Suboptimality of TDMA in the Low Power Regime

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***Abstract*— We consider multiaccess, broadcast and interference channels with additive Gaussian noise. Although the set of rate pairs achievable by time-division multiple-access (TDMA) is not equal to the capacity region, the TDMA achievable region converges to the capacity region as the power decreases. Furthermore, TDMA achieves the optimum minimum energy per bit.**

Despite those features, this paper shows that the growth of TDMA-achievable rates with the energy per bit is suboptimal in the low-power regime except in special cases: multiaccess channels where the users' energy per bit are identical and broadcast channels where the receivers have identical signal-to-noise ratios. For the additive Gaussian noise interference channel, we identify a small region of interference parameters outside of which TDMA is also shown to be suboptimal.

The effect of fading (known to the receiver) on the suboptimality of TDMA is also explored.

Keywords: Channel Capacity; TDMA; CDMA; Multiple-Access Channels; Broadcast Channels; Interference Channels; Wideband Regime; Low-power communication.

I. INTRODUCTION

For both multipoint-to-point (multiaccess) and point-to-multipoint (broadcast) links, the most important practical lesson drawn from multiuser information theory is that superposition strategies (e.g. code-division multiple-access (CDMA)) where users transmit simultaneously in time and frequency causing mutual interference offer, in general, higher capacity than orthogonal strategies (e.g. time-division multiple-access (TDMA)) provided that the inter-user interference is taken into account at the receiver (e.g. [1]). However, from several standpoints, TDMA is an attractive channel-sharing technology. Foremost among the attractive features of TDMA is the simplicity of the receiver design. Furthermore, the superiority of superposition over TDMA demonstrated by information theory is far from overwhelming. For example, in the absence of fading, the maximum total aggregate rate that superposition can achieve for a multiaccess channel subject to additive white Gaussian noise is no higher than that achieved by TDMA. The presence of fading tends to tilt the balance back in favor of superposition. Indeed, when users are affected by independent fading, they can achieve strictly higher total aggregate rate with superposition than with TDMA, as a simple consequence of the concavity of channel capacity as a function of signal-to-noise ratio [2]. Other practical effects that favor superposition include the presence of channel distortion (which destroys the orthogonality of the TDMA signals) and out-of-cell interference [3].

In multiaccess, the most common practical embodiment of the capacity-achieving superposition strategies is nonorthogonal CDMA. Thus, in practice, superposition is particularly relevant in the wideband low-power regime where the received energy per information bit may not be far from its minimum value. Therefore, it is of considerable practical interest to compare the capabilities of TDMA to the capabilities of superposition in the low-power regime.

Let us consider the standard two-user multiaccess and broadcast Gaussian channels [1]. Plotting the rate regions achieved by superposition and TDMA we see that for both the multiaccess channel (Figures 1 and 2) and for the broadcast channel (Figures 3 and 4) the proportion of the area achievable by TDMA to the area achievable by superposition goes to 1 as the signal-to-noise ratio goes to 0.

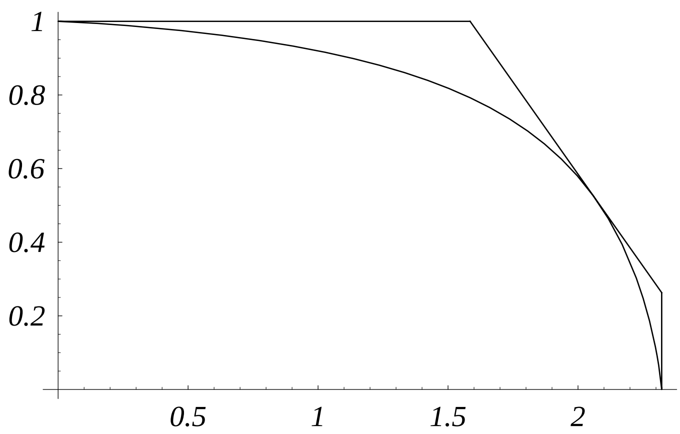

Fig. 1. Multiaccess channel capacity region and TDMA achievable region with with $\mathsf{SNR}_1 = 4$ and $\mathsf{SNR}_2 = 1$.

The intutive explanation for the behavior shown in these figures is simple. As the background thermal noise becomes the dominant component of the overall interference, the coupling between the users weakens. As a consequence of this, it is easy to prove that the minimum energy per bit (achieved at vanishing signal-to-noise ratio) required by TDMA for either multiaccess, broadcast or interference channels is the same as in the single-user channel. From this evidence, we would be justified to suspect that the purported advantage of superposition over TDMA may actually vanish in the low power regime. If this is the case, then the increase in receiver complexity required to realize the capacity achieved by superposition would be hardly justified unless some of the other factors mentioned above (fading, out-of-cell interference, channel distortion) come into play.

The main conclusion of this paper is that, except in some

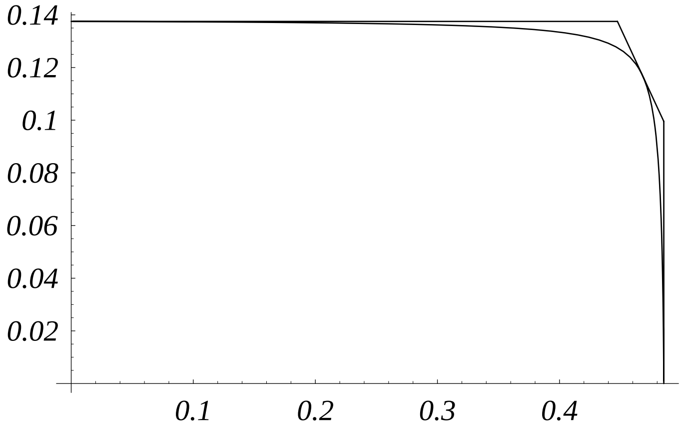

Fig. 2. Multiaccess channel capacity region and TDMA achievable region with with $\mathsf{SNR}_1 = 0.4$ and $\mathsf{SNR}_2 = 0.1$.

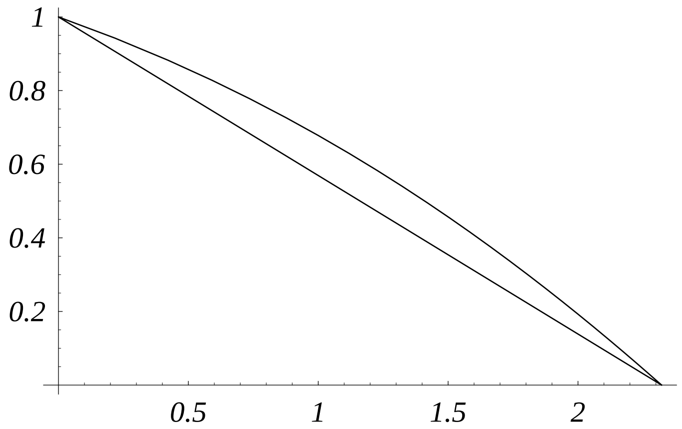

Fig. 3. Capacity region and TDMA-achievable rate region of broadcast channel with $\mathsf{SNR}_1 = 4$ and $\mathsf{SNR}_2 = 1$.

very special cases, and unless bandwidth is not a resource to be conserved (such as in the ultrawideband regime) TDMA does incur significant inefficiency. Our approach is to apply the low-power analysis tools introduced in [4] to investigate the power-bandwidth tradeoff of both superposition and TDMA. The minimum values of energy per bit are obtained in the limit of infinite bandwidth and therefore imply zero spectral efficiency. As argued in the recent work [4], in addition to the normalized minimum energy per bit $\frac{E_b}{N_0}_{\min}$ required for reliable communication, the key performance measure in the wideband regime is the slope of the spectral efficiency vs $\frac{E_b}{N_0}$ curve (b/s/Hz/3 dB) at $\frac{E_b}{N_0}_{\min}$. For a single-user channel, [4] shows that

$$\begin{aligned}\mathcal{S}_0 &\stackrel{\text{def}}{=} \lim_{\frac{E_b}{N_0}\downarrow\frac{E_b}{N_0}_{\min}} \frac{\mathsf{C}\left(\frac{E_b}{N_0}\right)}{10\log_{10}\frac{E_b}{N_0} - 10\log_{10}\frac{E_b}{N_0}_{\min}} 10\log_{10} 2 \\ &= \lim_{\Delta\downarrow 0}\frac{\mathsf{C}\left(2^{\Delta}\frac{E_b}{N_0}_{\min}\right)}{\Delta} \\ &= \frac{2\left[\dot{C}(0)\right]^2}{-\ddot{C}(0)} \qquad (1)\end{aligned}$$

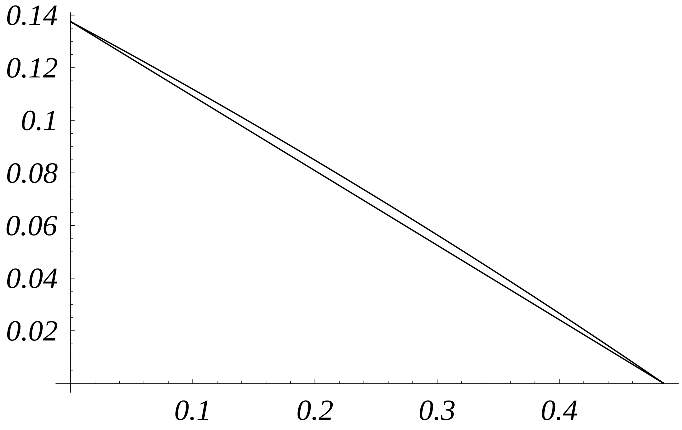

Fig. 4. Capacity region and TDMA-achievable rate region of broadcast channel with $\mathsf{SNR}_1 = 0.4$ and $\mathsf{SNR}_2 = 0.1$.

with

$$\frac{E_b}{N_0}_{\min} = \frac{\log_e 2}{\dot{C}(0)} \qquad (2)$$

where $\mathsf{C}(\frac{E_b}{N_0})$ and $C(\mathsf{SNR})$ denote the capacity as a function of $\frac{E_b}{N_0}$ and per-symbol SNR respectively, and $\dot{C}(0)$, $\ddot{C}(0)$ are the first and second derivatives of $C(\mathsf{SNR})$ evaluated in nats.

In the case of a single user white Gaussian noise channel

$$Y = cX + N \qquad (3)$$

where N is proper complex Gaussian noise with zero mean and variance

$$E[|N|^2] = \sigma^2 = N_0, \qquad (4)$$

subject to the power constraint

$$E[|X|^2] \le P = \mathsf{SNR}\,\sigma^2 \qquad (5)$$

and c a deterministic constant, we have

$$C(\mathsf{SNR}) = \log(1 + |c|^2\mathsf{SNR}) \qquad (6)$$

$$\frac{E_b}{N_0}_{\min} = \frac{\log_e 2}{|c|^2} \qquad (7)$$

and

$$\mathcal{S}_0 = 2 \text{ b/s/Hz/(3 dB)}. \qquad (8)$$

Note that (7) implies that the received energy per bit, $E_b^{\mathfrak{r}}$, satisfies

$$\frac{E_b^{\mathfrak{r}}}{N_0}_{\min} = \log_e 2 = -1.59 \text{ dB}.$$

Whereas the conventional capacity region supplies the tradeoff of rates for fixed powers, in the low-power regime, it is more illuminating to analyze both the minimum energy per bit required for reliable communication and the "slope region" $\mathbf{S}(\theta)$ that gives the tradeoff of individual user slopes for a fixed ratio θ with which the individual rates vanish. Although TDMA incurs no penalty in the minimum energy per bit, our comparative analysis of the slope regions achieved by TDMA and superposition reveals important differences.

II. The Multiple Access Channel

We consider the complex-valued multiaccess channel

$$Y = c_1 X_1 + c_2 X_2 + N \tag{9}$$

where N is complex Gaussian with independent real and imaginary components with variance (4); c_1 and c_2 are deterministic complex scalars. The inputs in (9) are constrained to satisfy

$$E[|X_k|^2] \leq P_k = \mathsf{SNR}_k\, \sigma^2. \tag{10}$$

The capacity region is the Cover-Wyner pentagon [1]:

$$\begin{aligned} \{R_1 &\leq \log_2\left(1 + |c_1|^2 \mathsf{SNR}_1\right) \\ R_2 &\leq \log_2\left(1 + |c_2|^2 \mathsf{SNR}_2\right) \\ R_1 + R_2 &\leq \log_2\left(1 + |c_1|^2 \mathsf{SNR}_1 + |c_2|^2 \mathsf{SNR}_2\right)\} \end{aligned} \tag{11}$$

In particular, we can conclude from (11) the celebrated result that the total capacity (maximum sum of rates) of the multiaccess channel is equal to the capacity of a single-user channel whose power is equal to the sum of the individual received powers, namely

$$R_1 + R_2 = \log_2\left(1 + |c_1|^2 \mathsf{SNR}_1 + |c_2|^2 \mathsf{SNR}_2\right). \tag{12}$$

As is well known, the boundary (or, more precisely, the Pareto-optimal points) of the capacity region (11) is achieved by superposition. In contrast, TDMA achieves the region described as the union of rectangles:

$$\bigcup_{0 \leq \alpha \leq 1} \begin{aligned} \{R_1 &\leq \alpha \log_2\left(1 + \frac{|c_1|^2 \mathsf{SNR}_1}{\alpha}\right) \\ R_2 &\leq (1-\alpha) \log_2\left(1 + \frac{|c_2|^2 \mathsf{SNR}_2}{1-\alpha}\right)\} \end{aligned} \tag{13}$$

where the parameter α is equal to the fraction of time that the first user is active. By letting the time sharing parameter be equal to

$$\alpha = \frac{|c_1|^2 \mathsf{SNR}_1}{|c_1|^2 \mathsf{SNR}_1 + |c_2|^2 \mathsf{SNR}_2}, \tag{14}$$

we obtain the well-known result that the total capacity achieved by (13) is also equal to (12) (cf. Figure 1). In particular, TDMA is optimal for the important special case, $|c_1|^2 \mathsf{SNR}_1 = |c_2|^2 \mathsf{SNR}_2$ and $R_1 = R_2$.

Moreover, as the noise level grows, we operate predominantly in the linear region of the logarithm, the multiaccess interference becomes a secondary factor and the achievable rates become decoupled. This is illustrated by comparing Figures 1 and 2, where we see that the TDMA achievable rate region occupies an increasingly large fraction of the capacity region as the noise level increases. This can be formalized by showing that the TDMA achievable region converges to the rectangle

$$\begin{aligned} \{R_1 &\leq \log_2\left(1 + |c_1|^2 \mathsf{SNR}_1\right) \\ R_2 &\leq \log_2\left(1 + |c_2|^2 \mathsf{SNR}_2\right)\} \end{aligned} \tag{15}$$

in the following sense:

$$\begin{aligned} \lim_{\mathsf{SNR}_1 \to 0} \frac{\alpha \log_2\left(1 + \frac{|c_1|^2 \mathsf{SNR}_1}{\alpha}\right)}{\log_2\left(1 + |c_1|^2 \mathsf{SNR}_1\right)} &= 1 \\ \lim_{\mathsf{SNR}_2 \to 0} \frac{(1-\alpha) \log_2\left(1 + \frac{|c_2|^2 \mathsf{SNR}_2}{(1-\alpha)}\right)}{\log_2\left(1 + |c_2|^2 \mathsf{SNR}_2\right)} &= 1. \end{aligned} \tag{16}$$

Define the transmitted and received energy per information bit relative to the noise spectral level of user $i = 1, 2$ by

$$\frac{E_i}{N_0} = \frac{\mathsf{SNR}_i}{R_i}, \tag{17}$$

and

$$\frac{E_i^{\mathrm{r}}}{N_0} = \frac{|c_i|^2 \mathsf{SNR}_i}{R_i}, \tag{18}$$

respectively. [1]

One of the fundamental limits of interest in this paper is the minimum energy per information bit, which is obtained with asymptotically low power. To that end, we can apply the general framework of capacity region per unit cost for multiaccess channels developed in [6]. However, in the particular case at hand it is instructive to give a self-contained derivation.

Several of the performance measures we will encounter later depend on the ratio θ with which both rates go to 0. As the following result shows, this is not the case for the multiaccess minimum energy per bit.

Theorem 1: For all $\theta = R_1/R_2$, the minimum energies per information bit for the multiaccess channel are equal to

$$\frac{E_1^{\mathrm{r}}}{N_0}_{\min} = \frac{E_2^{\mathrm{r}}}{N_0}_{\min} = \log_e 2 = -1.59 \text{ dB}. \tag{19}$$

Furthermore, (19) is achieved by TDMA.

Proof: Since the presence of interferers cannot lower the minimum energy per bit and (19) is the minimum received energy per bit, the result will follow by showing that TDMA achieves the single-user transmitted energies per bit (7):

$$\frac{E_i}{N_0}_{\min} = \frac{\log_e 2}{|c_i|^2}. \tag{20}$$

Consider a fixed time-sharing parameter $0 < \alpha < 1$. Using (13) we obtain

$$\frac{E_1}{N_0}_{\min} = \lim_{\mathsf{SNR}_1 \to 0} \frac{|c_1|^2 \mathsf{SNR}_1}{\alpha \log_2\left(1 + \frac{|c_1|^2 \mathsf{SNR}_1}{\alpha}\right)} = \frac{\log_e 2}{|c_1|^2}. \tag{21}$$

and

$$\frac{E_2}{N_0}_{\min} = \lim_{\mathsf{SNR}_2 \to 0} \frac{|c_2|^2 \mathsf{SNR}_2}{(1-\alpha) \log_2\left(1 + \frac{|c_2|^2 \mathsf{SNR}_2}{1-\alpha}\right)} = \frac{\log_e 2}{|c_2|^2}. \tag{22}$$

[1] Note that sometimes a "system" energy per bit is considered instead of the individual per-user energies per bit defined in (17). For example, when all the per-symbol energies are identical, [5] uses a system energy per bit which is equal to the harmonic mean of the individual energies per bit.

Since the convergence of the limits (21) and (22) is uniform over α, we can conclude that the result holds even if α is not held fixed and varies with the signal-to-noise ratio. (For example, in order to enforce the constraint $R_1 = \theta R_2$.) □

Let us turn our attention to the slope regions achieved by TDMA and superposition for the multiaccess channel. Fix the rate ratio $R_1/R_2 = \theta$. To define the slope region $\mathbf{S}(\theta)$ corresponding to a region of achievable rate pairs $\mathcal{A}(\mathsf{SNR}_1, \mathsf{SNR}_2)$, we use (17) to obtain the set of achievable rate pairs for given energies per bit. Because of the fixed ratio between the rates it is enough to consider the achievable segment of rates for user 1:

$$\begin{aligned}\mathcal{R}_\theta\left(\frac{E_1}{N_0}, \frac{E_2}{N_0}\right) = \{R_1 \in \mathbb{R}_+ \ \exists \ (\mathsf{SNR}_1, \mathsf{SNR}_2)\text{s.t.} \\ (R_1, R_1/\theta) \in \mathcal{A}(\mathsf{SNR}_1, \mathsf{SNR}_2) \\ \frac{\mathsf{SNR}_1}{R_1} = \frac{E_1}{N_0}, \frac{\theta \mathsf{SNR}_2}{R_1} = \frac{E_2}{N_0}\}\end{aligned} \tag{23}$$

The slope region $\mathbf{S}(\theta)$ is the set of slope pairs that result from

$$\mathcal{S}_1 = \lim_{\frac{E_1}{N_0} \downarrow \frac{E_1}{N_0}_{\min}} \frac{R_1}{10\log_{10}\frac{E_1}{N_0} - 10\log_{10}\frac{E_1}{N_0}_{\min}} 10\log_{10} 2$$

$$\mathcal{S}_2 = \frac{1}{\theta} \lim_{\frac{E_2}{N_0} \downarrow \frac{E_2}{N_0}_{\min}} \frac{R_1}{10\log_{10}\frac{E_2}{N_0} - 10\log_{10}\frac{E_2}{N_0}_{\min}} 10\log_{10} 2$$

for $(R_1, R_1/\theta)$ vanishing with $\frac{E_1}{N_0} \downarrow \frac{E_1}{N_0}_{\min}, \frac{E_2}{N_0} \downarrow \frac{E_2}{N_0}_{\min}$ respecting the membership $R_1 \in \mathcal{R}_\theta\left(\frac{E_1}{N_0}, \frac{E_2}{N_0}\right)$. It can be seen that this is equivalent to

$$\begin{aligned}\mathbf{S}(\theta) = \{(\mathcal{S}_1, \mathcal{S}_2) \in \mathbb{R}_+^2 : \exists \ \phi \in (0, \infty) \\ \mathcal{S}_1 \le \frac{d}{d\Delta} g_\theta(\Delta, \phi\Delta) \\ \mathcal{S}_2 \le \frac{\phi}{\theta}\frac{d}{d\Delta} g_\theta(\Delta, \phi\Delta)\}\end{aligned} \tag{24}$$

where

$$\begin{aligned} g_\theta(\Delta_1, \Delta_2) &= \\ \sup\{R_1 : R_1 &\in \mathcal{R}_\theta\left(2^{\Delta_1}\frac{E_1}{N_0}_{\min}, 2^{\Delta_2}\frac{E_2}{N_0}_{\min}\right)\}\end{aligned}$$

Theorem 2: For all $\theta = R_1/R_2$, the multiaccess slope region achieved by TDMA is:

$$\{(\mathcal{S}_1, \mathcal{S}_2) : \ 0 \le \mathcal{S}_1, \ 0 \le \mathcal{S}_2, \ \mathcal{S}_1 + \mathcal{S}_2 \le 2\}.$$

Proof: Fix $0 \le \alpha \le 1$. Applying (1) to the individual rate constraint equations in (13),

$$\begin{aligned} R_1(\mathsf{SNR}_1) &= \alpha \log_2\left(1 + \frac{|c_1|^2 \mathsf{SNR}_1}{\alpha}\right) \\ R_2(\mathsf{SNR}_2) &= (1-\alpha)\log_2\left(1 + \frac{|c_2|^2\mathsf{SNR}_2}{1-\alpha}\right)\}\end{aligned} \tag{25}$$

we obtain

$$\dot{R}_1(0) = |c_1|^2 \tag{26}$$

$$\dot{R}_2(0) = |c_2|^2 \tag{27}$$

$$\ddot{R}_1(0) = -\frac{|c_1|^4}{\alpha} \tag{28}$$

$$\ddot{R}_2(0) = -\frac{|c_2|^4}{1-\alpha} \tag{29}$$

Thus, if the rate pair belongs to the boundary of the achievable region, then $\mathcal{S}_1 = 2\alpha$ and $\mathcal{S}_2 = 2 - 2\alpha$. Taking the union over all possible time-sharing parameters gives the desired result. □

Theorem 3: Let the rates vanish while keeping $R_1/R_2 = \theta$. The optimum multiaccess slope region (achieved by superposition) is:

$$\begin{aligned}\mathbf{S}(\theta) = \{(\mathcal{S}_1, \mathcal{S}_2) \ :& \ 0 \le \mathcal{S}_1 \le 2, \ 0 \le \mathcal{S}_2 \le 2, \\ & \frac{1}{2} \le \left(\frac{\theta}{1+\theta}\right)^2 \frac{1}{\mathcal{S}_1} + \left(\frac{1}{1+\theta}\right)^2 \frac{1}{\mathcal{S}_2}\}.\end{aligned} \tag{30}$$

Furthermore,

$$\text{closure}\left\{\bigcup_{\theta>0} \mathbf{S}(\theta)\right\} = \{(\mathcal{S}_1, \mathcal{S}_2) : \ 0 \le \mathcal{S}_1 \le 2, \ 0 \le \mathcal{S}_2 \le 2\}. \tag{31}$$

Proof: As we showed in Theorem 1, when we let the powers and rates vanish, both received energies per bit approach the same value, and therefore (17) implies that in the limit

$$\frac{|c_1|^2 \mathsf{SNR}_1}{|c_2|^2 \mathsf{SNR}_2} = \frac{R_1}{R_2} = \theta. \tag{32}$$

In the rest of the proof we will assume that (32) holds. While this is only required in the limit, we can handle the more general case invoking uniform convergence in the same way as in the proof of Theorem 1.

Re-writing (11) as a union of rectangles

$$\begin{aligned}\bigcup_{0 \le \alpha \le 1} \{R_1 &\le \alpha \log_2\left(1 + |c_1|^2 \mathsf{SNR}_1\right) \\ &+ (1-\alpha)\log_2\left(1 + \frac{|c_1|^2\mathsf{SNR}_1}{|c_2|^2\mathsf{SNR}_2 + 1}\right) \\ R_2 &\le \alpha \log_2\left(1 + \frac{|c_2|^2\mathsf{SNR}_2}{|c_1|^2\mathsf{SNR}_1 + 1}\right) \\ &+ (1-\alpha)\log_2\left(1 + |c_2|^2\mathsf{SNR}_2\right).\}\end{aligned} \tag{33}$$

and using (32), the individual maximal achievable rates for fixed α and θ resulting from the Pareto-optimal segment of the Cover-Wyner pentagon become

$$\begin{aligned} R_1(\mathsf{SNR}_1) &= \alpha \log_2\left(1 + |c_1|^2\mathsf{SNR}_1\right) \\ &+ (1-\alpha)\log_2\left(1 + \frac{|c_1|^2\mathsf{SNR}_1}{|c_1|^2\mathsf{SNR}_1/\theta + 1}\right)\end{aligned} \tag{34}$$

$$\begin{aligned} R_2(\mathsf{SNR}_2) &= \alpha \log_2\left(1 + \frac{|c_2|^2\mathsf{SNR}_2}{\theta|c_2|^2\mathsf{SNR}_2 + 1}\right) \\ &+ (1-\alpha)\log_2\left(1 + |c_2|^2\mathsf{SNR}_2\right).\end{aligned} \tag{35}$$

The first and second derivatives of the functions in (34)-(35) at zero signal-to-noise ratio are equal to :

$$\dot{R}_1(0) = |c_1|^2 \quad (36)$$
$$\dot{R}_2(0) = |c_2|^2 \quad (37)$$
$$\ddot{R}_1(0) = -|c_1|^4 \left(1 + \frac{2(1-\alpha)}{\theta}\right) \quad (38)$$
$$\ddot{R}_2(0) = -|c_2|^4 \left(2\theta\alpha + 1\right). \quad (39)$$

Plugging these results into (1) we obtain

$$\mathcal{S}_1 = \frac{2\theta}{2 - 2\alpha + \theta} \quad (40)$$
$$\mathcal{S}_2 = \frac{2}{1 + 2\alpha\theta}. \quad (41)$$

We can solve for α in (40) and (41) and subtract the resulting equations in order to obtain:

$$1 = \frac{\theta}{\mathcal{S}_1} - \frac{\theta}{2} + \frac{1}{\theta \mathcal{S}_2} - \frac{1}{2\theta}, \quad (42)$$

which is equivalent to the boundary condition in (30). The conditions $\mathcal{S}_1 \leq 2$, $\mathcal{S}_2 \leq 2$ follow immediately from the fact that the existence of an interferer cannot improve the rate. Moreover, the points at which the lines $\mathcal{S}_1 = 2$ and $\mathcal{S}_2 = 2$ intersect with (42) correspond to $\alpha = 1$ and $\alpha = 0$, respectively, i.e. to the vertices to the Cover-Wyner pentagon.

To show (31) note that as either $\theta \to 0$ or $\theta \to \infty$ the third constraint in (30) becomes redundant. □

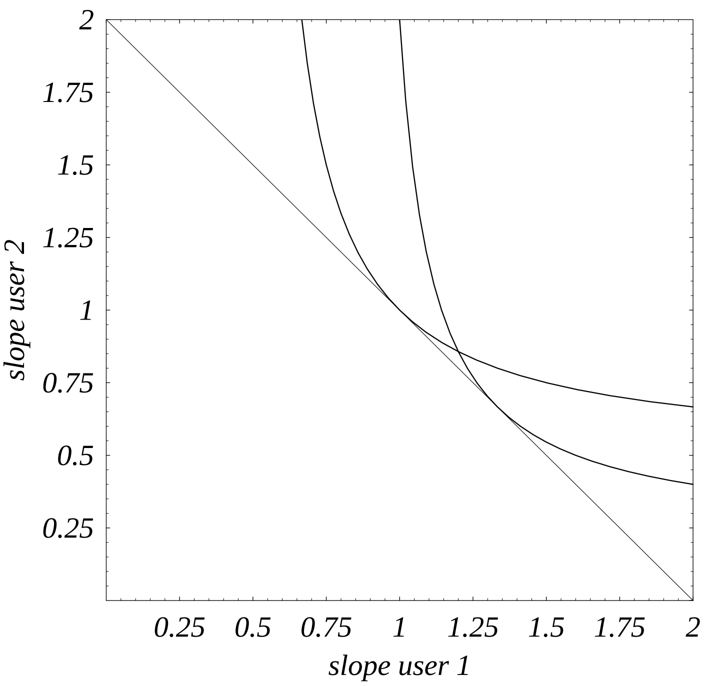

Fig. 5. Slope regions in the Gaussian multiaccess channel with TDMA and superposition $\theta = 1$ and $\theta = 2$.

Theorem 3 shows that *both* users can achieve slopes that are arbitrarily close to the single-user slopes provided they use superposition, optimum decoding, and their powers are sufficiently unbalanced. Comparing this to the triangular region achieved by TDMA we see that even in the simple setting of the two-user additive Gaussian multiaccess channel the low-power capabilities of TDMA are markedly suboptimal.

As a concrete example, let $|c_1| = |c_2| = 1$ and suppose we constrain user 1 to have a small rate $R_1 = \epsilon$ and

$$\frac{E_1}{N_0} = (3.01\epsilon - 1.59)dB$$

whereas

$$\frac{E_2}{N_0} = (3.01\frac{\epsilon}{4} - 1.59)dB,$$

then the highest rate achievable by TDMA is

$$R_2^{\mathsf{T}} = \frac{\epsilon}{4}$$

whereas superposition achieves

$$R_2 = \frac{\epsilon}{2},$$

operating at the point $\theta = 2$, $\mathcal{S}_1 = 1$, $\mathcal{S}_2 = 2$ (Figure 5).

Theorem 4: If both users are constrained to have the same received energy per bit, then TDMA achieves optimum slopes.

Proof: Identical received energies per bit imply that

$$\frac{\mathcal{S}_1}{\mathcal{S}_2} = \frac{R_1}{R_2} = \theta$$

which, when substituting the values found in (40) and (41), requires $\alpha = 1/2$. Furthermore, for every value of θ the superposition slope region "touches" the TDMA region at one point (Figure 5), which corresponds to the mid point $\alpha = 1/2$ in the Pareto-optimal segment of the Cover-Wyner pentagon. To see this, note that $\alpha = 1/2$ achieves the minimum sum (equal to 2) of the slopes in (40),(41):

$$\frac{2\theta}{2 - 2\alpha + \theta} + \frac{2}{1 + 2\alpha\theta}.$$

□

It is easy to extend the above results to the more general case where the channel is subject to fading known to the receiver. The expressions for the capacity region and the region achieved by TDMA simply boil down to expressions (33) and (13) respectively where each of the rate constraints is averaged with respect to c_1 and c_2. We assume henceforth that the fading coefficients have finite fourth moments. Unless c_1 and c_2 are deterministic, the total rate sum achieved by TDMA is no longer optimum, and its achievability region only intersects the capacity region at the trivial points where one of the users is silent. This property, which is one of the multiuser diversity mechanisms by means of which total capacity can be higher in the multiaccess channel than in the single user channel with the same aggregate power (e.g. [3]), is a straightforward consequence of Jensen's inequality and appears to have been pointed out for the first time in [2]. Nevertheless, even in the presence of fading with an arbitrary distribution, it is easy to show that

Theorem 1 holds, and thus TDMA is optimum as far as requiring the same minimum energies per bit as superposition.

In order to extend Theorem 2 to the fading channel all we need to do is take the expectation of the right sides of (26)-(29) with respect to c_1 and c_2. The resulting slope region for TDMA is the same as that in Theorem 2 except that the former individual slopes now become $\kappa(|c_1|)\mathcal{S}_1$, $\kappa(|c_2|)\mathcal{S}_2$ respectively, i.e.,

$$\left\{ (\mathcal{S}_1, \mathcal{S}_2) : \ 0 \le \mathcal{S}_1, \ 0 \le \mathcal{S}_2, \ \frac{\mathcal{S}_1}{\mathcal{S}_1^{(\mathsf{su})}} + \frac{\mathcal{S}_2}{\mathcal{S}_2^{(\mathsf{su})}} \le 1 \right\} \tag{43}$$

where the kurtosis of the fading coefficients is denoted by

$$\kappa(|c|) = \frac{E[|c|^4]}{E^2[|c|^2]} \tag{44}$$

and the slope of the coherent single-user fading channel found in [4], [5] is denoted by:

$$\mathcal{S}_k^{(\mathsf{su})} = \frac{2}{\kappa(|c_k|)}. \tag{45}$$

To generalize Theorem 3 we need to proceed a bit more carefully. Since we are assuming that the transmitters are not informed of the fading coefficients, the asymptotic equality of received energies per bit translates into

$$\frac{E[|c_1|^2]\mathsf{SNR}_1}{E[|c_2|^2]\mathsf{SNR}_2} = \frac{R_1}{R_2} = \theta. \tag{46}$$

Using (46), the fading counterparts of (34),(35) become

$$\begin{aligned} R_1(\mathsf{SNR}_1) &= \alpha E\left[\log_2\left(1 + |c_1|^2 \mathsf{SNR}_1\right)\right] \\ &+ (1-\alpha) E\left[\log_2\left(1 + \frac{|c_1|^2 \mathsf{SNR}_1}{|c_2|^2 \rho\, \mathsf{SNR}_1/\theta + 1}\right)\right] \end{aligned} \tag{47}$$

$$\begin{aligned} R_2(\mathsf{SNR}_2) &= \alpha E\left[\log_2\left(1 + \frac{|c_2|^2 \mathsf{SNR}_2}{\theta |c_1|^2 \mathsf{SNR}_2/\rho + 1}\right)\right] \\ &+ (1-\alpha) E\left[\log_2\left(1 + |c_2|^2 \mathsf{SNR}_2\right)\right]. \end{aligned} \tag{48}$$

where

$$\rho = \frac{E\left[|c_1|^2\right]}{E\left[|c_2|^2\right]}$$

Assuming that the users fade independently, the derivatives of (47) and (48) are

$$\dot{R}_1(0) = E\left[|c_1|^2\right] \tag{49}$$

$$\dot{R}_2(0) = E\left[|c_2|^2\right] \tag{50}$$

$$\ddot{R}_1(0) = -E\left[|c_1|^4\right] - \frac{2(1-\alpha)\left(E\left[|c_1|^2\right]\right)^2}{\theta} \tag{51}$$

$$\ddot{R}_2(0) = -E\left[|c_2|^4\right] - 2\theta\alpha\left(E\left[|c_2|^2\right]\right)^2. \tag{52}$$

This leads to the generalization of (41):

$$\mathcal{S}_1 = \frac{2\theta}{2 - 2\alpha + \theta\kappa(|c_1|)} \tag{53}$$

$$\mathcal{S}_2 = \frac{2}{\kappa(|c_2|) + 2\alpha\theta}. \tag{54}$$

which leads to the following generalization of (30):

$$\begin{aligned} \mathbf{S}(\theta) = \{ \quad & (\mathcal{S}_1, \mathcal{S}_2) : 0 \le \mathcal{S}_1 \le \mathcal{S}_1^{(\mathsf{su})}, \\ & 0 \le \mathcal{S}_2 \le \mathcal{S}_2^{(\mathsf{su})}, \\ 1 \ = \ & \theta\left[\frac{1}{\mathcal{S}_1} - \frac{1}{\mathcal{S}_1^{(\mathsf{su})}}\right] + \frac{1}{\theta}\left[\frac{1}{\mathcal{S}_2} - \frac{1}{\mathcal{S}_2^{(\mathsf{su})}}\right] \} \end{aligned} \tag{55}$$

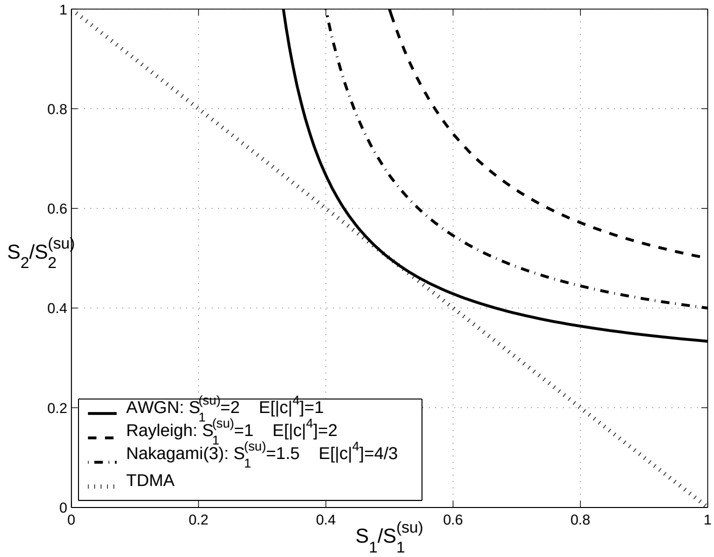

Fig. 6. Normalized slope region for the fading channel with $\theta = 1$ and different fading statistics.

Taking the closure of the union of (55) over all values of $\theta > 0$, we again get the result that single-user slopes are simultaneously achievable with superposition, in contrast to the triangular region achieved by TDMA.

As we can see in Figure 6, in the presence of fading TDMA no longer achieves optimum slopes when the energies per bit are required to be equal for nonzero rates. In the symmetric case of equal received energies per bit and equal spectral efficiencies (i.e., $\theta = 1$), we define the max-min slope of the system as

$$\overline{\mathcal{S}} = \max_{(\mathcal{S}_1, \mathcal{S}_2) \in \mathbf{S}(1)} \min\{\mathcal{S}_1, \mathcal{S}_2\} \tag{56}$$

where the slope region is given either by Theorem 2 or by Theorem 3 for TDMA and superposition coding, respectively. Clearly, $\overline{\mathcal{S}}$ is given by the intersection of the slope region boundary with the line $\mathcal{S}_1 = \mathcal{S}_2$. The *bandwidth expansion factor* incurred by TDMA in the low power regime is given by ratio of $\overline{\mathcal{S}}$ achieved by superposition over $\overline{\mathcal{S}}$ achieved by TDMA. It is easily seen from (43) and (55) that this is given by

$$\frac{2\left(\frac{1}{\mathcal{S}_1^{(\mathsf{su})}} + \frac{1}{\mathcal{S}_2^{(\mathsf{su})}}\right)}{1 + \frac{1}{\mathcal{S}_1^{(\mathsf{su})}} + \frac{1}{\mathcal{S}_2^{(\mathsf{su})}}}$$

which is strictly larger than 1 for any non-constant fading (i.e., with kurtosis larger than 1).

III. The Broadcast Channel

We consider the simple complex-valued two-user broadcast Gaussian channel where users 1 and 2 receive the same signal from the transmitter embedded in independent Gaussian noise with different signal-to-noise ratios:

$$\begin{aligned} Y_1 &= c_1 X + N_1 \\ Y_2 &= c_2 X + N_2 \end{aligned} \tag{57}$$

where c_1 and c_2 are deterministic and the various random variables are proper complex with

$$E[|X|^2] \leq P,$$

$$E[|N_i|^2] = \sigma^2,$$

and

$$\mathsf{SNR} = \frac{P}{\sigma^2}.$$

We will assume $|c_1|^2 > |c_2|^2$ as in the case $|c_1|^2 = |c_2|^2$ TDMA is trivially optimal. The capacity region of this channel (achieved by superposition and stripping) is equal to [1]

$$\bigcup_{0\leq\alpha\leq 1} \left\{ \begin{aligned} R_1 &\leq \log_2\left(1+\alpha\,|c_1|^2\mathsf{SNR}\right) \\ R_2 &\leq \log_2\left(1+\frac{(1-\alpha)|c_2|^2\mathsf{SNR}}{\alpha|c_1|^2\mathsf{SNR}+1}\right) \end{aligned} \right\} \tag{58}$$

whereas the region achievable by TDMA is

$$\bigcup_{0\leq\alpha\leq 1} \left\{ \begin{aligned} R_1 &\leq \alpha\log_2\left(1+|c_1|^2\mathsf{SNR}\right) \\ R_2 &\leq (1-\alpha)\log_2\left(1+|c_2|^2\mathsf{SNR}\right) \end{aligned} \right\} \tag{59}$$

As for multiaccess channels, it appears from Figures 3 and 4 that as the power decreases, the TDMA-achievable region occupies a larger fraction of the capacity region. This has been shown in [7] for a variety of broadcast channels in the sense that for every pair (R_1, R_2) in the boundary of the broadcast capacity region,

$$\limsup_{\mathsf{SNR}\to 0} \frac{R_1}{\log_2\left(1+|c_1|^2\mathsf{SNR}\right)} + \frac{R_2}{\log_2\left(1+|c_2|^2\mathsf{SNR}\right)} = 1. \tag{60}$$

Analogously to (17) we define the transmitted and received energies per bit as

$$\frac{E_i}{N_0} = \frac{\mathsf{SNR}}{R_i}. \tag{61}$$

$$\frac{E_i^{\mathrm{r}}}{N_0} = \frac{|c_i|^2\mathsf{SNR}}{R_i}. \tag{62}$$

Theorem 5: Suppose that $R_1/R_2 = \theta$. Then, the minimum received energies per bit achieved by both TDMA and superposition are:

$$\frac{E_1^{\mathrm{r}}}{N_0} = \left(1+\frac{|c_1|^2}{|c_2|^2\theta}\right)\log_e 2 \tag{63}$$

$$\frac{E_2^{\mathrm{r}}}{N_0} = \left(1+\frac{\theta|c_2|^2}{|c_1|^2}\right)\log_e 2 \tag{64}$$

Proof: Let us start with TDMA. Enforcing the constraint on the ratio of the rates in (59), pins down the value of the time-sharing parameter and we obtain that the rate achieved by user 1 is

$$R_1 = \frac{\theta\log_2\left(1+|c_1|^2\mathsf{SNR}\right)\log_2\left(1+|c_2|^2\mathsf{SNR}\right)}{\log_2\left(1+|c_1|^2\mathsf{SNR}\right)+\theta\log_2\left(1+|c_2|^2\mathsf{SNR}\right)}. \tag{65}$$

The reciprocal of the derivative of (65) with respect to SNR at $\mathsf{SNR} = 0$ is equal to

$$\frac{1}{\theta|c_2|^2} + \frac{1}{|c_1|^2}$$

which upon multiplication by $\log_e 2$ yields the transmitted energy per bit. Multiplying by $|c_1|^2$, we obtain the desired formula (63). Formula (64) is obtained in an entirely analogous way or simply by noticing from (62) that

$$\frac{E_1^{\mathrm{r}}}{E_2^{\mathrm{r}}} = \frac{|c_1|^2}{\theta|c_2|^2}. \tag{66}$$

Let us analyze now the capacity region (58). Define $\alpha_\theta(\mathsf{SNR})$ to be the solution to

$$\begin{aligned} R_1(\mathsf{SNR}) &\stackrel{\text{def}}{=} \log_2\left(1+\alpha_\theta(\mathsf{SNR})|c_1|^2\mathsf{SNR}\right) \\ &= \theta\log_2\left(1+\frac{(1-\alpha_\theta(\mathsf{SNR}))|c_2|^2\mathsf{SNR}}{\alpha_\theta(\mathsf{SNR})|c_2|^2\mathsf{SNR}+1}\right) \quad (67) \\ &\stackrel{\text{def}}{=} \theta R_2(\mathsf{SNR}) \quad (68) \end{aligned}$$

Although an explicit solution for $\alpha_\theta(\mathsf{SNR})$ does not seem feasible, we will be able to compute its value at $\mathsf{SNR} = 0$, as well as that of its derivative. By taking the first and second derivative at $\mathsf{SNR} = 0$ of $R_1(\mathsf{SNR})$ and $R_2(\mathsf{SNR})$ we get

$$\begin{aligned} \dot{R}_1(0) &= |c_1|^2\,\alpha_\theta(0) && (69) \\ \ddot{R}_1(0) &= -|c_1|^4\alpha_\theta(0)^2 + 2|c_1|^2\dot{\alpha}_\theta(0) && (70) \\ \dot{R}_2(0) &= |c_2|^2\,(1-\alpha_\theta(0)) && (71) \\ \ddot{R}_2(0) &= -|c_2|^4 + |c_2|^4\alpha_\theta(0)^2 - 2|c_2|^2\dot{\alpha}_\theta(0) && (72) \end{aligned}$$

By recalling that $R_1(\mathsf{SNR}) = \theta R_2(\mathsf{SNR})$ for all SNR and by equating the derivatives (69)-(71) and (70)-(72) we obtain

$$\alpha_\theta(0) = \frac{\theta\,|c_2|^2}{|c_1|^2+\theta\,|c_2|^2} \tag{73}$$

$$2\dot{\alpha}_\theta(0) = -\theta|c_2|^4|c_1|^2\frac{|c_1|^2(1-\theta)+2\theta|c_2|^2}{(|c_1|^2+\theta\,|c_2|^2)^3} \tag{74}$$

Multiplying the reciprocal of (69) by $|c_1|^2\log_e 2$, (63) follows, and so does (64) by applying (66). □

Let us direct our attention to the analysis of the slope regions for the broadcast channel. The definition of the slope region $\mathbf{S}(\theta)$ is parallel to that of the multiaccess channel. Starting from a region of achievable rate pairs $\mathcal{A}(\mathsf{SNR})$, we use (61) to define

$$\mathcal{R}_\theta\left(\frac{E_1}{N_0}\right) = \left\{R_1 \in \mathbb{R}_+ : \; (R_1, R_1/\theta) \in \mathcal{A}\left(R_1\frac{E_1}{N_0}\right)\right\} \tag{75}$$

The slope region $\mathbf{S}(\theta)$ is the set of slope pairs

$$\begin{aligned} \mathcal{S}_1 &= \lim_{\frac{E_1}{N_0}\downarrow\frac{E_1}{N_0}_{\min}} \frac{R_1}{10\log_{10}\frac{E_1}{N_0} - 10\log_{10}\frac{E_1}{N_0}_{\min}} 10\log_{10} 2 \\ \mathcal{S}_2 &= \frac{1}{\theta}\mathcal{S}_1 \end{aligned} \tag{76}$$

for R_1 vanishing with $\frac{E_1}{N_0} \downarrow \frac{E_1}{N_0}_{\min}$ within $R_1 \in \mathcal{R}_\theta\left(\frac{E_1}{N_0}\right)$. Alternatively we can write

$$\mathbf{S}(\theta) = \left\{(\mathcal{S}_1, \mathcal{S}_2) \in \mathbb{R}^2_+ : \mathcal{S}_1 = \theta\mathcal{S}_2,\ \ \mathcal{S}_1 \le \frac{d}{d\Delta} g_\theta(\Delta)\right\} \tag{77}$$

where

$$g_\theta(\Delta) = \sup\{R_1 : R_1 \in \mathcal{R}_\theta\left(2^\Delta \frac{E_1}{N_0}_{\min}\right)\} \tag{78}$$

Theorem 6: Let the rates vanish while keeping $R_1/R_2 = \theta$. The broadcast slope region achieved by TDMA is:

$$\{(\mathcal{S}_1, \mathcal{S}_2) :\ 0 \le \mathcal{S}_1 \le \frac{2\theta}{1+\theta},\ 0 \le \mathcal{S}_2 \le \frac{2}{1+\theta}\}. \tag{79}$$

Proof: The fact that if we operate on the boundary of the capacity region we get $\mathcal{S}_1 = \theta\mathcal{S}_2$ can be readily seen from the fact that the numerator in the definition of slope has a factor of θ because $R_1 = R_2\theta$, whereas the denominators are identical: the $\frac{E_b}{N_0}$'s differ by a multiplicative constant (66) which is immaterial in the definition of slope (left side of (1)).

As in the proof of Theorem 5, enforcing the constraint $R_1 = \theta R_2$, we obtain the value of the time sharing parameter and the value of the individual rates. Fix θ and let $\tau(\mathsf{SNR}) = \tau_1(\mathsf{SNR}) = 1 - \tau_2(\mathsf{SNR})$ be the solution of

$$\tau(\mathsf{SNR})\log_2(1+|c_1|^2\mathsf{SNR}) = \theta(1-\tau(\mathsf{SNR}))\log_2(1+|c_2|^2\mathsf{SNR}).$$

Although we are unable to find an explicit solution for $\tau(\mathsf{SNR})$, we are be able to compute its derivative at $\mathsf{SNR} = 0$. Taking the derivatives of the rate function

$$R_1(\mathsf{SNR}) = \tau_1(\mathsf{SNR})\log\left(1+|c_1|^2\,\mathsf{SNR}\right)$$

we obtain

$$\dot{R}_1(0) = |c_1|^2\,\tau_1(0) \tag{80}$$

$$\ddot{R}_1(0) = -\tau_1(0)|c_1|^4 + 2|c_1|^2\dot{\tau}_1(0) \tag{81}$$

For rate $R_2(\mathsf{SNR})$ just exchange subscripts 1 into 2 in the above expressions. Since $\tau_1(\mathsf{SNR}) + \tau_2(\mathsf{SNR}) = 1$ and $R_1(\mathsf{SNR}) = \theta R_2(\mathsf{SNR})$ for all SNR and by equating the derivatives we obtain

$$\tau(0) = \frac{\theta\,|c_2|^2}{|c_1|^2 + \theta\,|c_2|^2} \tag{82}$$

$$2\dot{\tau}(0) = \theta|c_1|^2|c_2|^2\frac{|c_1|^2 - |c_2|^2}{(|c_1| + \theta|c_2|^2)^2} \tag{83}$$

After substitution we get

$$\dot{R}_1(0) = \theta\dot{R}_2(0) = \theta\frac{|c_1|^2\,|c_2|^2}{|c_1|^2 + \theta|c_2|^2} \tag{84}$$

$$-\ddot{R}_1(0) = -\theta\ddot{R}_2(0) = \frac{\theta+1}{\theta}\left(\dot{R}_1(0)\right)^2 \tag{85}$$

The result now follows by applying (1) to (85). □

Theorem 7: Let the rates vanish while keeping $R_1/R_2 = \theta$. The optimum broadcast slope region (achieved by superposition) is:

$$\begin{aligned} \{(\mathcal{S}_1, \mathcal{S}_2) :\ 0 &\le \mathcal{S}_1 \le \frac{2\theta\,(\theta + |c_1|^2/|c_2|^2)}{\theta^2 + 2\theta + |c_1|^2/|c_2|^2}, \\ 0 &\le \mathcal{S}_2 \le \frac{\mathcal{S}_1}{\theta}\}. \end{aligned} \tag{86}$$

Proof: As before it is sufficient to justify the slope of user 1 because as we saw above $\mathcal{S}_1 = \theta\mathcal{S}_2$. We have already done the main calculation needed here in the proof of Theorem 5. From expressions (69), (70), (73) and (74) it follows that

$$\frac{2\dot{R}_1(0)}{-\ddot{R}_1(0)} = \frac{2\theta\,(\theta + |c_1|^2/|c_2|^2)}{\theta^2 + 2\theta + |c_1|^2/|c_2|^2}$$

and the theorem follows.

□

Comparing the results of Theorems 6 and 7 we see that unless $|c_1|^2 = |c_2|^2$ (in which case TDMA is optimal), TDMA is wasteful of channel resources. For given power and rates R_1 and $R_2 = R_1/\theta$, the bandwidth expansion factor incurred by TDMA in the low power regime is equal to the ratio of the slopes obtained in Theorems 6 and 7:

$$\frac{(1+\theta)(\theta + |c_1|^2/|c_2|^2)}{\theta^2 + 2\theta + |c_1|^2/|c_2|^2}, \tag{87}$$

a function which is monotonic in $|c_1|^2/|c_2|^2 \ge 1$ and achieves a maximum (over θ) equal to

$$\frac{|c_1|^2/|c_2|^2 + (1 + |c_1|^2/|c_2|^2)|c_1|/(\sqrt{2}|c_2|)}{|c_1|^2/|c_2|^2 + |c_1|/|c_2|}$$

Figure 7 plots the TDMA bandwidth expansion factor as a function of θ when the users are 10dB apart. Note that TDMA can be quite wasteful of bandwidth.

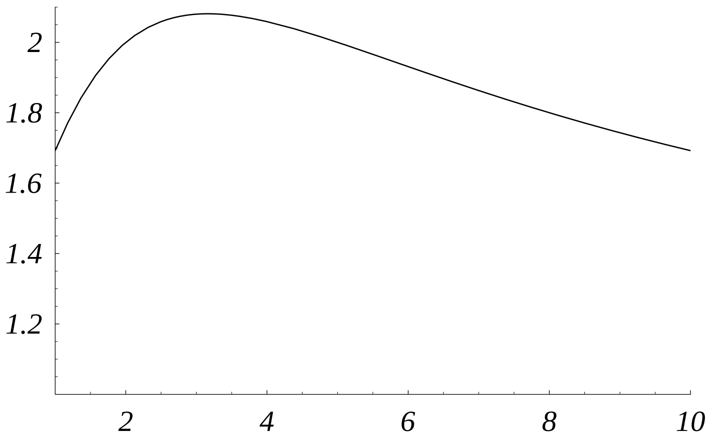

Fig. 7. Bandwidth factor penalty incurred by TDMA as a function of $R_1/R_2 = \theta$ for $|c_1|^2 = 10|c_2|^2$

The extension of the above results to the case of broadcast channels subject to fading known to the receivers only is not as straightforward as for multiaccess channels. When the information bearing signal X fades, the channel from the input to one of the outputs is no longer a *degraded* version [16] of, nor *more capable* [15] than, the channel from the input to the other output. The capacity region of general broadcast channels is still an open problem, and only inner and outer bounds are available. The best known inner bound was derived by Marton in [14]. Interestingly, the Marton region yields the capacity region for all the classes of broadcast channels for which a coding theorem is currently available.

In the following, we shall make use of an inner bound to the fading broadcast channel capacity region obtained by a particular choice of the auxiliary variables in the Marton region. We assume that c_1 and c_2 in (57) are ergodic random processes with finite fourth moment, perfectly known to the corresponding receiver. Then, we obtain the achievable region

$$\bigcup_{0\le\alpha\le1}\left\{\begin{array}{lcl} R_1 & \le & E\left[\log\left(1+|c_1|^2\alpha\mathsf{SNR}\right)\right] \\ R_1+R_2 & \le & E\left[\log\left(1+\dfrac{|c_2|^2(1-\alpha)\mathsf{SNR}}{1+|c_2|^2\alpha\mathsf{SNR}}\right)\right] \\ & & +E\left[\log\left(1+|c_1|^2\,\alpha\mathsf{SNR}\right)\right] \\ R_1+R_2 & \le & E\left[\log\left(1+|c_1|^2\,\mathsf{SNR}\right)\right] \end{array}\right. \quad (88)$$

by setting $W = V \sim \mathcal{N}(0,(1-\alpha)P)$, $U = X \sim \mathcal{N}(0,P)$ and $E[XV^*] = E[|V|^2] = (1-\alpha)P$ with $\alpha \in [0,1]$ in [14, Theorem 2]. On the other hand, TDMA achieves

$$\bigcup_{0\le\tau\le1}\left\{\begin{array}{lcl} R_1 & \le & \tau\, E\left[\log\left(1+|c_1|^2\mathsf{SNR}\right)\right] \\ R_2 & \le & (1-\tau)E\left[\log\left(1+|c_2|^2\mathsf{SNR}\right)\right] \end{array}\right. \quad (89)$$

Next, we show that the achievable regions in (88) and (89) yield the same minimum energies per bit (which, in absence of a converse coding theorem, cannot be claimed to be the minimum energies per bit required for reliable communication). Furthermore, we obtain the slope regions corresponding to the achievable regions (88) and (89), and we show that the TDMA slope region is strictly suboptimal.

Notice that the only effective sum-rate constraint in (88) is

$$E\left[\log\left(1+\frac{|c_2|^2(1-\alpha)\mathsf{SNR}}{1+|c_2|^2\alpha\mathsf{SNR}}\right)\right] + E\left[\log\left(1+|c_1|^2\,\alpha\mathsf{SNR}\right)\right]$$

if $\dot{C}_1(x) \ge \dot{C}_2(x)$ in the interval $x \in [0, \mathsf{SNR}]$ where

$$C_i(x) = E\left[\log\left(1+|c_i|^2\,x\right)\right]$$

is the single-user capacity for user i. Without loss of generality, we assume that in a sufficiently small right interval of $x = 0$ the single-user capacities satisfy $C_1(x) \ge C_2(x)$ and $\dot{C}_1(x) \ge \dot{C}_2(x)$ (otherwise swap the role of the users). [2] Therefore, in the low-power regime, it is enough to consider the simpler region defined by

$$\bigcup_{0\le\alpha\le1}\left\{\begin{array}{l} R_1 \le C_1(\alpha\,\mathsf{SNR}) \\ R_2 \le C_2(\mathsf{SNR}) - C_2(\alpha\,\mathsf{SNR}) \end{array}\right. \quad (90)$$

[2]The condition $C_1(x) \ge C_2(x)$ in a right interval of $x = 0$ is equivalent to $E[|c_1|^2] > E[|c_2|^2]$ or $E[|c_1|^2] = E[|c_2|^2]$ and $E[|c_1|^4] < E[|c_2|^4]$. If $E[|c_1|^2] = E[|c_2|^2]$ and $E[|c_1|^4] = E[|c_2|^4]$ then for the purpose of this analysis the channel to user 1 is *statistically equivalent in the low-power regime* to the channel to user 2. In this case the region (88) boils down to the TDMA region in the low-power regime.

It is worth pointing out that the region (90) is an inner bound for the general fading broadcast channel only in the interval of SNR on the right of 0 for which $\dot{C}_1(\mathsf{SNR}) \ge \dot{C}_2(\mathsf{SNR})$ and not for every SNR. It is an inner bound for every SNR if the overall channel is degraded and is the capacity region for the class of degraded fading broadcast channels analyzed in [7], as proved in [17].

In order to extend Theorems 5, 6 and 7 we need to take the expectation of the right hand sides of (69)-(72) and of (80)-(81) with respect to c_1 and c_2 with

$$\begin{aligned} \alpha_\theta(0) &= \tau(0) = \frac{\theta\, E[|c_2|^2]}{E[|c_1|^2] + \theta\, E[|c_2|^2]} \\ 2\dot{\alpha_\theta}(0) &= \frac{\alpha_\theta^2(0)\, E[|c_1|^4] - \theta(1-\alpha_\theta^2(0))\, E[|c_2|^4]}{E[|c_1|^2] + \theta\, E[|c_2|^2]} \\ 2\dot{\tau}(0) &= \frac{\tau(0)\, E[|c_1|^4] - \theta(1-\tau(0))\, E[|c_2|^4]}{E[|c_1|^2] + \theta\, E[|c_2|^2]} \end{aligned}$$

whose derivation follows that of (73)-(74) and of (82)-(83).

By substitution of the first and second derivatives at $\mathsf{SNR} = 0$ of the rate functions in (2) and (1), we see that the minimum received energies per bit are

$$\frac{E_1^{\mathrm{r}}}{N_0} = \left(1 + \frac{E[|c_1|^2]}{E[|c_2|^2]\theta}\right)\log_e 2 \quad (91)$$

$$\frac{E_2^{\mathrm{r}}}{N_0} = \left(1 + \frac{E[|c_2|^2]\theta}{E[|c_1|^2]}\right)\log_e 2 \quad (92)$$

achieved by both TDMA and (88). The slope region boundary achieved by TDMA is the following generalization of (79)):

$$S_1 = \theta S_2 = \frac{2\theta}{\theta\kappa(|c_1|) + \kappa(|c_2|)} \quad (93)$$

while the slope region boundary achieved by (88) is the following generalization of (86)

$$S_1 = \theta S_2 = \frac{2\theta(\theta+\rho)}{\theta^2\kappa(|c_1|) + (2\theta+\rho)\kappa(|c_2|)} \quad (94)$$

where

$$\rho = \frac{E[|c_1|^2]}{E[|c_2|^2]}.$$

The bandwidth expansion factor incurred by TDMA is given by

$$\frac{(\theta\kappa(|c_1|) + \kappa(|c_2|))(\theta+\rho)}{\theta^2\kappa(|c_1|) + (2\theta+\rho)\kappa(|c_2|)}$$

Interestingly, if the fading distributions have the same kurtosis this factor is independent of the fading distribution and coincides with the bandwidth expansion factor found in the absence of fading (87).

IV. The Interference Channel

Unlike the multiuser channels considered above, the capacity of the interference channel in additive white Gaussian noise remains unknown. However, using the available results we can prove the low-power suboptimality of TDMA in all but a small region of interfering parameters. The (non-canonical) channel model is

$$Y_1 = c_{11}X_1 + c_{12}X_2 + N_1 \quad (95)$$
$$Y_2 = c_{21}X_1 + c_{22}X_2 + N_2 \quad (96)$$

where the coefficients $\{c_{ij},\, i,j = 1,2\}$ are deterministic scalars, N_1, N_2 are proper-complex Gaussian with zero mean and variance σ^2 and

$$E[|X_i|^2] \leq \mathsf{SNR}_i\, \sigma^2.$$

The receiver that observes Y_i is only interested in recovering the information sent by user i. The canonical model assumes $c_{11} = c_{22} = 1$.

TDMA achieves the same region as for the multiaccess channel (13)

$$\bigcup_{0\leq\alpha\leq 1} \{R_1 \leq \alpha \log_2\left(1+\frac{|c_{11}|^2\mathsf{SNR}_1}{\alpha}\right)$$
$$R_2 \leq (1-\alpha)\log_2\left(1+\frac{|c_{22}|^2\mathsf{SNR}_2}{1-\alpha}\right)\} \quad (97)$$

Thus, Theorems 1 and 2 on the $\frac{E_b}{N_0}_{\min}$ and slope region achieved by TDMA also hold for the interference channel, namely the minimum received energies per bit are

$$\frac{E_1^{\mathrm{r}}}{N_0}_{\min} = \frac{E_2^{\mathrm{r}}}{N_0}_{\min} = \log_e 2 = -1.59 \text{ dB}$$

the minimum transmitted energies per bit are

$$\frac{E_i}{N_0}_{\min} = \frac{\log_e 2}{|c_{ii}|^2} \quad i = 1,2$$

and the slope region [3] is

$$\mathcal{S}_1 + \mathcal{S}_2 \leq 2$$

Since the received $\frac{E_b}{N_0}_{\min}$ achieved by TDMA does not depend on the interference parameters, it is equal to the optimum one. Equivalently, the convergence of the TDMA-achievable region to the (rectangular) capacity region in the sense of (16) holds verbatim for the interference channel.

Although a general expression for the optimum slope region of the interference channel is unknown at this time, we can use existing results on the capacity of interference channels to draw the following conclusions:

1) $\frac{|c_{12}|^2}{|c_{22}|^2} > 1$ and $\frac{|c_{21}|^2}{|c_{11}|^2} > 1$:
The capacity region is the intersection of the Cover-Wyner pentagons corresponding to both multiaccess channels [8]

$$R_1 \leq \log_2\left(1+|c_{11}|^2\mathsf{SNR}_1\right)$$
$$R_2 \leq \log_2\left(1+|c_{22}|^2\mathsf{SNR}_2\right)$$
$$R_1+R_2 \leq \min\{\log_2\left(1+|c_{11}|^2\mathsf{SNR}_1+|c_{12}|^2\mathsf{SNR}_2\right),\ \log_2\left(1+|c_{21}|^2\mathsf{SNR}_1+|c_{22}|^2\mathsf{SNR}_2\right)\} \quad (98)$$

Because the interference coefficients $|c_{ij}|$ are strictly larger than $|c_{ii}|$, $i = 1,2,\ j \neq i$, for sufficiently small SNR_1 and SNR_2 the constraint on R_1+R_2 in (98) is strictly larger than the sum of the single-user capacities and the capacity region reduces to a rectangle, regardless of ratio between the rates. Effectively, for $|c_{21}|/|c_{11}| > 1$ and $|c_{12}|/|c_{22}| > 1$ the *very large interference* condition [9] is always in effect in the low-power regime. Since the capacity region is a rectangle, the optimum slope region for any $\theta \in (0,\infty)$ is

$$\mathbf{S}(\theta) = [0,2]\times[0,2] \quad (99)$$

in contrast with the triangular slope region achieved by TDMA for any $\theta \in (0,\infty)$.

2) $\frac{|c_{12}|^2}{|c_{22}|^2} = 1$ and $\frac{|c_{21}|^2}{|c_{11}|^2} \geq 1$ or $\frac{|c_{12}|^2}{|c_{22}|^2} \geq 1$ and $\frac{|c_{21}|^2}{|c_{11}|^2} = 1$:
The capacity region is still given by (98) but in this case it reduces to the standard multiaccess region

$$R_1 \leq \log_2\left(1+|c_{11}|^2\mathsf{SNR}_1\right)$$
$$R_2 \leq \log_2\left(1+|c_{22}|^2\mathsf{SNR}_2\right)$$
$$R_1+R_2 \leq \log_2\left(1+|c_{11}|^2\mathsf{SNR}_1+|c_{22}|^2\mathsf{SNR}_2\right) \quad (100)$$

which, as we saw, leads to the slope region (30). Comparing (99) and (30), we see that $\mathbf{S}(\theta)$ is not continuous in the interference parameters.

3) $\frac{|c_{12}|^2}{|c_{22}|^2} > 1$ and $\frac{|c_{21}|^2}{|c_{11}|^2} < 1$:
The following rate-pair is achievable

$$R_1 = \log_2\left(1+|c_{11}|^2\mathsf{SNR}_1\right)$$
$$R_2 = \log_2\left(1+\frac{|c_{22}|^2\mathsf{SNR}_2}{1+|c_{21}|^2\mathsf{SNR}_1}\right) \quad (101)$$

as long as the receiver of user 1 can decode user 2, or equivalently if

$$\frac{|c_{22}|^2\mathsf{SNR}_2}{1+|c_{21}|^2\mathsf{SNR}_1} \leq \frac{|c_{12}|^2\mathsf{SNR}_2}{1+|c_{11}|^2\mathsf{SNR}_1}$$

which is guaranteed to hold for sufficiently small SNR_1 if

$$\frac{|c_{12}|^2}{|c_{22}|^2} > 1 \quad (102)$$

In fact, it was shown in [9], that (101) is a Pareto-optimal pair for the *degraded interference channel*, i.e, $\frac{|c_{12}|^2}{|c_{22}|^2}\frac{|c_{21}|^2}{|c_{11}|^2} = 1$. The the Pareto-optimality in the more general case (102) can be derived from [18, Theorem 2]. Clearly the optimal minimum energies per bit are achieved by this rate-pair.
By fixing the received SNR ratio to

$$\frac{|c_{11}|^2\mathsf{SNR}_1}{|c_{22}|^2\mathsf{SNR}_2} = \theta$$

and by defining

$$\rho = \frac{|c_{22}|^2}{|c_{11}|^2}$$

[3] For the interference channel, the slope region associated with a given achievable rate region is defined exactly as for the multiaccess channel.

the achievable rates become

$$R_1(\mathsf{SNR}_1) = \log\left(1+|c_{11}|^2\mathsf{SNR}_1\right)$$
$$R_2(\mathsf{SNR}_2) = \log\left(1+\frac{|c_{22}|^2\mathsf{SNR}_2}{1+|c_{21}|^2\,\rho\,\theta\,|c_{22}|^2\mathsf{SNR}_2}\right)$$

whose first and second order derivatives at zero SNR are (in nats)

$$\dot{R}_1(0) = |c_{11}|^2 \quad (103)$$
$$-\ddot{R}_1(0) = |c_{11}|^4 \quad (104)$$
$$\dot{R}_2(0) = |c_{22}|^2 \quad (105)$$
$$-\ddot{R}_2(0) = |c_{22}|^4+2|c_{21}|^2\,|c_{22}|^2\,\theta\,\rho \quad (106)$$

The corresponding (optimum) slope-pair is

$$\mathcal{S}_1=2,\quad \mathcal{S}_2=\frac{2}{1+2\frac{|c_{21}|^2}{|c_{11}|^2}\theta}$$

Notice that for every value of $\theta\geq 0$ this slope region is strictly superior to the TDMA slope region and the union over θ gives again $[0,2]\times[0,2]$

4) $\frac{|c_{12}|^2}{|c_{22}|^2}<1$ and $\frac{|c_{21}|^2}{|c_{11}|^2}>1$:
Proceeding entirely analogously to the previous case but with the role of the user exchanged, we obtain that the following (optimum) slope-pair is achievable

$$\mathcal{S}_1=\frac{2}{1+2\frac{|c_{12}|^2}{|c_{22}|^2}\frac{1}{\theta}},\quad \mathcal{S}_2=2$$

5) $\frac{|c_{12}|^2}{|c_{22}|^2}\leq 1$ and $\frac{|c_{21}|^2}{|c_{11}|^2}\leq 1$:
By treating the interfering signal as noise it is possible to achieve the rate-pair

$$R_1 = \log_2\left(1+\frac{|c_{11}|^2\mathsf{SNR}_1}{1+|c_{12}|^2\mathsf{SNR}_2}\right)$$
$$R_2 = \log_2\left(1+\frac{|c_{22}|^2\mathsf{SNR}_2}{1+|c_{21}|^2\mathsf{SNR}_1}\right)$$

which leads to optimum $\frac{E_b}{N_0}_{\min}$. By defining θ and ρ as before, by expressing R_1 as function of SNR_1 only and R_2 as function of SNR_2 only, the relevant derivatives are

$$\dot{R}_1(0) = |c_{11}|^2 \quad (107)$$
$$-\ddot{R}_1(0) = |c_{11}|^4+2|c_{12}|^2\,|c_{11}|^2\,\frac{1}{\theta\,\rho} \quad (108)$$
$$\dot{R}_2(0) = |c_{22}|^2 \quad (109)$$
$$-\ddot{R}_2(0) = |c_{22}|^4+2|c_{21}|^2\,|c_{22}|^2\,\theta\,\rho \quad (110)$$

The achievable slope-pair is then

$$\mathcal{S}_1=\frac{2}{1+2\frac{|c_{12}|^2}{|c_{22}|^2}\frac{1}{\theta}},\quad \mathcal{S}_2=\frac{2}{1+2\frac{|c_{21}|^2}{|c_{11}|^2}\theta}$$

By eliminating θ from the above expression we obtain the achievable slope region

$$\mathcal{S}_i\in[0,2],\quad \left(\frac{2}{\mathcal{S}_1}-1\right)\left(\frac{2}{\mathcal{S}_2}-1\right)=4\frac{|c_{12}|^2}{|c_{22}|^2}\frac{|c_{21}|^2}{|c_{11}|^2}$$

which is strictly larger than the TDMA slope region if

$$\frac{|c_{12}|^2}{|c_{22}|^2}\frac{|c_{21}|^2}{|c_{11}|^2}<\frac{1}{4} \quad (111)$$

On the contrary, if $\frac{|c_{12}|^2}{|c_{22}|^2}\leq 1$, $\frac{|c_{21}|^2}{|c_{11}|^2}\leq 1$ (excluding $\frac{|c_{12}|^2}{|c_{22}|^2}=\frac{|c_{21}|^2}{|c_{11}|^2}=1$) and $\frac{|c_{12}|^2}{|c_{22}|^2}\frac{|c_{21}|^2}{|c_{11}|^2}\geq\frac{1}{4}$ (see the region in Figure 8), we do not know of any strategy that achieves both the optimum $\frac{E_b}{N_0}$ and a slope pair outside the TDMA region.

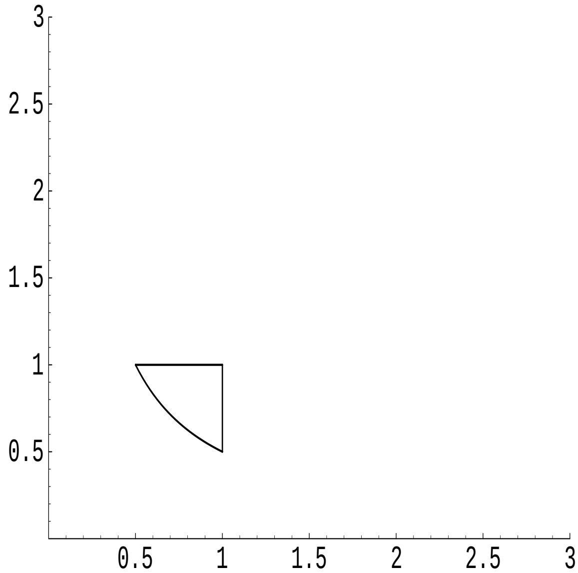

Fig. 8. Region of the interference parameters $\frac{|c_{12}|}{|c_{22}|}$, $\frac{|c_{21}|}{|c_{11}|}$ for which no strategy is known to provide better slopes than TDMA.

All the above results can be replicated in the case the coefficients $\{c_{ij},\,i,j=1,2\}$ are ergodic independent random processes with (c_{11},c_{12}) known at receiver 1 and (c_{21},c_{22}) known at receiver 2. As in the absence of fading, the minimum energy per bit and slope region achieved by TDMA are as those obtained for the multiple-access channel, i.e. the minimum energy per bit achieved by TDMA is optimum and the slope region is the triangle (43). A variety of multiaccess strategies are now shown to achieve slope pairs outside the TDMA region:

1) $\frac{E[|c_{12}|^2]}{E[|c_{22}|^2]}>1$ and $\frac{E[|c_{21}|^2]}{E[|c_{11}|^2]}>1$:
If the users transmit at the capacity of their respective hypothetical single-user channels in the absence of interference, they are still decodable with arbitrary reliability in the presence of interference for sufficiently small SNR_1 and SNR_2, because each interferer can be decoded with arbitrary reliability and then subtracted out. Thus, the capacity region is the rectangle composed of single-user capacities and the slope region is the rectangle

$$\mathbf{S}(\theta)=[0,\mathcal{S}_1^{(\mathsf{su})}]\times[0,\mathcal{S}_2^{(\mathsf{su})}]$$

where $\mathcal{S}_i^{(\mathsf{su})}=2/\kappa(|c_{ii}|)$, for $i=1,2$, are the individual single-user slopes.

2) $\frac{E[|c_{12}|^2]}{E[|c_{22}|^2]}>1$ and $\frac{E[|c_{21}|^2]}{E[|c_{11}|^2]}\leq 1$:
The following rate-pair is achievable

$$R_1 = E\left[\log_2\left(1+|c_{11}|^2\mathsf{SNR}_1\right)\right]$$

$$R_2 = E\left[\log_2\left(1+\frac{|c_{22}|^2\mathsf{SNR}_2}{1+|c_{21}|^2\mathsf{SNR}_1}\right)\right] \quad (112)$$

as long as the receiver of user 1 can decode user 2, which is guaranteed to hold for sufficiently small SNR_1. This strategy achieves optimum minimum energy per bit and the (not necessarily optimal) slope-pair

$$\mathcal{S}_1 = \frac{2}{\kappa(|c_{11}|)}, \quad \mathcal{S}_2 = \frac{2}{\kappa(|c_{22}|)+2\frac{E[|c_{21}|^2]}{E[|c_{11}|^2]}\theta},$$

which lies outside the TDMA triangle.

3) $\frac{E[|c_{21}|^2]}{E[|c_{11}|^2]} > 1$ and $\frac{E[|c_{12}|^2]}{E[|c_{22}|^2]} \le 1$:
analogous to the previous case, by switching the indices 1 and 2.

4) c_{1i} and c_{2i} have identical distribution (including scale) for $i = 1, 2$:
The channels seen by both receivers are statistically identical and the optimal slope region is that of the multiple-access channel with fading, which, as we saw, is strictly larger than the TDMA slope region.

5) $\frac{E[|c_{21}|^2]}{E[|c_{11}|^2]} \le 1$ and $\frac{E[|c_{12}|^2]}{E[|c_{22}|^2]} \le 1$: by treating the interfering user as noise, the rate-pair

$$R_1 = E\left[\log_2\left(1+\frac{|c_{11}|^2\mathsf{SNR}_1}{1+|c_{12}|^2\mathsf{SNR}_2}\right)\right]$$
$$R_2 = E\left[\log_2\left(1+\frac{|c_{22}|^2\mathsf{SNR}_2}{1+|c_{21}|^2\mathsf{SNR}_1}\right)\right]$$

is achievable. This leads to optimum $\frac{E_b}{N_0}_{\min}$ and to the slope region

$$\mathcal{S}_i \in [0, \mathcal{S}_i^{(\mathsf{su})}],$$
$$\left(\frac{1}{\mathcal{S}_1}-\frac{1}{\mathcal{S}_1^{(\mathsf{su})}}\right)\left(\frac{1}{\mathcal{S}_2}-\frac{1}{\mathcal{S}_2^{(\mathsf{su})}}\right) = \frac{E[|c_{21}|^2]}{E[|c_{11}|^2]}\frac{E[|c_{12}|^2]}{E[|c_{22}|^2]} \quad (113)$$

The TDMA slope region

$$\frac{\mathcal{S}_1}{\mathcal{S}_1^{(\mathsf{su})}} + \frac{\mathcal{S}_2}{\mathcal{S}_2^{(\mathsf{su})}} \le 1$$

is strictly included in (113) if

$$\frac{E[|c_{21}|^2]}{E[|c_{11}|^2]}\frac{E[|c_{12}|^2]}{E[|c_{22}|^2]} < \frac{1}{\mathcal{S}_1^{(\mathsf{su})}\mathcal{S}_2^{(\mathsf{su})}} \quad (114)$$

which necessarily holds in the case

$$\mathcal{S}_1^{(\mathsf{su})}\mathcal{S}_2^{(\mathsf{su})} < 1.$$

6) Rayleigh fading ($c_{i,j}$ proper Gaussian random variables):
Since in this case $\mathcal{S}_1^{(\mathsf{su})} = \mathcal{S}_2^{(\mathsf{su})} = 1$, depending on the relative strength of the interference coefficients, one of the above cases must hold. Thus, in this case, TDMA is strictly suboptimal.

7) Unsolved case:
No transmission strategy is known to be better than TDMA in the low power regime for

$$\frac{E[|c_{21}|^2]}{E[|c_{11}|^2]} \le 1, \quad \frac{E[|c_{12}|^2]}{E[|c_{22}|^2]} \le 1,$$
$$\frac{E[|c_{21}|^2]}{E[|c_{11}|^2]}\frac{E[|c_{12}|^2]}{E[|c_{22}|^2]} \ge \frac{1}{\mathcal{S}_1^{(\mathsf{su})}\mathcal{S}_2^{(\mathsf{su})}}$$

except in those cases of identical fading (4 and 6) for which TDMA is indeed suboptimal.

V. Conclusion

In the hypothetical ultrawideband regime where bandwidth is not a commodity to be conserved, minimum energy per bit is achieved by avoiding interference altogether by assigning users to nonoverlapping frequency bands. Not surprisingly, we have seen that the same result can be obtained using TDMA for multiaccess, broadcast and interference channels. References [10], [7] analyze the broadcast channel in the wideband regime and based on a first order analysis, such as (60), conclude that in low-power wideband channels very little is to be gained from the complexity incurred by superposition schemes. However, we have shown in this paper that this conclusion is unwarranted as long as bandwidth is not a free commodity. In that case, the minimum energy per bit is not the only figure of merit of interest; indeed, one needs to assess the growth of the achievable rates as the energy per bit grows from its minimum value. For multiuser channels, we have seen that, revealing differences in efficiency that are transparent to other figures of merit, the slope region is a convenient analysis tool in the low-power regime.

Interestingly, the information-theoretic suboptimality of TDMA is more pronounced in the near-far scenario where users have imbalanced signal-to-noise ratios. In that case, superposition and a stripping receiver can achieve essentially single-user capacity for both users simultaneously in marked contrast to TDMA. In terms of the Cover-Wyner pentagon, consider the case where user 1 (in the x-axis) is much more powerful than user 2; then as far as the rate achieved by user 2 is concerned it is much more preferable to operate at the upper vertex of the pentagon than at the TDMA-achieved maximum rate-sum rate pair.

Only when the received energies per bit are required to be not only close to -1.59dB but identical for all users, is TDMA as good as superposition in the multiaccess channel, but then only in the absence of fading. With fading, the TDMA slope region is strictly inside the optimum region. Other results on the suboptimality of TDMA for fading multiaccess channels in the presence of delay constraints are given in [11].

To translate the conclusions of this paper into practical lessons that apply to real-world embodiments of TDMA and CDMA, it should be emphasized that the advantages of superposition strategies over orthogonal strategies we have shown may not hold unless the receiver uses multiuser detection to take into account inter-user interference.

For the broadcast channel without fading, our analysis was based on the well-known capacity of the degraded Gaussian channel. If the receivers see identical signal to noise ratios, then TDMA is trivially capacity-achieving for all signal-to-noise ratios. Otherwise, TDMA incurs a bandwidth expansion whose severity increases with the power imbalance. In the presence of fading, although the capacity region is unknown we were able to obtain the minimum energy per bit and optimum slope region. In particular we showed that the TDMA bandwidth expansion penalty does not depend on the fading distribution if the users are subject to the same fading distribution.

For the interference channel, the TDMA slope region is the same triangular region as in the multiaccess channel. In the case of large interference, the optimum single-user slopes are simultaneously achievable. In the imbalanced case where one of the interference coefficients is larger than 1 and the other is smaller than 1, we can also identify optimum slope pairs that lie strictly outside the TDMA triangle regardless of the desired rate ratio. In the case of small interference, neglecting the interference at both receivers results in slope pairs outside the TDMA triangle. However, depending on the fading kurtosis, a small region of interference parameters remains for which we do not know of any strategy that beats TDMA in the low-power regime. Interestingly, in the important case of independent Rayleigh fading, TDMA is suboptimal regardless of the strength of the interference coefficients.

Finally, we note that for the degraded Gaussian relay channel [12], it is easy to show that TDMA does not achieve $\frac{E_b}{N_0}_{\min}$ [13]. On the contrary, finding $\frac{E_b}{N_0}_{\min}$ for more general networks with relays appears to be a challenging problem.

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