmitter. This information then corresponds to partial channel state information at the transmitter.

We investigate the ergodic capacity of the channel given in (4) when $\boldsymbol{A}$ and $\boldsymbol{B}$ are fixed.

2.2 Channel models for limited reciprocity

Assume that the physical channel is reciprocal between uplink and downlink, and the transmitter knows the uplink channel $\boldsymbol{W}^T$. The overall channel in downlink including the cabling and electronic devices for both ends is therefore

$$\boldsymbol{H} = \boldsymbol{D}_1 \boldsymbol{W} \boldsymbol{D}_2,$$

where $\boldsymbol{D}_1$ and $\boldsymbol{D}_2$ are diagonal matrices. These matrices reflect the amplitude and phase shifts that arise when the roles of transmitter and receiver are reversed in case of no or limited calibration. We use three different models for the matrices $\boldsymbol{D}_1$ and $\boldsymbol{D}_2$

Model 1 Only phase shifts: Diagonal elements contain i.i.d. phases ($\boldsymbol{D}_1 = \text{diag}\{e^{j\phi_1^1}, \ldots, e^{j\phi_N^1}\}$ and $\boldsymbol{D}_2 = \text{diag}\{e^{j\phi_1^2}, \ldots, e^{j\phi_M^2}\}$, where ϕ_l^i are i.i.d. and uniformly distributed on $[0, 2\pi]$)

Model 2 Case of complete absence of calibration: Diagonal elements of D_1 and D_2 are i.i.d. zero mean complex circularly symmetric Gaussian with variance 1.

Model 3 Case of imperfect calibration: The diagonal matrices are given by $\boldsymbol{D}_1 = \sqrt{1-\epsilon_1^2}\boldsymbol{I} + \epsilon_1 \boldsymbol{DN}_1$ and $\boldsymbol{D}_2 = \sqrt{1-\epsilon_2^2}\boldsymbol{I} + \epsilon_2 \boldsymbol{DN}_2$, where ϵ_i are small and $\boldsymbol{DN}_1$ and $\boldsymbol{DN}_2$ are diagonal matrices with i.i.d. diagonal elements that are zero mean complex circularly symmetric Gaussian with variance 1.

3 Results for pathwise channel model

In the case of pathwise model, the ergodic capacity for a given transmit covariance matrix $P\boldsymbol{Q}$ is

$$\mathcal{C} = E_C \left\{ \log\det \left[I + \rho \boldsymbol{ACBQB}^H \boldsymbol{C}^H \boldsymbol{A}^H \right] \right\}. \tag{5}$$

For arbitrary SNR (ρ), the optimal $\boldsymbol{Q}$ can be given by direct numerical solution as described later in this paper. In what follows, we first calculate approximations for low and high SNR scenarios.

3.1 Low SNR

When $\rho << 1$, we may approximate (5) by

$$\begin{aligned}\mathcal{C} &\approx E\text{tr}\left\{\rho \boldsymbol{ACBQB}^H \boldsymbol{C}^H \boldsymbol{A}^H\right\} \\ &= \rho E\text{tr}\left\{\boldsymbol{BQB}^H \boldsymbol{C}^H \boldsymbol{A}^H \boldsymbol{AC}\right\} \\ &= \rho\text{tr}\left\{\boldsymbol{BQB}^H \text{diag}\{\boldsymbol{A}^H\boldsymbol{A}\}\right\} \\ &= \rho\text{tr}\left\{\boldsymbol{QB}^H\boldsymbol{B}\right\}.\end{aligned}$$

Note that $\text{diag}\{\boldsymbol{A}^H\boldsymbol{A}\} = \boldsymbol{I}$ due to the normalization. Write $\boldsymbol{B}^H\boldsymbol{B} = \boldsymbol{U\Lambda U}^H$ according to the spectral decomposition, and let $\boldsymbol{Q}' = \boldsymbol{U}^H\boldsymbol{QU}$. Note that $\text{tr}\{\boldsymbol{Q}'\} = \text{tr}\{\boldsymbol{Q}\} = 1$. Now

$$\text{tr}\left\{\boldsymbol{QB}^H\boldsymbol{B}\right\} = \text{tr}\left\{\boldsymbol{Q}'\boldsymbol{\Lambda}\right\}. \tag{6}$$

For any $\boldsymbol{\Lambda}$, the matrix $\boldsymbol{Q}'$ maximizing (6) is given by

$$\boldsymbol{Q}' = \text{diag}\{0, \ldots, 0, 1, 0, \ldots, 0\},$$

where the only nonzero diagonal element is in the position corresponding to the largest diagonal element of $\boldsymbol{\Lambda}$ (if there is no unique maximum, we may choose a position of any of the "maximum" elements).

We have thus shown that for $\rho << 1$, the optimal transmit covariance matrix maximizing 5 is given by

$$\boldsymbol{Q} = \boldsymbol{uu}^H, \tag{7}$$

where $\boldsymbol{v}$ is the eigenvector corresponding to the maximum eigenvalue of the channel covariance matrix

$$\boldsymbol{\Sigma} = \boldsymbol{B}^H \boldsymbol{B}. \tag{8}$$

The optimal covariance matrix thus depends only on the channel covariance matrix at the transmitter.

We note that in this case, the capacity without any channel knowledge ($\boldsymbol{Q} = \frac{1}{M}\boldsymbol{I}$) is given by

$$\frac{\rho}{M}\sum_{i=1}^{M}\lambda_i,$$

where λ_i are the eigenvalues of the matrix given in (8). Hence the ratio between the capacity with partial channel knowledge and the capacity without channel knowledge is given by

$$\frac{\max\{\lambda_i\}}{M^{-1}\sum_{i=1}^{M}\lambda_i} \geq 1.$$

As a conclusion, the gain obtained by using the partial knowledge can be very significant.

3.2 High SNR

When $\rho >> 1$, giving a general solution is not possible, because the optimal covariance matrix $\boldsymbol{Q}$ depends on the dimensions N, M and L, more specifically on the minimum dimension. We now derive the solution for two different possibilities for the minimum dimension.

1. When $L \leq \min\{M, N\}$,

$$\begin{aligned}\mathcal{C} &= \log\det\left[\boldsymbol{I}_L + \rho\boldsymbol{B}\boldsymbol{Q}\boldsymbol{B}^H\boldsymbol{C}^H\boldsymbol{A}^H\boldsymbol{A}\boldsymbol{C}\right] \\ &\approx E_C\left\{\log\det\left[\rho\boldsymbol{B}\boldsymbol{Q}\boldsymbol{B}^H\boldsymbol{C}^H\boldsymbol{A}^H\boldsymbol{A}\boldsymbol{C}\right]\right\} \\ &= \log\det\left[\rho\boldsymbol{B}\boldsymbol{Q}\boldsymbol{B}^H\right] + E_C\log\det\left[\boldsymbol{C}^H\boldsymbol{A}^H\boldsymbol{A}\boldsymbol{C}\right]\end{aligned}$$

   Therefore the solution is given by

$$\boldsymbol{Q} = \frac{1}{L}\boldsymbol{U}\boldsymbol{U}^H, \tag{9}$$

   where $\boldsymbol{U}$ is the matrix of the eigenvectors of $\boldsymbol{\Sigma}$ corresponding to the nonzero eigenvalues.

2. If $M \leq \min\{N, L\}$, by using the same technique as above we get that

$$\mathcal{C} \approx \log\det\{\boldsymbol{Q}\} + \text{constant}. \tag{10}$$

   Hence the solution is given by

$$\boldsymbol{Q} = \frac{1}{M}\boldsymbol{I} \tag{11}$$

In these cases, the difference between the capacity with channel and the capacity without any channel knowledge is given by

$$\min\{M, L\}\log\frac{M}{\min\{M, L\}}.$$

Therefore, when $\rho >> 1$, the gain obtained by using partial knowledge is important especially for large number of transmit and receive antennas and small number of multipaths.

When N is the minimum dimension, it is not possible to isolate $\boldsymbol{Q}$ from the random part of the channel, because the approximation used in the previous cases gives

$$C \approx E_C\log\det\left[\rho\boldsymbol{A}\boldsymbol{C}\boldsymbol{B}\boldsymbol{Q}\boldsymbol{B}^H\boldsymbol{C}^H\boldsymbol{A}^H\right].$$

Since N is the minimum dimension, this expression can not be decomposed any further.

3.3 Waterfilling solution for the channel covariance matrix

Since $\log\det$ is a concave on the set of positive definite matrices, the ergodic capacity for any transmit covariance matrix $\boldsymbol{Q}$ may be upper bounded by

$$\begin{aligned}\mathcal{C} &= E_C\left\{\log\det\left[I + \rho\boldsymbol{A}\boldsymbol{C}\boldsymbol{B}\boldsymbol{Q}\boldsymbol{B}^H\boldsymbol{C}^H\boldsymbol{A}^H\right]\right\} \\ &\leq \log\det\left[I + \rho\boldsymbol{Q}\boldsymbol{B}^H E_C\{\boldsymbol{C}^H\boldsymbol{A}^H\boldsymbol{A}\boldsymbol{C}\}\boldsymbol{B}\right] \\ &= \log\det\left[I + \rho\boldsymbol{Q}\boldsymbol{B}^H\boldsymbol{B}\right].\end{aligned}$$

The optimal $\boldsymbol{Q}$ maximizing this upper bound corresponds to the waterfilling solution applied to $\rho\boldsymbol{\Sigma}$ [5]. It can be shown that the waterfilling solution for $\rho << 1$ and $\rho >> 1$ matches the solutions given in equations (7),(9) and (11).

3.4 Optimal solution

As mentioned above, $\log\det$ is concave on the set of positive definite matrices. The set of positive semidefinite matrices with trace equal to 1 is a convex set. Therefore, the optimum transmit covariance matrix may be found by using numerical methods. In practice, the object function has to be formed by averaging over sufficient number of Monte Carlo realizations. Note that the averaging preserves the concavity of the objective function. We demonstrate the usage numerical methods in Section 5. The applied method is based on projected gradient descent algorithm [7].

3.5 Solution for spatially separable channel model

The MIMO channel is often modeled as a spatially separable channel model. In this channel model the channel is given by

$$\boldsymbol{H} = \boldsymbol{\Sigma}_1^{1/2} \boldsymbol{W} \boldsymbol{\Sigma}_2^{1/2},$$

where $\boldsymbol{W}$ is an $N \times M$ random matrix of i.i.d. complex circular Gaussian elements with mean 0 and variance 1. The matrix $\boldsymbol{\Sigma}_1$ is the receive array covariance matrix and $\boldsymbol{\Sigma}_2$ is the transmit array covariance matrix. It can be shown that the pathwise channel model converges in distribution to the spatially separable model with appropriate covariance matrices, as the number of multipaths tends to infinity [11]. The ergodic capacity for this channel model for the case $\boldsymbol{\Sigma}_1 = \boldsymbol{I}$ has been considered e.g in [8, 9, 3, 10]. It has been shown that for this case the optimal transmit covariance matrix $\boldsymbol{Q}$ has the same eigenvectors as $\boldsymbol{\Sigma}_2$. The capacity achieving power allocation (the eigenvalues of optimal $\boldsymbol{Q}$) have to be calculated by using numerical methods (e.g. gradient descend algorithm). The method used in [3] to show that the eigenvectors of $\boldsymbol{Q}$ correspond to those of $\boldsymbol{\Sigma}_2$ is complex. Here we give a simpler proof of that fact. Let $\boldsymbol{\Sigma}_2 = \boldsymbol{U}\boldsymbol{D}\boldsymbol{U}^H$ be the spectral decomposition of $\boldsymbol{\Sigma}_2$. The ergodic capacity for covariance matrix $\boldsymbol{Q}$ is then given by

$$E \log\det[\boldsymbol{I} + \rho \boldsymbol{\Sigma}_1^{\frac{1}{2}} \boldsymbol{W}\boldsymbol{U}\boldsymbol{D}^{\frac{1}{2}}\boldsymbol{U}^H\boldsymbol{Q}\boldsymbol{U}\boldsymbol{D}^{\frac{1}{2}}\boldsymbol{U}^H\boldsymbol{W}^H\boldsymbol{\Sigma}_1^{\frac{H}{2}}]$$

Since for any $M \times M$ unitary matrix $\boldsymbol{U}$, the distribution of $\boldsymbol{W}$ is the same as the distribution of $\boldsymbol{W}\boldsymbol{U}$, the ergodic capacity may also be written as

$$E_\phi E_W \log\det\left[\boldsymbol{I} + \rho \Sigma_1^{\frac{1}{2}} \boldsymbol{W}\boldsymbol{\Phi}\boldsymbol{D}^{\frac{1}{2}}\boldsymbol{Q}'\boldsymbol{D}^{\frac{1}{2}}\boldsymbol{\Phi}^H\boldsymbol{W}^H\boldsymbol{\Sigma}_1^{\frac{H}{2}}\right],$$

where $\boldsymbol{\Phi} = \mathrm{diag}\{e^{j\phi_1}, e^{j\phi_2}, \ldots, e^{j\phi_M}\}$ with ϕ_i i.i.d. and uniformly distributed on $[0, 2\pi)$, and $\boldsymbol{Q}' = \boldsymbol{U}^H\boldsymbol{Q}\boldsymbol{U}$. Note that $\mathrm{trace}(\boldsymbol{Q}') = \mathrm{trace}(\boldsymbol{Q})$. Since $\log\det$ is concave,

$$\begin{aligned}
&E_\phi E_W \log\det\left[\boldsymbol{I} + \rho \Sigma_1^{\frac{1}{2}} \boldsymbol{W}\boldsymbol{\Phi}\boldsymbol{D}^{\frac{1}{2}}\boldsymbol{Q}'\boldsymbol{D}^{\frac{1}{2}}\boldsymbol{\Phi}^H\boldsymbol{W}^H\boldsymbol{\Sigma}_1^{\frac{H}{2}}\right] \\
&\le E_W \log\det\left[\boldsymbol{I} + \rho \Sigma_1^{\frac{1}{2}} \boldsymbol{W} E_\phi\{\boldsymbol{\Phi}\boldsymbol{D}^{\frac{1}{2}}\boldsymbol{Q}'\boldsymbol{D}^{\frac{1}{2}}\boldsymbol{\Phi}^H\}\boldsymbol{W}^H\boldsymbol{\Sigma}_1^{\frac{H}{2}}\right] \\
&= E_W \log\det\left[\boldsymbol{I} + \rho \Sigma_1^{\frac{1}{2}} \boldsymbol{W}\boldsymbol{D}\mathrm{diag}\{\boldsymbol{Q}'\}\boldsymbol{W}^H\boldsymbol{\Sigma}_1^{\frac{H}{2}}\right].
\end{aligned}$$

The equality is achieved if and only if $\boldsymbol{Q}'$ is a diagonal matrix, and the result follows.

4 Results for channel models with limited reciprocity

In the case of limited reciprocity, the ergodic capacity for transmit covariance matrix $P\boldsymbol{Q}$ is

$$\mathcal{C} = E\left\{\log\det\left[I + \rho \boldsymbol{D}_1\boldsymbol{W}\boldsymbol{D}_2\boldsymbol{Q}\boldsymbol{D}_2^H\boldsymbol{W}^H\boldsymbol{D}_1^H\right]\right\}, \tag{12}$$

where the expectation is calculated with respect to $\boldsymbol{D}_1$ and $\boldsymbol{D}_2$.

By using the technique described in Section 3.5, it is straightforward to show that in the case of Model 1 or Model 2 (only phases or Gaussian zero mean diagonal entries), the optimal transmit covariance matrix has to be diagonal: $\boldsymbol{Q} = \boldsymbol{D}_Q$. For the Model 1, the optimum solution may hence be derived by numerically maximizing

$$\mathcal{C} = \log\det\left[I + \rho \boldsymbol{W}\boldsymbol{D}_Q\boldsymbol{W}^H\right], \tag{13}$$

which is a concave on $\boldsymbol{D}_Q$. We note that for given $\boldsymbol{D}_Q$, (13) is an upper bound of the ergodic capacity for Model 2.

For Model 2, the optimal solution can be found by using numerical methods described in Section 3.4, but the optimization is simpler because it has to be done only for diagonal matrices. For Model 3, optimization is performed as described in Section 3.4.

In addition to the optimal solutions, sub-optimal solutions may be derived by considering the upper bound on ergodic capacity as was done in the case of pathwise model in Section 3.3. For Models 1 and 2, this leads to waterfilling on

$$\rho\mathrm{diag}\{\boldsymbol{W}^H\boldsymbol{W}\},$$

when for Model 3, it leads to waterfilling on

$$\rho\left((1-\epsilon_1^2)\boldsymbol{W}^H\boldsymbol{W} + \epsilon_1^2\mathrm{diag}\{\boldsymbol{W}^H\boldsymbol{W}\}\right).$$

For Model 2, a tighter upper bound is given by (13). Therefore, a better solution may be given by applying the optimal solution for Model 1. For Model 3, waterfilling on $\rho\boldsymbol{W}^H\boldsymbol{W}$ can also be used.

5 Simulation results

5.1 Pathwise model

We first present results of a simulation study for pathwise model. In the simulations, we used Uniform Linear Arrays (ULAs) with half wavelength inter element spacing both at the transmitter and the receiver side. The path variances were generated randomly from exponential distribution with mean 1. At the receiver, the Directions of Arrival (DOA) were generated from uniform distribution on the interval $[-\pi, \pi]$. At the transmitter side, the directions of departure were generated from Gaussian distribution with mean $0°$ (array broadside) and standard deviation $\sigma = 5°$. In all the simulations the trace of the channel covariance matrix at the transmitter was normalized to be equal to 1.

We compare seven different cases.

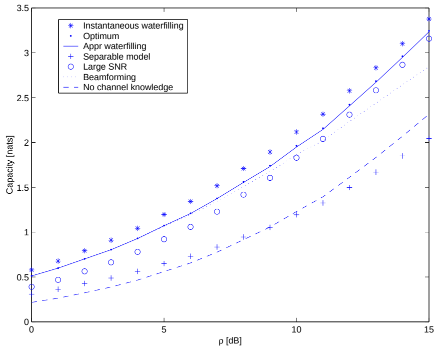

Figure 1: Result for $M = N = 4$, $L = 2$.

1. Instantaneous waterfilling: waterfilling solution for every realization of the channel. This gives an upper bound for the ergodic capacity with any transmit covariance matrix.

2. Optimum: solution obtained by the numerical method described in Section 3.4.

3. Approximate waterfilling: waterfilling on the channel covariance matrix (Section 3.3).

4. Separable model: solution based on the spatially separable channel model.

5. Large SNR: large SNR approximation in (9) or (11) depending on the dimensions.

6. Beamforming: optimal solution for low SNR in (7).

7. No channel knowledge: $\boldsymbol{Q} = \frac{1}{M}\boldsymbol{I}$.

In the first experiment, the number of paths is small (poor scattering environment). We use $M = N = 4$ and $L = 2$. Figure 1 presents the result averaged over 100 Monte-Carlo realizations for the angles, and for each set of angles, 1000 Monte-Carlo realizations for the path amplitudes. The results show that the approximate waterfilling gives nearly optimal results, especially for small values of ρ. It can also be seen that the difference between the high SNR approximation and optimum solution decreases as ρ increases. The capacity for the transmit covariance matrix that is optimal for separable channel model is very low. In the second experiment the number of paths is changed to 10 (rich scattering environment). In this case the capacity for the transmit covariance matrix obtained from separable channel model is much better than in the previous experiment. This is due to the fact that the pathwise channel model converges in distribution to the spatially separable channel model as L tends to infinity [11].

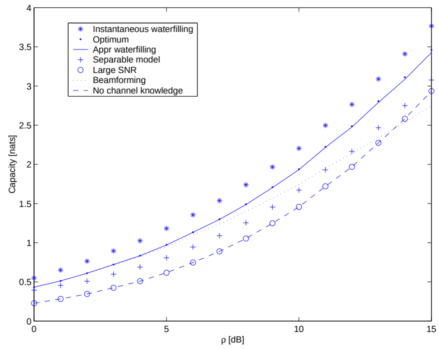

Figure 2: Result for $M = N = 4$, $L = 10$.

5.2 Limited reciprocity

In case of limited reciprocity, we use $N = M = 4$ for all simulations. The presented results are averaged over 100 realizations for $\boldsymbol{W}$, for which every element was generated independently from $\mathcal{CN}(0, 1)$ distribution. For every realization of $\boldsymbol{W}$ the capacities were averaged over 1000 Monte-Carlo realizations for $\boldsymbol{D}_1$ and $\boldsymbol{D}_2$. For Model 3, we use $\epsilon_1^2 = \epsilon_2^2 = 0.1$.

Simulation results are presented in Figure 3. It can be seen that for Model 1 and Model 2, approximated waterfilling gives near optimal results. Therefore, as in the case of pathwise model, waterfilling on the covariance matrix seen from the transmitter is almost sufficient. The same observation can be made also from the result for Model 3.

6 Conclusion

We studied the ergodic capacity of two models for partial channel knowledge: the pathwise channel model with knowledge of the slow varying parameters at the transmitter and the limited reciprocity channel model. The simulation studies and the theoretical results show that waterfilling on the channel covariance matrix at the transmitter leads to almost optimal capacity. As a conclusion we may therefore state that the additional information obtained seems not to be very significant; to achieve closely optimal capacity, only the covariance matrix information is required at the transmitter.

The simulation results for the pathwise model also show that the use of the spatially separable channel model to optimize the transmit covariance matrix results in loss of performance especially for small number of multipaths. Beamforming used in the multipath environment gives close to optimum performance for low

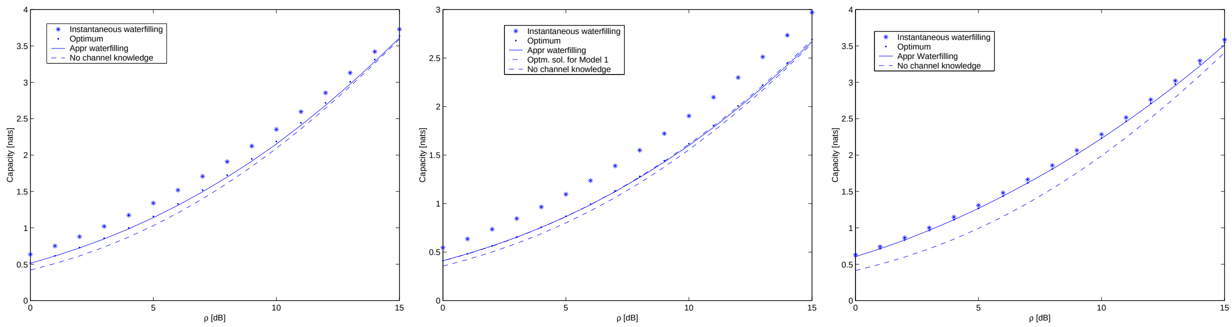

Figure 3: Results for limited reciprocity, $N = M = 4$. From left to right: Model 1, Model 2 and Model 3.

and middle range SNRs.

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